MFSK FREQUENCY ACQUISITION AND SYNCHRONIZATION FOR THE JUPITER PROBE-TO-RELAY COMMUNICATION LINK

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Summary. This paper discusses the coarse frequency acquisition problem and the fine frequency tracking problem for a communication link between a spacecraft and a probe entering the atmosphere of Jupiter. A coded noncoherent MFSK modulation format is assumed along with a severely fading link. Fine frequency tracking is shown to be a more serious problem than coarse acquisition.

Introduction. In the early 1980's NASA Ames plans to send a spacecraft past the planet Jupiter. As it passes by Jupiter it will launch a small probe which will descend into the atmosphere. The probe will take various kinds of scientific data and telemeter it back to the spacecraft for later retransmission to an earth station. Although it is possible that the data could be sent back to Earth without demodulation, it would simplify the data storage and telemetry requirements of the spacecraft significantly if the data could be demodulated at the spacecraft.

The probe will send approximately 44 data bits/sec for more than 20 minutes. The link will be subjected to severe fading induced by the ionosphere of Jupiter and will be power limited. The initial frequency uncertainty is approximately 100 KHz due to uncertainties in the Doppler frequency at the time of initial data transmission. This is primarily due to apriori uncertainty in the position of the probe and spacecraft at time of initial data transmission. The probe design presently has no ranging or angle tracking capability. The trajectories are picked such that the spacecraft flies through the probes antenna pattern at the correct time.

Modestino and Mui[1] have shown that an ideal (i.e. perfect frequency tracking and bit synchronization) coded noncoherent M-ary frequency shift keyed (MFSK) receiver would provide adequate bit error rate performance in a severe (Rayleigh) fading environment. A number of papers have been written dealing with bit and frequency synchronization for a noncoherent MFSK system[2,3,4]. However, there appears to be no published results on the performance of these frequency synchronization schemes. This paper addresses the frequency acquisition and tracking problem assuming a noncoherent MFSK ($M = 8$) modulation format.

Error rate performance is very sensitive to the form of the fading amplitude distribution. However, tracking performance is more sensitive to the fading bandwidth than the exact form of the amplitude distribution. This paper assumes that the instantaneous received signal amplitude is a log normal random variable. For severe fading the phase is a uniform random variable $[0, 2\pi]$.

The first problem to be considered, which is actually somewhat independent of the choice of modulation format, is coarse acquisition. Coarse acquisition is defined here to be the reduction of frequency uncertainty from 100 KHz to a few hundred Hz. Note that the coarse acquisition processor does not locate the individual MFSK tones.

Coarse Acquisition. Initial studies indicated that sweeping a single tracking loop (phase or frequency) through the entire 100 KHz Doppler uncertainty would result in excessive acquisition time requirements (on the order of minutes). Although a bank of analog loops might be used, an attractive alternate approach[2,3,4] is to use a digital FFT processor. The FFT algorithm fixes the relationship between the number of samples N used and the frequency resolution of the FFT. Since it may be desirable to take samples over a very long period T to reduce the effect of noise but keep a fixed FFT length to reduce computer memory and logic speed requirements, it is necessary to consider the case where the period T is divided into n intervals, a separate FFT computed in each interval, then all the FFT's are averaged together in some manner.

Let r(t) be the received signal given by r(t) = s(t) + n(t)

where $$s(t) = \sqrt{2P_s}\,A(t)\cos(wt + \theta(t) + \psi(t)) \qquad (1)$$

- P_s is the mean signal power in watts.
- A(t) is amplitude modulation due to fading.
- $\theta(t)$ is the phase modulation due to fading.
- $\psi(t)$ is the data modulation.

n(t) is band limited Gaussian noise with zero mean and a flat spectrum over the region of interest (two-sided power spectral density $N_o/2$ watt/Hz.):

$$n(t) = n_c(t)\cos wt - n_s(t)\sin wt \qquad (2)$$

Let σ_n^2 be the variance of n(t).

The received signal can be written in terms of its quadrature components and normalized by σ_n :

$$\frac{r(t)}{\sigma_n} = \left[\frac{\sqrt{2P_s}A(t)\cos[\theta(t)+\psi(t)]}{\sigma_n} + \frac{n_c(t)}{\sigma_n}\right]\cos wt$$

$$- \left[\frac{\sqrt{2P_s}A(t)\sin[\theta(t)+\psi(T)]}{\sigma_n} + \frac{n_s(t)}{\sigma_n}\right]\sin wt \qquad (3)$$

$$\frac{r(t)}{\sigma_n} \triangleq x(t)\cos wt + y(t)\sin wt.$$

Let $x_{\ell,i}$ and $y_{\ell,i}$ be samples of x(t) and y(t) at times $t = \ell\Delta T$ during the ith transform interval, where ΔT is the sampling interval. Then the kth discrete Fourier coefficient of the complex sequence

$$x_{\ell,i} + jy_{\ell,i}$$

during the ith transform interval is given by

$$S_{k,i} \triangleq a_{k,i} + jb_{k,i} = \frac{1}{N}\sum_{\ell=(i-1)N}^{iN-1}(x_{\ell,i} + jy_{\ell,i})\, e^{-j2\pi\ell k/N} \qquad (4)$$

$$k = 0, 1, 2, \ldots, N-1$$
$$i = 1, 2, \ldots, n$$

To obtain approximate analytical results on coarse acquisition, it is assumed that the signal has no data modulation. Since A(t) is log normal, the probability density function (pdf) of A(t) is given by

$$p(A) = \frac{1}{A\sqrt{2\pi\sigma^2}}\exp\left[-(\ln A + \sigma^2)^2/2\sigma^2\right] \qquad (5)$$

where σ^2 is the variance of the log amplitude. $\theta(t)$ is assumed to be uniform on $(0, 2\pi]$. (Note that the mean of $A^2(t)$ is normalized to 1.)

Slow Fading. Consider first the slow fading case where the bandwidth B_o of A(t) and $\theta(t)$ is much less than 1/T. An attractive decision algorithm is to choose the value of k which maximizes

$$L = \sum_{i=1}^{n}(a_{k,i}^2 + b_{k,i}^2)$$

Assume the signal frequency, w, corresponds to one of the computed FFT components, w_k. Then L has a chi-square distribution with 2n degrees-of-freedom for the values of k for which $w_k \neq w$ (the nonsignal components) and has a noncentral chi-square distribution for $w_k = w$ (the signal component), for the slow fading case. In reality, none of the w_k's will exactly correspond to w and thus the following are best case results.

The probability of correct acquisition with no fading is then given by

$$P_{CA}(\lambda) = \int_0^{\infty} [P(x;\ 2n)]^{N-1} f(x;\ 2n|\lambda)\ dx \tag{7}$$

where P(x; 2n) is the chi-square cumulative distribution function with 2n degrees-of-freedom and f(x; 2n|λ) is the noncentral chi-square probability density function with 2n degrees-of-freedom given λ the noncentrality parameter:

$$P(x;\ 2n) = 1 - \sum_{j=0}^{n-1} \frac{(x/2)^j}{j!} e^{-x/2} \tag{8}$$

$$f(x;\ 2n|\lambda) = \frac{1}{2} (x/\lambda)^{\frac{n-1}{2}} I_{n-1}(\lambda x)\ e^{-(\lambda + x)/2} \tag{9}$$

$$\lambda = n\lambda_o$$

$$\lambda_o = (2P_s)/\sigma_n^2$$

N, the total number of FFT coefficients, is given by

$$N = \frac{BT}{n}$$

assuming Nyquist sampling rates and total uncertainty rf bandwidth B (100 KHz for our case). Thus for a fixed acquisition time Nn is a constant.

$P_{CA}(\lambda)$ can be approximated by assuming that $[P(x;\ 2n)]^{N-1}$ is essentially a step function. This approximation allows the integral for $P_{CA}(\lambda)$ to be expressed in terms of the noncentral chi-square distribution function:

$$P_{CA}(\lambda) = 1 - \sum_{j=0}^{\infty} \left(\frac{\lambda}{2}\right)^j e^{-\lambda/2} P[x_o;\ 2n + 2j] \tag{10}$$

where x_o is chosen such that $[P(x_o; 2n)]^{N-1} = 1/2$. If it is further assumed that the noncentral chi-square density can be approximated by a Gaussian density, then

$$P_{CA}(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\gamma}^{\infty} e^{-t^2/2}\, dt \triangleq Q(-\gamma) \tag{11}$$

where

$$\gamma = \frac{\frac{\lambda_o \sqrt{n}}{2} - x_o'}{\sqrt{1 + \lambda_o}},$$

and x_o' is chosen such that $[Q(x_o')]^{N-1} = ½$.

Figure 1 shows 1 - $P_{CA}(\lambda_o)$ for a fixed number of points (i.e. fixed nN). Note that for a fixed nN, increasing N the length of the FFT improves performance.

With log normal fading the probability of correct acquisition in the slow fading case is P_{CA} where

$$P_{CA} = \int_{0}^{\infty} P_{CA}(\lambda) \frac{1}{\lambda\sqrt{8\pi\sigma^2}} \exp\left[-\left(\ell n\ \lambda + 2\sigma^2 - \ell n\left(\frac{2n\ P_s}{\sigma_n^2}\right)\right)/8\sigma^2\right] d\lambda \tag{12}$$

Figure 2 shows a plot of Eq. 17 for σ = .49 and .75 where $P_{CA}(\lambda)$ was approximated using the Gaussian assumption described above. These values of a correspond to very severe fading.

Practical Moderate Fading Case. The previous theoretical results were for a constant frequency (unmodulated) signal exactly aligned with one of the FFT harmonics and with slow fading. For the actual case of interest the signal carrier frequency varies due to Doppler, the signal contains data modulation and the signal amplitude and phase varies during the decision interval due to fading. To compare the theoretical predictions with the more practical case, a computer simulation was performed using estimates of the Jupiter probe relay link parameters. Figures 3 and 4 show simulation data on probability of error assuming a fading bandwidth of 2 Hz and a Doppler rate of 30 Hz/sec. The signal has MFSK modulation with M = 8 and a tone spacing of 58 Hz. The bit rate is 88 bps which with a rate 1/2 code would correspond to 44 data bps. E_b is the energy per coded bit, i.e. $E_b = P_s/88$. In Figure 3, N = 256 and n = 960 corresponds to having 1 FET harmonic spaced over the entire signal spectrum. The degradation from the ideal, no fading case due to the modulation, fading, and misalignment appears to be about 2 dB. In Figure 4, n = 240 and N = 1024 corresponds to having 4 FFT harmonics spaced over the signal spectrum. This alone might be expected to reduce performance by 6 dB from ideal but the advantage of having four correct answers in the decision process offsets some of this 6 dB and of the loss due to misalignment and fading. The long total averaging time also eliminates much of the effect of fading. Figure 4 indicates that for E_b/N_o = -0.4 dB, the loss compared with the ideal, unmodulated, nonfading case is only about 4 dB.

λ_o can be related to E_b/N_o by noting that

$$\lambda_o = \frac{2P_B}{\sigma_n^2} = \frac{2E_b/T_b}{\Delta f\ N_o} = \frac{E_b}{N_o}\frac{2}{\Delta f\ T_b} \qquad (13)$$

where T_b is the bit period in sec.

Modestino and Mui's[1] results indicate that good bit error rate performance requires E_b/N_o on the order of 10 dB. Figure 4 indicates that for E_b/N_o of 10 dB coarse acquisition in 2 seconds is no problem.

Fine Acquisition for MFSK. The technique used for fine acquisition depends upon the choice of modulation format. For MFSK, fine acquisition is the process of locating the M individual tones within a bandwidth not a great deal larger than the total signal bandwidth. Assuming 8 tones spaced 58 Hz apart, the total signal bandwidth is 464 Hz. The coarse acquisition technique discussed in the previous section reduces the frequency uncertainty to less than 1000 Hz. The fine frequency tracking loop must be able to track with an error small compared to 30 Hz.

The Doppler rate can be as large as 30 Hz/sec and the expected fine acquisition time is greater than 20 secs, thus the acquisition loop must have a tracking capability. Since bit synchronization is not being assumed at this point, the frequency discriminator (i.e. the device making the instantaneous frequency error estimates) must look over at least two-tone intervals to make sure it has seen at least one complete tone. Another approach would be to lock the bit synchronizer either before or at the same time as the fine frequency tracking loop.

Assuming a second order tracking loop and a worst case Doppler rate rate of .03 Hz/sec^2, a loop bandwidth of 0.1 Hz would yield a static tracking loop error of less than 1.0 Hz. Whether VCO instability will actually allow a loop bandwidth this low is open to question. To achieve this a VCO must be used which is tunable over 100 KHz with short term stability of around 1 part in 10^5.

Decision Directed Technique. Springett[4] suggested a technique for determining frequency error which first determines an FFT over each successive two-tone period. It then chooses the FFT component with the maximum power during each two-tone period. The frequency of the maximum for the first period is stored and all subsequent frequency estimates are subtracted from the first one. The frequency difference is computed modulo the tone spacing. Thus, during each two-tone period an integer and a fraction is computed. The fraction is an estimate of how much the present transmitted tone lacks of being an integer number of tone spacings from the first tone estimate. These fractions are averaged (or filtered in the closed loop case) to obtain an estimate of the distance the first estimate is from a true-tone. Figure 5 shows our closed loop version of the decision directed technique which attempts to hold one tone at a fixed reference frequency (or an integer number of tone spacings from the reference frequency). The basic problem with this approach is that it doesn't really know what tone it is tracking. Springett attempts to reconcile this ambiguity by remembering the maximum and minimum of the integers discarded during the Mod operations. If the difference between the maximum and minimum is M (the number of tones), the ambiguity is probably resolved. If it is not, there is a problem. Note that for this to have a chance of working both the end frequencies must be seen regularly. Like all synchronization loops for periodic signals, this loop can "slip cycles" which in this case means it changes the tone it is trying to hold at the reference frequency. It appears that cycle slipping must not be a significant problem if this approach is to be attractive. The main advantage of this approach is that its detection bandwidth is small or, in other words, during one-half the time of a frequency error estimate, the signal does not change frequency.

Loop Tracking Error. Error in estimating loop frequency error results from both finite frequency spacing in the FFT and from errors in choosing the correct frequency component due to noise and fading. Figure 6 shows a plot of probability of error ($N = 64$, $n = 1$) as a function of λ_o for slow fading assuming no data modulation.

To determine the effect of data modulation and Doppler changes on the performance of the tracking loop, the loop was simulated and the rms tracking error experimentally measured. Figure 7 shows the rms frequency tracking error as a function of E_b/N_o (for a loop with 0.2 Hz bandwidth) measured by computer simulation.

Correlator Technique. An alternate (Figure 8) to the decision directed technique is to compute an FFT over an interval corresponding to many tones. This interval must be long enough to see both end tones. To simplify computational requirements this will probably be accomplished (as in the coarse acquisition case) by averaging a number of FFT's each of which taken over a period of several tones. At this point there is an estimate of the signal spectrum available. To convert this to an estimate of the location of the M tones, it is necessary to correlate the signal spectrum estimate with the average signal spectrum determined analytically.

This approach has the disadvantage that in the calculation of the signal spectrum estimate at any given time there is only one signal tone present, so that all but one of the true tone frequencies is being averaged with noise. This reduces the quality of the spectral estimate. However this approach does take care of the ambiguity problem of the previous technique in a more satisfying manner. Figure 9 shows the rms frequency tracking error for the correlator approach measured by simulation.

Note that the correlator approach gives large rms frequency tracking error, and that the decision directed approach is significantly superior to the correlator approach in rms tracking error. The correlator approach has a linear tracking range on the order of 1000 Hz whereas the linear tracking range of the decision directed technique is on the order of 60 Hz. This results in more erratic performance and a requirement for somewhat wider loop bandwidths during acquisition for the decision directed approach. A significant probability of "tone slippage" by the decision directed tracking loop makes the ambiguity resolution problem serious.

Conclusions. Coarse acquisition within two seconds appears readily achievable using the present "best guess" for probe communication link parameters. Fine frequency acquisition and tracking to an acceptable accuracy appears difficult but possible.

References

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3. H. D. Chadwick and J. C. Springett, "The Design of a Low Data Rate MFSK Communication System," IEEE Trans. Commun. Tech., Vol. COM-18, No. 6, pp. 740-750, December 1970

4. J. C. Springett, "Summary Report JPL Pioneer-Venus Telemetry Analysis Task," October 1973

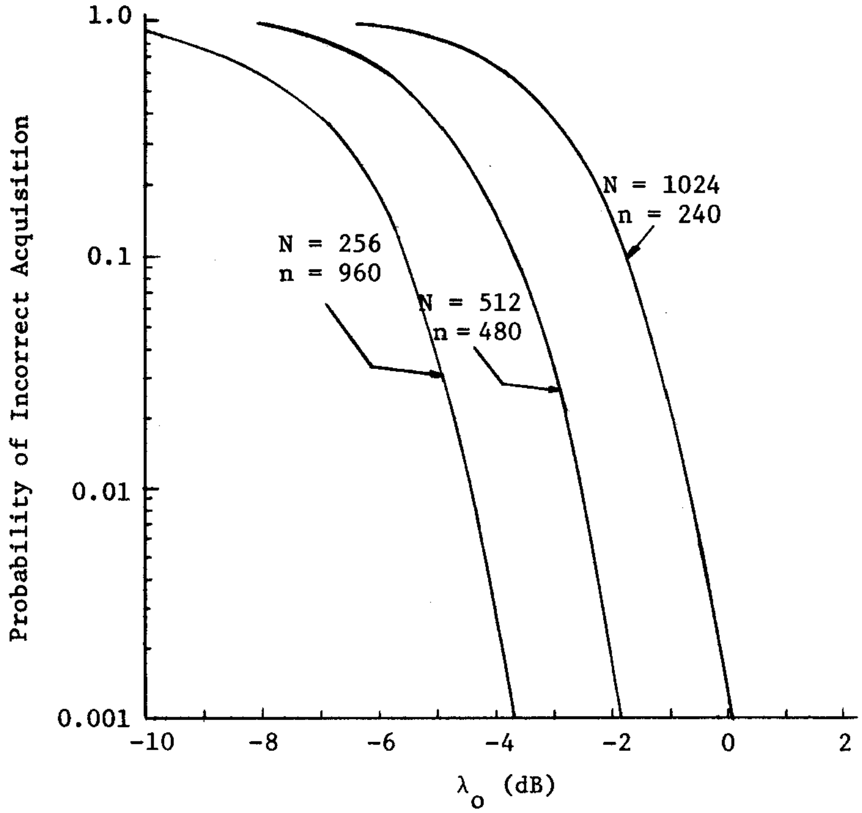

Fig. 1 . Probability of False Acquisition for the Nonfading Case

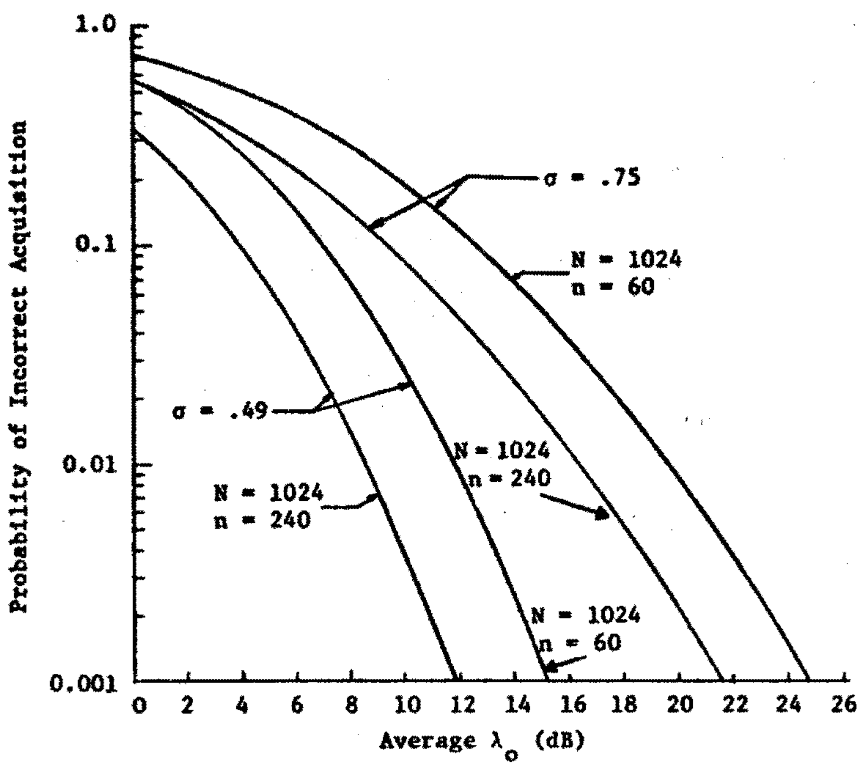

Fig. 2. Probability of False Acquisition For the Slow Fading Case

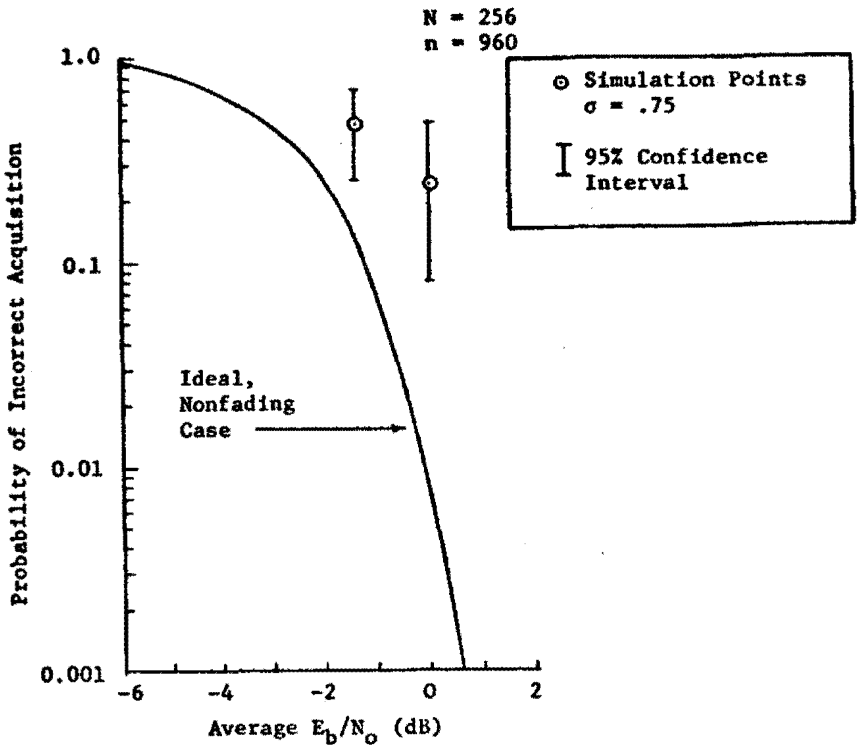

Fig. 3. Simulation Results for FFT Spacing = MFSK Signal Bandwidth

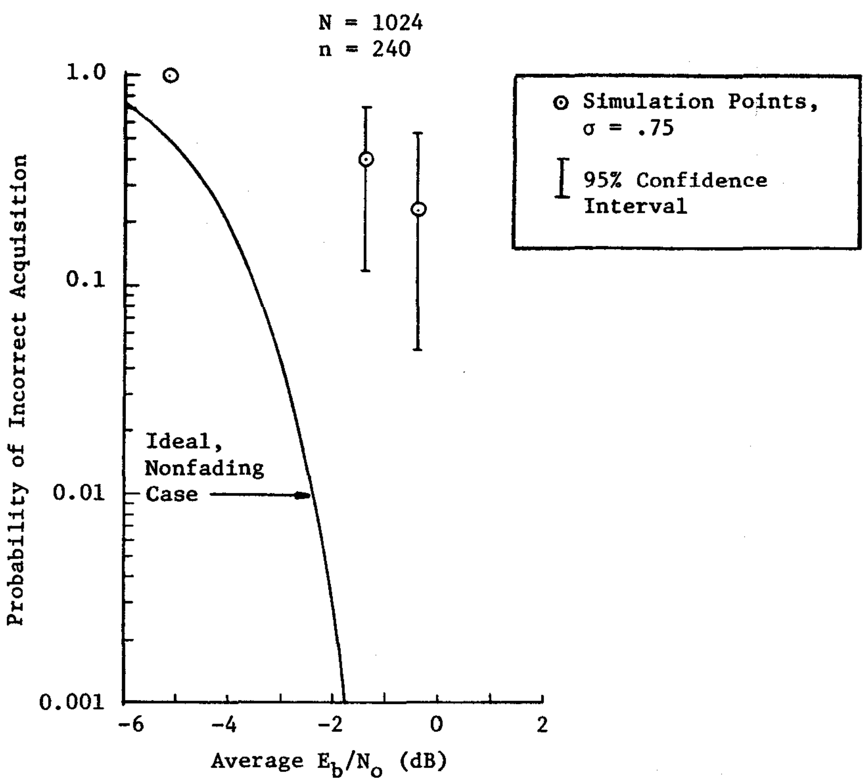

Fig. 4. Simulation Results for FFT Spacing = MFSK Signal Bandwidth/4

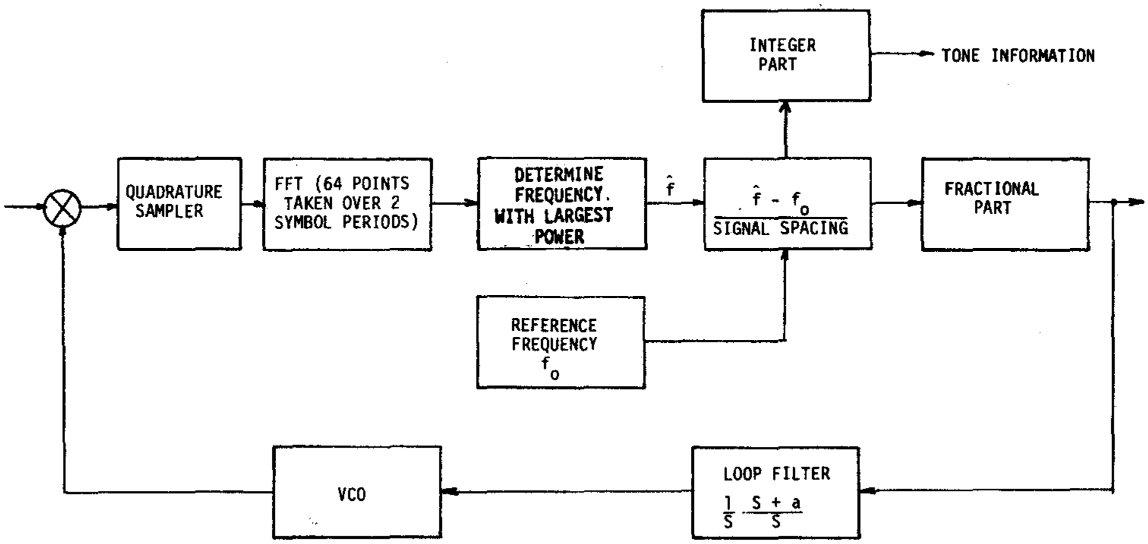

Fig. 5. Decision Directed Frequency Tracking Block Diagram

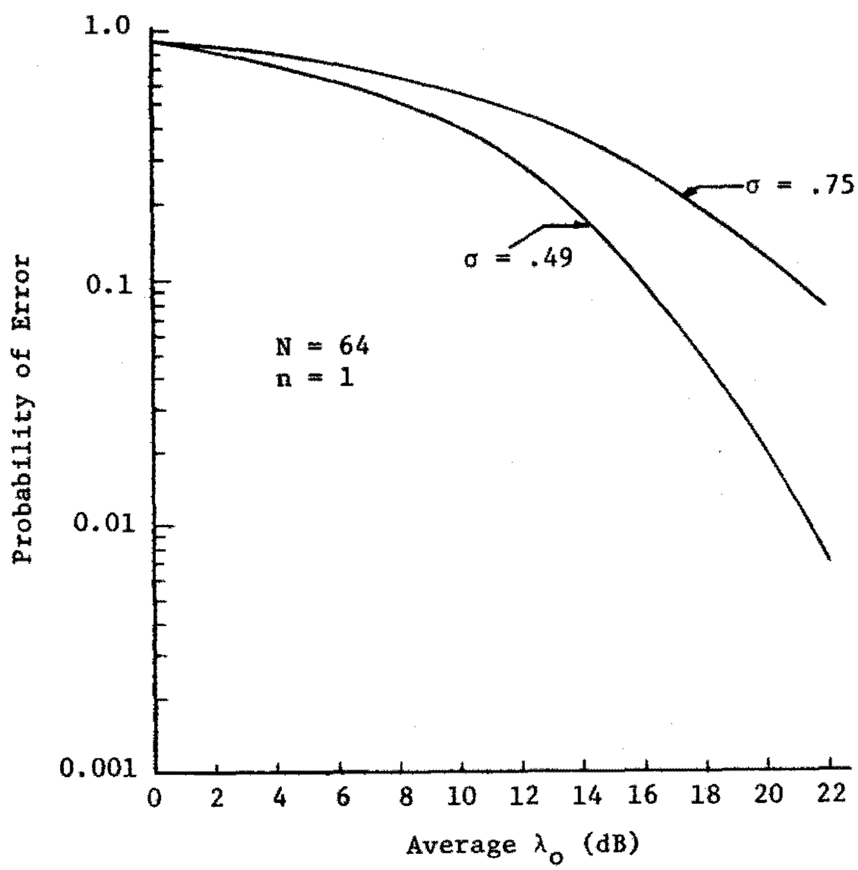

Fig. 6. Probability of Decision Error for Decision Directed Tracking Parameters (Slow Fading Assumed)

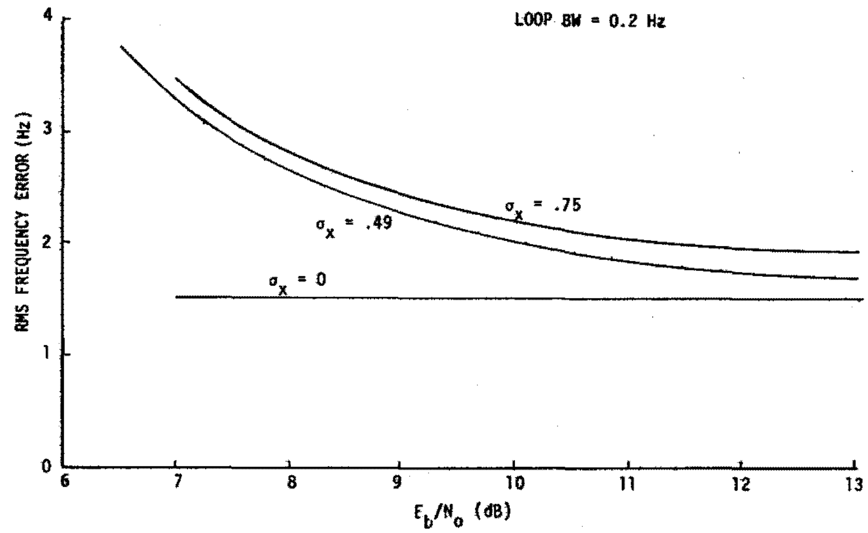

Fig. 7. RMS Frequency Error for Decision Directed Tracking Loop

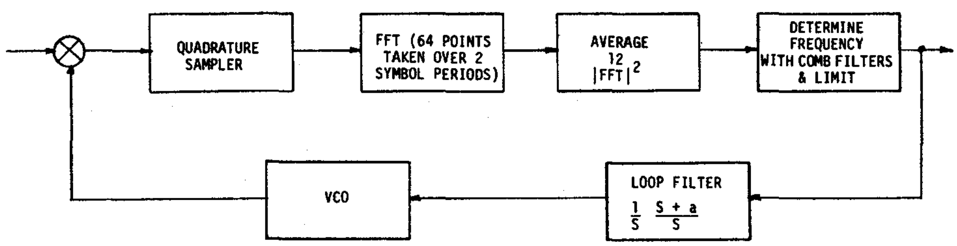

Fig. 8 . Correlator Frequency Tracking Block Diagram

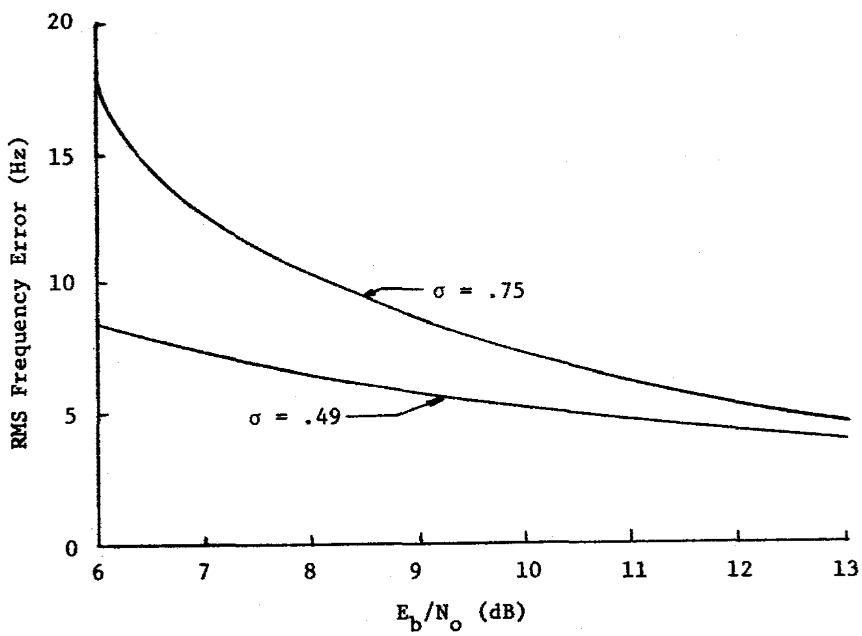

Fig. 9. RMS Frequency Error for Correlator Tracking Loop