INTERLEAVING OF REED-SOLOMON VITERBI CONCATENATED CODING CHANNEL

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ABSTRACT

In this paper, two interleaving schemes are discussed and several R-S code array synchronization configurations are investigated.

A procedure for obtaining synchronization sequences, for the R-S code array, under specified conditions is suggested and it is followed by the identification of sequences with desirable properties.

Several graphs are presented, e.g., false synchronization probability versus various bit error rates for the number of errors permitted, and also, the missed synchronization probability versus various bit error rates for number of errors permitted and for various lengths of synchronization sequences.

For the interleaving schemes discussed there is no analytical advantage, with respect to array synchronization, for selecting one scheme over the other.

INTRODUCTION

A concatenated coding standard is currently being written at the Goddard Space Flight Center. The need for this standard developed when experimenter data reliability (confidence level) was widely different, and their data were required to be time multiplexed. An advantage of concatenated coding is that each experimenter can be provided with his own confidence level. In addition, the telecommunication channel may be designed for the lowest required confidence level, with extra parity for the experimenter requiring higher confidence.

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The inner code was specified by an existing standard as a short constraint length convolutional code. Because of the burst characteristics of the Viterbi decoder, a Reed-Solomon (R-S) code with interleaving was considered as the outer code, the requirement to interleave the transmitted symbols led to a study of R-S code array synchronization. This study was performed under a two-month summer fellowship grant.

Following a summary of a literature search, the remainder of this paper discusses two interleaving schemes and several R-S code array synchronization configurations. Also given is a procedure for obtaining synchronization sequences which is followed by several graphs showing false and missed synchronization statistics for various bit error rates.

TWO INTERLEAVING APPROACHES FOR REED-SOLOMON VITERBI CONCATENATED CODING CHANNEL

A R-S Viterbi concatenated coding channel can be described as follows and a block diagram is shown in Figure 1.

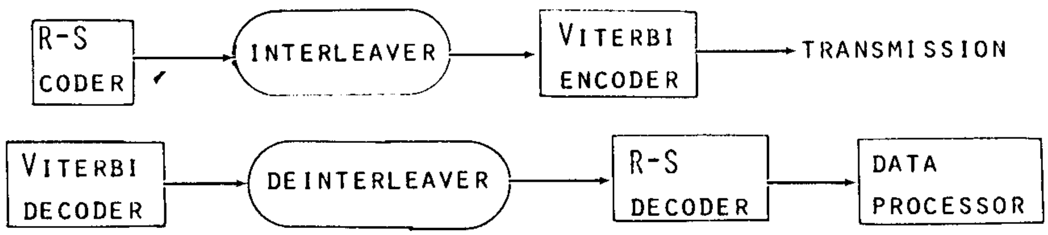


FIGURE 1 - R-S VITERBI CONCATENATED CODING CHANNEL

With Additive White Gaussian Noise on the transmission channel, the Viterbi decoder error events will normally occur in bursts. At signal to noise ratios of interest (between 2.0 and 2.5 db), without data interleaving, the Viterbi decoder burst error events would tend to occur within one R-S code word. The effect of interleaving is to spread these burst error events over many code words so as to create a guard space. With a guard space the R-S decoder will tend to operate uniformly on all the coded data.

As shown in Figure 2, the consecutive numbers 1, 2, 3..., 3568 are information symbols from GF(2^8) which are to be coded into 16 R-S code words.

	INFORMATION SYMBOLS	32 PARITY SYMBOLS
CODE WORD NO. 1	1, 2....223	1............. 32
CODE WORD NO. 2	224, 225....446	....
	....	....
CODE WORD NO. 16	3346, 3347...3568	....

FIGURE 2 - TYPE 1 R-S CODE ARRAY

Without interleaving, these information symbols would be transmitted over the channel in sequential order (code word 1 is followed by code word 2, etc.). Thus, a long burst error event from a Viterbi decoder would tend to affect the symbols of one entire code word. With interleaving, the R-S information symbols will not be transmitted in sequence. Thus, a long burst error event would affect only one symbol of several code words.

TWO KINDS OF INTERLEAVING APPROACHES ARE DESCRIBED BRIEFLY BELOW

The type 1 interleaving approach assumes the R-S code array is as shown in Figure 2. The order of R-S information symbols transmission is 1, 224,...3346; 2, 225,...3347;...223, 446,... 3568 (column 1 followed by column 2 .... etc.). The parity symbols would follow in the same manner. In this arrangement, a burst of errors that spans $n \leq 16$ R-S symbols will be distributed among n different code words, affecting only one symbol of each code word.

The type 2 interleaving approach assumes the R-S code array is as shown in Figure 3. In this case, the information symbols for code word 1 are made of information symbols 1, 17, 33...3553.

	INFORMATION SYMBOLS	32 PARITY SYMBOLS
CODE WORD NO. 1	1, 17,.....3553	1..............32
CODE WORD NO. 2	2, 18,.....3554	....
	....	....
CODE WORD NO. 16	16, 32,.....3568	....

FIGURE 3 - TYPE 2 R-S CODE ARRAY

Again, the first column of the code word is transmitted first and followed by column 2, etc. However, as a result, the order of transmission of R-S information symbols is exactly the same as they appear in the information block. The parity symbols would follow in the same manner. The affect of a decoding error on a particular code word is spread throughout the information block. For type 1 interleaving, the decode errors are constrained to a consecutive string of 223 symbols (refer to reference 7).

In both types of interleaving, the number of rows in the R-S code array is termed the degree of interleaving. The degree should be large enough to ensure statistical independence between R-S symbols of individual code words before decoding.

For both types of interleaving, the probability of an R-S code word error is given by

$$P_{ps} = P_r \{ \text{more than 16 independent symbol errors} \} = \sum_{k=17}^{255} \binom{255}{k} \Pi^k (1-\Pi)^{255-k} \quad (1)$$

where Π denotes the average probability of an R-S symbol error leaving the Viterbi decoder.

R-S CODE ARRAY SYNCHRONIZATION

The basic idea of array synchronization (sync) is to find and maintain the correct location of the starting point of R-S code blocks in a long bit stream containing many such R-S code blocks. Synchronization can be accomplished by periodically inserting a known sync pattern into the data stream and estimating the starting point of the array based on the cross-correlation between the received subsequence and a stored copy of the sync pattern.

The correlation between two patterns is determined by comparing the 8-bit symbols which made up the pattern.

1. Configurations

Techniques for inserting the sync pattern for the above described array configurations have been proposed and the performance of each configuration, for interleaving degrees 8 and 16, were investigated by Odenwalder (refer to reference 6). The performance of each configuration was based on the probability of missed sync and false lock during the acquisition and the tracking modes.

2. Sync Pattern Selection

The sync pattern must be carefully chosen so that cyclic shifts of the sync pattern do not have a high correlation. Otherwise, only a few errors might result in a decision to lock-up on a shifted version of the sync pattern.

Burst Error Statistics and its Effect in Selecting Sync Pattern With Good Correlation Properties - A literature research has shown that the efficient way of obtaining burst error statistics for a Viterbi decoder output is to simulate a Viterbi decoding algorithm or to measure error patterns from a Viterbi decoder. It has been stated that randomly occurring burst errors carry no weight in selection of a sync pattern, because the error events are statistically independent.

Probability of Missed Sync and Probability of False Sync - There are two criteria to be used in the selection of the sync pattern. The analysis of these criteria follows.

Probability of Missed Sync - When a known pattern is to be sent, the probability that it is received with more than k errors is

$$\sum_{X=k+1}^{n} C_x^n \quad (1-p)^{n-x} \quad p^x \qquad (2)$$

where n is the length of the sync pattern,
k is the number of errors permitted,
p is the bit error rate.

Hence the missed sync probability is

$$mp = (S) \; (1-\sum_{X=0}^{k} C_x^n \quad (1-p)^{n-x} p^x) \qquad (3)$$

where S is the a priori probability that the synchronizing signal was sent.

mp can be approximated by

$$\sum_{X=k+1}^{\infty} \frac{e^{-np} (np)^x}{X!} \qquad (4)$$

where np is the number of bit errors in a sync pattern.

Figure 4 presents a family of missed sync probability versus the number of errors permitted and np. Figure 5 presents the np versus number of errors permitted and the probability of missed sync. These two figures will be used to find a suitable n and its associated number of errors permitted, to meet the missed sync probability constraint.

Probability of False Sync - At ground stations, sync recognition is accomplished by cross correlation in which the code recognizer examines the incoming data stream for the sync pattern code. The pattern seen by the recognizer in the absence of noise is either made up of random data bits, the true sync pattern bits, or a combination of both the random data bits and one or more bits of the sync pattern (this will be termed overlap region).

In the overlap region, the probability of a false sync indication is dependent upon the particular sync pattern used. False sync probability is determined by measuring the number of bits in agreement between the incoming bit stream and the stored copy of the sync pattern. This calculation is made with the sync code recognizer at each stage of overlap (refer to references 2 and 5).

The probability of false sync indication occurring at λ number bits of overlap can be computed by using the following formula:

$$u(P) = \sum_{i=0}^{\substack{\epsilon \text{ if } \epsilon \le \lambda \\ \lambda \text{ if } \epsilon > \lambda}} \left[\sum_{\substack{j = i - (\lambda - A(\lambda)) \text{ if } i - (\lambda - A(\lambda)) \ge 0 \\ j = 0 \quad \text{ if } i - (\lambda - A(\lambda)) < 0}}^{\substack{i \text{ if } i \le A(\lambda) \\ A(\lambda) \text{ if } i > A(\lambda)}} \binom{A(\lambda)}{j} \binom{\lambda - A(\lambda)}{i - j} (1 - p)^{(A(\lambda)+i-2j)} \, p^{(\lambda - A(\lambda) - i + 2j)} \right] \times \left[\frac{1}{2^{n-\lambda}} \sum_{K=0}^{\substack{\epsilon - i \text{ if } \epsilon - i \le n - \lambda \\ n - \lambda \text{ if } \epsilon - i > n - \lambda}} \binom{n - \lambda}{K} \right] \qquad (5)$$

where p = bit error rate (p = 0.1 in this context)
ϵ = number of errors allowed by the code recognizer
A(λ) = number of agreements at λ symbol of overlap.

Note when ϵ = 0

$$\mu(\lambda) = (1 - p)^{A(\lambda)} (p)^{\lambda - A(\lambda)} (1/2)^{n - \lambda}$$

The total probability of false sync indication in the entire region of overlap can be approximated by:

$$X_n = \sum_{L-1}^{n-1} \mu(\lambda) \qquad (6)$$

where n is the length of the sync pattern in bits.

PROCEDURE FOR SELECTING FRAME SYNCHRONIZATION PATTERN TO MEET THE SPECIFIED REQUIREMENTS

The following procedure was developed to select a sync pattern with a given bit error rate that met the probability of missed sync and the total probability of false sync indication.

Determine the length, n, of the sync pattern necessary to meet the probability of missed sync requirement. Figures 4 and 5 can be used to serve this purpose. As seen in Figure 5, for a given missed sync probability, one can find many np’s and their associated number of errors permitted by the recognizer. Observe that the more errors permitted by the recognizer, the smaller the missed sync probability for a given np. Also note that smaller k values are associated with smaller np. Because the computation of false sync probability is the most time consuming part of the computation, the smaller n value should be computed first.

If additional constraints are imposed (e.g., n has to be a multiple of 8) then the next larger n values can be used.

Determine the sync pattern necessary to meet the imposed probability of false sync constraint. When n is small, say ≤ 17, the exhaustion technique can be used to select the sync pattern which minimizes the total probability of false sync (hence meets the reasonable requirement). This technique is used to examine the total probabilities of false sync of all possible patterns with length n, with the exception of patterns with the same agreement vector. Refer to TABLE I for the 8-bit patterns that have the lower probability of false sync.

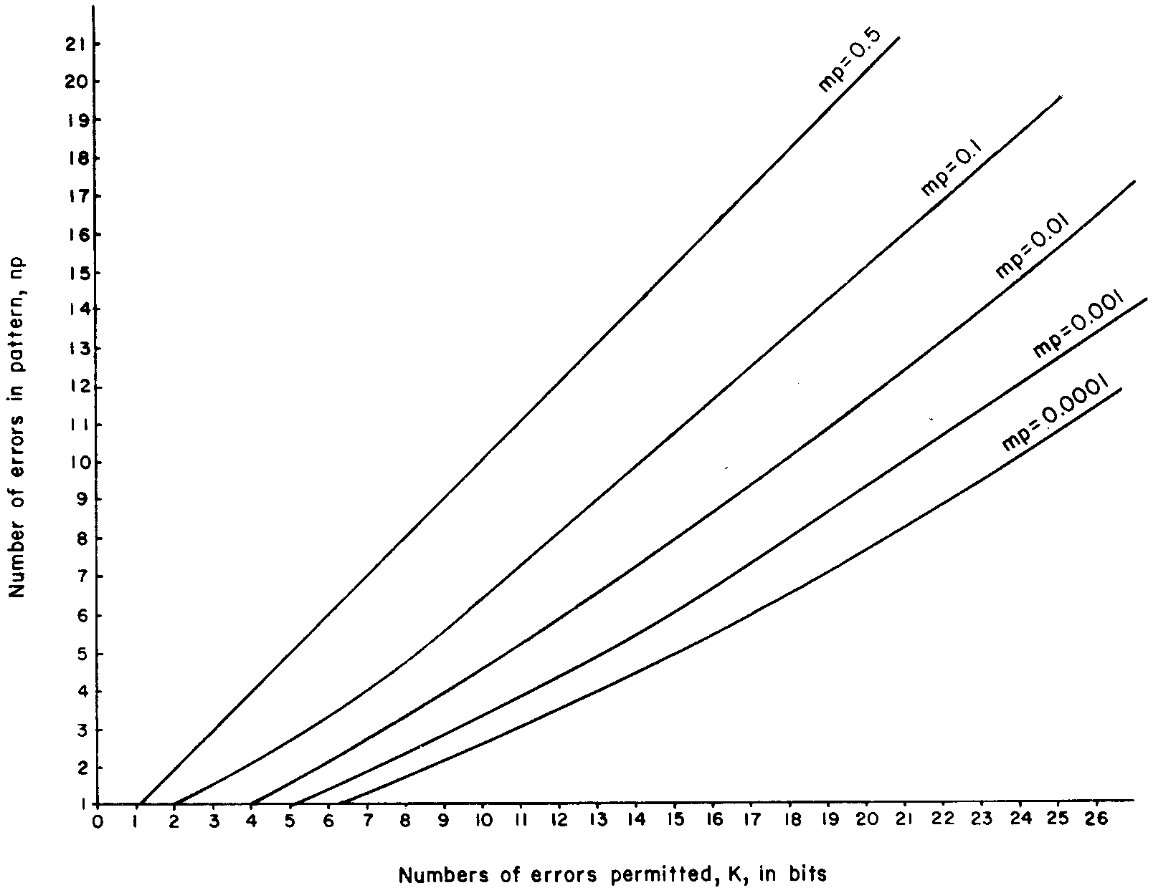
When n is large, select a pattern consisting of distinct 8-bit symbols which minimizes the total probability of false sync and contains approximately an equal number of 1's and 0's. A smaller false sync probability may be obtained by switching around the symbols or by replacing one or more of these symbols with other symbols that have lower total probability of false sync. However, if a desirable pattern cannot be found, the next larger n value which meets the missed sync probability requirement should be selected.

For an n value greater than or equal to 128, the total probability of false sync can be replaced by partial probability of false sync without significantly affecting the quality of the selected pattern. For large n values the difference between the probabilities of false sync in a good pattern and in a poor pattern, at lower degrees of overlap stages, is a very small number. For example, with p=0.1, k=0, and n=128, the probabilities of false sync for the worst pattern at overlap degrees 127, 50, and 2 are approximately 0.5, 10^{-25}, and 10^{-38} respectively.

The partial probability of false sync is defined as

$$\sum_{\lambda=b}^{n-1} \mu(\lambda) \qquad (7)$$

where b is the lowest degree of overlap to be investigated. For any magnitude of $\mu(b)$, b can be obtained by examining the false sync probabilities for the worst case, where the number of agreements equals the degree of overlap. For example $\mu(b) \geq$ 10-25, b can be obtained by solving the following equation.



$$mp = \sum_{X=K+1}^{\infty} \frac{e^{-np}(np)^{X}}{X!}$$

Figure 4 - Missed Sync Probability as a Function of Number of Errors Permitted, K, and Number of Errors in Pattern.

$$b = \frac{-25-n\log 2}{\log 2p} \qquad (8)$$

where n and p are defined as before.

Two good patterns of 128 bits with their total probability for k=0, 1, 2,... 26 are:

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pattern 1=  11011000, 00100111, 11100100, 00011011,
            10111000, 01000111, 11100010, 00011101
            11101000, 00010111, 10110000, 01001111
            11110010, 00001101, 11010000, 00101111

pattern 2=  00000011, 11111010, 10100110, 01110111
            01001011, 00011011, 11011010, 11011001
            00100011, 10000101, 11110010, 10111001
            10100010, 01111000, 10100001, 10000010
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Refer to TABLE II for the probability of false sync as a function of several numbers of errors permitted for the above two patterns.

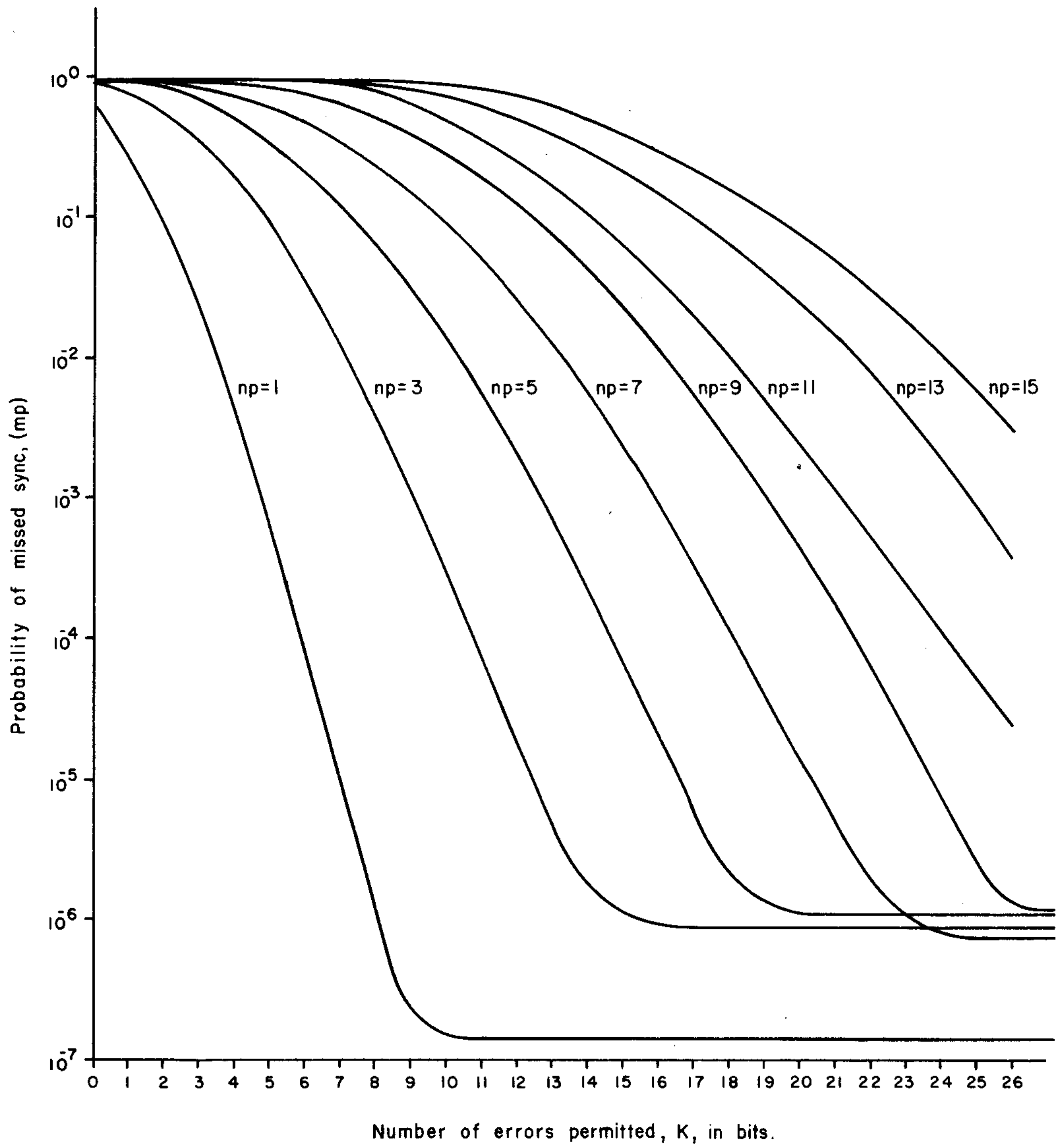

Figure 5- A Family of Error Per Pattern (mp) Curves as a Function of Probability of Missing Sync and the Number of Errors Permitted.

TABLE I

Eight Bit Patterns Having The Lowest Total Probability Of False Sync Indication With p=0.1 (Range 5 To 263)

Pattern	Number of Errors Allowed by Recognizer E=0	E=1
11011000	$.29845 \times 10^{-2}$	$.66529 \times 10^{-1}$
11100100	$.29845 \times 10^{-2}$	$.66529 \times 10^{-1}$
00011011	$.29845 \times 10^{-2}$	$.66529 \times 10^{-1}$
00100111	$.29845 \times 10^{-2}$	$.66529 \times 10^{-1}$
10111000	$.34968 \times 10^{-2}$	$.70114 \times 10^{-1}$
11100010	$.34968 \times 10^{-2}$	$.70114 \times 10^{-1}$
00011101	$.34968 \times 10^{-2}$	$.70114 \times 10^{-1}$
01000111	$.34968 \times 10^{-2}$	$.70114 \times 10^{-1}$
11101000	$.35445 \times 10^{-2}$	$.77969 \times 10^{-1}$
00010111	$.35445 \times 10^{-2}$	$.77969 \times 10^{-1}$
10110000	$.39465 \times 10^{-2}$	$.80064 \times 10^{-1}$
11110010	$.39465 \times 10^{-2}$	$.80064 \times 10^{-1}$
00001101	$.39465 \times 10^{-2}$	$.80064 \times 10^{-1}$
01001111	$.39465 \times 10^{-2}$	$.80064 \times 10^{-1}$
11010000	$.41545 \times 10^{-2}$	$.90960 \times 10^{-1}$
00101111	$.41545 \times 10^{-2}$	$.90960 \times 10^{-1}$

TABLE II

Probability of False Sync As A Function Of Numbers Of Errors Permitted For Two Good Patterns With p=0.1

Number of Errors Permitted	Total Probability of False Sync Indication Pattern I	Pattern II
0	0.71711×10^{-36}	0.85396×10^{-38}
1	0.12051×10^{-33}	0.11993×10^{-35}
2	0.10745×10^{-31}	0.83125×10^{-34}
3	0.67418×10^{-30}	0.37930×10^{-32}
4	0.33005×10^{-28}	0.12824×10^{-30}
5	0.13214×10^{-26}	0.34286×10^{-29}
6	0.44378×10^{-25}	0.75528×10^{-28}
7	0.12708×10^{-23}	0.14106×10^{-26}
8	0.31427×10^{-22}	0.22808×10^{-25}
9	0.67814×10^{-21}	0.32444×10^{-24}
10	0.12883×10^{-19}	0.41119×10^{-23}
11	0.21715×10^{-18}	0.46912×10^{-22}
12	0.32684×10^{-17}	0.48592×10^{-21}
13	0.44177×10^{-16}	0.46025×10^{-20}
14	0.53865×10^{-15}	0.40107×10^{-19}
15	0.59475×10^{-14}	0.32324×10^{-18}
16	0.59652×10^{-13}	0.24206×10^{-17}
17	0.54485×10^{-12}	0.16909×10^{-16}
18	0.45408×10^{-11}	0.11058×10^{-15}
19	0.34582×10^{-10}	0.67911×10^{-15}
20	0.24097×10^{-8}	0.39278×10^{-14}
21	0.15375×10^{-8}	0.21448×10^{-13}
22	0.89896×10^{-8}	0.11083×10^{-12}
23	0.48190×10^{-7}	0.54301×10^{-12}
24	0.23697×10^{-6}	0.25276×10^{-11}
25	0.10694×10^{-5}	0.11196×10^{-10}
26	0.44314×10^{-5}	0.47270×10^{-10}

CONCLUSION

Because of the burst nature of the Viterbi decoder, some form of interleaver is considered necessary for the R-S Viterbi concatenated coded channel. The type 2 interleaving scheme has an advantage over the type 1 scheme in that a smaller memory is required for the R-S encoder. The type 2 scheme also has an advantage in that the information symbols are transmitted sequentially. But, as far as the code array synchronization is concerned, there is no analytical advantage for selecting one scheme over the other. Once the synchronization configuration has been determined, the sync pattern length, the number of errors permitted, and the pattern must be carefully chosen so that specific requirements can be met. The procedure in the text serves as a guideline for selecting the pattern with the above given parameters. The synchronization patterns of length eight which minimize the total probability of false sync indication are presented in TABLE I. TABLE II presents two good sync patterns of length 128 bits and their associated total probability of false sync indication.

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