ANTI-JAM PERFORMANCE OF SEVERAL DIVERSITY COMBINERS

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ABSTRACT

The relative anti-jam (AJ) performance of several diversity combiners are investigated. The modulation is 8-ary frequency-shift-keying (FSK), the demodulation process consists of energy detection of the eight frequency bins at each hop and the subsequent combining of detector outputs. Three combiners are considered : the linear combiner, where the detector outputs of each hop (corresponding to the same frequency bin) are summed without any processing; the self-normalized combiner, where the eight detector outputs of any particular hop are normalized so that they add to unity; and the max-normalized combiner, where the eight detector outputs of any hop are divided by the maximum value among those eight outputs. Results indicate that under worst-case tone jamming, the self-normalized combiner performs the best, the max-normalized combiner second best, and the linear combiner performs the worst among the three.

KEY WORDS

AJ Performance, Diversity Combining, Digital Communications.

INTRODUCTION

In the presence of sophisticated jamming, it is known that the performance of fast frequency hopped communication systems can be significantly degraded. Various techniques have been devised to ameliorate such degradation, among them is the usage of nonlinear hop combining, which is the subject of the present investigation. Another anti-jam measure that has been implemented in practice is fine-granularity hopping, where the minimum bandwidth between hop frequencies is smaller (usually much smaller) than the frequency separation of the FSK signal set. Fine-granularity hopping effectively spreads out the in-band jammer power throughout the bandwidth of the signal set, thus eliminating the advantage of a ‘smart’ tone jammer.

Since the primary focus of this study is the AJ capability of diversity combiners, fine-granularity hopping is not assumed; the analysis of the combined performance of these and other AJ measures can be quite involved and will be investigated at a later date. Accordingly, the simple hopping scheme assumed here is fairly easy to jam, and is therefore not a realistic candidate for an actual implementation; however, it does facilitate the analysis of the relative AJ performance of the diversity combiners of interest. Related references in this area include the papers by J. S. Lee and associates [1, 2, 3], and Robertson and Ha [4]; these papers analyzed combiner performance in the presence of partial-band noise jamming (PBNJ) for BFSK.

SYSTEM MODEL

A block diagram of the system model used in this study is shown in Figure 1. The transmission signal set is standard 8-ary FSK, where one of eight tones (equally-spaced in the frequency domain with separation R), at frequencies $\{f_i\}$, i = 1, 2, ..., 8, is selected (according to some pre-determined mapping) to represent three information bits. These eight frequencies are positioned around the center (or carrier) frequency f_c as shown in Figure 2, and are assumed to be equally probable.

Prior to transmission, the center frequency is pseudo-randomly hopped among a set of possible hop frequencies $\{f_{c,i}\}$ where $| f_{c,i} - f_{c,j} | \geq 8R$ for $i \neq j$. (This implies that the hopping signal sets do not overlap.) Also, the ordering of the signal set $\{f_i\}$ around f_c is not changed from hop to hop (i.e., no scrambling or shuffling within the signal set). At the receiver, the signal is dehopped and energy detected at the eight frequency bins, the outputs of which are then processed and combined in some manner to arrive at a final test statistic for each bin. The frequency corresponding to the bin with the greatest test statistic is selected as the combiner output. Detailed descriptions of the combiner processing are contained in later sections.

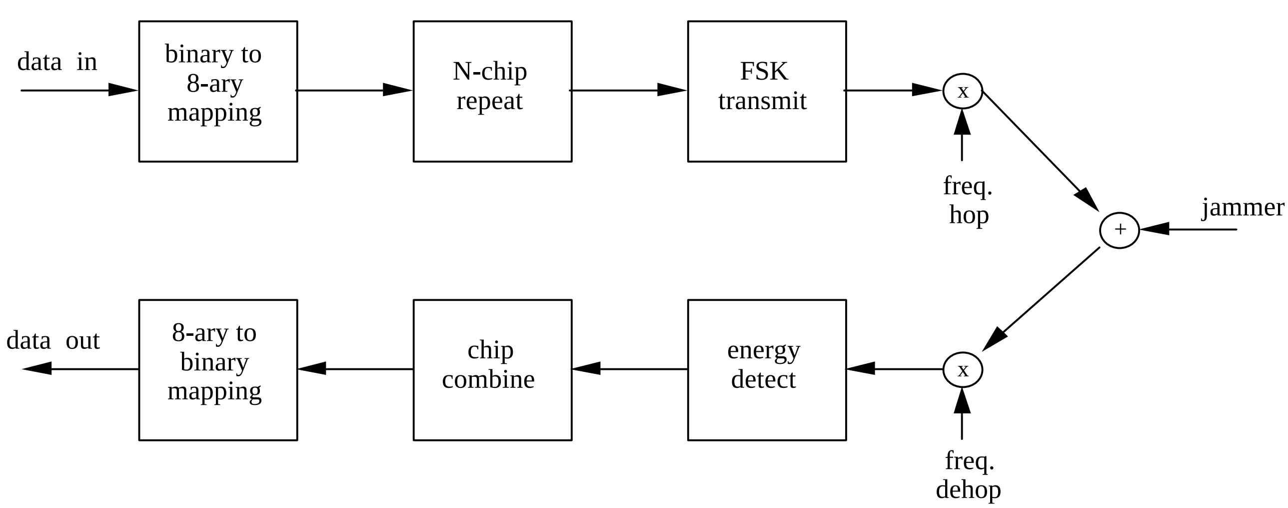

Figure 1: Fast Frequency Hopped System Model .

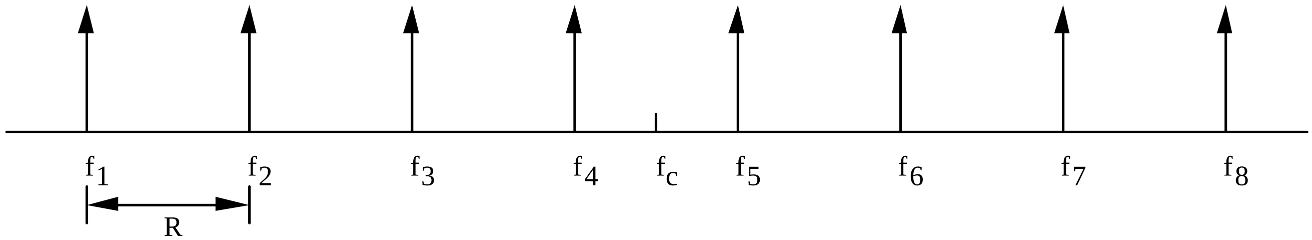

Figure 2: 8FSK Signal Set .

JAMMER MODEL

The jammer assumed here is a ‘smart’ multi-tone jammer which has complete knowledge of the signal set and hopping structure, except for the hopping sequence. The jammer has finite total power J, and over a hop period T_h has total energy JT_h, which, if spread evenly among the total number of possible hop frequencies, N_{ss}, yields the ‘spectral density’ of the jammer, denoted as J_0.

Rather than jamming the entire hop bandwidth (i.e., emitting energy in all N_{ss} hop frquencies), the jammer instead jams only a fraction ρ, $0 < \rho \leq 1$, of the band, with effective spectral density J_0/ρ. The ‘optimized’ or ‘worst-case’ jammer is that jammer which chooses a ρ such that the resulting bit error rate (BER) of the communicator is greatest.

Given the hopping and signal structure as described above, the jammer can concentrate its attack on one particular 8-ary symbol (say f_2) by choosing the jammed frequencies such that they have constant frequency separation (- 2.5 R in this example, see Fig. 2) from the closest hop frequency.

PERFORMANCE ANALYSIS

For simplicity, it will be assumed that the only disruption that the signal encounters is energy from the jammer, in the form of a randomly-phased tone at the jammed frequency. To facilitate the discussion, often-used variables and parameters are defined below.

N	---	order of diversity.
ρ	---	jammed band fraction; also equal to the probability of a hop being jammed.
J_0	---	jammer ‘spectral density’.
E_c	---	signal energy detected in one hop.
γ	---	E_c/J_0
$\{z_{i,j}\}$	---	energy detector output for hop i (i = 1, 2, ..., N), frequency bin j (j = 1, 2, ..., 8).

LINEAR COMBINER

The linear combiner simply adds the detector outputs of all the hops to arrive at a final test statistic for each frequency bin, specifically, the combiner forms the following,

$$u_j = \sum_{i=1}^{N} z_{i,j} \quad , \qquad j = 1, 2, ..., 8. \tag{1}$$

The output of the combiner (i.e., the receiver's symbol decision) is f_k whenever $u_k = \max\{u_j\}$.

In a no-ambient-noise environment, and assuming no signal attenuation or fading, it is clear that, at any jammed hop, the detected energy from the jammer must be no less than that from the signal in order for an error to occur; otherwise the jammer could hit every hop (and at the same symbol) and still not have enough total energy after summing to cause an error. Thus, it is assumed that $J_0/\rho \geq E_c$, or, defining $\gamma \equiv E_c/J_0$, $\rho\gamma \leq 1$.

Denote by N_J the number of hops (out of a maximum of N) that are jammed. There is a probability of 1/8 that the jammed symbol is the transmitted symbol, and probability of 7/8 that it is not. An error can only occur for the latter case. Without loss of generality, assume that the signal symbol is f_1 and the jammed symbol is f_2, the symbol test statistics are then,

$$\begin{aligned} u_1 &= N E_c , \\ u_2 &= N_J \frac{J_0}{\rho} , \\ u_i &= 0, \quad i = 3,4,\ldots,8 . \end{aligned} \tag{2}$$

Thus, an error occurs if and only if (iff) $N_J J_0/\rho \geq N E_c$, or equivalently, when $N_J \geq N\rho\gamma$. (Note that it is implicitly assumed that when $u_1 = u_2$, an error occurs, this slight bias towards the jammer is for computational convenience and does not affect the worst-case performance analysis.) The probability of a symbol error is then,

$$P_{s1}(\rho,\gamma) = \frac{7}{8} \sum_{k=k_1}^{N} p(N_J = k), \quad k_1 = \lceil N\rho\gamma \rceil, \tag{3}$$

where

$$p(N_J = k) = \binom{N}{k} \rho^k (1-\rho)^{N-k}.$$

Note that $p(N_J = k)$, the probability that $N_J = k$, is an implicit function of ρ.

MAX-NORMALIZED COMBINER

This nonlinear combiner offers a greater degree of jamming resistance by attempting to prevent a very large detector output value of any single hop from dominating or skewing the combined test statistics. Specifically, the combiner forms the following,

$$u_j = \sum_{i=1}^{N} \frac{z_{i,j}}{\max_j \{z_{i,j}\}}, \qquad j = 1, 2, ..., 8, \tag{4}$$

where $\{z_{i,j}\}$ are the energy detector outputs as defined previously; the combiner output decision logic is the same as that for the linear combiner, namely, choose f_k whenever $u_k = \max\{u_j\}$.

The approach used to analyze the linear combiner may now be applied to the max-normalized combiner. Conditioned on N_J out of N hops being jammed, and assuming f_1 is the signal symbol and f_2 is the jammed symbol, the test statistics after combining are ($\rho\gamma \le 1$ is again assumed),

$$\begin{aligned} u_1 &= N_J \frac{E_c}{(J_0/\rho)} + (N - N_J), \\ u_2 &= N_J \ , \\ u_i &= 0 \ , \qquad i = 3, 4, ..., 8. \end{aligned} \tag{5}$$

Therefore, a symbol error occurs iff $N_J \ge N_J \frac{E_c}{(J_0/\rho)} + (N - N_J)$, or equivalently, iff. $N_J \ge \frac{N}{2 - \rho\gamma}$.

The probability of a symbol error is then,

$$P_{s2}(\rho,\gamma) = \frac{7}{8} \sum_{k=k_2}^{N} p(N_J = k), \quad k_2 = \left\lceil \frac{N}{2 - \rho\gamma} \right\rceil, \tag{6}$$

where $p(N_J = k)$ is as defined in (3).

SELF-NORMALIZED COMBINER

This is another nonlinear combiner with AJ capability. It suppresses strong jammer hits and prevents a single high-energy jammed hop from dominating the test statistics and thus causing an error. This is accomplished by normalizing the detector outputs of each hop by the sum of the energies detected, specifically, the combined test statistics are,

$$u_k = \sum_{i=1}^{N} \frac{z_{i,k}}{\sum_{j=1}^{8} z_{i,j}}, \qquad k = 1, 2, ..., 8. \tag{7}$$

Following the same logic and reasoning as applied to the other combiners, it is easy to see that,

$$u_1 = N_J \left(\frac{E_c}{E_c + \frac{J_0}{\rho}} \right) + (N - N_J),$$

$$u_2 = N_J \left(\frac{\frac{J_0}{\rho}}{E_c + \frac{J_0}{\rho}} \right), \tag{8}$$

$$u_i = 0, \qquad i = 3, 4, ..., 8.$$

Rearranging terms, it can be seen that $u_2 \geq u_1$ is equivalent to $N_J \geq N\left(\frac{1+\rho\gamma}{2}\right)$, therefore, the symbol error probability is,

$$P_{s3}(\rho,\gamma) = \frac{7}{8} \sum_{k=k_3}^{N} p(N_J = k), \quad k_3 = \left\lceil N\left(\frac{1+\rho\gamma}{2}\right) \right\rceil, \tag{9}$$

where $p(N_J = k)$ is as defined in (3).

PERFORMANCE COMPARISON

The most direct way to compare the relative AJ performance of these combiners is simply to plot their respective BER curves and make a comparison based on worst-case (with respect to ρ) BER. This is done later on. A less involved (and perhaps a more insightful) comparison may be made by examining each combiner's error threshold for a given N, ρ, and γ. That is, the number of jammed hops required to cause an error for a given set of (N, ρ, γ)is a measure of AJ effectiveness. The error thresholds for the three combiners considered are summarized below,

$$
\begin{array}{lll}
\text{linear} & : & N_J \geq N\rho\gamma \;, \\
\text{max-normalized} & : & N_J \geq \dfrac{N}{2-\rho\gamma} \;, \\
\text{self-normalized} & : & N_J \geq N\left(\dfrac{1+\rho\gamma}{2}\right).
\end{array}
\qquad (10)
$$

The ‘thresholds’ are the quantities on the right hand sides of the inequalities; a higher threshold implies greater AJ capability. Noting that N is a common factor of all of the threshold expressions and that ρ and γ appear only as a product, the comparison may be made by plotting the fraction N_J/N as a function of $\alpha \equiv \rho\gamma$, where " (0 < " ≤ 1) is the ratio between the detected signal energy and detected jammer energy of a jammed hop. Figure 3 shows the resulting plot of the three curves. From Figure 3, it is seen that the curves do not cross one another, and merge only at $\alpha = 0$ and at $\alpha = 1$ (only the nonlinear thresholds merge at $\alpha = 0$); at all other values of α, the self-normalized threshold has the greatest value, the linear threshold has the lowest value, and the max-normalized threshold lie somewhere in between. In general, the nonlinear threshold curves are similar in appearance and therefore the respective combiners are expected to perform similarly, with the self-normalized version slightly superior than the max-normalized one. Thus, judging from the behavior of their error thresholds alone, the relative AJ performance of the respective combiners may be deduced.

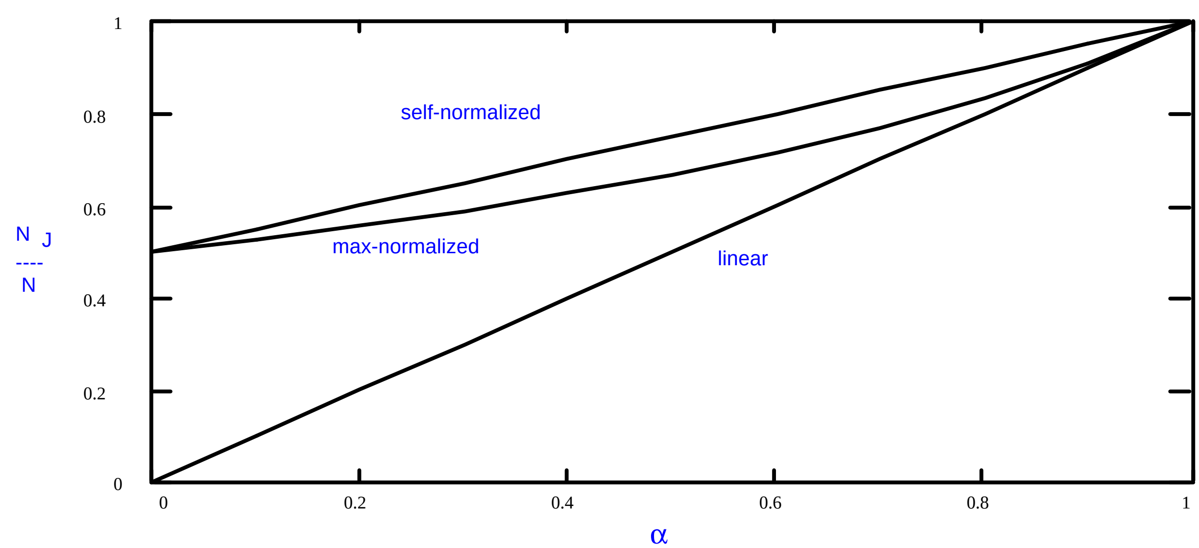

Figure 3 : Error Thresholds.

To obtain a quantitative comparison, one needs to evaluate a more-precise performance measure such as the BER, as expressed in equations (3), (6), and (9). One commonly-used criterion is the ‘worst-case’ BER, obtained either analytically by differentiation with respect to ρ, or numerically by evaluating the error expression for the entire range of ρ–values and determining the highest BER. Due to the discrete nature of the expressions in

(3), (6), and (9), they are not amenable to differentiation, therefore, worst-case BER results were obtained numerically. A typical BER plot is shown in Figure 4, for the self-normalized combiner. In this figure, the worst-case BER 'curve' is the straight line drawn to delineate (approximately) the upper envelope of the individual curves. Note that the individual curves are actually staircase-shaped, which is due to the no-ambient-noise assumption.

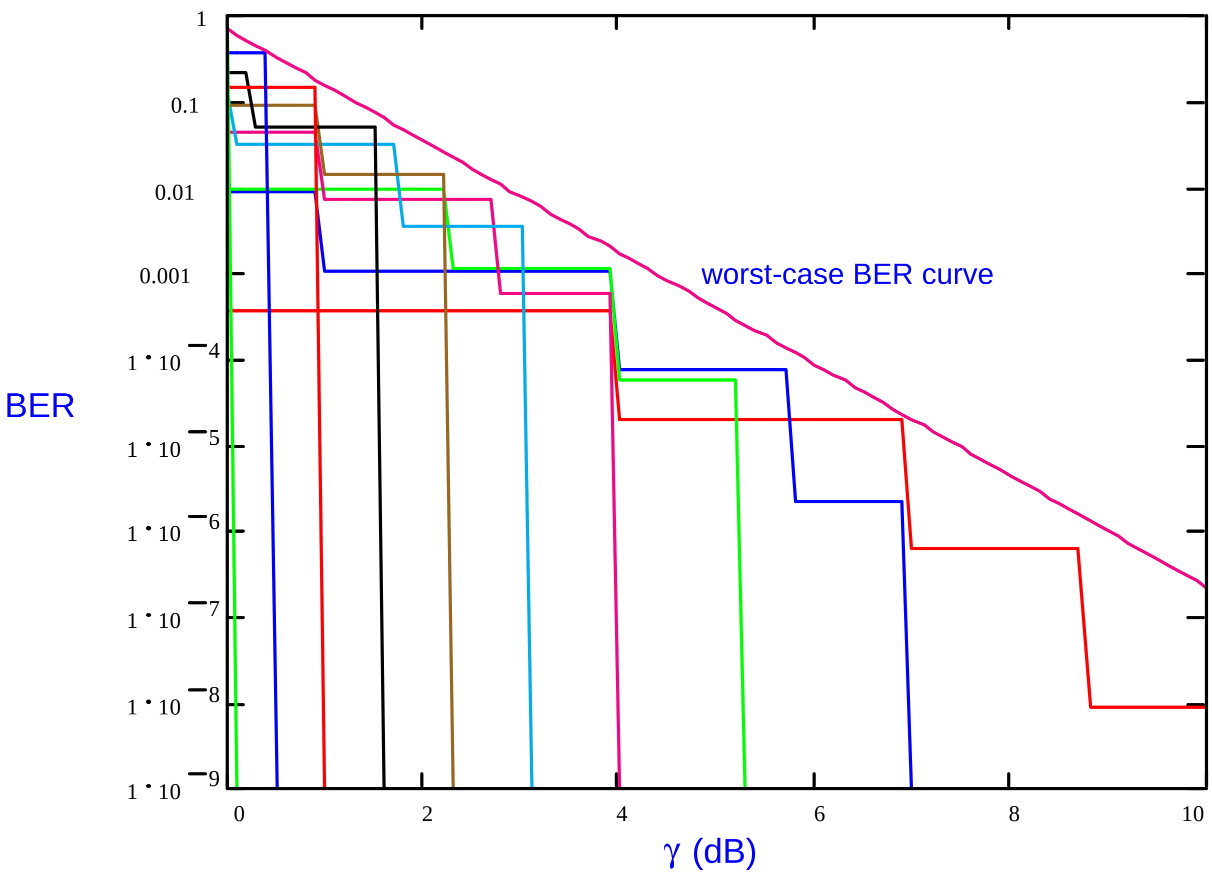

Figure 4: Worst-case BER, self-normalized combiner, N=8.

Rather than comparing the actual worst-case BER's, a more convenient way to assess AJ effectiveness is to examine the (asymptotic) slope or fall-off of the worst-case BER curves. From plots such as Figure 4, it is found that for the linear combiner, the asymptotic slope is 10 dB/decade for all values of N (order of diversity). For the max-normalized and self-normalized combiners, the asymptotic slopes increase monotonically (and approximately linearly) with N, with the self-normalized combiner performing slightly better than the max-normalized combiner in each case, which is consistent with the deduction made earlier based on the relative magnitudes of their error thresholds. Table I below lists the asymptotic slopes of each combiner for several values of N.

Table I : Asymptotic fall-off of worst-case BER (dB/decade).

N	linear	max-normalized	self-normalized
2	10	20	20
4	10	29	35
8	10	55	65
16	10	100	120

CONCLUSIONS/SUMMARY

The relative AJ performance of three diversity combiners for fast frequency hopped 8FSK has been investigated, assuming a 'coarse' hopping structure and no shuffling of the signal set. The three combiners considered, in descending order of AJ effectiveness, are the self-normalized combiner, the max-normalized combiner, and the linear combiner.

The worst-case BER for the linear combiner is found to improve at 10 dB/decade for all orders of diversity, N; those for the max-normalized and self-normalized combiners improve at roughly 7N to 8N dB/decade. By comparison, a combiner with jammer state information (usually considered to be the best-achievable combiner) in partial-band-noise jamming decays at 10N dB/decade [5].

REFERENCES

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