SUBCARRIER PHASE RECOVERY PERFORMANCE IN BENT-PIPE MODE OF SHUTTLE DATA TRANSMISSION

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SUMMARY

The subcarrier phase recovery is analyzed for the bent-pipe mode of Space Shuttle detached-payload data transmission on the Tracking and Data Relay Satellite System (TDRSS) Ku-band return link. The high-power component of the subcarrier modulation is unrestored payload data, either at baseband or modulating another subcarrier. At the receiver a Costas loop recovers the subcarrier phase. To analyze its performance in the baseband case, we obtain the loop S-curve, the power spectral density of the equivalent noise process, and the loop phase error variance.

INTRODUCTION

In the bent-pipe mode of Shuttle detached payload data transmission, the data are not detected at the Shuttle receiver. They go through the Shuttle repeater, where they are low-pass filtered and hard-limited. Then they are modulated onto the high-power component of the 8.5 MHz subcarrier (Reference 1). This is shown in Figure 1.

For the analysis of the 8.5 MHz subcarrier recovery, we may assume that the carrier has been recovered perfectly. This leads to the link model of Figure 2.

We wish to characterize the phase recovery performance of the Costas loop. The theory is developed in the case of baseband data for an arbitrary signal format at the Shuttle repeater output. The results are then applied to two different implementations for the Shuttle repeater: a hard-limiter preceded by a wide low-pass filter which does not affect the data signal and an arbitrary low-pass filter without hard-limiter.

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COSTAS LOOP MODEL (Following Reference 2)

The input signal to the Costas loop can be written

$$w(t) = \sqrt{2}a(t) \sin \Phi(t) + \sqrt{2}b(t) \cos \Phi(t) \tag{1}$$

for some random processes a(t) and b(t) , where $\Phi(t) \equiv \omega_2 t + \theta$ and θ is the input phase to be estimated. Let $\Phi(t)$ be the loop estimate of $\Phi(t)$. In the upper arm of the loop, w(t) is multiplied by $\sqrt{2K_1} \cos \hat{\Phi}(t)$, in the lower arm by $\sqrt{2K_1} \sin \hat{\Phi}(t)$. The multiplier units have gain $\sqrt{K_m}$ and are insensitive to the double-frequency terms. Each output passes through a filter corresponding to G(p), where p is the Heaviside operator, thus yielding the upper-arm signal $z_c(t)$ and the lower-arm signal $z_s(t)$.

$$z_c(t) = \sqrt{K_1 K_m}\, G(p)[a(t) \sin \varphi(t) + b(t) \cos \varphi(t)] \tag{2}$$

$$z_s(t) = \sqrt{K_1 K_m}\, G(p)[a(t) \cos \varphi(t) - b(t) \sin \varphi(t)] \tag{3}$$

where $\varphi(t) \equiv \Phi(t) - \hat{\Phi}(t)$ is the loop phase error. The two signals are multiplied together, with unit gain, to produce the dynamic error signal

$$z(t) = z_c(t) z_s \tag{4}$$

The instantaneous frequency of the VCO output is related to z(t) by

$$\frac{d\hat{\Phi}(t)}{dt} = K_V F(p) z(t) + \omega_2 \tag{5}$$

where K_Y is a gain constant. Hence, the stochastic integro-differential equation of operation of the loop is

$$2 \frac{d\varphi(t)}{dt} = -2K_V F(p) z(t) \tag{6}$$

Conditioned on φ, z(t) can be partitioned into a nonrandom part and a zero-mean random process:

$$z(t) = S(\varphi) + n_z(t, \varphi) \tag{7}$$

where

$$S(\varphi) = E\{z(t) | \varphi\} \quad \text{and} \quad n_z(t, \varphi) = z(t) - S(\varphi) \tag{8}$$

This allows us to rewrite Equation (6) as

$$2 \frac{d\varphi(t)}{dt} = -2K_V F(p) S[\varphi(t)] - 2K_V F(p) n_z(t, \varphi) \tag{9}$$

This equation describes a non-Markovian diffusion process. However, under suitable conditions (in particular, if the process $n_z(t,\varphi)$ is considerably faster than the process $\varphi(t)$)

it can be approximated by a Markov process and Fokker-Planck techniques can be applied to characterize the stationary distribution of the modulo-π reduced phase noise process as well as the cycle-slipping rate. For this analysis the S-curve $S(\varphi)$ and the statistics of the equivalent-noise process $n_z(t,\varphi)$ are needed.

COSTAS LOOP S-CURVE

We first obtain the input signal to the H(p) filter. We will denote a narrowband signal a(t) with center frequency ω_2 by

$$a(t) = \sqrt{2}\,\mathrm{Re}[a'(t)e^{j\omega_2 t}] \tag{10}$$

where a'(t) is the baseband equivalent to a(t). Defining the payload signal as

$$s_i(t) = \sqrt{2P_2}\; y(t)\, \sin(\omega_2 t) \tag{11}$$

where y(t) is normalized such that $E\{y^2(t)\} = 1$, we find

$$s_i'(t) = -j\sqrt{P_2}\, y(t) \tag{12}$$

For the low-power signal we have

$$s_q(t) = -\sqrt{2P_1}\, d_1(t)\cos(\omega_2 t) \quad \text{and} \quad s_q'(t) = -\sqrt{P_1}\, d_1(t) \tag{13}$$

Now we consider the noise u(t), a narrowband Gaussian process with center frequency ω_2.

$$u(t) = \sqrt{2}u_1(t)\cos(\omega_2 t) - \sqrt{2}u_2(t)\sin(\omega_2 t) \tag{14}$$

The spectral density of u(t) is

$$S_u(f) = S_{u'}(f-f_2) + S_{u'}(-f-f_2) \tag{15}$$

for some real function $S_u(f)$, where $f_2 = \omega_2/(2\pi)$. We define

$$R_{u_i}(\tau) = E[u_i(t)u_i(t+\tau)],\ i = 1,2 \tag{16}$$

$$R_{u_{ik}}(\tau) = E[u_i(t)u_k(t+\tau)],\ i \neq k \tag{17}$$

ik

We assume that $S_{u'}(f)$ is symmetric. Then letting R_u be the inverse Fourier transform of S_u, we have

$$R_{u_1} = R_{u_2} = R_{u'} \quad \text{and} \quad R_{u_{12}} = -R_{u_{21}} = 0 \tag{18}$$

We assume that the H'(p) filter is conjugate-symmetric, so that $H'(p)u_1(t)$ and $H'(p)u_2(t)$ are real-valued processes.

Now we go into the Costas loop. In the upper arm the input signal is multiplied by $\sqrt{2K_1 K_m}\cos(\omega_2 t-\varphi)$ and the double-frequency terms are dropped. The same result is obtained if the baseband equivalent of the signal is multiplied by $\sqrt{K_1 K_m}e^{j\varphi}$ and the real part is taken. In the lower arm of the loop, the input signal is multiplied by $\sqrt{2K_1 K_m}\cos(\omega_2 t-\varphi)$ and the double-frequency terms are dropped. This is the same as multiplying the basehand equivalent $-\sqrt{K_1 K_m}e^{j\varphi}$ and taking the imaginary part.

Since z_c and z_s are each linear functions of the input, we may write

$$z_c(t) = z_c(t;s_i) + z_c(t;s_q) + z_c(t;u) \tag{19}$$

$$z_c(t) = z_c(t;s_i) + z_c(t;s_q) + z_c(t;u) \tag{20}$$

where, for example, $z_c(t;s_i)$ is $z_c(t)$ when the loop input is just $H(p)s_i(t)$. We introduce the notation

$$\bar{a}(t) = G(p)H'(p)a(t) \tag{21}$$

for any signal a(t). Let $C=K_1K_m$. Then we have

$$z_c(t;s_i) = \sqrt{C}\sqrt{P_2}\,\sin(\varphi)\bar{y}(t) \tag{22}$$

$$z_c(t;s_q) = -\sqrt{C}\sqrt{P_1}\,\cos(\varphi)\bar{d}_1(t) \tag{23}$$

$$z_c(t;u) = \sqrt{C}(\cos(\varphi)\bar{u}_1(t)-\sin(\varphi)\bar{u}_2(t)) \tag{24}$$

$$z_s(t;s_i) = \sqrt{C}\sqrt{P_2}\,\cos(\varphi)\bar{y}(t) \tag{25}$$

$$z_s(t;s_q) = \sqrt{C}\sqrt{P_1}\,\sin(\varphi)\bar{d}_1(t) \tag{26}$$

$$z_s(t;u) = \sqrt{C}(-\sin(\varphi)\bar{u}_1(t)-\cos(\varphi)\bar{u}_2(t)) \tag{27}$$

Now we can calculate S(φ).

$$
\begin{aligned}
S(\varphi) &\equiv E[z(t)|\varphi] = E[z_c(t)z_s(t)|\varphi] \\
&= E[z_c(t;s_i)z_s(t;s_i)|\varphi] + E[z_c(t;s_q)z_s(t;s_q)|\varphi] + E[z_c(t;u)z_s(t;u)|\varphi] \\
&= \frac{C}{2}\left(P_2R_{\bar{y}}(0) - P_1R_{\bar{d}_1}(0)\right)\sin(2\varphi)
\end{aligned} \tag{28}
$$

where we have used the assumptions that s_i and s_q and u are independent and zero-mean and that u_1 and u_2 are independent and identically distributed.

SPECTRAL DENSITY OF THE EQUIVALENT NOISE

Now we will obtain the spectral density of z for fixed φ. We have

$$
\begin{aligned}
&\frac{1}{C}\left(z(t) - E[z(t)|\varphi]\right) \\
&\quad = \frac{1}{2}P_2(\bar{y}^2(t)-R_{\bar{y}}(0))\sin(2\varphi) - \frac{1}{2}P_1(\bar{d}_1^2(t)-R_{\bar{d}_1}(0))\sin(2\varphi) - \sqrt{P_1P_2}\cos(2\varphi)\bar{y}(t)\bar{d}_1(t) \\
&\quad + \sqrt{P_2}\cos(2\varphi)\bar{y}(t)\bar{u}_1(t) - \sqrt{P_2}\sin(2\varphi)\bar{y}(t)\bar{u}_2(t) + \sqrt{P_1}\sin(2\varphi)\bar{d}_1(t)\bar{u}_1(t) \\
&\quad + \sqrt{P_1}\cos(2\varphi)\bar{d}_1(t)\bar{u}_2(t) - \frac{1}{2}\sin(2\varphi)\bar{u}_1^2(t) - \cos(2\varphi)\bar{u}_1(t)\bar{u}_2(t) + \frac{1}{2}\sin(2\varphi)\bar{u}_2^2(t)
\end{aligned} \tag{29}
$$

so

$$
\begin{aligned}
&\frac{1}{C^2}R_z(\tau|\varphi) \\
&\quad = \frac{1}{4}P_2^2(E[\bar{y}^2(t)\bar{y}^2(t+\tau)]-R_{\bar{y}}^2(0))\sin^2(2\varphi) + \frac{1}{4}P_1^2(E[\bar{d}_1^2(t)\bar{d}_1^2(t+\tau)] - R_{\bar{d}_1}^2(0))\sin^2(2\varphi)) \\
&\quad + P_1P_2\cos^2(2\varphi)R_{\bar{y}}(\tau)R_{\bar{d}_1}(\tau) + P_2\cos^2(2\varphi)R_{\bar{y}}(\tau)R_{\bar{u}_1}(\tau) + P_2\sin^2(2\varphi)R_{\bar{y}}(\tau)R_{\bar{u}_1}(\tau) \\
&\quad + P_1\sin^2(2\varphi)R_{\bar{d}_1}(\tau)R_{\bar{u}_1}(\tau) + P_1\cos^2(2\varphi)R_{\bar{d}_1}(\tau)R_{\bar{u}_1}(\tau) + \frac{1}{4}\sin^2(2\varphi)E[\bar{u}_1^2(t)\bar{u}_1^2(t+\tau)] \\
&\quad - \frac{1}{4}\sin^2(2\varphi)R_{\bar{u}_1}^2(0) + \cos^2(2\varphi)R_{\bar{u}_1}^2(\tau) - \frac{1}{4}\sin^2(2\varphi)R_{\bar{u}_1}^2(0) + \frac{1}{4}\sin^2(2\varphi)E[\bar{u}_1^2(t)\bar{u}_1^2(t+\tau)] \\
&\quad = \frac{1}{4}\sin^2(2\varphi)[P_2^2R_{\bar{y}^2}(\tau)+P_1^2R_{\bar{d}_1^2}(\tau)] + P_1P_2\cos^2(2\varphi)R_{\bar{y}}(\tau)R_{\bar{d}_1}(\tau) + P_2R_{\bar{y}}(\tau)R_{\bar{u}_1}(\tau) \\
&\qquad + P_1R_{\bar{d}_1}(\tau)R_{\bar{u}_1}(\tau) + R_{\bar{u}_1}^2(\tau)
\end{aligned} \tag{30}
$$

where we have used the fact that

$$
E[\bar{u}_1^2(t)\bar{u}_1^2(t+\ )] = R_{\bar{u}_1}^2(0) + 2R_{\bar{u}_1}^2(\tau) \tag{31}
$$

The power spectral density evaluated at 0, written $S_z(0|\varphi)$, is then the integral over all τ of $R_z(\tau|\varphi)$. We use the estimate that Sz $(f|\varphi) = S_z(0|\varphi)$ for f near 0.

NONLINEAR COSTAS LOOP THEORY

We are now in a position to investigate the Costas loop performance. To keep the notation manageable we will develop the theory for a first-order loop only.

The equation of loop operation (9) can be rewritten for a first-order loop as

$$2[(d\varphi(t))/dt] = -2K_V S(\varphi) - 2K_V \sigma_n(\varphi) n_z(t) \tag{32}$$

where $n_z(t)$ is a unit-variance zero-mean random process. Approximating $n_z(t)$ by a delta-correlated Gaussian process the above equation can be rewritten as a diffusion equation

$$2d\varphi(t) = \left[-2K_V S(\varphi) + 2K_V^2 \sigma_n(\varphi)\frac{d\sigma_n(\varphi)}{dt}\right]dt - 2K_V \sigma_n(\varphi)\,dW(t) \tag{33}$$

where W(t) is a Brownian motion process. Introducing the notation

$$K_1(\varphi) = -K_V S(\varphi) + K_V^2 \sigma_n(\varphi)\frac{d\sigma_n(\varphi)}{d\varphi} \qquad \text{(Drift Coefficient)} \tag{34}$$

$$K_2(\varphi) = -K_V \sigma_n(\varphi) \qquad \text{(Diffusion Coefficient)} \tag{35}$$

the diffusion equation takes the form

$$d\varphi(t) = K_1(\varphi)dt + K_2(\varphi)dW(t) \tag{36}$$

and we may use the standard techniques to characterize the stationary behavior of φ. In particular, the p.d.f. of ϕ, the modulo-π reduced phase error, is given by

$$p(\phi) = \frac{1}{K_2(\phi)} \exp[-U_0(\phi)]\cdot\left[C-2J\int_{-\pi/2}^{\phi} \exp U_0(s)ds\right] \tag{37}$$

where

$$U_0(s) = -\int_{-\pi/2}^{s} \frac{2K_1(x)}{K_2(x)}dx = -\beta s + \alpha\int_{-\pi/2}^{s} g(x)dx \tag{38}$$

C is a normalization constant and J describes the average rate of cycle slips N_+-N_-. In the absence of loop stress the density function is therefore

$$p(\phi) = \frac{C'}{K_2(\phi)} \exp[-U_0(\phi)] \tag{39}$$

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VARIANCE OF LOOP PHASE ERROR

From Reference 3 we have for loop phase error φ,

$$\sigma_\varphi^2 \doteq \frac{N_0' B_L}{(S'(0))^2} \tag{40}$$

where $N_0'/2 = S_z(f=0|\varphi=0)$ and B_L is the loop bandwidth, when the bandwidth of the equivalent noise is much wider than B_L. It can be shown that $R_2 S_z(0|0)$ does not depend on R_2 but only on R_1/R_2 if the bandwidth of the G(f)H'(f) filter is taken as a multiple of R_2, where R_i is the data rate for channel i, i=1,2. We find that

$$\sigma_\varphi^2 \doteq \frac{2R_2\left[\int_{-\infty}^{\infty} R_{\bar{y}}(t)\,\frac{P_1}{P_2}\,R_{\bar{d}_1}(t)dt + \int_{-\infty}^{\infty} R_{\bar{y}}(t)\,\frac{1}{P_2}\,R_{\bar{u}_1}(t)dt + \int_{-\infty}^{\infty} \frac{P_1}{P_2}\,R_{\bar{d}_1}(t)\frac{1}{P_2}R_{\bar{u}_1}(t)dt + \int_{-\infty}^{\infty}\left(\frac{1}{P_2}R_{\bar{u}_1}(t)\right)^2 dt\right]}{\left[R_{\bar{y}}(0) - \frac{P_1}{P_2}\,R_{\bar{d}_1}(0)\right]^2} \cdot \frac{B_L}{R_2} \tag{41}$$

HARD-LIMITING SHUTTLE REPEATER

The hard-limiting repeater implementation is illustrated in Figure 3a.

Assume that signal + noise before and after the hard-limiter are, respectively,

$$r(t) = \sqrt{P}\,d_2(t) + n(t) \quad \text{and} \quad y(t) = \operatorname{sgn} r(t) \tag{42}$$

From Reference 4 we obtain the correlation function of y(t),

$$\begin{aligned} R_y(\tau) &\equiv E[y(t)y(t+\tau)] \\ &= \operatorname{erf}^2\left(\frac{\sqrt{P}}{\sqrt{2}\sigma_n}\right) R_{d_2}(\tau) + \frac{1}{\pi}\exp\left(\frac{-P}{\sigma_n^2}\right)\sum_{k=1}^{\infty}\left(\frac{R_n(\tau)}{\sigma_n^2}\right)^k \frac{1}{k!}\, He_{k-1}^2\left(\frac{\sqrt{P}}{\sigma_n}\right)[(1+R_{d_2}(\tau)) \\ &\qquad - (-1)^k(1-R_{d_2}(\tau))] \end{aligned} \tag{43}$$

where $\sigma_n^2 = R_n(0)$, He_k is the $k^{\underline{th}}$ Hermite polynomial,

$$\operatorname{erf}(x) = \frac{2}{\sqrt{2\pi}}\int_0^{\sqrt{2}x} e^{-u^2/2}du = \frac{2}{\sqrt{\pi}}\int_0^{x} e^{-u^2}du \tag{44}$$

$$R_{d_2}(\tau) = \begin{cases} 1-|\tau|/T_2, & |\tau| \le T_2 \\ 0, & |\tau| > T_2 \end{cases} \quad \text{(NRZ)} \quad \text{or} \quad = \begin{cases} 1-3|\tau|/T_2, & |\tau| \le T_2/2 \\ |\tau|/T_2-1, & T_2/2<|\tau|\le T_2 \\ 0, & |\tau|>T_2 \end{cases} \quad \text{(Biphase)} \tag{45}$$

with T**2** = $1/R_2$. $R_{d_1}(\tau)$ has the same form as the biphase case of $R_{d_2}(\tau)$. The data rates are R_1 = 192 Kbps and R_2 in the range 16 Kbps to 2 Mbps (Reference 1). We can now obtain $S(\varphi)$. The power spectral density at 0 is difficult to calculate in general, but when $G(p)H'(p)$ is identity it has the simple form below.

$$S_z(0|\varphi) = C^2[P_1P_2\cos^2(2\varphi)\int_{-\infty}^{\infty} R_y(t)R_{d_1}(t)dt + P_2\int_{-\infty}^{\infty} R_y(t)R_{u_1}(t)dt + P_1\int_{-\infty}^{\infty} R_{d_1}(t)R_{u_1}(t)dt + \int_{-\infty}^{\infty} R_{u_1}^2(t)dt] \tag{46}$$

LINEAR SHUTTLE REPEATER

Now consider the case where the hard-limiter is not used and M(p) is general, illustrated in Figure 3b. Then before and after the LPF we have, respectively,

$$r(t) = \sqrt{P}\, d_2(t) + n(t) \quad \text{and} \quad y(t) = \frac{1}{\sqrt{R_b(0)+R_v(0)}}\, b(t) + \frac{1}{\sqrt{R_b(0)+R_v(0)}}\, v(t) \tag{47}$$

where

$$b(t) = \sqrt{P}M(p)d_2(t) \quad \text{and} \quad v(t) = M(p)n(t) \tag{48}$$

and we have scaled y(t) so that $E[y^2(t)] = 1$. Then

$$R_y(\tau) = \frac{1}{R_b(0)+R_v(0)}\,(R_b(\tau)+R_v(\tau))\ . \tag{49}$$

$$R_{\bar{y}}(\tau) = \frac{1}{R_b(0)+R_v(0)}\,(R_{\bar{b}}(\tau)+R_{\bar{v}}(\tau))\ . \tag{50}$$

$R_{d_1}(\tau)$ and $R_{d_2}(\tau)$ are as specified in the previous section. We can now obtain $S(\varphi)$. To calculate $S_z(0|\varphi)$ we need statistics of $\bar{y}^2$.

$$\begin{aligned} R_{\bar{y}^2}(\tau) &= E[\bar{y}^2(t)\bar{y}^2(t+\tau)] - R_{\bar{y}}^2(0) \\ &= \frac{1}{(R_b(0)+R_v(0))^2}\{E[\bar{b}^2(t)\bar{b}^2(t+\tau)] + 2R_{\bar{b}}(0)R_{\bar{v}}(0) + 4R_{\bar{b}}(\tau)R_{\bar{v}}(\tau) \\ &\qquad + E[\bar{v}^2(t)\bar{v}^2(t+\tau)] - (R_{\bar{b}}(0)+R_{\bar{v}}(0))^2\} \\ &= \frac{1}{(R_b(0)+R_v(0))^2}\{E[\bar{b}^2(t)\bar{b}^2(t+\tau)] - R_{\bar{b}}^2(0) + 4R_{\bar{b}}(\tau)R_{\bar{v}}(\tau) + 2R_{\bar{v}}^2(\tau)\} \end{aligned} \tag{51}$$

To obtain $E[\bar{b}^2(t)\bar{b}^2(t+\tau)] - R_{\bar{b}}^2(0)$, we note that

$$\bar{b}(t) = \sum_i c_i q(t-iT) \tag{52}$$

where $c_i = \sqrt{P}$ or $-\sqrt{P}$ each with probability 1/2 and q(t) is the response of the G(p)H'(p)M(p) filter to a pulse (NRZ or biphase) of duration T_2 and absolute height 1.

$$\bar{b}^2(t) = P\sum q^2(t-iT) + \sum_i \sum_{k\neq i} c_i c_k q(t-iT)q(t-kT) \tag{53}$$

$$\begin{aligned} R_{\bar{b}^2}(\tau) &= E[\bar{b}^2(t)\bar{b}^2(t+\tau)] - R_{\bar{b}}^2(0) \\ &= P^2\Big\langle \sum_i q^2(t-iT)\sum_\ell q^2(t+\tau-\ell T)\Big\rangle - P^2\Big(\Big\langle \sum_i q^2(t-iT)\Big\rangle\Big)^2 + 2P^2\Big\langle\sum_i\sum_{k\neq i} q(t-iT)q(t-kT) \\ &\qquad \cdot q(t+\tau-iT)q(t+\tau-kT)\Big\rangle \end{aligned} \tag{54}$$

where <·> denotes the average over any time interval of length T. A similar expression holds for $R_{\bar{d}_1^2}(\tau)$, with q(t) replaced by the response of the G(p)H'(p) filter to a biphase pulse of duration T_1.

$$\begin{aligned} S_z(0|\varphi) = C^2 \Bigg\{ &\frac{1}{4}\sin^2(2\varphi)\Bigg[\left(\frac{P_2}{R_b(0)+R_v(0)}\right)^2 \int_{-\infty}^{\infty} [R_{\bar{b}^2}(t) + 4R_{\bar{b}}(t)R_{\bar{v}}(t) + 2R_{\bar{v}}^2(t)]dt \\ &+ P_1^2\int_{-\infty}^{\infty} R_{\bar{d}_1^2}(t)dt\Bigg] + \frac{P_1P_2}{R_b(0)+R_v(0)}\cos^2(2\varphi)\int_{-\infty}^{\infty}(R_{\bar{b}}(t)+R_{\bar{v}}(t))R_{\bar{d}_1}(t)dt \\ &+ \frac{P_2}{R_b(0)+R_v(0)}\int_{-\infty}^{\infty}(R_b(t)+R_{\bar{v}}(t))R_{\bar{u}_1}(t)dt + P_1\int_{-\infty}^{\infty} R_{\bar{d}_1}(t)R_{\bar{u}_1}(t)dt + \int_{-\infty}^{\infty} R_{\bar{u}}^2(t)dt\Bigg\} \end{aligned} \tag{55}$$

NUMERICAL RESULTS

Figures 4 through 9 are plots of the S-curve amplitude and of loop phase-error variance for both Shuttle repeaters studied, in the case where channel-2 data has NRZ format. Three different R_2 values are used. Specifically, Figures 4, 5 and 6 are plots of

$$S_a = \frac{2}{CP_2}[S(\varphi)/\sin 2\varphi] = R_{\bar{y}}(0) - (P_1/P_2)R_{\bar{d}_1}(0)$$

and Figures 7, 8 and 9 are plots of

$$\frac{N_0' R_2}{(S'(0))^2} \doteq \frac{\sigma_\varphi^2}{B_L/R_2}$$

The values of R_2 are 2000, 192 and 16 Kbps, and R_1 = 192 Kbps. The quantities are plotted as functions of repeater-input E_b/N_0. Individual curves correspond to different values of the bandwidth, in units of R_2, of the combined LPF G(f)H'(f). Following are the assumptions made in obtaining the curves. The G(f)H'(f) filter is a one-pole Butterworth filter. In the case of HL repeater, the spectrum of the repeater input noise is rectangular with a bandwidth of 1.5 R_2. In the case of LPF repeater, the input white noise and signal are both filtered by a four-pole Butterworth filter of bandwidth 1.5 R_2. In both cases, P_1/P_2 = .25. Additionally for the variance curves, E_b/N_0 = 10 dB for the s_i and u processes and the spectrum of u is flat over its bandwidth.

CONCLUSIONS

The LPF Shuttle repeater is better than the HL repeater in that the S-curves of the former are flatter and at a higher level and the phase-error variance curves are flatter and at a lower level than the curves for the latter repeater. The curves for the case where channel-2 data is biphase format (not shown) make the HL repeater look even somewhat worse compared to the LPF repeater than Figures 4-9 show it to be for the NRZ case.

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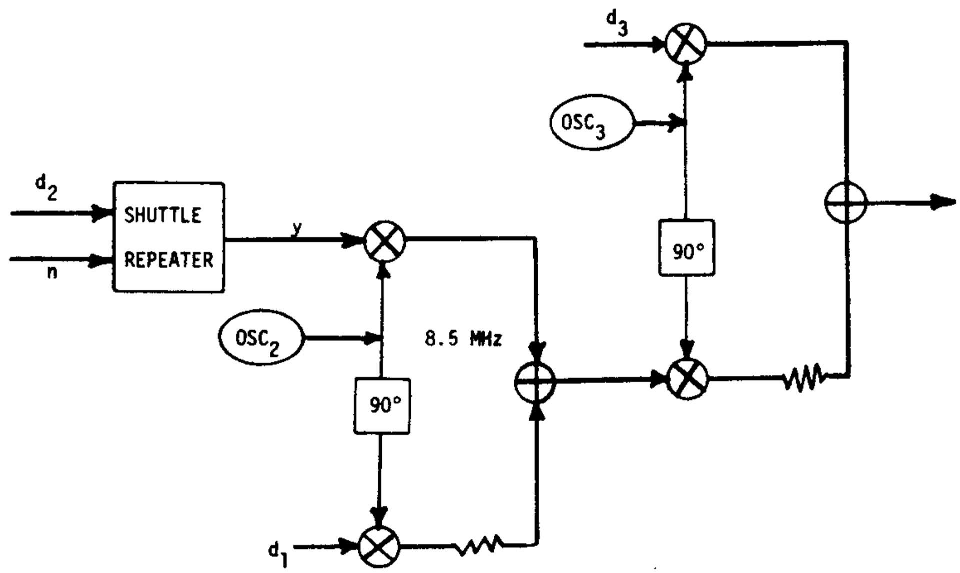

Figure 1. Three-Channel Transmitter in Bent-Pipe Mode.

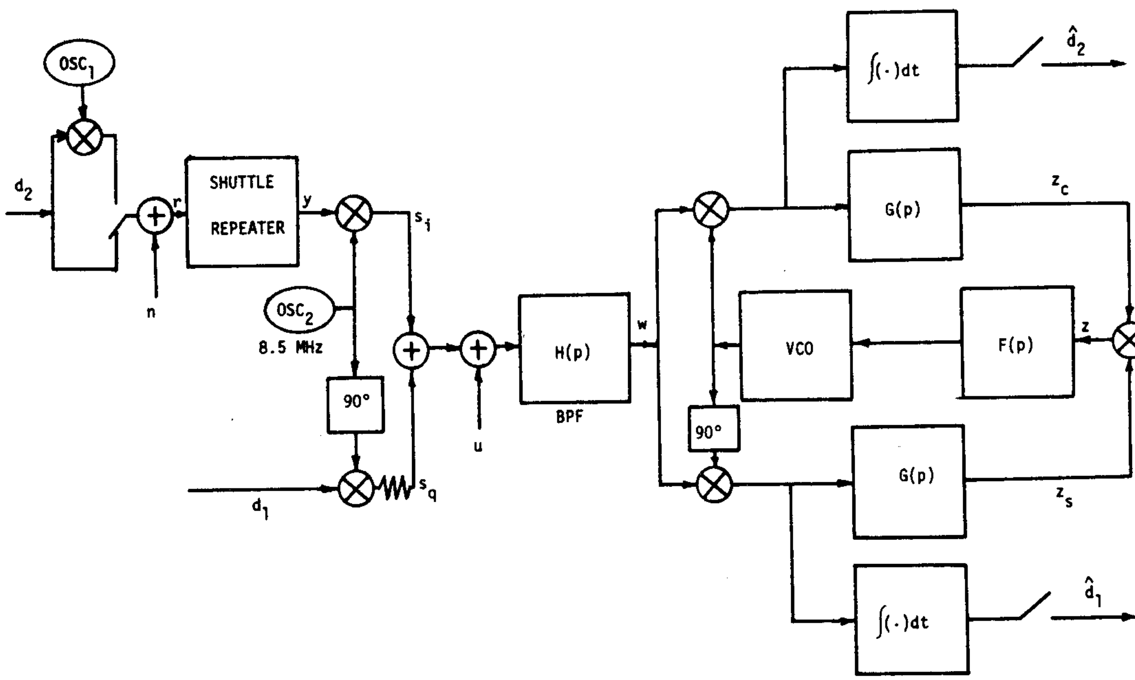

Figure 2. Simplified Link Diagram.

Figure 3. Two Shuttle Repeater Implementations Studied.

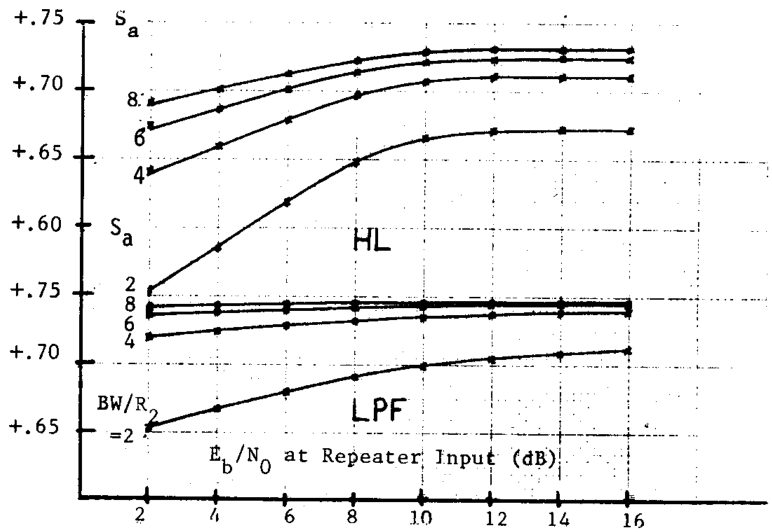

Figure 4. S_a for R_2 = 2000 Kbps, Channel 2 NRZ.

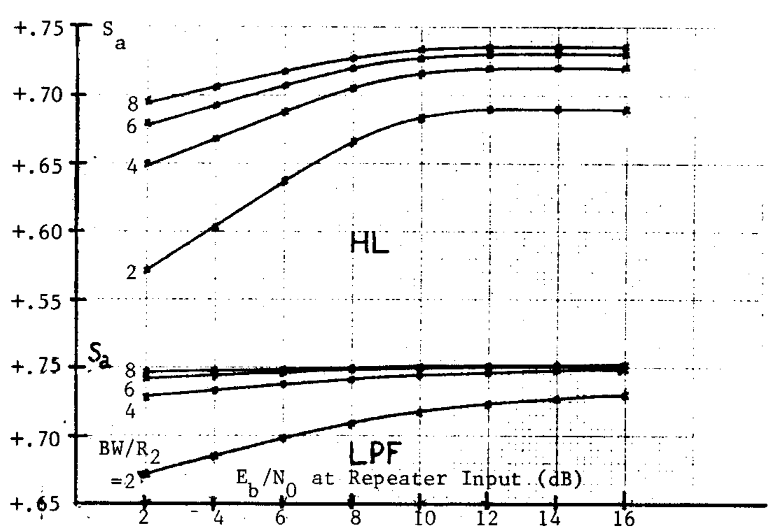

Figure 5. S_a for R_2 = 192 Kbps, Channel 2 NRZ.

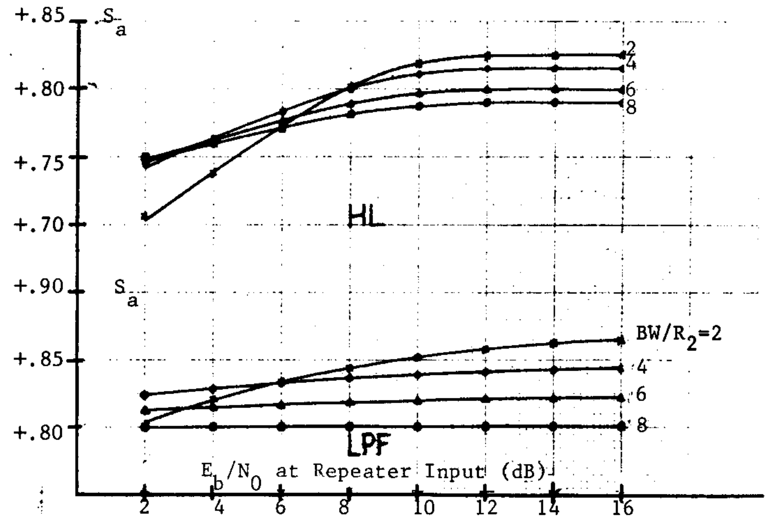

Figure 6. S_a for R_2 = 16 Kbps, Channel 2 NRZ.

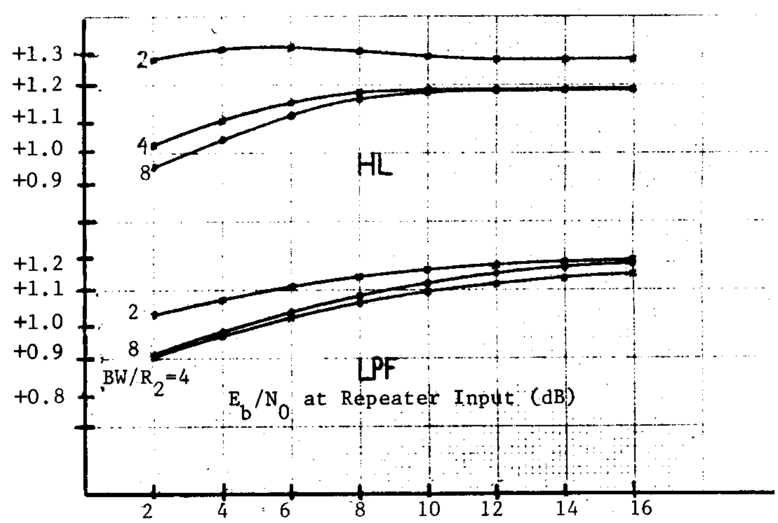

Figure 7. $\sigma_\varphi^2/(B_L/R_2)$ for R_2= 2000Kbps, Channel 2 NRZ, E_b/N_0=10 dB for s_i and u.

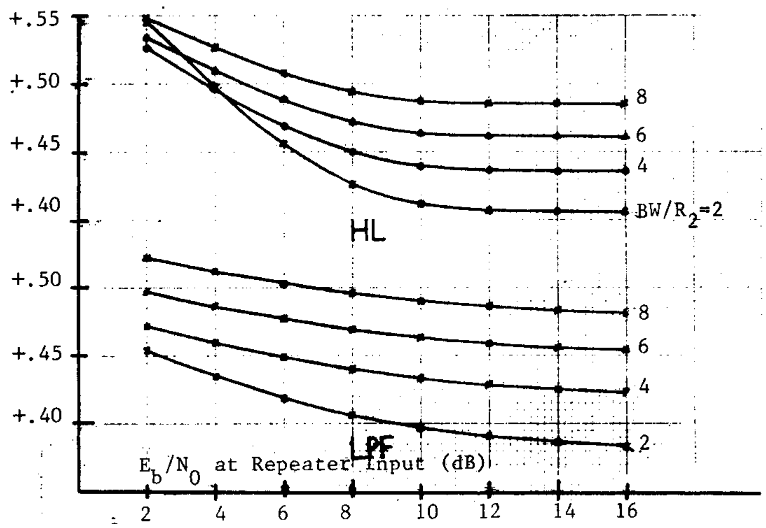

Figure 8. $\sigma_\varphi^2/(B_L/R_2)$ for R_2=192 Kbps, Channel 2 NRZ, E_b/N_0= 10dB for s_i and u.

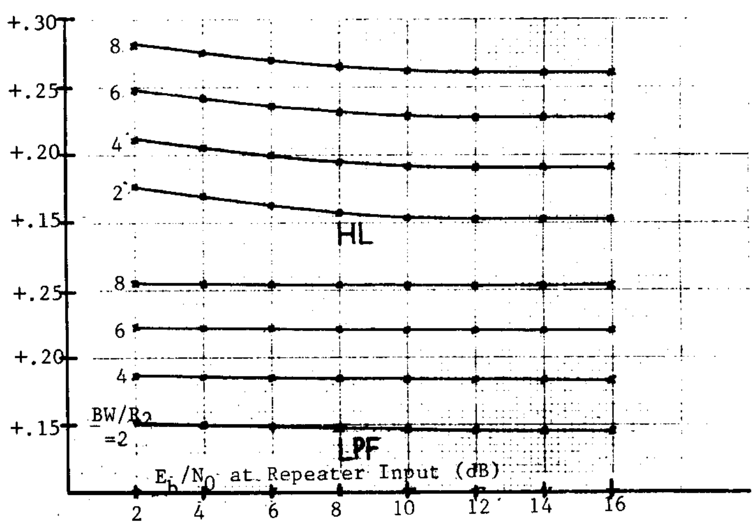

Figure 9. $\sigma_\varphi^2/(B_L/R_2)$ for R $_2$= 16 Kbps, Channel 2 NRZ, E_b/N_0=10 dB for s_i and u.