A COHERENT RECEIVER FOR QPSK AND SQPSK SIGNALS

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Summary - A demod-remod type of coherent tracking loop for conventional QPSK and staggered QPSK (SQPSK) is presented. The phase detector characteristic (S-curve) is determined. The effects of power unbalance and arm gain unbalance on the S-curve are presented. The steady state rms phase error is shown as a function of the signal-to-noise ratio at the output of the arm filters.

Introduction - Several coherent tracking loops for balanced and unbalanced QPSK and SQPSK have recently been analyzed in the literature, among which are [1-3]. The tracking loops receiving primary attention are the conventional biphase Costas loop and the 4th power loop. The optimality of the quadriphase Costas loop for conventional QPSK using the maximum a posteriori (MAP) criterion has recently been demonstrated by Simon [4]. Although this is an intuitively satisfying motivating force and does lead to easily implementable receiver structures, the MAP criterion does not imply optimality from the rms error viewpoint. The demod-remod loop in a TDMA environment was first described in [5].

The demod-remod type receiver is quite attractive from the standpoint that the entire suppressed carrier power spectral density of the QPSK signal is despread (or folded) to dc and used to generate the tracking error signal. Equivalently stated, self-noise in the demod-remod receiver occurs only via tracking errors and is not an inherent problem for QPSK signals as it is for the fourth power loop.

The demod-remod receiver is described herein and the S-curve (phase detector characteristic) of the loop generated. The effects of power imbalance between the quadriphase signals as well as the arm gain imbalance are also discussed. The steady state rms phase jitter is finally computed. For simplicity of calculations, the system parameters are chosen to be ideal and not necessarily optimum.

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Receiver Description - Consider the demod-remod receiver illustrated in Figure 1. The input signal at IF is

$$y(t) = s(t) + n(t) \tag{1}$$

where s(t) is a balanced conventional QPSK or SQPSK signal, given by

$$s(t) = \sqrt{S}\,\{d_1(t)\sin[\omega_0 t + \theta_0(t)] + d_2(t)\cos[\omega_0 t + \theta_0(t)]\} \tag{2}$$

where $d_1(t)$ is a sequence of ±1's, equally likely, with symbol time T_s, d_2 (t) is a nonsynchronoul sequence of equally likely ±1's, also with symbol time T_s , and S is the average received signal power.

The phase modulation is

$$\theta_0(t) = \Omega_0 t + \theta_0 \tag{3}$$

where Ω_0, is the total frequency difference (loop stress), including doppler, in rad/sec between the carrier RF of the received signal and the rest frequency of the receiver VCXO in Figure 1, and θ_0 is an arbitrary reference phase.

In (1), n(t) is broadband receiver front-end Gaussian noise with one-sided spectral density N_0 watts/Hz after passage through the IF filter (see Figure 1). The IF filter H(p) is assumed to have single-sided noise bandwidth of B_{IF} Hz about the center frequency f_0. The narrowband quadrature representation of the noise n(t) is

$$n(t) = \sqrt{2}\,[N_c(t)\cos(\omega_0 t + \theta_0(t)) + N_s(t)\sin(\omega_0 t + \theta_0(t))] \tag{4}$$

where $N_c(t)$ and $N_s(t)$ are independent zero mean Gaussian random processes with single-sided power spectral density N_0 watts/Hz, normalized autocorrelation function $\mu(\tau)$, and single-sided noise bandwidth B_{IF} Hz.

The autocorrelation function of N_c and N_s is $R_N(\tau) = \sigma_N^2\,\mu(\tau)$, and the total noise power in n(t) is $P_n \triangleq 2N_0B_{IF}$. The noise power is both $N_c(t)$, and$N_s(t)$ is designated as $\sigma_N^2 \triangleq N_0B_{IF}$

The signal-to-noise ratio (SNR) at the output of the IF filter is given by $p \triangleq S/(\sigma_N^2) = S/P_n$

Three subsystems can be distinguished in Figure 1: the demodulator or data wipeoff, the remodulator, and the tracking loop. The reference signals from the VCXO for the QPSK data wipeoff are

$$r(t) = \sqrt{2}\cos[\omega_0 t + \hat{\theta}_0(t)] \tag{5a}$$

and

$$r'(t) = \sqrt{2} \sin [\omega_0 t + \hat{\theta}_0(t)] \tag{5b}$$

where $\hat{\theta}(t)$ is an estimate of $\theta_0(t)$. The output signals of the data wipeoff, after phase detecting in the two data channels and the lowpass filters, are given by

$$\begin{aligned} \varepsilon_I(t) &= \sqrt{\frac{S}{2}} \left\{ \hat{d}_1(t) \sin \phi(t) + \hat{d}_2(t) \cos \phi(t) \right\} \\ &+ \hat{N}_c(t) \cos \phi(t) + \hat{N}_s(t) \sin \phi(t) \end{aligned} \tag{6a}$$

for the in-phase channel, and

$$\begin{aligned} \varepsilon_Q(t) &= \sqrt{\frac{S}{2}} \left\{ \hat{d}_1(t) \cos \phi(t) - \hat{d}_2(t) \sin \phi(t) \right\} \\ &- \hat{N}_c(t) \sin \phi(t) + \hat{N}_s(t) \cos \phi(t) \end{aligned} \tag{6b}$$

for the quadrature channel. The receiver loop phase error is defined as $\phi(t) \triangleq \theta_0(t) - \hat{\theta}_0(t)$

In (6), the ^ denotes filtering of the baseband signals and noises, namely

$$\hat{d}_i(t) = G_0(p)\, d_i(t), \quad i = 1,2 \tag{7a}$$

and

$$\hat{N}_\alpha(t) = G_0(p)\, N_\alpha(t), \quad \alpha = c,s \tag{7b}$$

where p is the Heaviside operator.

The filter $G_0(p)$ accounts for the filtering of both G(p) and H(p), and is given by

$$G_0(p) = G(p)\; L[H(p)] \tag{8}$$

where L[H(p)] designates the variation of the IF filter H(p) about the IF frequency, and G(p) is the low pass arm filter with one-sided noise bandwidth B_{ARM}.

The signal power at the output of each of the arm filters is given by

$$S_0/2 = \gamma^2 S/2 \tag{9a}$$

where

$$\gamma \triangleq \sqrt{2\int_0^\infty |G_0(j2\pi f)|^2 S_{d_i}(f)\, df} \tag{9b}$$

is normalized amplitude suppression factor at the output of the arm filter, and $S_{d_i}(f)$ is the two-sided PSD of the data signals i = 1, 2. The noise power in $\hat{N}_\alpha(t)$ is $\sigma^2_{\hat{N}} \triangleq N_0B_0$, where B_0 is the one-sided noise bandwidth of the equivalent filter G_0, i.e.,

$$B_0 \triangleq \int_0^\infty |G_0(j2\pi f)|^2\, df \text{ Hz}. \tag{10}$$

The SNR at the output of each of the arm filters is, therefore, given by

$$\rho_0 \triangleq \frac{S_0}{2N_0B_0} = \rho\left(\frac{\gamma^2 B_{IF}}{B_0}\right) = \left(\frac{\mathscr{E}_s}{N_0}\right)\left(\frac{\gamma^2}{B_0T_s}\right) \tag{11}$$

where $\mathscr{E}_s = ST_s/2$ is the received signal energy in T_s sec at the output of each of the arm filters. The factor $\gamma^2/(B_0T_s)$ is the attenuation in SNR at the output of each of the arm filters.

The video data signals $\hat{d}_i(t)$ are sent to the symbol sync tracking loop and to the matched filter data demodulators as indicated in Figure 1. The inputs to the symbol sync tracking loop and the matched filter data demodulators may come from either before or after the arm filters given by G(p). Both alternatives are shown in Figure 1. The demodulator and remodulator portions of the coherent receiver operate instantaneously on the received signal and do not require symbol synchronization as do the matched filter data demodulators.

The reconstructed waveform at the output of the remodulator, $K_C(t)$, assuming that the gain matching adjustment is performing ideally, is

$$K_C(t) = \sqrt{2}\,\{[\operatorname{sgn} \varepsilon_I(t)] \sin(\omega_0 t + \hat{\theta}_0(t)) - [\operatorname{sgn} \varepsilon_Q(t)] \cos(\omega_0 t + \hat{\theta}_0(t))\} \tag{12}$$

where $\operatorname{sgn} x \triangleq |X|/X$ accounts for the presence of the hard limiter.

The reconstructed signal, $K_C(t)$, is the reference signal for the tracking loop phase detector. The input to the phase detector is a delayed version of the received signal at IF, namely y(t). The delay line compensates for the time it takes for y(t) to be processed by the demodulator/remodulator. In the analysis, we assume this circuit delay time is zero.

The dynamic phase error, which is defined as the video (or baseband) portion of the output of the phase detector, is then

$$\varepsilon(t) \triangleq [y(t)\ K_C(t)]\Big|_{\text{baseband}} \tag{13}$$

<u>The Phase Detector Characteristic</u> - The loop phase detector characteristic (S-curve) g(ϕ) is defined as

$$g(\phi) \triangleq E[\varepsilon(t) \mid \phi(t)] \tag{14}$$

Assume that

(i) The phase error process θ(t) is essentially constant over large number of symbol intervals. If the phase reference θ_0 is essentially constant over many symbol times and the phase estimate $\hat{\theta}_0(t)$ satisfies the same restrictions, then θ(t) will satisfy the above requirements.

(ii) A direct consequence of this assumption is that the response of the tracking loop be very slow with respect to the symbol time T_s. This is equivalent to $B_L T_s \ll 1$ where B_L is the one-sided receiver tracking loop noise bandwidth.

Then, the phase detector characteristic of the demod-remod loop is derived in [6,7] to be

$$\begin{aligned} g(\phi) &= E[\varepsilon(t) \mid \phi(t)] \\ &= \sqrt{2S}\Big(\sin\phi\,\{1 - \operatorname{erfc}[\sqrt{\rho_0}\,(A(\phi))] - \operatorname{erfc}[\sqrt{\rho_0}\,(B(\phi))]\} \\ &\quad + \cos\phi\,\{\operatorname{erfc}[\sqrt{\rho_0}\,(A(\phi))] - \operatorname{erfc}[\sqrt{\rho_0}\,(B(\phi))]\}\Big) \end{aligned} \tag{15}$$

where

$$A(\phi) \triangleq \cos\phi + \sin\phi \tag{16a}$$

$$B(\phi) \triangleq \cos\phi - \sin\phi \tag{16b}$$

and erfc is the complementary error function defined as

$$\operatorname{erfc}(u) \triangleq \int_u^\infty \frac{\exp(-v^2/2)}{\sqrt{2\pi}}\, dv \tag{17}$$

and

$$r'(t) = \sqrt{2}\sin[\omega_0 t + \hat{\theta}_0(t)] \tag{5b}$$

where $\hat{\theta}(t)$ is an estimate of $\theta_0(t)$. The output signals of the data wipeoff, after phase detecting in the two data channels and the lowpass filters, are given by

$$\begin{aligned}\varepsilon_I(t) &= \sqrt{\frac{S}{2}}\left\{\hat{d}_1(t)\sin\phi(t) + \hat{d}_2(t)\cos\phi(t)\right\} \\ &\quad + \hat{N}_c(t)\cos\phi(t) + \hat{N}_s(t)\sin\phi(t)\end{aligned} \tag{6a}$$

for the in-phase channel, and

$$\begin{aligned}\varepsilon_Q(t) &= \sqrt{\frac{S}{2}}\left\{\hat{d}_1(t)\cos\phi(t) - \hat{d}_2(t)\sin\phi(t)\right\} \\ &\quad - \hat{N}_c(t)\sin\phi(t) + \hat{N}_s(t)\cos\phi(t)\end{aligned} \tag{6b}$$

for the quadrature channel. The receiver loop phase error is defined as $\phi(t) \triangleq \theta_0(t) - \hat{\theta}_0(t)$

In (6), the ^ denotes filtering of the baseband signals and noises, namely

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where L[H(p)] designates the variation of the IF filter H(p) about the IF frequency, and G(p) is the low pass arm filter with one-sided noise bandwidth B_{ARM}.

The signal power at the output of each of the arm filters is given by

$$S_0/2 = \gamma^2 S/2 \tag{9a}$$

The normalized phase-detector characteristic, which is obtained from dividing $g(\phi)$ by the rms of the noise in the IF bandwidth, is plotted in Figure 2, for various values of ρ_0. For negative ϕ, $g_n(-\phi) = -g_n(\phi)$, that is $g_n(\phi)$ is an odd function. As in any quadriphase system, the tracking loop nonlinearity has four stable lock points located at 0, ±90, and 180 degrees. Differential encoding of the data or a sync burst is, therefore, required to resolve this ambiguity. The improvement in the slope of the S-curve as the signal strength increases, is apparent by inspection of Figure 2.

Effect of Power Unbalance and Receiver Gain Variations - So far, it was assumed that the received signal is a balanced QPSK signal and that the gain matching adjustment in Figure 1 is performing ideally. In this section, an unbalanced power factor $0 \leq \alpha \leq 1$ is introduced, so as to make the received signal an Unbalanced QPSK. Then,

$$s(t) = \sqrt{2\alpha S}\, d_1(t) \sin(\omega_0 t + \theta_0(t)) + \sqrt{2(1-\alpha)S}\, d_2(t) \cos(\omega_0 t + \theta_0(t)) \qquad (18)$$

where all the parameters are described in (2) and $\alpha = 1/2$ corresponds to the balanced QPSK case.

Furthermore, in the remod part of the receiver, the reconstructed signal with β accounting for an error in gain matching adjustment is given by

$$K_C(t) = \sqrt{2}\left\{\beta[\mathrm{sgn}\ \varepsilon_I(t)] \sin(\omega_0 t + \hat{\theta}_0(t)) - [\mathrm{sgn}\ \varepsilon_Q(t)] \cos(\omega_0 t + \hat{\theta}_0(t))\right\} \qquad (19)$$

The receiver phase detector characteristic as a function of α and β is derived in [6]. Figures 3 and 4 illustrate two typical S-curves. By varying α and β over a wide range of values, it is noted that the more critical parameter is the power balance α. Receiver tracking undergoes negligible variation in performance if α is maintained within ±1 dB of its correct value of 0.5. When α is in error by ±1 dB, this corresponds to approximately a 60 per cent to 40 percent power split in the two channels of the QPSK waveform. If α is maintained within ±1 dB of its correct value, the tolerance on β is not severe. Also, a ±6 dB variation in β can be easily tolerated, if the variation in α is maintained within ±1 dB. In conclusion, the tolerances on α and β are not severe in order to maintain a specified system performance.

Tracking Performance - The power spectral density of the equivalent noise in the loop, as well as the phase error steady state probability density function $p(\phi)$is derived in [7]. It is shown that in the absence of loop stress Ω_0 the rest frequency of the VCXO is identical to the carrier frequency of the received signal.

The rms phase error, which is a measure of the tracking performance of the loop, can be given as

$$\sigma_\phi = \left\{ \int_{-\pi/4}^{\pi/4} [\phi - E(\phi)]^2 p(\phi) d\phi \right\}^{1/2} \text{ radians} \tag{20}$$

where $E(\phi)$ denotes the expected value of ϕ. Assuming no unbalances, Figure 5 shows the rms phase error of the demod-remod loop as a function of the signal-to-noise ratio (ρ_0) at the IF filter with the single sided loop bandwidth B_L = 50 khz and symbol time T_s = 2.5 usec. It is observed that there is a threshold effect in rms phase error in the vicinity of 8 to 10 dB in ρ_0 when the reciprocal of the time bandwidth product is $[B_L T_s]^{-1} = 8$. It should be noted that this is a relatively small value of $(B_L T_s)$

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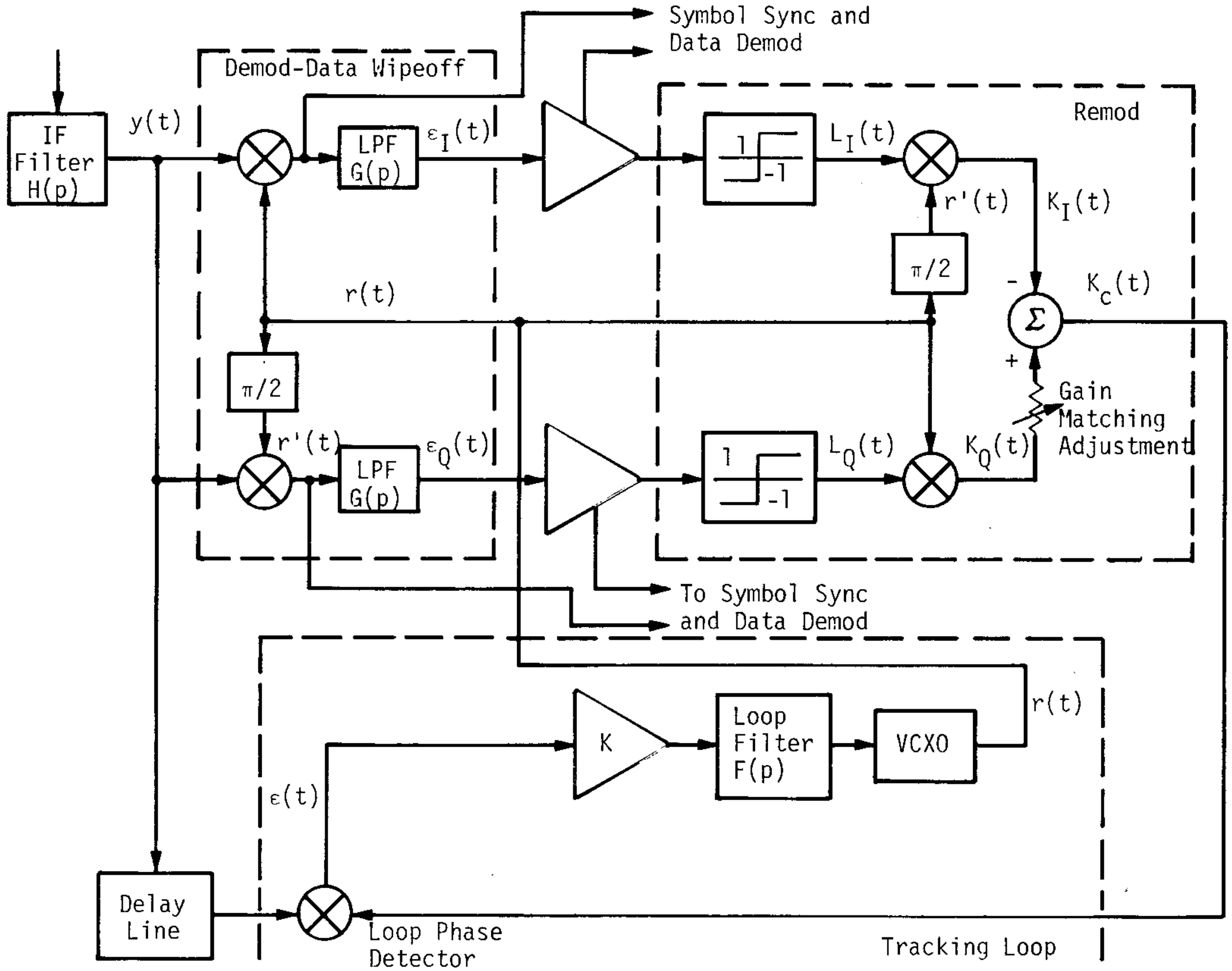

Figure 1. Demod-Remod Receiver Block Diagram

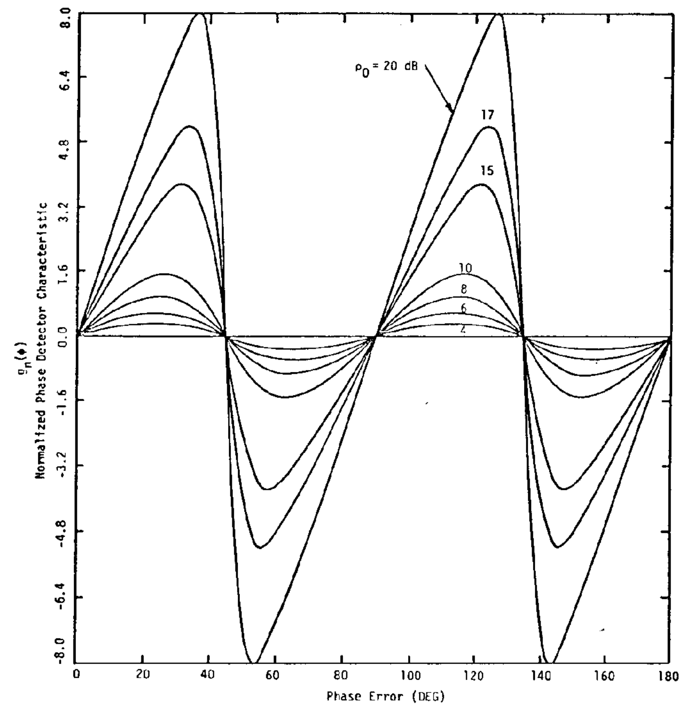

Figure 2. Normalized Phase Detector Characteristic vs. ϕ for Various Signal-to-Noise Ratios, ρ_0.

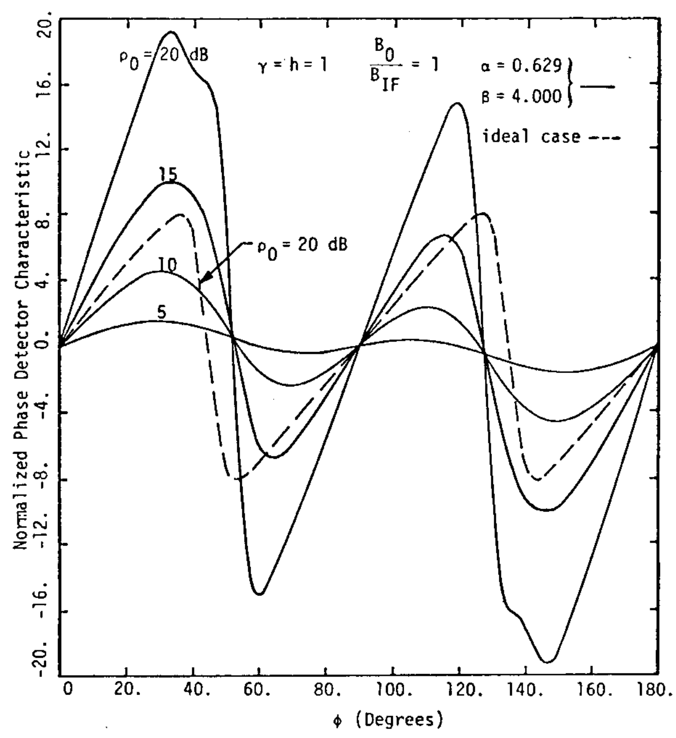

Figure 3. Unbalanced Normalized Phase Detector Characteristic vs. ϕ for Various Sianal-to-Noise Ratios, ρ_0.

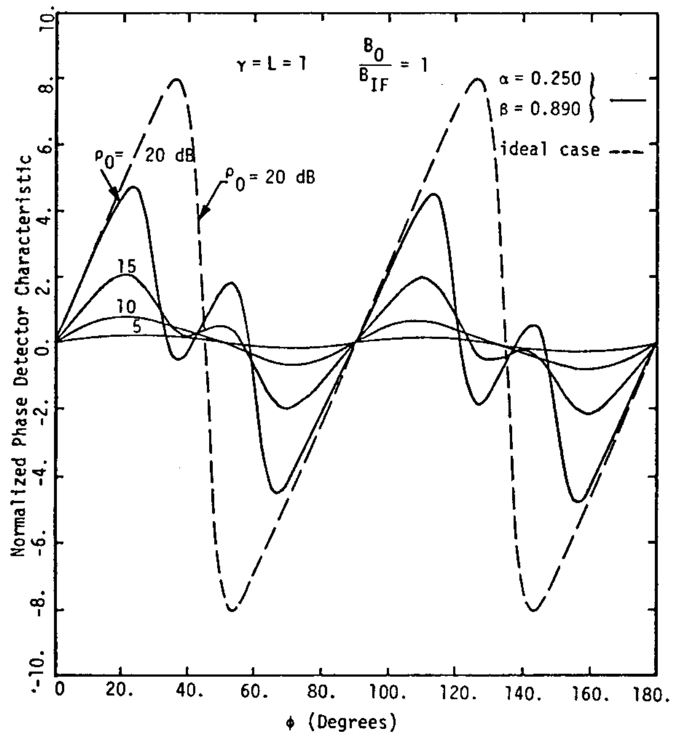

Figure 4. Unbalanced Normalized Phase Detector Characteristic vs. ϕ for Various Signal-to-Noise Ratios, ρ_0.

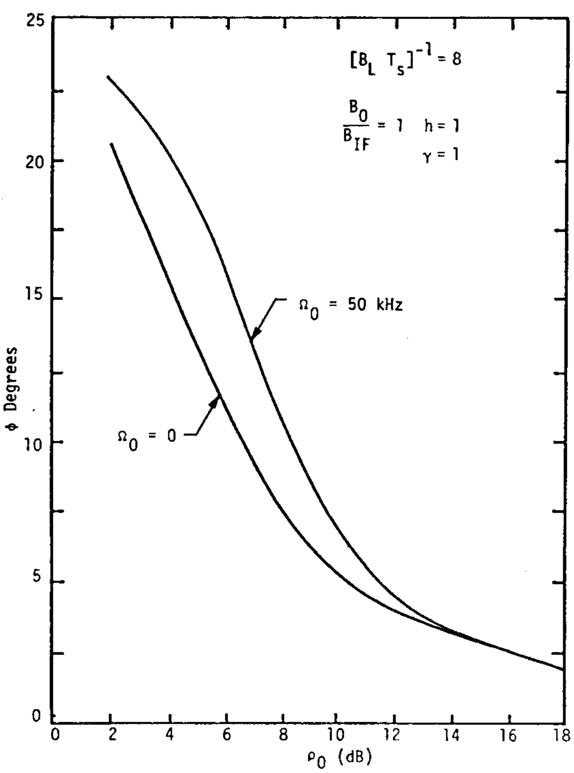

Figure 5. RMS Phase Error vs. Signal-to-Noise Ratio ρ_0.