PERFORMANCES OF REGENERATIVE AND NON-REGENERATIVE SATELLITE REPEATERS WITH MPSK SIGNALLING

K. T. Woo
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

ABSTRACT

Linear (translation), hard-limited, and demod/remod types of satellite repeaters are considered in this paper. Both uncoded and coded multiple phase shift keyed (MPSK) signals are assumed to be transmitted through these repeaters. Relative performances of these repeaters in the presence of uplink and downlink noises are then compared quantitively. Probabilities of bit errors and the computational cutoff rates are computed for 2, 4, and 8 phases PSK signals, with uplink and downlink SNR's as parameters.

I. Introduction

The communication system to be considered in this paper is shown in Figure 1. Either coded or uncoded binary bits are first grouped into k-bit groups and then transmitted to the satellite repeater by a MPSK ($M = 2^k$, k = 1, 2, 3, ...) transmitter which transmits one of the M phase-shift keyed symbols per k-bits. Three types of satellite repeaters are considered in this paper: the linear repeater, the hard-limited repeater, and the demod/remod repeater. The linear repeater is a simple repeater that retransmits the received uplink signal plus noise, with a translation in carrier frequency, to the ground receiver. This assumes that the TWT amplifier in the satellite is operating in the linear region. The hardlimited repeater is not much different than the linear repeater in construction. However, the uplink signal is first passed through a bandpass limiter before it is translated in carrier frequency and amplified by the TWT amplifier. In this paper we assume the bandpass limiter is actually a hard limiter. The demod/remod repeater, which is the most complex of the three, requires a MPSK demodulator on the satellite. The

---

This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology under contract NAS7-100 sponsored by the National Aeronautics and Space Administration. A portion of this work was performed earlier during the author's previous employment at LinCom Corporation under contract DCA 100-76-C-0062, sponsored by the Defense Communications Agency.

demodulated MPSK signal is then remodulated on a downlink carrier, amplified by the TWT amplifier and transmits to the ground receiver.

Additive Gaussian noise is introduced on both the uplink (repeater noise) and the downlink (receiver noise). The demodulator is assumed to be a coherent MPSK detector which heterodynes the received MPSK signal with a coherent reference and decides one of M phases was sent by computing, in principle, the inverse tangent of the ratio of the sampled baseband outputs of the in-phase and quadrature channels in the demodulator. The sampling is performed once per symbol. Figure 2 illustrates the operations of the MPSK coherent receiver.

Our major concern in this paper is the effect of uplink and downlink noises on the performance of the MPSK communication system of Figure 1. Thus we have omitted the effects of intersymbol interferences created by the transmit filter, the receive filter, and the bandpass filters in the satellite repeater. In other words, all the filters in Figure 1 will be assumed zonal filters that pass the desired bands of signals without introducing intersymbol interferences, while limiting noise powers to finite values. Both coded and uncoded systems will be considered.

II. Uncoded System Performances

Linear Repeaters:

A natural performance criteria for uncoded systems is the probability of bit error. Consider first the linear repeater. Let ϕ_k be the transmitted phase of the kth symbol, which can assume any one of the M values: $2\pi m/M;\ m = 0, \pm 1, \pm 2, \ldots, \pm \frac{M-2}{2}$, M for MPSK signalling. The received signal at the repeater, after passing through a bandpass filter of bandwidth B_1, can be written as

$$X_1(t) = \sqrt{2P_1} \sum_{k=-\infty}^{\infty} a(t - kT) \cos(\omega_1 t + \phi_k) + n_1(t) \tag{1}$$

where P_1 is the uplink carrier power, a(t) is the rectangular pulse function with duration T, which is the symbol duration, ω_1 is the uplink angular carrier frequency, and $n_1(t)$ is a narrow band Gaussian process:

$$n_1(t) = \sqrt{2}\ N_{c1}(t) \cos \omega_1 t - \sqrt{2}\, N_{s1}(t) \sin \omega_1 t \tag{2}$$

The processes N_{c1} N_{s1} are independent, slowly varying zero mean Gaussian processes with variance N_1B_1, where N_1 is the one-sided noise power spectral density in the front end of the repeater. Since the spectral occupancy of the MPSK signals are inversely proportional to T, the BPF bandwidth B_1 is selected such that B_1T is equal to a constant typically of value greater than 1. As mentioned in Section 1, the filtering distortions will be assumed negligible.

The downlink signal of the linear repeater is an amplified version of $X_1(t)$ with a translation in carrier frequency:

$$X_2(t) = g\left\{\sqrt{2P_1}\sum_{k=-\infty}^{\infty} a(t-kT)\cos(\omega_2 t + \phi_k) + n_1(t)\right\} \tag{3}$$

where g is the repeater gain and ω_2 is the angular downlink frequency. The downlink power is seen from (3) to be

$$P_2 = g^2[P_1 + N_1B_1] \tag{4}$$

Let B_2 be the bandwidth of the BPF of the receiver of Figure 2, and let N_2 be the one-sided power spectral density of the receiver front end noise $n_2(t)$, which can be represented in the same way as $n_1(t)$ of (2), then the received downlink signal after the BPF can be written as, during the kth symbol:

$$y_2(t) = \nu_2(t)\cos[\omega_2 t + \eta_2(t)] \tag{5}$$

where the envelope and phase ν_2 and η_2 will have the following joint distribution:

$$P(\nu_2, \eta_2) = \frac{\nu_2}{2\pi\sigma^2}\exp\left[-\frac{\nu_2^2 + g^2P_1 - 2g\sqrt{P_1}\cos(\eta_2 - \phi_k)}{2\sigma^2}\right] \tag{6}$$

and where σ^2 is the variance:

$$\sigma^2 = g^2 N_1B_1 + N_2B_2 \tag{7}$$

Integrating (6) over ν_2 from 0 to ∞ we obtain the distribution of η_2 (see [3]):

$$P(\eta_2) = \frac{1}{2\pi}\sum_{n=0}^{\infty}\epsilon_n \frac{\rho^n}{n!}\Gamma\left(\frac{n}{2}+1\right){}_1F_1\left[\frac{n}{2}; n+1; -\rho^2\right]\cos n(\eta_2 - \phi_k) \tag{8}$$

where ${}_1F_1(x)$ is the confluent hypergeometric function; ϵ_n equals to 1 if n is zero, 2 otherwise; and

$$\rho^2 \equiv \frac{g^2P_1}{2\sigma^2} \tag{9}$$

which is evaluated from (4) and (7) to be

$$\rho^2 = \frac{\rho_1^2 \rho_2^2}{1 + \rho_1^2 + \rho_2^2} \tag{10}$$

where ρ_1^2, ρ_2^2 are uplink and downlink SNR's defined by

$$\rho_1^2 = \frac{P_1}{N_1 B_1} \; ; \quad \rho_2^2 = \frac{P_2}{N_2 B_2} \tag{11}$$

Without loss of generality we can assume $\phi_k = 0$ to be the transmitted phase. The transition probabilities of deciding $\phi_k = j2\pi/M$, $j = \pm 1, \pm 2, \ldots, \pm \frac{M-2}{2}, \frac{M}{2}$, while $\phi_k = 0$ is sent is then computed from (8) to be

$$P_M(j) = \int_{(2j-1)\frac{\pi}{M}}^{(2j+1)\frac{\pi}{M}} p(\eta_2)\, d\eta_2$$

$$= \frac{1}{M} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{M}}{n} \cdot \frac{\Gamma\left(\frac{n}{2}+1\right)}{\Gamma(n+1)} \rho^n \; {}_1F_1\left[\frac{n}{2} ; \; n+1; \; -\rho^2\right] \cos n\left(j\frac{2\pi}{M}\right) \tag{12}$$

When j = 0 the above expression is the probability of correct symbol detection of MPSK signals with equivalent SNR ρ^2.

Further simplification of (12) is possible. This can be done by a procedure similar to that suggested in [5]. By the Kummer's transformation [6]:

$${}_1F_1(a, b, z) = e^z \; {}_1F_1(b-a, b, -z) \tag{13}$$

and the relationship [6]:

$$\frac{d}{dz} \; {}_1F_1(a, b, z) = \frac{a}{b} \; {}_1F_1(a+1, b+1, z) \tag{14}$$

the function ${}_1F_1(\frac{n}{2} ; n+1; -z)$ can be written as

$${}_1F_1(\tfrac{n}{2}; \; n+1; -z) = 2 \, e^{-z} \frac{d}{dz} \; {}_1F_1(\tfrac{n}{2}; \; n; \; z) \tag{15}$$

Because of the known identity [7]:

$${}_1F_1(\tfrac{m}{2}; \; m; \; z) = \Gamma\left(\frac{m+1}{2}\right) \left(\frac{z}{4}\right)^{-\frac{m-1}{2}} e^{\frac{z}{2}} \; I_{\frac{m-1}{2}}\left(\frac{z}{2}\right)$$

(15) can be directly evaluated to be:

$$ {}_1F_1\left(\frac{n}{2};\ n+1;\ -z\right) = \left(\frac{2}{\sqrt{z}}\right)^{n-1} e^{-\frac{z}{2}}\, \Gamma\left(\frac{n+1}{2}\right) \left[I_{\frac{n-1}{2}}\left(\frac{z}{2}\right) + I_{\frac{n+1}{2}}\left(\frac{z}{2}\right) \right] \tag{16} $$

Substituting (16) into (12) the transition probabilities can be evaluated to be:

$$ P_M(j) = \frac{1}{M} + \rho\, \frac{e^{-\rho^2/2}}{\sqrt{\pi}} \sum_{n=1}^{\infty} \frac{\mathrm{Sin}\,\frac{n\pi}{M}}{n} \left[I_{\frac{n-1}{2}}\left(\frac{\rho^2}{2}\right) + I_{\frac{n+1}{2}}\left(\frac{\rho^2}{2}\right) \right] \cos\left(\frac{nj2\pi}{M}\right) \tag{17} $$

It should be noted here that when M = 2, Eq. (17) reduces to

$$ P_2(1) = P_2(\mathrm{error}) = \frac{1}{2}\,\mathrm{erfc}\,\sqrt{\rho^2} \tag{18} $$

which is already discussed in the earlier work of Jain and Blachman [1].

(ii) Hard-limited Repeater:

Next we consider the performance of the hard-limited repeater. This case has been considered by several authors (see [1], [2], [4], among others). The case of BPSK signalling transmitted through a hard-limited repeater is first reported in [1], and this work was later generalized to MPSK signalling transmitted through a hard-limited repeater [2] and to a general memoryless nonlinearity exhibiting both AM to AM and AM to PM characteristics [4]. We shall consider the hard-limited repeater case here. The expression for $P_M(j)$ for this case was given in [2], [4] as

$$ \begin{aligned} P_M(j) = \frac{1}{M} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\mathrm{Sin}\,\frac{n\pi}{M}}{n} \left[\frac{\Gamma\left(\frac{n}{2}+1\right)}{\Gamma(n+1)}\right]^2 \rho_1^n\ \rho_2^n\ \cos\left(\frac{nj2\pi}{M}\right) \\ \times\ {}_1F_1\left[\frac{n}{2};\ n+1;\ -\rho_1^2\right] \cdot {}_1F_1\left[\frac{n}{2};\ n+1;\ -\rho_2^2\right] \end{aligned} \tag{19} $$

where ρ_1^2 and ρ_2^2 are the uplink and downlink SNR's:

$$ \rho_1^2 = \frac{P_1}{N_1 B} \quad ; \quad \rho_2^2 = \frac{P_2}{N_2 B} \tag{20} $$

Using the procedure discussed earlier, (19) can also be written in terms of modified Bessel functions. The was done for the BPSK case in [1]. However, to the best knowledge of the author, the probability of symbol errors for general MPSK signals passing through the hard limited repeater has not been expressed in terms of modified Bessel functions in literature.

Using (16) we now can write the transition probabilities $P_M(j)$ of (19) in the terms of modified Bessel functions as follows:

$$P_M(j) = \frac{1}{M} + \frac{\rho_1\rho_2 \, e^{-\frac{\rho_1^2+\rho_2^2}{2}}}{2} \sum_{n=1}^{\infty} \frac{\text{Sin}\,\frac{n\pi}{M}}{n} \cdot \cos\left(\frac{nj2\pi}{M}\right)$$
$$\times \left[I_{\frac{n-1}{2}}\left(\frac{\rho_1^2}{2}\right) + I_{\frac{n+1}{2}}\left(\frac{\rho_1^2}{2}\right)\right]\left[I_{\frac{n-1}{2}}\left(\frac{\rho_2^2}{2}\right) + I_{\frac{n+1}{2}}\left(\frac{\rho_2^2}{2}\right)\right] \tag{21}$$

When M is 2 only terms with odd n's remain; (21) then reduces to the BPSK result reported earlier in [1].

(iii) Demod/Remod Repeater:

Without loss of generality we can assume, again, the transmitted phase ϕ_k is zero. The probability of demodulating this phase to be j2πM instead of 0 at the demodulator on board of the satellite is readily seen to be the one link MPSK transition probability $P_M(j)$ with ρ_1^2 as the parameter. This phase information is re-modulated onto the downlink carrier. The ground receiver will then decide which one of the M-phases was originally sent. The composite transition probability that the receiver decides that phase J2π/M was sent while indeed the zero phase was transmitted is given by

$$P_M(j) = \sum_k P_M^{(1)}(k)\, P_M^{(2)}(j-k) \tag{22}$$

where $P_M^{(1)}(j)$ and $P_M^{(2)}(\ell)$ are the one link MPSK transition probabilities of Eq. (17), with uplink SNR ρ_1^2 and downlink SNR ρ_2^2 as parameters respectively.

From these transition probabilities we can easily arrive at the probabilities of bit errors. Assume Gray code is used in the mapping from each group of k bits to one of the M phases. Then the probabilities of bit errors for 2, 4 and 8 phase PSK signals are given, respectively, in terms of these transition probabilities by

M	Probability of Bit Error
2	$p_2 = P_2(1)$
4	$p_4 = P_4(2) + \frac{1}{2}[P_2(1) + P_2(-1)]$
8	$p_8 = P_8(-3) + \frac{2}{3}[P_8(2) + P_8(4) + P_8(-2)] + \frac{1}{3}[P_8(1) + P_8(-1) + P_8(3)]$ (23)

III. Numerical Computations of Bit Error Rates

To compute the bit error probabilities in (24), we need to evaluate the symbol transition probabilities (17), (21), and (22). Because of the recurrence formula [6] of Bessel functions

$$I_{\nu+1}(x) = -\frac{2\nu}{x} I_{\nu}(x) - I_{\nu-1}(x) \tag{24}$$

for all odd integers n in (17) or (21) it is required only to evaluate $I_0(x)$ and $I_1(x)$, while all other Bessel functions of integer orders can be evaluated in terms of these two through the recurrence formulae (24). When n is an even integer, Bessel functions with fractional order appear. These Bessel functions can be expressed in finite sums [6] as follows:

$$I_{m+\frac{1}{2}}(x) = \frac{1}{\sqrt{2\pi x}} \sum_{k=0}^{m} \frac{(m+k)!\,[(-1)^k e^x - (-1)^m e^{-x}]}{k!\,(m-k)!\,(2x)^k} \tag{25}$$

Nevertheless the recurrence relationship (24) still holds. Thus to compute these Bessel functions it is required only to evaluate $I_{1/2}(x)$ and $I_{3/2}(x)$.

Figure 3 shows the bit error rates of 2, 4, 8 phase PSK signals transmitted through the three kinds of repeaters mentioned, with ρ_1^2 fixed at 10 dB and ρ_2^2 varies from 6 to 12dB. Also shown in figure 3 are the performances of 2, 4, 8 phases PSK signals with downlink noise alone (i.e., one link systems). Since in figure 3 the bit error rates are plotted v.s. the same ρ_1^2 and ρ_2^2 for various MPSK signals with identical symbol durations T, we are indeed comparing them under the assumptions of equal P/N_o's and equal bandwidths (which is proportional to 1/T), while letting the throughputs to vary. In other words, the throughputs of QPSK signals with the bit error rates shown in Figure 3 is twice that of the BPSK signals shown in the same figure. Similarily, the throughputs of the 8-PSK signals will be three times that of the BPSK signals. Thus even though at identical transmitted powers and bandwidth occupancies QPSK or 8-PSK have worse bit error rates than BPSX, coding can be used to improve their performances. This is discussed in the next section.

IV. Coded System Performance

For coded communication systems the probability of bit (or symbol) error is no longer a valid performance criterion [8], since decoding is not performed on a symbol by symbol basis. Since the works of [8], [9] it has became an accepted fact that the cut-off rate R_o is a valid performance measure of coded communication systems. As pointed out by Massey in [8], R_o criterion will lead to the design of modulation systems which are compatible with effective coding systems.

Consider Figure 1. Suppose block codes are used. Every K bits of the data source will be mapped into N M-ary symbols to be transmitted through the satellite channel. Let x's stand for the M-ary symbols to be transmitted and let y's be the demodulated information. Assume hard decision is used, then the demodulator output y's will also be M-ary symbols. It is known from random coding bound techniques that the average probability of bit error of a coded communication system will be bounded by (see, e.g., References [8], [9]), assuming the channel is memoryless:

$$\overline{P}_e \leq 2^{K - N R_o(M)} \tag{26}$$

In (26) $\overline{P}_e$ is the average probability of bit error, in the sense of averaging over all block codes with code rate

$$R \equiv \frac{K}{N} \tag{27}$$

and $R_o(M)$ is the cut-off rate, in units of bits/symbol, given by:

$$R_o(M) = -\log_2 \left\{ \min_{Q(x)} \sum_y \left[\sum_x \sqrt{p(y|x)}\, Q(x) \right]^2 \right\} \tag{28}$$

where Q(x) is the probability distribution on the input symbols x. When Q(x) is uniformly distributed, i.e., when Q(x) = 1/M for all x, assuming a M-ary PSK signal set is used, then (28) is simplified to be

$$R_o(M) = -\log_2 \left\{ \frac{1}{M} \left[\sum_{k=0}^{M-1} \sqrt{P_M(k)} \right]^2 \right\} \tag{29}$$

where $P_M(k)$ is the transition probabilities defined in the last section, for M-ary PSK modulation, over the three types of satellite channels involved. Define $r \equiv R/T$ to be the information bit rate where T is the M-ary symbol duration, and $r_o(M) \equiv R_o(M)/T$, both in units of bits/second, then (26) can be written as

$$\overline{P}_e \leq 2^{K[1 - \frac{R_o(M)}{R}]} = 2^{K[1 - \frac{r_o(M)}{r}]} \tag{30}$$

Suppose convolutional code is used. It is shown in [10] that a similar bound on $\overline{P}_e$ can be obtained. Suppose K is the constraint length of the code, and suppose for each b information bits the encoder will output N M-ary channel symbols, so that the information bit rate r is b/NT. From [10] we have:

$$\overline{P}_e \leq A\, 2^{-K \frac{R_o(M)}{R}} = A\, 2^{-K \frac{r_o(M)}{r}} \tag{31}$$

where A is a constant defined by [11]

$$A = \frac{b^{(2^b - 1)}}{\left\{1 - 2^{-b[r_o(M)/r - 1]}\right\}^2} \tag{32}$$

It is thus apparent from (30) and (31) that the attainable throughput r, at which the coded communication system is able to maintain a specified bit error probability, for various satellite channels with various MPSK signalling at equal bandwidth occupancies and equal up/down link SNR's, and for equal coding complexity measures such as either the number of information bits in each block of the block codes, or the constraint lengths of a convolutinal code, is completely determined by the respective cut-off rates $r_o(M)$.

Moreover, R_o is also the computational cut-off rate "R_{COMP}" of sequential decoding [9], i.e., the rate above which the average number of decoding steps per decoded bit becomes infinite. Since sequential decoding is a powerful decoding technique that can handle long constraint length codes and therefore have arbitrarily small decoding bit error probabilities, $r_o(M)$ also represents a practical maximum data rate below which we can have arbitrary small data bit error probabilities.

Figure 4 plots the various values of $R_o(M)$, as functions of ρ_2^2, with ρ_1^2 = 10 dB, for M = 2, 4, 8, and for the three kinds of repeaters discussed previously.

V. Conclusions

We have presented an analytical procedure to evaluate the bit error rate performances of coded or uncoded communication systems using MPSK signalling over three kinds of satellite repeater channels: linear, hard limited and demod/remod, in the presence of uplink and downlink noises.

The ordering of these channels in terms of either coded or uncoded link performances is quite clear from Figures 3 and 4. The demod/remod channel offers the best performance. The hard limited channel is second in performance. The linear channel is the worse of the three. The main reason of this is due to the fact that in the case of the linear channel both uplink noise as well as uplink signal are amplified by the TWT and transmitted to the downlink, thus the total power of the downlink is "robbed". Within the range of interest of up/down link SNR's (ρ_1^2 = 10 dB, ρ_2^2 from 6 to 14 dB), the hard limited channel performs very closely to the demod/remod channel when BPSK signalling is used. However, for the same range of SNR's, the hard limited channel's performance as far as bit error rates are concerned is further degraded from that of the demod/remod channel when multiple phase PSK (M = 4, 8, ...) signalling is applied.

Assuming equal symbol times, thus also equal bandwidth occupancies, the cut-off rates R_o of these channels using 2, 4, and 8 phase PSK signalling are computed, for the same range of SNR's of interest. R_o is directly proportional to the attainable throughputs given a specified bit error probability. It is thus concluded from Figure 4 that within the range of SNR's discussed it is apparent that QPSK offers better attainable throughputs than BPSK. However, 8 phase PSK, while at higher SNR's is able to afford higher throughputs than that of QPSK, does not offer a much higher attainable throughput than that of the QPSK case in the range of SNR's mentioned.

References

(1) Jain, P. C. and Blachman, N. M., "Detection of a PSK Signal Transmitted Through a Hard-Limited Channel," IEEE Trans. on Information Theory, Vol. IT-9, No. 5, Sept. 1973.

(2) Mizuno, T., Morinaga, N., and Namekawa, T., "Transmission Characteristics of an M-ary Coherent PSK Signal via a Cascade of N Bandpass Hardlimiters," IEEE Transection on Communications, Vol. COM-24, No. 5, May, 1976.

(3) Middleton, D., <u>Introduction to Statistical Communication Theory</u>, McGraw Hill Book Company, 1960.

(4) Jain, P. C., Lindsey, W. C., Woo, K. T., Huang, T. C., and Omura, J. K., "Detection of MPSK Signals Transmitted Through a Nonlinear Satellite Repeater," National Telecommunication Conference Proceedings, Dec. 1977.

(5) Butman, S. A., and Lesh, J. R., "The Effects of Bandpass Limiters on n-Phase Tracking Systems," IEEE Trans. on Communication, Vol. COM-25, No. 6, June 1977.

(6) Abramowitz, M. and Stegun, I., <u>Handbooks of Mathematical Functions</u>, Dover Publications, 1972.

(7) Gradshteyn, I. and Ryzhik, I., <u>Tables of Integrals, Series, and Products</u>. New York: Academic Press, 1965.

(8) Massey, J. L., “Coding and Modulation in Digital Communications,” Proc. International Zurich Seminar on Digital Comm., Switzerland, March, 1974.

(9) Wozencraft, J. M. and Kennedy, R., “Modulation and Demodulation for Probabilistic Coding,” IEEE Trans. on Information Theory, Vol. IT-12, July, 1966-

(10) Viterbi, A. J., “Convolutional Codes and Their Performance in Communication Systems,” IEEE Trans. on Communications, Vol. COM-19, No. 5, Oct. 1971.

(11) Lindsey, W. C., Omura, J. K., Woo, K. T., Huang, T. C., and Biederman, L., “Investigation of Modulation/Coding Trade-Off for Military Satellite Communications,” LinCom Report TR-7807-1276.

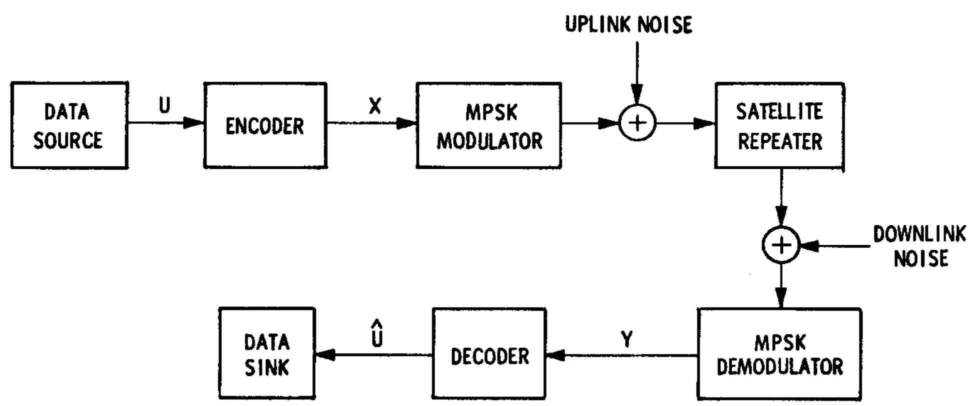


Figure 1 THE SATELLITE COMMUNICATION CHANNEL MODEL

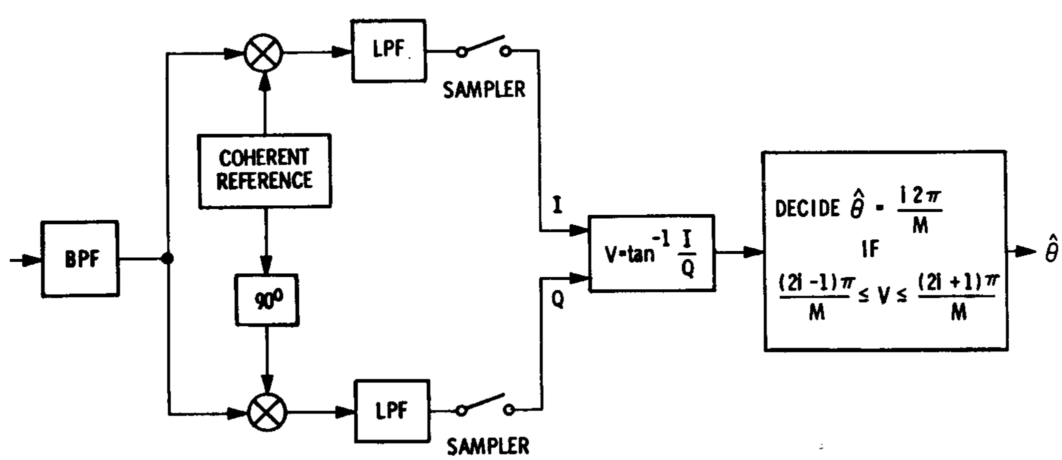
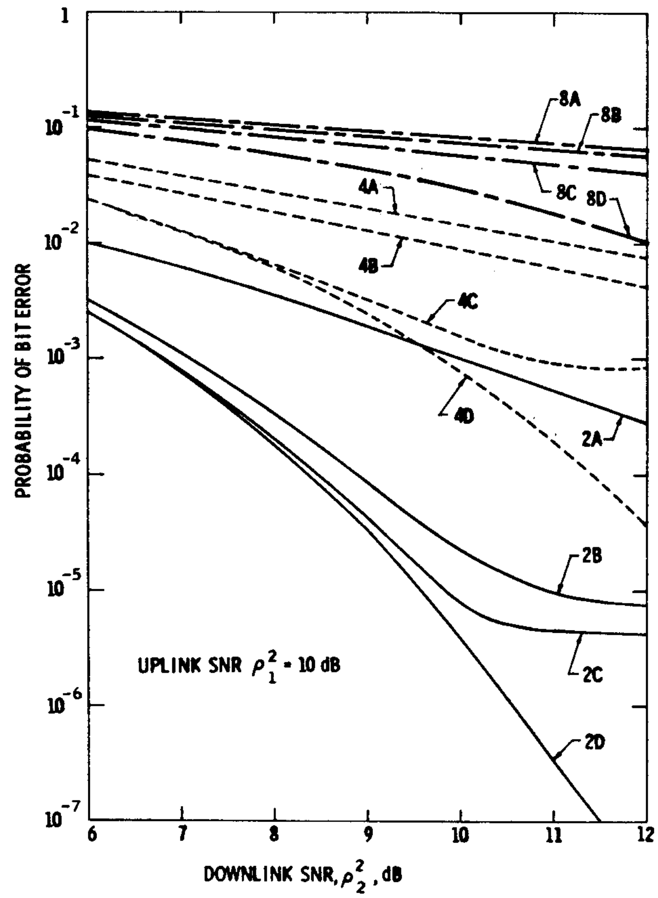


Figure 2 MPSK COHERENT DEMODULATOR

Figure 3:

Probabilities of Bit Errors for 2, 4, 8 phase PSK signals transmitted through the satellite repeater channels (Notations = 2, 4, 8 stand for BPSK, QPSK and 8 PSK, A = Linear Channel, B = Hardlimited Channel, C = Demod/Remod Channel, D = No Uplink Noise Case).



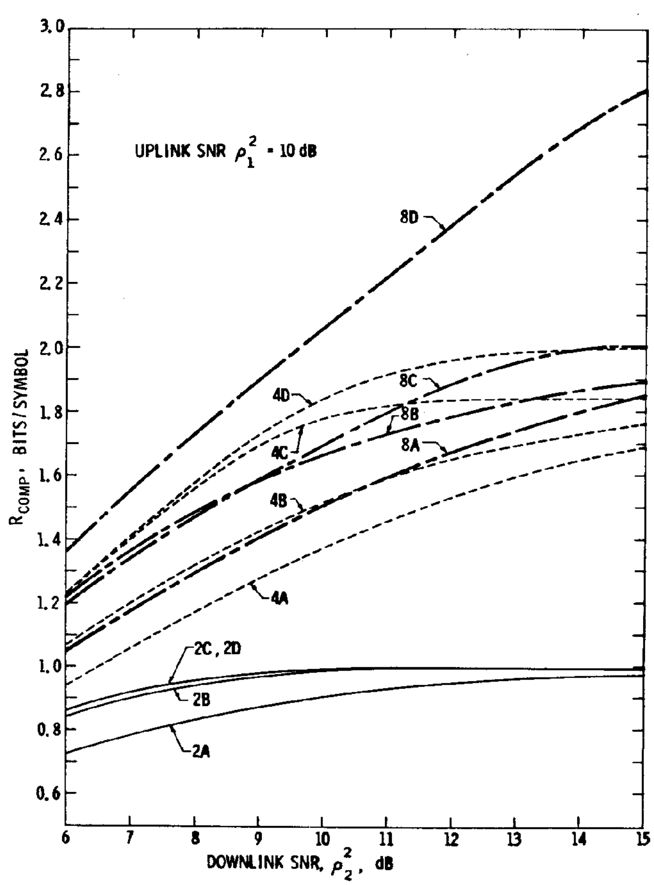


Figure 4:

Cutoff rates for 2, 4, 8 phase PSK signals transmitted through the Satellite Repeater Channels (same notations as those used in Figure 3).