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Geothermal Reservoir*

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U.S. Geological Survey

Technical Reports on
Hydrology and Water Resources

The University of Arizona
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ABSTRACT

A management model is developed that determines the optimum economic recoverability of a particular hot-water geothermal reservoir undergoing exploitation for electric power generation. The management model integrates a physical model of the reservoir that predicts the areas of pressure decline due to withdrawals, and pressure rise due to reinjection of spent fluid, with a model of a two-stage steam turbine power plant that determines the quantity of electricity generated for a rate of hot-water extraction. Capital costs, variable costs and annual fixed costs are obtained for the reservoir development, extraction and reinjection, the transmission system, and the power plant. Revenues are determined for electrical power production. Application of the management model to a simplified, yet realistic example reservoir demonstrates that the methodology developed in this report can be used for analyzing the management of an integrated geothermal reservoir-power plant system.

INTRODUCTION

Efforts to achieve a large measure of energy self-sufficiency have played a significant role in stimulating research and public interest in alternative energy resources. Among the energy resource options being considered are those converting geothermal energy into electric power. In this report a mathematical model is developed to demonstrate a technique for determining the amount and rate of conversion to electric power for one particular source of geothermal energy.

The ultimate source of all geothermal energy is heat energy generated and stored within the earth. Potential sources of geothermal energy can be divided into three major system types: hydrothermal, geopressured and hot dry rock. In hydrothermal systems heat from near surface sources such as magmatic bodies is transferred to a porous media and the fluid within that media by conductive and convective processes. These systems can be further subdivided as either liquid dominated or vapor dominated depending on fluid pressure responses. In geopressured systems fluid is trapped in geosynclinal accumulations where it is subjected to extreme pressures that produce substantial increases in fluid temperatures. Finally, in hot rock systems low-permeable, igneous rocks are heated by sources similar to those associated with hydrothermal systems. These systems are by nature, fluid independent. A similar classification of geothermal energy sources is presented in a report by White and Williams (1975). Assessments of hydrothermal and geopressured

systems in the United States are presented in reports by Renner, White and Williams (1975) and Papadopoulos and others (1975), respectively.

In this report only geothermal energy from hydrothermal, liquid-dominated systems is considered. When such a system is utilized for its heat energy it is called a liquid dominated geothermal reservoir. Hot fluid is extracted from the reservoir by means of wells and is transmitted to a power plant by means of insulated pipe. The fluid is generally under high pressure so that the wells need not be pumped. The best known liquid-dominated, geothermal reservoir is the Wairakei field in New Zealand.

Within the power plant the fluid's thermal energy is converted to electric power. The electricity is produced either by indirect or direct methods. When indirect methods are used the hot fluid from the wells is passed through a series of heat exchangers where its heat energy is transferred to a secondary fluid. The heated secondary fluid is then used to drive a system of turbines. When direct methods are used the geothermal fluid itself is passed through the turbine system. Indirect methods produce heat losses and so reduce the conversion efficiency of geothermal energy to electrical energy. These losses are critical in that geothermal fluid temperatures are considerably less than those of superheated steam used in a conventional, steam electric power plant. In this report geothermal energy conversion to electric power by a direct method is considered. Direct methods, although they do reduce heat losses, are not without drawbacks in that they may have problems with

fouling. The geothermal fluid may contain dissolved solids which produce scaling, and may chemically attack the system of turbines.

Recently researchers have begun to consider the above ground aspects of delivering and converting energy from geothermal wells, and have begun to develop economic models for geothermal reservoirs and power plants. Battelle-Northwest laboratories have been extremely active in this regard. They have developed a model, GEOCOST, (Huber and others, 1975; Bloomster and Knutsen, 1975; and Bloomster, 1975a and 1975b) that is a simulation model, which estimates a cost for generating electric power from geothermal energy. GEOCOST is composed of two principal parts: a reservoir model that provides cost estimates for exploration, development and operation of a geothermal reservoir; and a power plant model that provides cost estimates for design, construction, and operation of a power plant. Most emphasis is placed on the power plant model, with optional power plant types, using both direct and indirect methods. The reservoir model treats the reservoir as a "black box" that yields fluids of specified characteristics at the well head.

Nathenson (1975) and Nathenson and Muffler (1975) develop generalized recoverability factors and conversion efficiencies to estimate the potential of electrical generation from various hydrothermal systems. The main concern of the two papers is to determine the reservoir and fluid properties that most strongly affect geothermal energy utilization.

In the above references, no attempt was made to develop a management model to integrate the reservoir and power plant systems. Economic recoverability depends in part on fluid temperatures, reservoir depth,

recharge conditions, the porous media having sufficient volume, porosity and permeability to yield adequate quantities of fluid, the technology incorporated in the power plant, the costs of developing and producing fluids from the reservoir, the costs of designing, building and operating the power plant, and on the price of the output product--electricity. Thus to determine an "optimal" economic recovery an integrated model is extremely desirable. One such optimization model is presented by Scherer (1975) and Scherer and Golabi (1976). Their reservoir model consists of an analytical solution to the doublet problem, where the reservoir is of infinite extent, the fluid is single phase hot water, and the well system is comprised of a pumped production well and a recharge well. The fluid is withdrawn and injected at the same constant rate but at different temperatures, so that with time the reservoir cools. At the surface the hot water is used for space heating. The objective of the studies is to determine the time sequence of withdrawal-injection rates that maximizes total discounted net benefits.

In this report a management model is developed that demonstrates a technology for determining the economic recoverability of a liquid-dominated geothermal reservoir undergoing exploitation for electric power generation. Pressures and temperatures in the reservoir are such that the fluid is single phase, water. The hot water may or may not flash to steam in the wells, in either case the fluid is transported to a power plant that uses a direct method of power production. The spent water is then reinjected into the reservoir. In the vicinity where hot water is extracted from the reservoir, pressures in the field decline

and where it is reinjected in the reservoir, pressures in the field rise. The management model integrates a physical model of the reservoir with a power plant model that determines the quantity of electricity generated for a rate of hot fluid extraction. Capital costs, variable costs and annual fixed costs are determined for the reservoir development, extraction and reinjection, the transmission system and the power plant. Revenues are determined for electric power production. The management model determines the maximum profits for the integrated system over a specified economic design horizon. Specifically, the management model determines, for the geothermal reservoir:

1. The spatial and temporal distribution of extraction and injection wells,
2. The annual rate of fluid extracted by withdrawal wells over the design horizon,
3. The annual rate of fluid injected into recharge wells over the design horizon,

for the power plant:

1. The turbine configuration over the design horizon,
2. The annual electric power generation,

and for the integrated system the total cost, revenues and profits over the design period.

It should be noted that none of the numbers utilized in the report are considered sacred; this is particularly true of numbers used for costs. The management model demonstrates a technology and

is not intended to be applied to actual field problems without careful consideration of its assumptions and determination of applicable parameters.

RESERVOIR MODEL

In general, the fluid flow and heat transport in a porous geothermal reservoir can be described by a set of nonlinear, partial differential equations (Appendix A or Faust and Mercer, 1979). Considerable effort has been expended recently to develop numerical models that solve these equations, and hence, that simulate the physical behavior of geothermal reservoirs. A summary of this effort is presented by Witherspoon and others (1975).

The purpose of this report is not to present a complex, nonlinear simulation model but to demonstrate the methodology of coupling a management model with a geothermal reservoir model. Hence, a simplified, yet realistic reservoir model is developed based on the following assumptions.

Assumptions

1. The reservoir is liquid dominated (that is, the liquid phase controls the pressure response). White (1970) notes that most geothermal reservoirs are liquid dominated.

2. The reservoir contains only pure water. Figure 1 shows the effect of NaCl concentrations on a pressure-temperature phase diagram (data from Haas, 1975a, 1975b) and it is apparent that, for weight concentrations less than 2 percent, the pure water assumption is adequate. Although geothermal reservoirs contain other dissolved solids in addition to NaCl, qualitatively the effects would be similar for other impurities.

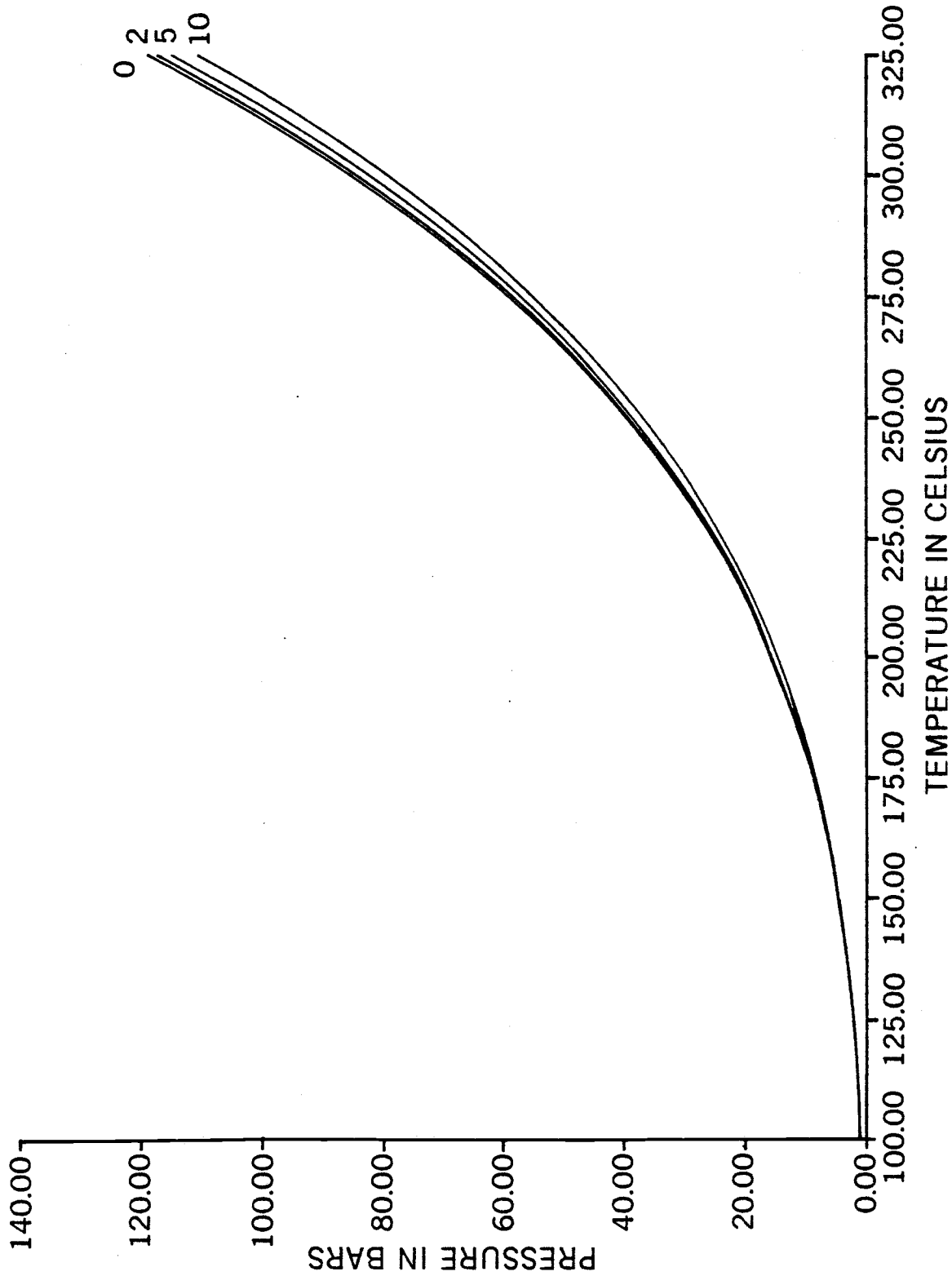


Figure 1. Pressure-temperature diagram for pure water and NaCl solutions (pure water, 2, 5, 10 percent weight concentrations shown; based on data from Haas, 1975a and Haas, 1975b).

For geothermal reservoirs such as those at Wairakei, New Zealand, Larderello, Italy and The Geysers, California, the dissolved solid concentrations are less than 1 percent (Koenig, 1973).

3. The spatial distribution of temperature for the reservoir is known and is invariant with time. Field data for liquid-dominated systems support this assumption (Grindley, 1965). Figure 2 shows a time series of temperature data for the Wairakei geothermal reservoir from 1953 to 1962. During this period, it is believed that the reservoir remained liquid-dominated, and as shown, the temporal average of temperature was approximately invariant with time. This assumption holds as a good approximation even with reinjection provided that the spent fluid is injected at a site with the same reservoir temperature as the fluid.

4. Viscosity is a function of temperature only and, as a result of the assumption for temperature, is invariant with time. The relation between viscosity and temperature is (Meyer and others, 1968),

$$\mu(\hat{x}) = 10^{-6} \{ 241.4 \times 10^{[247.8/(T(\hat{x}) + 133.15)]} \} \quad (1)$$

where $\mu(\hat{x})$ and $T(\hat{x})$ are the viscosity [g/cm-s] and temperature [°C] at point $\hat{x} = (x,y)$ respectively. Equation (1) is valid for liquid water along the saturation line from 0°C to 300°C.

5. Density is a function of both temperature and pressure in the following form,

$$\rho(\hat{x},t) = \rho_0(\hat{x}) + \beta \rho_0(\hat{x}) \left(p(\hat{x},t) - p_0(\hat{x}) \right) \quad (2)$$

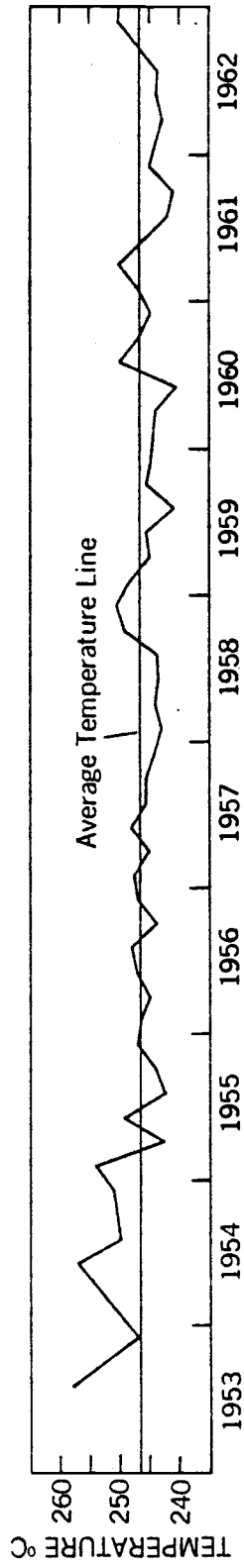


Figure 2. Time series of temperature data for Mairakei, N.Z., 1953-1962 (after Grindley, 1965).

where

$$\rho_0(\hat{x}) = 1.00606 - 2.46020 \times 10^{-4} T(\hat{x}) - 2.31633 \times 10^{-6} T^2(\hat{x}) \quad (3)$$

and where $\rho(\hat{x},t)$ and $\rho_0(\hat{x})$ are density and initial density respectively [g/cm^3], β is liquid compressibility [cm^2/dyne], $p(\hat{x},t)$ and $p_0(\hat{x})$ are pressure and initial pressure respectively [dyne/cm^2], and $T(\hat{x})$ is temperature [$^{\circ}\text{C}$]. Equation (3) is valid for liquid saturated temperatures between 100°C and 280°C (I.G. Donaldson, written communication, 1972). The initial density distribution is a function of temperature, and temporal changes in density are a function of temporal changes in pressure.

6. The reservoir is a porous medium, which is confined, and horizontal. Furthermore all wells are fully penetrating. Therefore, a two-dimensional model may be used in conjunction with vertically averaged reservoir and fluid properties.

7. Liquid compressibility, β , and porosity ϕ , are constants.

8. Spent fluid is injected into wells on sites with the reservoir temperature the same as the spent fluid's.

Equation of Flow

Invoking these assumptions, the set of nonlinear, partial differential equations developed in Faust and Mercer (1979) is reduced to the linear equation (see Appendix A)

$$\nabla \cdot (m(\hat{x}) \nabla p(\hat{x}, t)) = a(\hat{x}) \frac{\partial p}{\partial t}(\hat{x}, t) + Q'(\hat{x}, t), \quad (4)$$

which is the equation of flow, where ∇ is defined over the two horizontal dimensions x and y , and where $Q'(\hat{x}, t)$ are source-sink terms. No energy equation is required because the temperature distribution is assumed known and invariant with time. The storage quality of the aquifer is measured by

$$a(\hat{x}) = b(\hat{x}) \phi \rho_0(\hat{x}) \beta, \quad (5)$$

where $b(\hat{x})$ is the aquifer thickness. The storage quality is independent of time and is dependent only on the space variables.

The transmissive quality of the aquifer is approximated by

$$m(\hat{x}) = \frac{b(\hat{x}) \rho_0(\hat{x}) k(\hat{x})}{\mu(\hat{x})} \quad (6)$$

where $k(\hat{x})$ is the local intrinsic permeability. Note that $m(\hat{x})$ is also only dependent on the space variables. Werner (1946) observed that for small liquid compressibilities such as those associated with water, this approximation is valid.

The source term in equation (4) is composed of two parts: a point source-sink term, $Q_1(\hat{x}, t)$ and a vertical leakage source term, $Q_2(\hat{x}, t)$. The point source-sink term is approximated by

$$Q_1(\hat{x}, t) = - \sum_{j=1}^M \rho_0(\hat{x}_j) q(\hat{x}_j, t) \delta(\hat{x} - \hat{x}_j) \quad (7)$$

where $q(x_j, t)$ is the discharge from the j^{th} well for positive values and is recharge to the j^{th} well for negative values, M is the total number of wells, and $\delta(x-x_j)$ is a Dirac delta function. $Q_2(\hat{x}, t)$ represents the transient vertical leakage through a confining bed which is assumed to be caused by a stepwise change in pressure within the aquifer, and is approximated by (Trescott, Pinder and Larson, 1976)

$$Q_2(\hat{x}, t) = \rho_0(\hat{x}) (p_0(\hat{x}) - p(\hat{x}, t)) B(\hat{x}) \quad (8)$$

where

$$B(\hat{x}) = \frac{K'}{\rho_0(\hat{x})gb'} \left(\frac{\pi K' \eta}{3b'^2 S_s} \right)^{\frac{1}{2}} \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp \left[- \frac{3n^2 b'^2 S_s}{K' \eta} \right] \right\} \quad (9)$$

where K' , S_s and b' are the hydraulic conductivity, the specific storage and the thickness of the confining bed respectively, g is gravitational acceleration (constant) and η is the time interval over which the discharge from wells is considered constant.

Substitution of equations (7) and (8) into equation (4) yields,

$$\begin{aligned} \nabla \cdot \left(m(\hat{x}) \nabla p(\hat{x}, t) \right) + B(\hat{x}) p(\hat{x}, t) - a(\hat{x}) \frac{\partial p}{\partial t}(\hat{x}, t) \\ = B(\hat{x}) p_0(\hat{x}) - \sum_{j=1}^M \rho_0(\hat{x}_j) q(\hat{x}_j, t) \delta(\hat{x} - \hat{x}_j) \end{aligned} \quad (10)$$

Assuming that the pressure distribution is initially at steady state,

$$\nabla \cdot \left(m(\hat{x}) \nabla p_0(\hat{x}) \right) = 0 \quad (11)$$

and subtracting equation (10) from equation (11) gives,

$$\begin{aligned} \nabla \cdot \left(m(\hat{x}) \nabla p_D(\hat{x}, t) \right) - B(\hat{x}) p_D(\hat{x}, t) - a(\hat{x}) \frac{\partial p_D}{\partial t}(\hat{x}, t) \\ = \sum_{j=1}^M \rho_o(\hat{x}_j) q(\hat{x}_j, t) \delta(\hat{x} - \hat{x}_j) \end{aligned} \quad (12)$$

where

$$p_D(\hat{x}, t) = p_o(\hat{x}) - p(\hat{x}, t) \quad (13)$$

Equation (12) is the equation of fluid flow to be solved and is subject to the initial conditions,

$$p_D(\hat{x}, 0) = 0 \quad (14)$$

and boundary conditions,

$$\frac{\partial p_D}{\partial n}(\Gamma, t) = 0 \quad t \geq 0 \quad (15)$$

where Γ is the boundary curve and \hat{n} is normal to Γ . Equation (15) represents an unperturbed boundary condition since it implies that the natural recharge and discharge through the boundary curve Γ are unperturbed by the withdrawals from wells, that is,

$$\frac{\partial}{\partial n} p_D(\Gamma, t) = \frac{\partial}{\partial n} p_o(\Gamma) - \frac{\partial p}{\partial n}(\Gamma, t) = 0 \quad (16)$$

or

$$\frac{\partial p_o}{\partial n}(\Gamma) = \frac{\partial p}{\partial n}(\Gamma, t) \quad (17)$$

Method of Solution

Equation (12) and its associated initial conditions and boundary conditions given by equation (14) and (15) respectively,

are linear and thus provide for the existence of a Green's function (Morse and Feshbach, 1953). The Green's function, $G(\hat{x}, \hat{x}', t)$, is determined by solving the equation

$$\nabla \cdot [m(\hat{x}) \nabla G(\hat{x}, \hat{x}', t)] - B(\hat{x})G(\hat{x}, \hat{x}', t) - a(\hat{x}) \frac{\partial G}{\partial t}(\hat{x}, \hat{x}', t) = \delta(\hat{x} - \hat{x}') \delta(t) \quad (18)$$

subject to the causality condition

$$G(\hat{x}, \hat{x}', t - \tau) = 0 \quad \tau > t \quad (19)$$

and the boundary condition

$$\frac{\partial G}{\partial n}(\Gamma, t) = 0 \quad (20)$$

By definition (Morse and Feshbach, 1953)

$$p_D(\hat{x}, t) = \int_{\substack{\hat{x} \\ \text{in } \Gamma}} \int_0^t G(\hat{x}, \hat{x}', t - \tau) F(\hat{x}', \tau) d\tau d\hat{x}' \quad (21)$$

where

$$F(\hat{x}', \tau) = \sum_{j=1}^M \rho_0(\hat{x}_j) q(\hat{x}_j, \tau) \delta(\hat{x}' - \hat{x}_j) \quad (22)$$

The design period for the geothermal system is to consist of N equal duration time periods of length η . The discharges or recharges $q(x_j, \tau)$, $j=1, \dots, M$, are constants within a time period but may vary over the N intervals. Thus the pressure at the k^{th} well at the end of the n^{th} time period, written $p(k, n)$ is given by

$$p(k,n) = p_o(k) - \sum_{j=1}^M \sum_{i=1}^n Q(j,i) \int_{(i-1)\eta}^{i\eta} G(\hat{x}_k, \hat{x}_j, n\eta - \tau) d\tau \quad (23)$$

where

$$Q(j,i) = \rho_o(\hat{x}_j) q(\hat{x}_j, i\eta) \quad (24)$$

is the mass-rate of water withdrawn from (if positive) or injected into (if negative) the j^{th} well in the i^{th} time period, and

$$p_o(k) = p_o(\hat{x}_k). \quad (25)$$

Define

$$R(k,j,n-i+1) \triangleq \int_{(i-1)\eta}^{i\eta} G(\hat{x}_k, \hat{x}_j, n\eta - \tau) d\tau \quad (26)$$

and equation (23) becomes

$$p(k,n) = p_o(k) - \sum_{j=1}^M \sum_{i=1}^n Q(j,i) R(k,j,n-i+1) \quad (27)$$

The $R(k,j,n-i+1)$ are response coefficients (Maddock, 1972) and are constants independent of withdrawals and pressure. The coefficient $R(k,j,n-i+1)$ measures the increment in pressure drop at the k^{th} well at the end of the n^{th} time period due to withdrawal of a unit mass at the j^{th} well during the i^{th} time period ($i \leq n$). The coefficients R are related to the transmissive quality of the aquifer, $m(\hat{x})$, the leakage coefficient, $B(\hat{x})$, the storage quality of the aquifer, $a(\hat{x})$, the unperturbed boundary conditions (equation (15)), the initial

conditions (equation (14)), the distances between wells, the well radii and the form of the partial differential equation. In practice the R 's are determined by a finite-difference or a finite-element simulation model because irregularly shaped boundaries (Γ) and non-homogeneous parameters ($m(\hat{x})$, $B(\hat{x})$ and $a(\hat{x})$) make analytical solutions impossible (Maddock, 1974).

A further assumption is needed when a finite difference or finite element scheme is used to calculate the response functions. The pressure values calculated by these methods apply over a grid or an element and represent an average discharge or recharge over an area. If the area is large then more than one well may be present and the $Q(j,i)$ in equation (27) is the aggregated withdrawal (if positive) or injection (if negative) from a number of wells. The area is thought of as a site, and the site may have more than one well. For simplicity if a site has more than one well, then these wells are assumed to be distributed equal distance from each other within the area and the aggregated withdrawals or injections are divided equally amongst the wells.

In this report, $Q(j,i)$ has the site interpretation with $Q_E(j,i)$ designating the aggregated withdrawals from the j^{th} extraction site in the i^{th} time period and $Q_R(j,i)$ designating the aggregated injection in the j^{th} reinjection site in the i^{th} time period. The number of potential extraction site is M_E , the number of potential reinjection sites is M_R , and the sum of M_R and M_E is M (equation (27)).

Equation (27) provides a constraint set that controls the interaction between withdrawals from wells and the pressure drop at the wells due to those withdrawals. This constraint set will be utilized in the management model and will simulate the pressure response in the reservoir to fluid withdrawals. An additional constraint is provided by the assumption that the reservoir is liquid dominated, which requires that the pressure within the wells does not drop below the saturation pressures (a function of temperature). The reason for this constraint is to maintain a linear system so that the approach outlined in this paper is valid. The saturation pressure $p_s(k)$ for a well is determined from the equation

$$p_s(k) = 3.5968 \times 10^7 - 5.3667 \times 10^{-3} H(k) + 5.4014 \times 10^{-23} H^3(k) \\ - 6.8971 \times 10^{16} \left(\frac{1}{H(k)}\right) - 2.8585 \times 10^{-1} \left(\frac{1}{H(k)}\right)^3 \quad (28)$$

where

$$H(k) = c_v T(k), \quad k = 1, \dots, M \quad (29)$$

and c_v is the specific heat capacity of the fluid at constant volume [ergs/g°C] and $T(k)$ is the temperature [°C] within the well. Equation (28) was obtained using a least squares regression and data from steam tables (Meyer and others, 1968), and is valid over the temperature range of 0-300°C. Thus the total pressure at the well sites is constrained by the condition that,

$$p(k,n) \geq p_s(k) \quad k = 1, \dots, M \text{ and } n = 1, \dots, N \quad (30)$$

POWER PLANT MODEL

The power plant model is based on the operation of an appropriately idealized steam-flash turbine system. As with the reservoir model the attempt is to present a simplified, yet realistic, description of a geothermal power plant. Specifically, the model predicts the intensive electrical power output, the intensive cooling water requirements and the steam quality mass fractions to various components (flashers, separators, condensers, etc.) of a steam-flash, two-stage, turbine-condenser power plant for each of the sites designated for well development in the reservoir model. These predictions are a function of the site design pressures, temperatures, and efficiencies specified for the power plant.

A schematic diagram of the power plant model is shown in Figure 3. This schematic may actually represent a more complex system in that the turbines could be considered a series of turbines operating at the same inlet and outlet pressure levels. Likewise the inlet separator-flasher may represent a system of separator-flasher in the power plant, several separator-flashers at the reservoir sites, or even flashing in the well bores (but not in the reservoir). The power plant model calculates a material-energy balance for the system shown in the schematic at each potential site in the geothermal reservoir. These balances provide the predictions for intensive power output, intensive cooling water requirements and steam fractions. The intrinsic power output, intensive cooling water requirements and the steam fractions

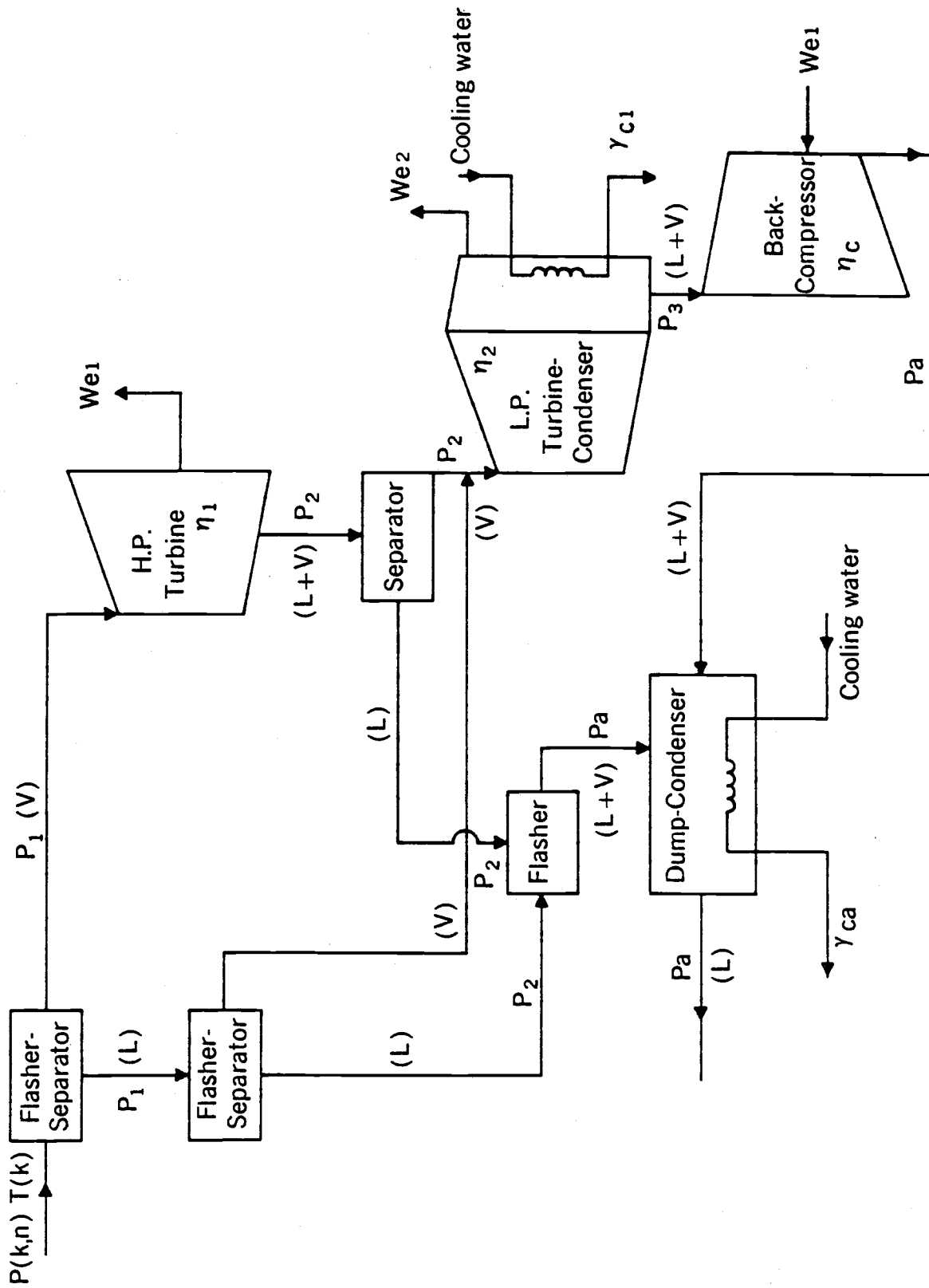


Figure 3. Schematic diagram of the power plant model showing operating pressures (P_1 , P_2 , P_3 , P_a).

are utilized in the objective function and constraint set of the management model presented in the next section.

Several assumptions are invoked in the power plant model. For demonstration of the overall management model, these assumptions simplify the material-energy balance calculations. They can, however, be relaxed for other applications.

Assumptions

1. The geothermal fluid is pure water thus allowing the use of standardized steam tables (for example, Meyer and others, 1968).
2. Thermal equilibrium exist between phases in the separator-flashers, turbines and compressors.
3. Change of phase in the separator-flashers occur under adiabatic conditions.
4. Heat losses from all pipelines are neglected. Heat losses in the well bore are also neglected. Pressure drops occur under isoenthalpic conditions, hence the enthalpy value for the hot water remains constant at the sites and is taken to be the saturation value for the reservoir temperature at the site.
5. The inlet and outlet pressures for turbines, compressors and flashers, the inlet and outlet temperatures for condensers, and the efficiencies of various processes are specified. No attempt is made to determine an optimum set of these parameters.
6. The temperature reference base is 0°C .

7. The mechanical power available from utilizing the geothermal pressure is negligible compared to thermal power potential. For example, if the initial pressure in a well is 150×10^6 dyne/cm², the temperature is 200°C and the well discharges at a rate of 70,000 g/s the mechanical power available is $2.51 \times 10^5 \frac{\text{cal}}{\text{s}}$ and the thermal power available is $140.0 \times 10^5 \frac{\text{cal}}{\text{s}}$.

8. The operating pressure for various components of the power plant are less than the saturation pressures in the reservoir.

These assumptions provide the basis for computing the material-energy balance of the power plant. As shown in Figure 3, hot water from wells at the kth reservoir site with temperature T(k) and pressure p(k,n) enters a primary separator-flasher system. The outlet pressure of the primary separator-flasher system is specified at P₁. One fraction of the input liquid leaves the primary separator-flasher as saturated vapor and enters the high-pressure turbine system, while the other fraction of the input liquid travels to the secondary separator-flasher system. Note that the value of these fractions are dependent on the site temperature. A mixture of liquid and vapor leaves the high pressure turbine system at pressure P₂ and enters a separator. The vapor fraction from this separator is fed to the low pressure turbine system while the liquid fraction is fed to a flasher, reduced to atmospheric pressure P_a, and fed to the dump condenser. The vapor fraction from the secondary separator-flasher system is also fed to the low pressure turbine system; and the liquid fraction is fed to a flasher, reduced to atmospheric pressure and fed to

the dump condenser. The outlet pressure P_3 of the low pressure turbine system is less than atmospheric, hence a back compressor system is used to restore atmospheric pressure. The liquid and vapor fractions exiting the back compressor are fed to the dump condenser. As shown in the schematic, cooling waters are required for the low pressure turbine system which use a condenser system and the dump condenser. Process waters exit the dump condenser in the liquid phase and are transported to reinjection sites.

Power Plant Material-Energy Balance Calculation

The computational procedure for individual components are as follows.

Separator-Flashers: The steam fraction leaving a separator-system is given by

$$x_s^F = \frac{h_i^F - h_{l0}^F}{h_{v0}^F - h_{l0}^F} \quad (31)$$

where h_i^F is the enthalpy of the input fluid (liquid only into flasher-separators, and liquid and vapor into separators), h_{l0}^F is the saturated liquid enthalpy at the outlet pressure, and h_{v0}^F is the saturated vapor enthalpy at the outlet pressure.

Turbines: The energy balance for the turbines is illustrated in the temperature-entropy and pressure-enthalpy diagrams shown in Figure 4, in which P_i and P_o represent the inlet and outlet pressure, respectively, for the turbines. The energy balance for the turbines is given by (see Figure 4)

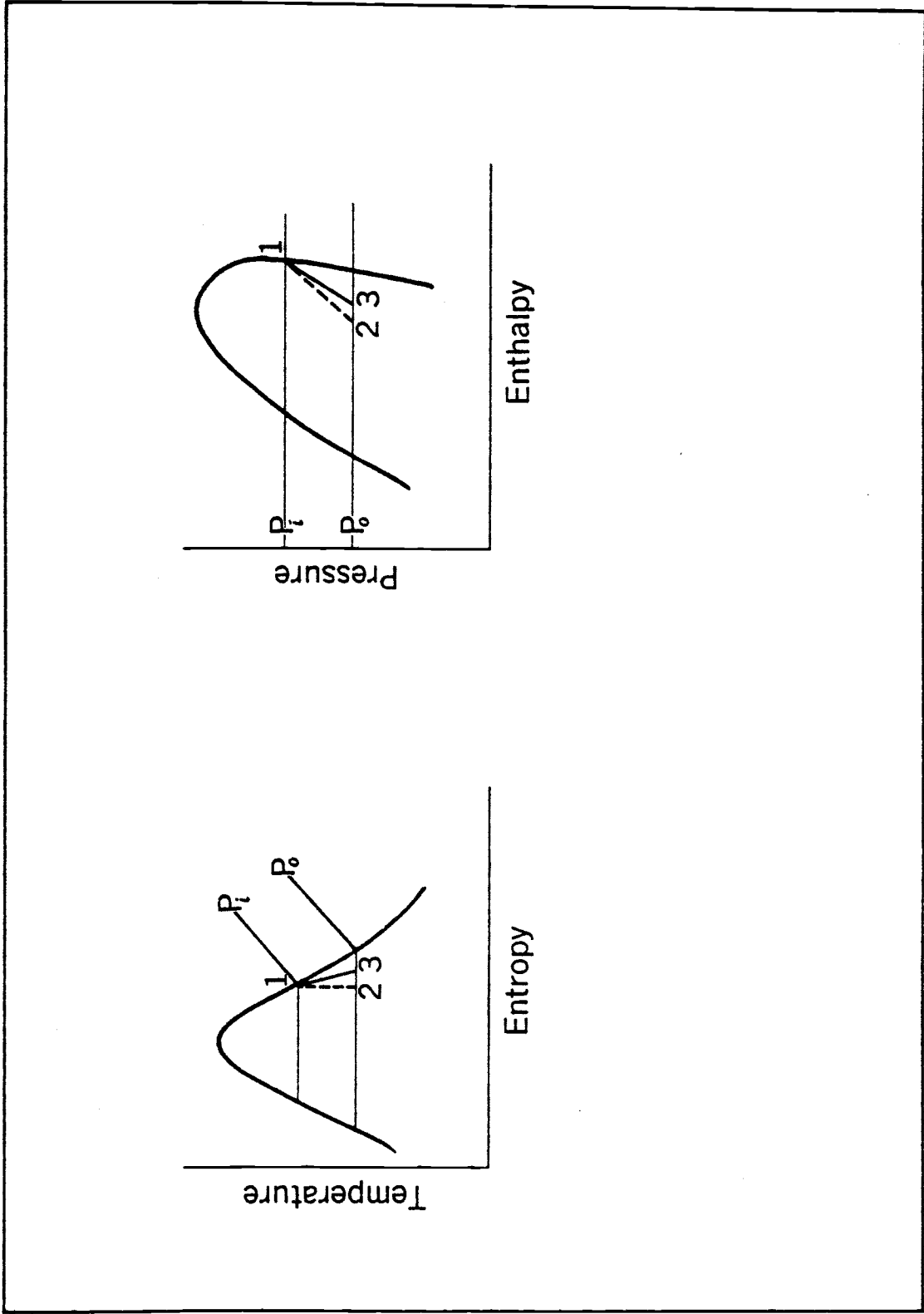


Figure 4. Temperature-entropy and pressure-enthalpy diagrams for pure water indicating inlet pressure (P_i), outlet pressure (P_o) and points (1-3) used in turbine energy balance calculations.

$$W_T = h_i^T - h_3^T \quad (32)$$

where h_i^T is the enthalpy of the inlet steam. The enthalpy h_3^T is determined by first calculating the steam fraction and enthalpy at point 2 (Figure 4) for an isentropic process

$$S_{2s}^T = \frac{S_2^T - S_{10}^T}{S_{vo}^T - S_{10}^T}, \quad (33)$$

and

$$h_2^T = h_{10}^T + X_{2s}^T (h_{vo}^T - h_{10}^T), \quad (34)$$

where S is the entropy and subscripts 10 and vo refer to the values at the outlet pressures for saturated liquid and vapor, respectively. The energy balance for the turbine may be rewritten as,

$$W_T = \eta_s (h_i^T - h_2^T), \quad (35)$$

where η_s is the isentropic efficiency for the turbine (which has been specified, by assumption). Thus

$$h_3^T = h_i^T - \eta_s (h_i^T - h_2^T). \quad (36)$$

The steam fraction at the outlet is obtained as

$$X_{s3}^T = \frac{h_3^T - h_{10}^T}{h_{vo}^T - h_{10}^T}. \quad (37)$$

Compressor: The energy-material balance calculations for the back pressure compressor are similar to those for the turbines. As with the turbine calculations the first step is to determine the steam fraction and the enthalpy of the fluid at the outlet pressure

(atmospheric) for an isentropic process. Expressions analogous to equations (33) and (34) are used for this purpose. To obtain the final enthalpy and steam fraction the isentropic efficiency of the process is used in equation analogous to equations (36) and (37).

Condenser: The cooling water requirement per gram of fluid being cooled is given by

$$\gamma_c = \frac{h_i^c - h_o^c}{C_p \Delta T_c} \quad (38)$$

where subscripts i and o refer to inlet and outlet values of material being cooled, C_p is the heat capacity of the cooling water, and ΔT_c is the temperature rise allowed for the cooling water.

The intensive electrical work output is determined by multiplying the intensive mechanical work W_T by an efficiency factor, that is,

$$W_{ek} = \eta_{ek} \times W_{Tk} \quad k = 1,2 \quad (39)$$

where the k subscript indicates the type of turbine system; k = 1 for high pressure and k = 2 for low pressure.

The power plant model is run for each of the M_E potential sites for extraction well development. If $W_{e1}(j)$ and $W_{e2}(j)$ are the intensive electrical work produced from the high pressure turbines and low pressure turbines, respectively, for the j^{th} site, then the electrical power produced in the power plant for the i^{th} time period, $A_{MW}(i)$ is

$$A_{MW}(i) = \sum_{j=1}^{M_E} (W_{e1}(j) + W_{e2}(j)) Q_E(j,i) \quad (40)$$

where $Q_E(j,i)$ is the total withdrawal rate from extraction wells at the j^{th} site in the i^{th} year. Likewise if $\gamma_{c1}(j)$ and $\gamma_{c2}(j)$ are the intensive cooling water requirements for the condenser on the low pressure turbine and the dump condenser, respectively, for the j^{th} site, then the rate of cooling water required for the i^{th} time period, $\Gamma(i)$ is

$$\Gamma(i) = \sum_{j=1}^{M_E} (\gamma_{c1}(j) + \gamma_{c2}(j)) Q_E(j,i) \quad (41)$$

Steam rates entering or exiting the various components of the power plant are calculated in the same fashion. For example, if $X_1^F(j)$ is fraction of steam exiting the primary separator-flasher system for waters from the j^{th} site, then the total rate of steam exiting that system in the i^{th} time period, $S_{sp}(i)$, is

$$S_{sp}(i) = \sum_{j=1}^{M_E} X_1^F(j) Q_E(j,i) \quad (42)$$

Finally, it should be noted, that $W_{ek}(j)$'s, $\gamma_{ck}(j)$'s and the $X_k(j)$'s are independent of the $Q_E(j,i)$'s, the pressure drops and the time periods; but are dependent on the reservoir temperatures, which

remain invariant, and on the operating temperatures or pressures specified for each component of the power plant. This is a direct result of assumption 5 in this section.

MANAGEMENT MODEL

The reservoir model simulates the response of the physical system under development while the power plant model provides a means of evaluating the energy resource of the resource. The management model incorporates these two models to aid in determining the feasibility and profitability of development. It is based on the relevant costs for producing and the revenues acquired from selling a unit of electricity. To arrive at a unit cost of electricity both capital and operating cost (annual fixed and variable costs) are estimated for the reservoir development (withdrawal and reinjection), the transmission system and the power plant. The reservoir, the transmission system and the power plant are assumed to be operated by the same firm as an integrated unit. Because the operating costs are dependent on the rates of extraction, rates of reinjection and the distribution of wells, this assumption simplifies the analysis.

There is a large degree of uncertainty associated with geothermal field costs and power generation costs at the present time due to the limited experience in operations and the experimental nature of much of the equipment. Moreover, what little cost and reservoir performance information exists are generally proprietary in nature. Rather than developing cost relationships for potentially numerous possible engineering designs, some which have no existing prototypes, cost relationships developed by Battelle Institute (Huber and others, 1975; Bloomster and Knutsen, 1975) are used. These are representative of

cost prevailing during 1974. The cost functions presented in the Battelle reports were simplified to facilitate their use. In particular, nonlinear relationships are approximated by linear functions that produced little error for the range of reservoir conditions and power plant sizes under consideration.

The following set of assumptions are invoked in the management model. In the assumptions designed capacity flow and megawatt power rating are to be distinguished from actual flow and actual megawatt power production, respectively. The former two refer to equipment design and are fixed, the latter two refer to production quantities and vary with operating conditions.

Assumptions

1. The basic unit of time is the year, however expansion and construction occur in time intervals called construction intervals that may be greater than a single year.

2. The capital cost for development of extraction wells in the ℓ^{th} construction interval, $CC_{EW}(\ell)$ is based on costs associated with lease acquisition; exploration; and drilling, testing and completion of wells. It is expressed as a linear function of the number of extraction wells added during the ℓ^{th} construction interval, $N_{DWE}(\ell)$

$$CC_{EW}(\ell) = a_{11} N_{DWE}(\ell) + b_1 \quad (43)$$

3. The capital cost of injection wells in the ℓ^{th} construction interval, $CC_{\text{RW}}(\ell)$ is based on costs associated with drilling, testing and completion of wells; and procurement of pumping equipment for injection. It is expressed as a linear function of the number of injection wells added during the ℓ^{th} construction interval, $N_{\text{DWR}}(\ell)$.

$$CC_{\text{RW}}(\ell) = a_{21}N_{\text{DWR}}(\ell) + b_2 \quad (44)$$

4. The capital cost for the transmission system in the ℓ^{th} construction interval, $CC_{\text{TS}}(\ell)$ is based on the cost of piping and fittings for wells. It is expressed as a linear function of the following items added during the ℓ^{th} construction interval: $N_{\text{DWE}}(\ell)$, $N_{\text{DWR}}(\ell)$, the design withdrawal rate (steam and liquid) from wells, $Q_{\text{DE}}(\ell)$ and the design injection rate to wells, $Q_{\text{DR}}(\ell)$,

$$CC_{\text{TS}}(\ell) = a_{31}N_{\text{DWE}}(\ell) + a_{32}N_{\text{DWR}}(\ell) + a_{33}Q_{\text{DE}}(\ell) + a_{34}Q_{\text{DR}}(\ell) + b_3 \quad (45)$$

The siting of the power plant is not included as a decision variable in the management model, thus transmission length from extraction well head to power plant and back to injection well head is not explicitly included in variable factors affecting cost.

5. The capital cost for the power plant in the ℓ^{th} construction interval, $CC_{\text{pp}}(\ell)$, excluding separator-flashers, is based on costs associated with cranes, turbine-generator electrical equipment, buildings and installations, switch yards, piping, condenser, instrumentation, cooling water system, design and administration. It is

expressed as linear functions of the megawatt-power rating of the turbine system added during the ℓ^{th} construction interval, $N_{\text{DMW}}(\ell)$ and the design capacity of steam for the turbines added during the ℓ^{th} construction interval, $S_{\text{DT}}(\ell)$

$$CC_{\text{pp}}(\ell) = a_{41}N_{\text{DMW}}(\ell) + a_{42}S_{\text{DT}}(\ell) + b_4 \quad (46)$$

6. The capital cost of the separator-flasher system in the ℓ^{th} construction interval, $CC_{\text{SF}}(\ell)$ is expressed as a linear function of the design capacity of steam outflow from the separator-flasher system added during the ℓ^{th} construction interval, $S_{\text{DF}}(\ell)$

$$CC_{\text{SF}}(\ell) = a_{51}S_{\text{DF}}(\ell) + b_5 \quad (47)$$

7. Engineering design and administration cost for the power plant during the ℓ^{th} construction interval, $CC_{\text{DA}}(\ell)$, is a fixed fraction α of the sum of capital costs $CC_{\text{pp}}(\ell)$ and $CC_{\text{SF}}(\ell)$

$$CC_{\text{DA}}(\ell) = \alpha (CC_{\text{pp}}(\ell) + CC_{\text{SF}}(\ell)) \quad (48)$$

8. Variable costs are computed on an annual basis.

9. The annual fixed and variable cost for development of extraction wells in the i^{th} year, $CV_{\text{EW}}(i)$ is based on costs associated with well maintenance, replacement, abandonment and depreciation. It is expressed as a linear function of the total number of extraction wells present in the i^{th} year, $T_{\text{WE}}(i)$

$$CV_{EW}(i) = a_{61} T_{WE}(i) + b_6 \quad (49)$$

10. The annual fixed and variable costs for development of injection wells in the i^{th} year, $CV_{RW}(i)$ is based on cost associated with well maintenance, injection pressure, replacement, abandonment and depreciation. It is expressed as a linear function of the total number of injection wells present in the i^{th} year, $T_{WR}(i)$, and the total flow to injection wells in the i^{th} year, $Q_{TR}(i)$,

$$CV_{RW}(i) = a_{71} T_{WR}(i) + a_{72} \frac{P_{\max}}{\rho_T} Q_{TR}(i) + b_7 \quad (50)$$

where P_{\max} is the injection pressure, and ρ_T is the density of the fluid at the injection site with temperature T .

11. The annual fixed and variable cost for the transmission system in the i^{th} year, $CV_{TS}(i)$ is based on costs associated with transmission line maintenance, replacement, taxes and depreciation. It is a linear function of $T_{WE}(i)$, $T_{WR}(i)$, the total flow from extraction wells during the i^{th} year, ($Q_{TE}(i)$ and $Q_{TR}(i)$).

$$CV_{TS}(i) = a_{81} T_{WE}(i) + a_{82} T_{WR}(i) + a_{83} Q_{TE}(i) + a_{84} Q_{TR}(i) + b_8 \quad (51)$$

12. The annual fixed and variable costs for the power plant in the i^{th} year are based on costs associated with operation, maintenance,

investment capital, depreciation and purchase of makeup cooling water. The operation cost in the i^{th} year, $CV_{OP}(i)$ is a linear function of the design capacity for megawatt-power production in the i^{th} year, $D_{MW}(i)$

$$CV_{OP}(i) = a_{91} D_{MW}(i) + b_9 \quad (52)$$

Maintenance, investment capital and depreciation cost in the i^{th} year, $CV_{MID}(i)$ is a linear function of the actual quantity of steam exiting the separator-flasher system during the i^{th} year, $S_{AF}(i)$, the actual quantity of steam entering the turbine system during the i^{th} year, $S_{AT}(i)$, and $D_{MW}(i)$

$$CV_{MID}(i) = a_{101} S_{AF}(i) + a_{102} S_{AT}(i) + a_{103} D_{MW}(i) + b_{10} \quad (53)$$

The cost of purchasing makeup cooling water in the i^{th} year, $CV_{CW}(i)$ is a linear function of the quantity of makeup cooling water necessary to replace evaporative cooling or blowdown losses in the i^{th} year,

$Q_{CW}(i)$

$$CV_{CW}(i) = a_{111} Q_{CW}(i) + b_{11} \quad (54)$$

13. A depletion allowance for the reservoir, extraction wells and injection wells is not considered.

14. No residual modification other than reinjection of spent fluid is considered.

15. Revenue for the i^{th} year, $R_V(i)$ is a linear function of the actual megawatt-power production for the i^{th} year, $A_{MW}(i)$

$$R_V(i) = a_{121} A_{MW}(i) \quad (55)$$

These assumptions provide the basis for computing the capital, annual-fixed and variable costs of developing wells; extracting, transporting and reinjecting fluid; and producing electrical power. The assumptions also provide a basis for computing revenues. Table 1 and 2 summarizes the variables used in the assumptions.

Cost, Revenue and Profit Calculations

Invoking the assumptions detailed in the previous section the following aggregated cost functions are produced.

The capital cost for reservoir development for the ℓ^{th} construction interval, $CC_{RS}(\ell)$ is defined as

$$CC_{RS}(\ell) = CC_{EW}(\ell) + CC_{RW}(\ell). \quad (56)$$

The capital cost for the power plant system for the ℓ^{th} construction interval, $CC_{PS}(\ell)$ is defined as

$$CC_{PS}(\ell) = CC_{PP}(\ell) + CC_{SF}(\ell) + CC_{DA}(\ell) = (1+\alpha)(CC_{PP}(\ell) + CC_{SF}(\ell)). \quad (57)$$

The total capital cost incurred during the ℓ^{th} construction interval, $CC_T(\ell)$ is given by the sum of capital costs for reservoir development, transmission system and power plant.

$$CC_T(\ell) = CC_{RS}(\ell) + CC_{TS}(\ell) + CC_{PS}(\ell), \quad (58)$$

Table 1

Independent variables used in cost functions.

- $N_{DWE}(\ell)$ = number of extraction wells added during the ℓ^{th} construction interval from all sites.
- $N_{DWR}(\ell)$ = number of injection wells added during the ℓ^{th} construction interval from all sites.
- $Q_{DE}(\ell)$ = the added design withdrawal rate for the ℓ^{th} construction interval.
- $Q_{DR}(\ell)$ = the added design reinjection rate for the ℓ^{th} construction interval.
- $N_{DMW}(\ell)$ = the megawatt-power rating of the turbine systems added during the ℓ^{th} construction interval.
- $S_{DT}(\ell)$ = the design capacity of steam to the turbines added during the ℓ^{th} construction interval.
- $S_{DF}(\ell)$ = the design capacity of steam outflow from the separator-flasher systems added during the ℓ^{th} construction interval.
- $T_{WE}(i)$ = the total number of extraction wells present in the i^{th} year.
- $T_{WR}(i)$ = the total number of injection wells present in the i^{th} year.
- $Q_{TE}(i)$ = the total flow from extraction wells during the i^{th} year.
- $Q_{TR}(i)$ = the total flow to injection wells during i^{th} year.
- $D_{MW}(i)$ = the design capacity for megawatt-power production for the i^{th} year.
- $S_{AF}(i)$ = the actual quantity of steam exiting the separator-flasher system during the i^{th} year.

Table 1 - continued

$S_{AT}(i)$ = the actual quantity of stream entering the turbine systems during the i^{th} year.

$Q_{CW}(i)$ = the amount of makeup cooling water necessary to replace evaporative or blowdown losses for the i^{th} year.

$A_{MW}(i)$ = the actual megawatt-power productions for the i^{th} year.

The annual fixed and variable cost for reservoir development in the i^{th} year, $CV_{RS}(i)$ is defined as

$$CV_{RS}(i) = CV_{EW}(i) + CV_{RW}(i) \quad (59)$$

The annual fixed and variable cost for the i^{th} year for the power plant, $CV_{PS}(i)$ is defined as

$$CV_{PS}(i) = CV_{OP}(i) + CV_{MID}(i) + CV_{CW}(i) . \quad (60)$$

The total annual fixed and variable cost for the i^{th} year, $CV_T(i)$ is given by

$$CV_T(i) = CV_{RS}(i) + CV_{TS}(i) + CV_{PS}(i) \quad (61)$$

The total cost over a design period of N years, C_T is

$$C_T = \sum_{\ell=1}^L CC_T(\ell) R(\ell, r) + \sum_{i=1}^N \frac{CV_T(i)}{(1+r)^i} \quad (62)$$

where L is the number of construction intervals in the N year design period, and

$$R(\ell, r) = \sum_{k=I_t(\ell-1)+1}^{I_t \cdot \ell} \left(\frac{1}{1+r}\right)^k , \quad (63)$$

where I_t is the number of years in a construction interval and r is an interest rate that reduces future cost to present value.

The total revenue over a design period of N years, R_T is

$$R_T = \sum_{i=1}^N R_V(i) \frac{1}{(1+r)^i} \quad (64)$$

and the total profit, P_T is

$$P_T = R_T - C_T \quad (65)$$

The independent variables in equations (43) through (55)(Table 1) can all be expressed as a function of one or more of six decision variables. These variables are:

1. The number of extraction wells from the j^{th} site added during the ℓ^{th} construction interval, $\delta_{DWE}(j,\ell)$,
2. the number of injection wells from the j^{th} site added during the ℓ^{th} construction interval, $\delta_{DWR}(j,\ell)$,
3. the number of high pressure turbines added in the ℓ^{th} construction interval, $\delta_{DTH}(\ell)$,
4. the number of low pressure turbines added in the ℓ^{th} construction interval, $\delta_{DTL}(\ell)$,
5. the withdrawal rate from extraction wells at the j^{th} site in the i^{th} year, $Q_E(j,i)$ and
6. the injection rate to reinjection wells at the j^{th} site in the i^{th} year, $Q_R(j,i)$.

The variables δ_{DWE} , δ_{DWR} , δ_{DTH} and δ_{DTL} take on only integer values while the variables Q_E and Q_R take on continuous values.

The quantity $Q_E(j,i)$ is the aggregated withdrawal from a j^{th} site in the i^{th} year. It is distributed equally among the total number of wells present at the j^{th} site during that year, $\delta_{WE}(j,i)$

Table 2

Cost, revenue and profit functions

$CC_{EW}(\ell)$	= capital cost for development of extraction wells in the ℓ^{th} construction interval
$CC_{RW}(\ell)$	= capital cost of injection wells in the ℓ^{th} construction interval
$CC_{TS}(\ell)$	= capital cost for the transmission system in the ℓ^{th} construction interval
$CC_{PP}(\ell)$	= capital cost for the power plant (excluding separator flashers) in the ℓ^{th} construction interval
$CC_{SF}(\ell)$	= capital cost of the separator-flasher system in ℓ^{th} construction interval
$CC_{DA}(\ell)$	= engineering design and administration cost for power plant during the ℓ^{th} construction interval
$CV_{EW}(i)$	= annual fixed and variable cost for development of extraction wells in the i^{th} year
$CV_{RW}(i)$	= annual fixed and variable cost for injection well in the i^{th} year
$CV_{TS}(i)$	= annual fixed and variable cost for the transmission system in the i^{th} year
$CV_{OP}(i)$	= annual fixed and variable cost for operations in the i^{th} year
$CV_{MID}(i)$	= fixed annual and variable cost for maintenance, investment capital and depreciation in the i^{th} year

Table 2 - continued

$CV_{CW}(i)$	=	fixed annual and variable cost for purchasing makeup cooling water in the i^{th} year
$R_V(i)$	=	revenue for the i^{th} year
$CC_{RS}(\ell)$	=	capital cost for reservoir development for the ℓ^{th} construction interval
$CC_{PS}(\ell)$	=	capital cost for power plant system for the ℓ^{th} construction interval
$CC_T(\ell)$	=	total capital cost incurred during the ℓ^{th} construction interval
$CV_{RS}(i)$	=	annual fixed and variable cost for reservoir development in the i^{th} year
$CV_{PS}(i)$	=	annual fixed and variable cost for the power plant system in the i^{th} year
$CV_T(i)$	=	total annual fixed and variable cost for the i^{th} year
C_T	=	total cost over a design period of N years
R_T	=	total revenue over a design period of N years
P_T	=	total profits over a design period of N years

$$\delta_{WE}(j,i) = \sum_{\ell=1}^{L^*} \delta_{DWE}(j,\ell) \quad (66)$$

where i is contained in the L^* construction interval.

The quantity $Q_R(j,i)$ is associated in the same manner with a variable $\delta_{WR}(j,i)$, defined

$$\delta_{WR}(j,i) = \sum_{\ell=1}^{L^*} \delta_{DWR}(j,\ell) \quad (67)$$

The functions are as follows.

The number of extraction wells added during the ℓ^{th} construction interval from all sites is

$$N_{DWE}(\ell) = \sum_{j=1}^{M_E} \delta_{DWE}(j,\ell) \quad (68)$$

where M_E is the number of extraction sites considered.

The number of injection wells added during the ℓ^{th} construction interval from all sites is

$$N_{DWR}(\ell) = \sum_{j=1}^{M_R} \delta_{DWR}(j,\ell) \quad (69)$$

where M_R is the number of reinjection sites considered.

The added design reinjection rate for the ℓ^{th} construction interval over all sites is

$$Q_{DE}(\ell) = \sum_{j=1}^{M_E} U_E \delta_{DWE}(j,\ell) \quad (70)$$

where U_E is the design capacity for a single injection well.

The added design reinjection rate for the ℓ^{th} construction interval over all sites is

$$Q_{DR}(\ell) = \sum_{j=1}^{M_R} U_R \delta_{DWR}(j, \ell) \quad (71)$$

where U_R is the design capacity for a single injection well.

The megawatt-power rating of the turbine system added during the ℓ^{th} construction interval is

$$N_{DNW}(\ell) = T_H \delta_{DTH}(\ell) + T_L \delta_{DTL}(\ell) \quad (72)$$

where T_H and T_L are the megawatt rating per turbine for high and low pressure turbines, respectively.

The design capacity of steam to the turbines added during the ℓ^{th} construction interval is

$$S_{DT}(\ell) = \sum_{j=1}^{M_E} \{X_1^T(j) + X_2^T(j)\} U_E \delta_{DWE}(j, \ell) \quad (73)$$

where $X_1^T(j)$ and $X_2^T(j)$ are the steam fractions from the j^{th} site entering the high and low pressure system turbines, respectively.

The design capacity of steam outflow from the separator-flasher system added during the ℓ^{th} construction interval is

$$S_{DF}(\ell) = \sum_{j=1}^{M_E} \{X_1^F(j) + X_2^F(j) + X_3^F(j) + X_4^F(j)\} U_E \delta_{DWE}(j, \ell) \quad (74)$$

where $X_1^F(j)$, $X_2^F(j)$, $X_3^F(j)$ and $X_4^F(j)$ are the steam fractions exiting the primary flasher system, the secondary flasher system, the flasher

system to the dump condenser, and the separator system to the low pressure turbine, respectively.

The total number of extraction wells present in the i^{th} year is

$$T_{WE}(i) = \sum_{j=1}^{M_E} \delta_{WE}(j,i) \quad (75)$$

The total number of injection wells present in the i^{th} year is

$$T_{WR}(i) = \sum_{j=1}^{M_R} \delta_{WR}(j,i) \quad (76)$$

The total flow from extraction wells during the i^{th} year is

$$Q_{TE}(i) = \sum_{j=1}^{M_E} Q_E(j,i) \quad (77)$$

The total flow to injection wells during the i^{th} year is

$$Q_{TR}(i) = \sum_{j=1}^{M_R} Q_R(j,i) \quad (78)$$

The design capacity for power production for the i^{th} year

$$D_{MW}(i) = T_H \delta_{TH}(i) + T_L \delta_{TL}(i) \quad (79)$$

Where $\delta_{TH}(i)$ is the total number of high pressure turbines present in the i^{th} year,

$$\delta_{TH}(i) = \sum_{\ell=1}^{L^*} \delta_{DTH}(\ell) \quad (80)$$

where the i^{th} is contained in the L^* construction interval. Likewise $\delta_{\text{TL}}(i)$ is the total number of low pressure turbines present in the i^{th} year,

$$\delta_{\text{TL}}(i) = \sum_{\ell=1}^{L^*} \delta_{\text{DTL}}(\ell) \quad (81)$$

The actual quantity of steam exiting the separator-flasher system during the i^{th} year is

$$S_{\text{AF}}(i) = \sum_{j=1}^{M_E} \{X_1^{\text{F}}(j) + X_2^{\text{F}}(j) + X_3^{\text{F}}(j) + X_4^{\text{F}}(j)\} Q_E(j,i) \quad (82)$$

The actual quantity of steam entering the turbine system during the i^{th} year is

$$S_{\text{AT}}(i) = \sum_{j=1}^{M_E} \{X_1^{\text{T}}(j) + X_2^{\text{T}}(j)\} Q_E(j,i) \quad (83)$$

The amount of makeup cooling water necessary to replace evaporative or blowdown losses for the i^{th} year is

$$Q_{\text{CW}}(i) = \alpha_{\text{CW}} \sum_{j=1}^{M_E} \{\gamma_{\text{C1}}(j) + \gamma_{\text{C2}}(j)\} Q_E(j,i) \quad (84)$$

where $\gamma_{\text{C1}}(j)$ and $\gamma_{\text{C2}}(j)$ are the intensive cooling water requirements for the condenser on the low pressure turbine and dump condenser, respectively, for the j^{th} site; and α_{CW} is the fractional loss.

The actual megawatt-power production for the i^{th} year, $A_{\text{MW}}(i)$ is given by equation (40).

Equations (68) through (84), equation (40) and equations (56) through (64), allows equation (65) to be written as a function of the six decision variables, δ_{DWE} , δ_{DWR} , δ_{DTH} , δ_{DTL} , Q_E and Q_R and the four variables δ_{WE} , δ_{WR} , δ_{TH} , and δ_{TL} defined in equation (66), (67), (80) and (81), respectively. Thus the total profit is

$$\begin{aligned}
 P_T = & \sum_{i=1}^N \sum_{j=1}^{M_E} \frac{1}{(1+r)^i} \{ a_{121} (w_{e1}(j) + w_{e2}(j)) - a_{83} - a_{101} (X_1^F(j) \\
 & + X_2^F(j) + X_3^F(j) + X_4^F(j)) - a_{102} (X_1^T(j) + X_2^T(j)) - a_{111} \alpha_{\text{CW}} \\
 & (\gamma_{c1}(j) + \gamma_{c2}(j)) \} Q_E(j,i) \\
 & - \sum_{i=1}^N \sum_{j=1}^{M_R} \frac{(a_{72} P_{\text{max}} + a_{84})}{(1+r)^i} Q_R(j,i) - \sum_{i=1}^N \sum_{j=1}^{M_E} \frac{[a_{61} + a_{81}]}{(1+r)^i} \delta_{\text{WE}}(j,i) \\
 & - \sum_{i=1}^N \sum_{j=1}^{M_R} \frac{a_{71} + a_{82}}{(1+r)^i} \delta_{\text{WR}}(j,i) - \sum_{i=1}^N \frac{(a_{91} + a_{103})}{(1+r)^i} T_H \delta_{\text{TH}}(i) - \\
 & - \sum_{i=1}^N \frac{(a_{91} + a_{103})}{(1+r)^i} T_L \delta_{\text{TL}}(i)
 \end{aligned}$$

$$\begin{aligned}
& - \sum_{\ell=1}^L \sum_{j=1}^{M_E} R(\ell, r) \{ a_{11} + a_{31} + a_{33} U_E + (1+\alpha) a_{42} (X_1^T(j) + \\
& \quad X_2^T(j)) U_E + (1+\alpha) a_{51} (X_1^F(j) + X_2^F(j) + X_3^F(j) + \\
& \quad X_4^F(j)) U_E \} \delta_{DWE}(j, \ell) \\
& - \sum_{\ell=1}^L \sum_{j=1}^{M_R} R(\ell, r) \{ a_{21} + a_{32} - a_{34} U_R \} \delta_{DWR}(j, \ell) \\
& - \sum_{\ell=1}^L R(\ell, r) (1+\alpha) a_{41} T_H \delta_{DTH}(\ell) - \sum_{\ell=1}^L R(\ell, r) (1+\alpha) a_{41} T_L \delta_{DTH}(\ell) \\
& - \sum_{k=1}^6 \frac{b_k}{N} \frac{(1+r)^{N-1}}{r(1+r)^N} - \sum_{k=7}^{11} b_k \frac{(1+r)^{N-1}}{r(1+r)^N} \tag{85}
\end{aligned}$$

Note that

$$\sum_{\ell=1}^L R(\ell, r) = \sum_{i=1}^N \frac{1}{(1+r)^i} = \frac{(1+r)^N - 1}{r(1+r)^N}$$

Mixed Integer Program

The fundamental assumption of the management model is that the geothermal reservoir, the transmission system and the power plant are operated as an integrated unit by a single firm for profit. The objective of the firm is to maximize profit,

$$\max \{P_f = R_T - C_T\} \quad (86)$$

Equation (86) with a suitably defined constraint set, constitutes the management model. The model as formulated is solved as a mixed integer program with δ_{DWE} , δ_{DWR} , δ_{DTH} , δ_{DTL} as integer variables and Q_E and Q_R as continuous variables. The variables δ_{WE} , δ_{WR} , δ_{TH} and δ_{TL} may be defined as continuous variables because subsequent constraint sets will restrict them to integer values.

One additional integer variable is introduced, δ_F , which has the value one if the power plant, transmission system and reservoir are developed and zero if they are not. δ_F is multiplied by the sum of the fixed constant terms present in equation (85). With δ_F present there are no constant terms in the objective function. The constraint set is as follows

1. pressure-withdrawal -- the relation between extraction and injection at sites and pressure

$$\begin{aligned}
 p(k,n) = p_0(k) - \sum_{j=1}^{M_E} \sum_{i=1}^N R(k,j,n-i+1) Q_E(j,i) + \\
 \sum_{\sigma=1}^{M_R} \sum_{i=1}^N R(k,\sigma + M_E,n-i+1) Q_R(\sigma,i) \quad k=1,\dots,M; n=1,\dots,N
 \end{aligned}$$

$$(M = M_E + M_R) \quad (87)$$

2. recharge-discharge -- no more fluid may be injected than is extracted.

$$(1-\theta) \sum_{k=1}^{M_E} Q_E(k,n) - \sum_{\sigma=1}^{M_R} Q_R(\sigma,n) = 0 \quad n=1,\dots,N \quad (88)$$

where θ is the fraction of water lost to evaporation and any consumptive use.

3. equal power -- provides for equal annual power generation within a construction interval

$$\sum_{j=1}^{M_E} (w_{e1}(j) + w_{e2}(j)) Q_E(j,i) - \sum_{j=1}^{M_E} (w_{e1}(j) + w_{e2}(j)) Q_E(j,i+1) = 0$$

for all i except modulus $(i, I_t) = 0$ (89)

4. high pressure turbine capacity -- the production of electricity from the high pressure turbines may not exceed the production capacity

$$T_{H\delta TH}(i) - \sum_{j=1}^{M_E} w_{e2}(j) Q_E(j,i) \geq 0 \quad i = 1, \dots, N \quad (90)$$

5. low pressure turbine capacity -- the production of electricity from the low pressure turbine may not exceed the production capacity

$$T_L \delta_{TL}(i) - \sum_{j=1}^{M_E} w_{e2}(j) Q_E(j,i) \geq 0 \quad i=1, \dots, N \quad (91)$$

6. extraction well capacity -- the rate of withdrawal from extraction sites may not exceed the site's production capacity limitation

$$U_E \delta_{WE}(j,i) - Q_E(j,i) \geq 0 \quad j=1, \dots, M_E; i=1, \dots, N \quad (92)$$

7. injection well capacity -- the rate of injection into reinjection sites may not exceed the site's injection capacity limitation

$$U_R \delta_{WR}(\sigma,i) - Q_R(\sigma,i) \geq 0 \quad \sigma=1, \dots, M_R; i=1, \dots, N \quad (93)$$

8. aggregated number of high pressure turbines -- the functional relation between $\delta_{DTH}(\ell)$ and $\delta_{TH}(i)$ is

$$\delta_{TH}(i) = \sum_{\ell=1}^{L^*} \delta_{DTH}(\ell) \quad \text{where the construction interval } L^* \text{ contains } i \quad (94)$$

9. aggregated number of low pressure turbines -- the functional relation between $\delta_{DTL}(\ell)$ and $\delta_{TL}(i)$

$$\delta_{TL}(i) = \sum_{\ell=1}^{L^*} \delta_{DTL}(\ell), \quad L^* \text{ as above} \quad (95)$$

10. aggregated number of extraction wells

$$\delta_{WE}(j,i) = \sum_{\ell=1}^{L^*} \delta_{DWE}(j,\ell), \quad L^* \text{ as above; } j=1, \dots, M_e \quad (96)$$

11. aggregated number of injection wells

$$\delta_{WR}(\sigma,i) = \sum_{\ell=1}^{L^*} \delta_{DWR}(\sigma,\ell), \quad L^* \text{ as above; } \sigma=1, \dots, M_R \quad (97)$$

12. fixed cost -- provides that δ_F is one if the geothermal project is built

$$U_{\max} \delta_F \geq \delta_{TH}(N) + \delta_{TL}(N) \quad (98)$$

where U_{\max} is a large number (i.e., greater than $L \cdot (B_{TNH} + B_{TNL})$)

13. single phase -- the reservoir is restricted to a single phase hot water system (equation (30))

$$p(k,n) \geq p_s(k) \quad k=1, \dots, M; \quad n=1, \dots, N \quad (99)$$

where $p_s(k)$ is given by equation (28)

14. extraction well number -- provides an upper bound for the number of extraction wells added to a site

$$\delta_{DWE}(k,\ell) \leq B_{NWE}, \quad k=1, \dots, M_E; \quad \ell=1, \dots, L \quad (100)$$

15. injection well number -- provides an upper bound for the number of injection wells added to a site

$$\delta_{DWR}(j,\ell) \leq B_{NWR} , \quad j=1,\dots, M_R; \ell=1,\dots, L \quad (101)$$

16. high pressure turbine number -- provides an upper bound for the number of high pressure turbines added

$$\delta_{DTH}(\ell) \leq B_{NTH} \quad \ell=1,\dots, L \quad (102)$$

17. low pressure turbine number -- provides an upper bound for the number of low pressure turbines added

$$\delta_{DTL}(\ell) \leq B_{NTL} \quad \ell=1,\dots, L \quad (103)$$

18. injection pressure -- insures that the reservoir pressure does not rise above the injection pressure

$$p(\sigma,n) \leq P_{\max} \quad \sigma=1,\dots, M_R; n=1,\dots, N \quad (104)$$

19. fixed cost -- set an upper bound of unity on the fixed costs-note that δ_F is a zero or one variable

$$\delta_F \leq 1 \quad (105)$$

All the decision variables are greater than or equal to zero. Table 3 presents a summary of all parameters used in the objective function (equation (85)) and the constraint set (equations (87) through (105)).

Table 3

Summary of parameters

- U_E = the design capacity rate for an extraction well
 U_R = the design capacity rate for an injection well
 T_H = the megawatt rating per turbine for a high pressure turbine system
 T_L = the megawatt rating per turbine for a low pressure turbine system
 $x_1^T(j)$ = the steam fraction entering the high pressure turbine system from the j^{th} site
 $x_2^T(j)$ = the steam fraction entering the low pressure turbine system from the j^{th} site
 $x_1^F(j)$ = the steam fraction exiting the primary flasher from the j^{th} site
 $x_2^F(j)$ = the steam fraction exiting the secondary flasher from the j^{th} site
 $x_3^F(j)$ = the steam fraction exiting the dump condensor flasher from the j^{th} site
 $x_4^F(j)$ = the steam fraction exiting the flasher to the low pressure turbine from the j^{th} site
 $\gamma_{c1}(j)$ = the intensive cooling water requirement for the condenser on the low pressure turbine from the j^{th} site
 $\gamma_{c2}(j)$ = the intensive cooling water requirement for the dump condenser

Table 3 - continued

α_{CW}	= fraction of cooling water lost to blowdown or evaporation each year
M_E	= number of sites for extraction wells
M_R	= number of sites for injection wells
α	= the fraction of the sum of capital costs $CC_{PP}(\lambda)$ and $CC_{SF}(\lambda)$ that constitutes engineering and administrative costs
$R(k,j,L)$	= a response function
$W_{e1}(j)$	= the intensive electrical work produced from the high pressure turbine for the j^{th} site
$W_{e2}(j)$	= the intensive electrical work produced from the low pressure turbine for the j^{th} site
U_{\max}	= a large number in the constraint given by equation (98) that forces δ_F to be 1.0 if a power plant is built
B_{NWE}	= upper bound on number of extraction wells added to a site during a construction interval
B_{NWR}	= upper bound on number of injection wells added to a site during a construction interval
B_{NTH}	= upper bound on the number of high pressure turbines
B_{NTL}	= upper bound on the number of low pressure turbines
θ	= fraction of water extracted that is lost to evaporation and consumptive use and is unavailable for reinjection
$p_s(k)$	= the pressure below which site k becomes two phase
$p_0(k)$	= initial pressure at site k
P_{\max}	= injection pressure

MANAGEMENT MODEL EXAMPLE

A hypothetical geothermal reservoir is developed for electrical power production. Hot water is extracted from the reservoir, used to generate electrical power by a direct method, and reinjected into the reservoir. The characteristics of the reservoir, power plant, and a prevailing pricing structure (based on 1974 prices) are presented in the following sections. A management model using the reservoir, power plant and pricing data is developed for three types of reinjection policies, 1) no reinjection, 2) reinjection of available extracted fluid (given that there is a 10 percent, 15 percent, or 20 percent loss of fluid to evaporation) into the aquifer and 3) unrestricted reinjection where not all the available extracted fluid is reinjected into the aquifer and thus part of the fluid, will by necessity, be disposed of by injection into some other aquifer or by some other means (waste water treatment).

The Reservoir

A vertical cross-sectional view of the Hot Water Basin geothermal reservoir is shown in Figure 5. The reservoir is an aquifer (composed of porous material) that is bounded above by a relatively impermeable layer and is bounded below by a confining layer capable of vertical leakage. The aquifer is 500 m in thickness and the lower confining

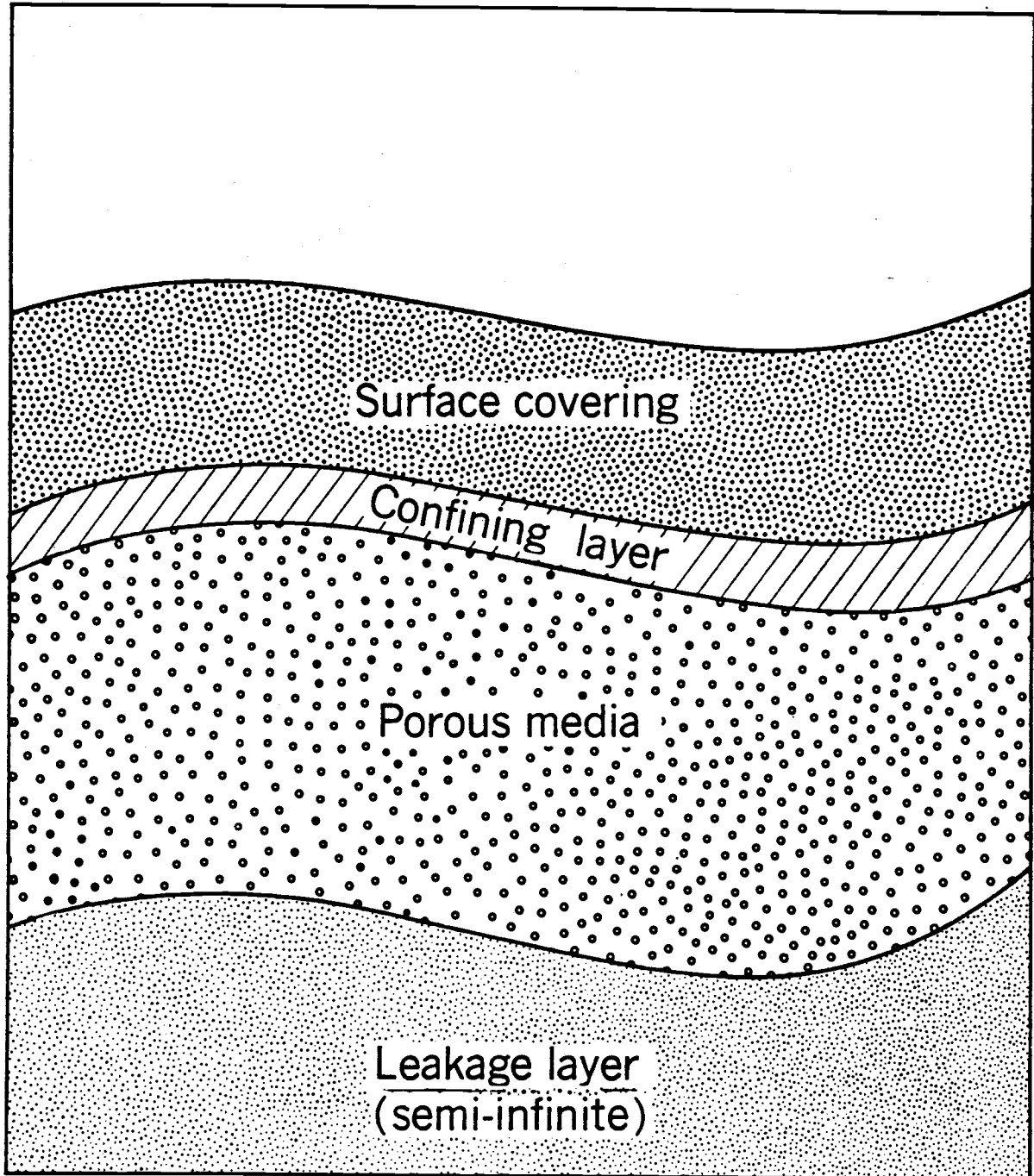


Figure 5. Geologic cross-section of a hypothetical geothermal reservoir.

layer is assumed to be semi-infinite in thickness. There is a thick layer of surface material above the confining layer that does not interact in any fashion with the aquifer. The aquifer underlies an area of $7.2 \times 10^7 \text{ m}^2$. The area is rectangular in shape with north-to-south dimensions of 9000 m and east-to-west dimensions of 8000 m. The aquifer's porosity and intrinsic permeability are 0.2 and $0.1 \times 10^{-9} \text{ cm}^2$, respectively. The compressibility coefficient of water is taken to be $.768 \times 10^{-10} \text{ cm}^2/\text{dy}$.

Figure 6 presents the vertically averaged temperature distribution within the aquifer. Temperatures range from 50°C to 230°C. Figure 7 presents the initial pressure distribution based on a reference level near the top of the reservoir which ranges from $1.15 \times 10^8 \text{ dy/cm}^2$ to $1.60 \times 10^8 \text{ dy/cm}^2$. Utilizing equation (1) to calculate the viscosity distribution, equation (3) to calculate the initial density distribution, the transmissive quality and storage quality distributions are calculated using equation (6) and (5), respectively. Figure 8 presents the transmissive quality distribution while Figure 9 presents the storage distribution.

The semi-infinite leaky layer is composed of homogeneous material and has a hydraulic conductivity of $1.0 \times 10^{-8} \text{ cm/s}$ and a specific storage of $0.1 \times 10^{-7} / \text{cm}$.

A finite-difference technique (Maddock, 1974) is used to calculate values of $R(k,j,i)$ by superimposing a 72 node, square grid system (Figure 10) over a plan view of the aquifer, and by determining average

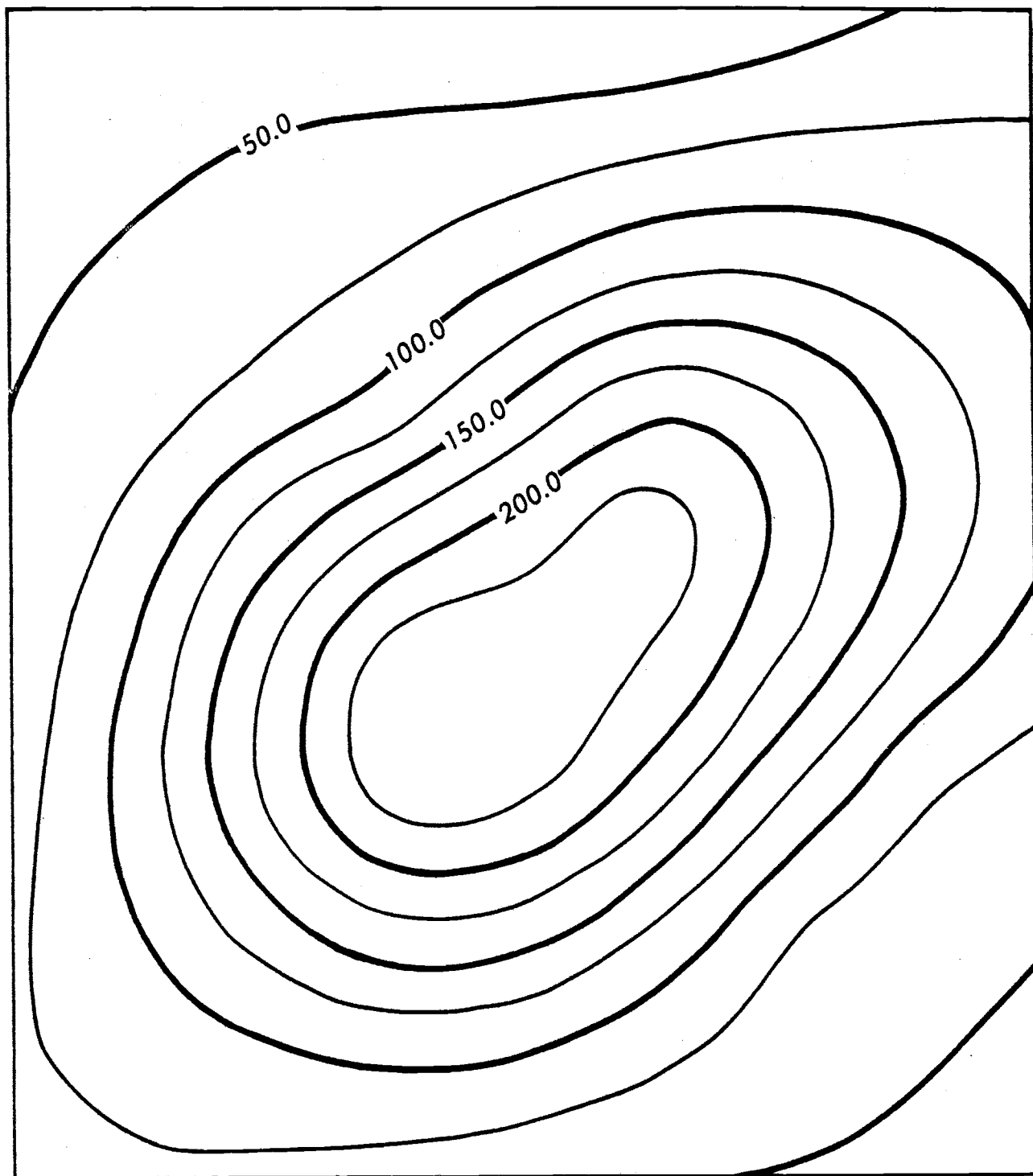


Figure 6. Vertically averaged temperature distribution, C° (averaged over the reservoir thickness) for the reservoir.

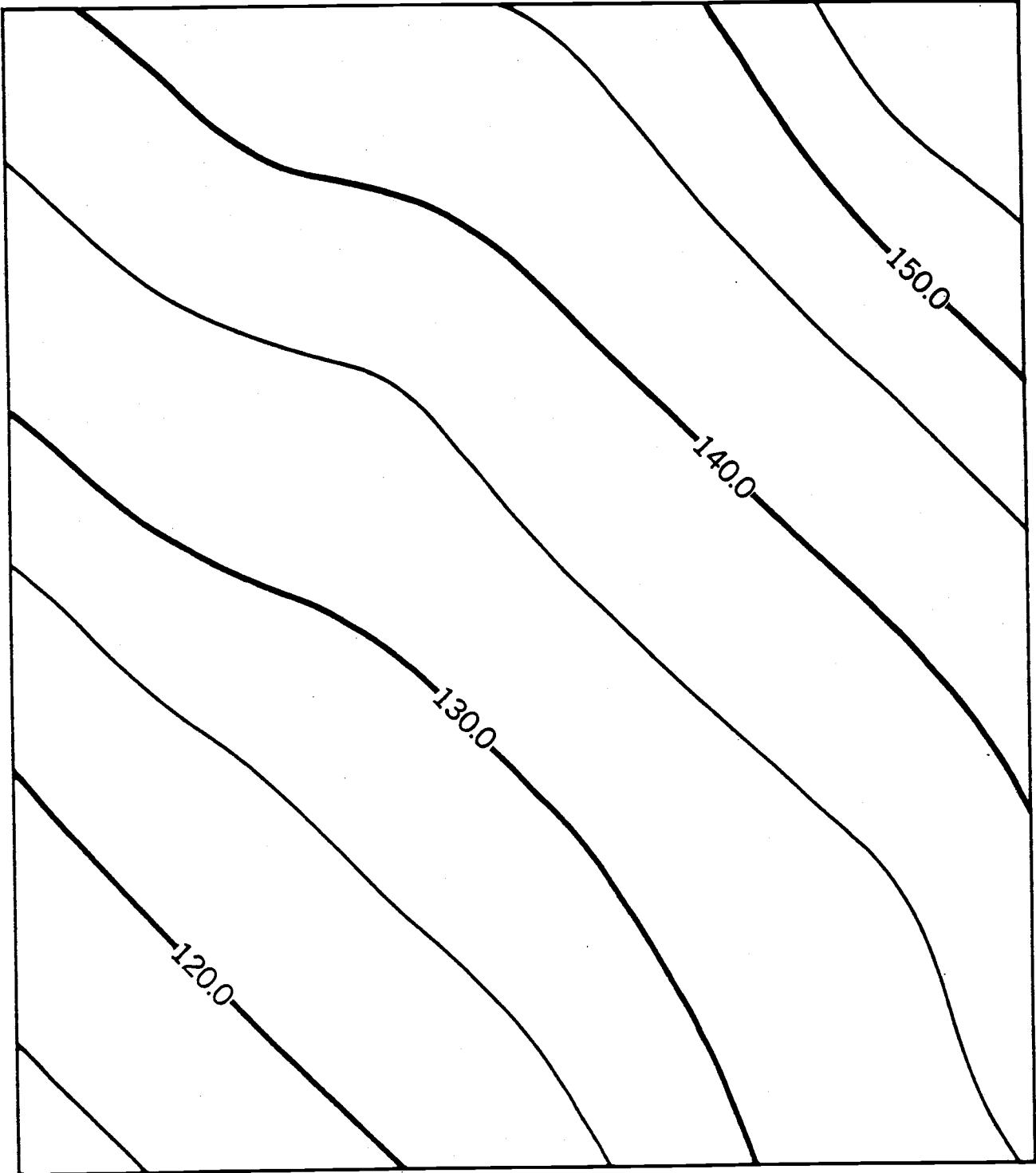


Figure 7. Initial pressure distribution, $\text{dyne/cm}^2 \times 10^6$, at a reference level near the top of the reservoir.

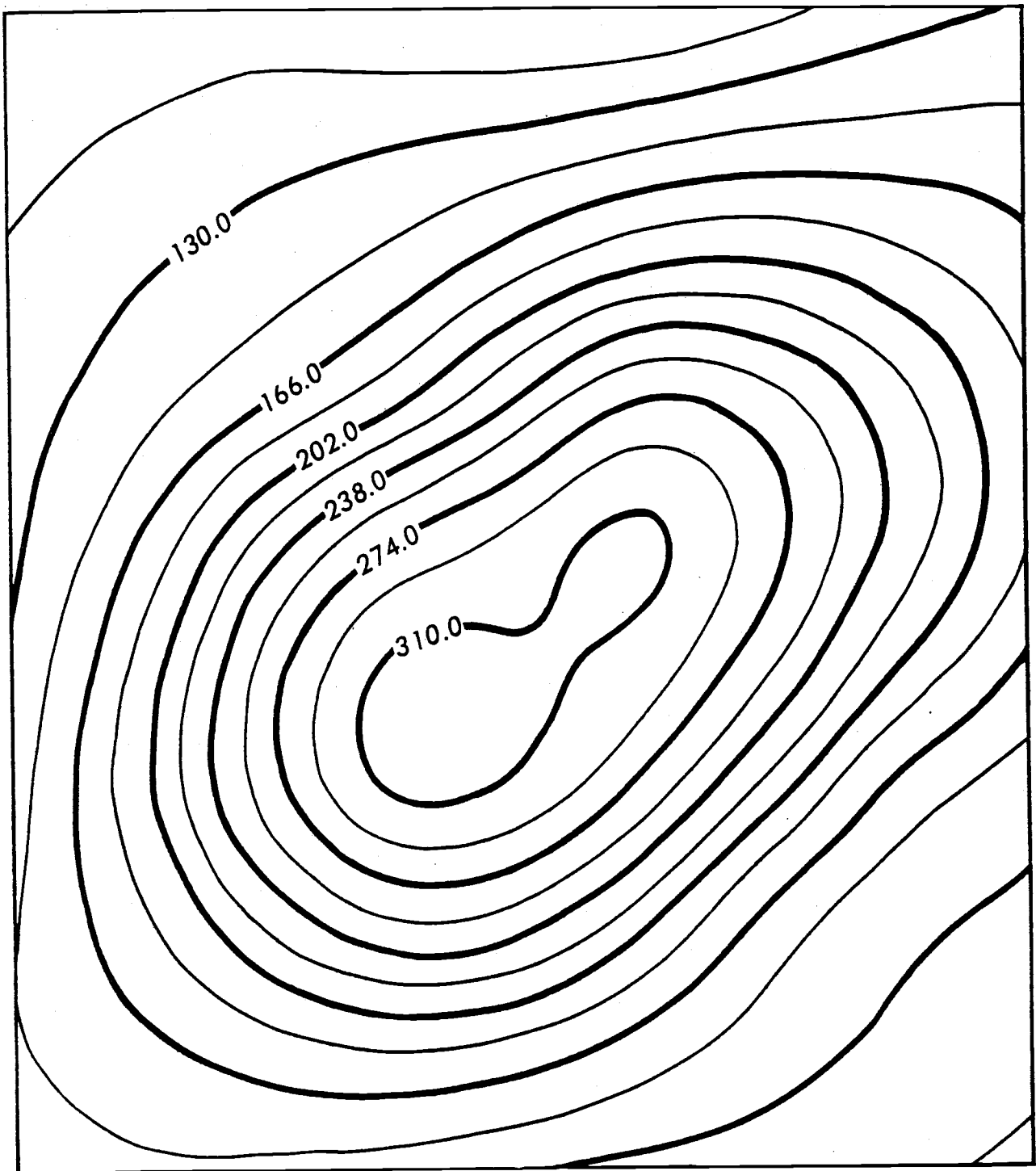


Figure 8. Distribution over the reservoir of the transmissive quality, $\text{cm-s} \times 10^{-8}$.

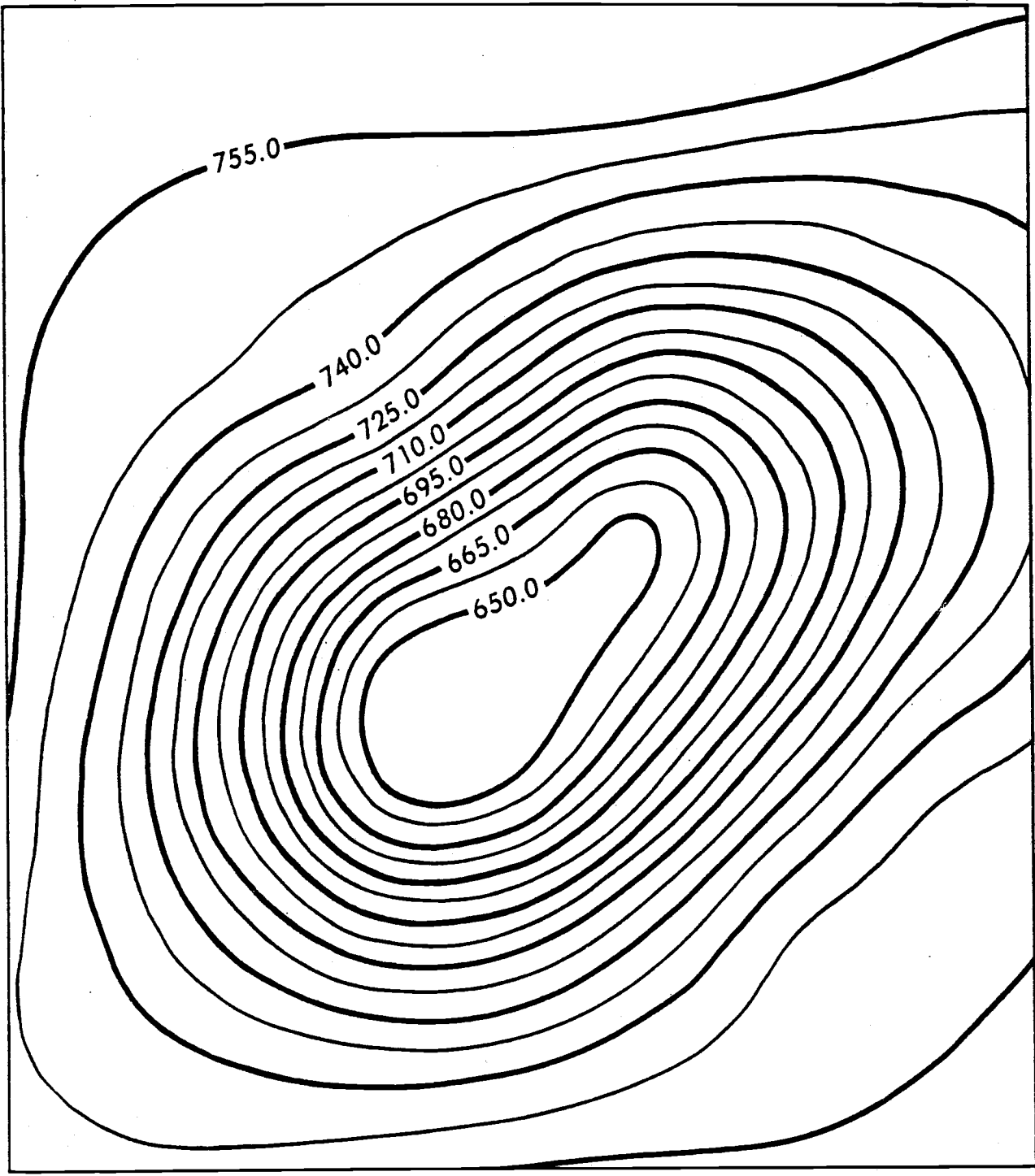


Figure 9. Distributing over the reservoir of the storativity, $s^2/\text{cm} \times 10^{-9}$.

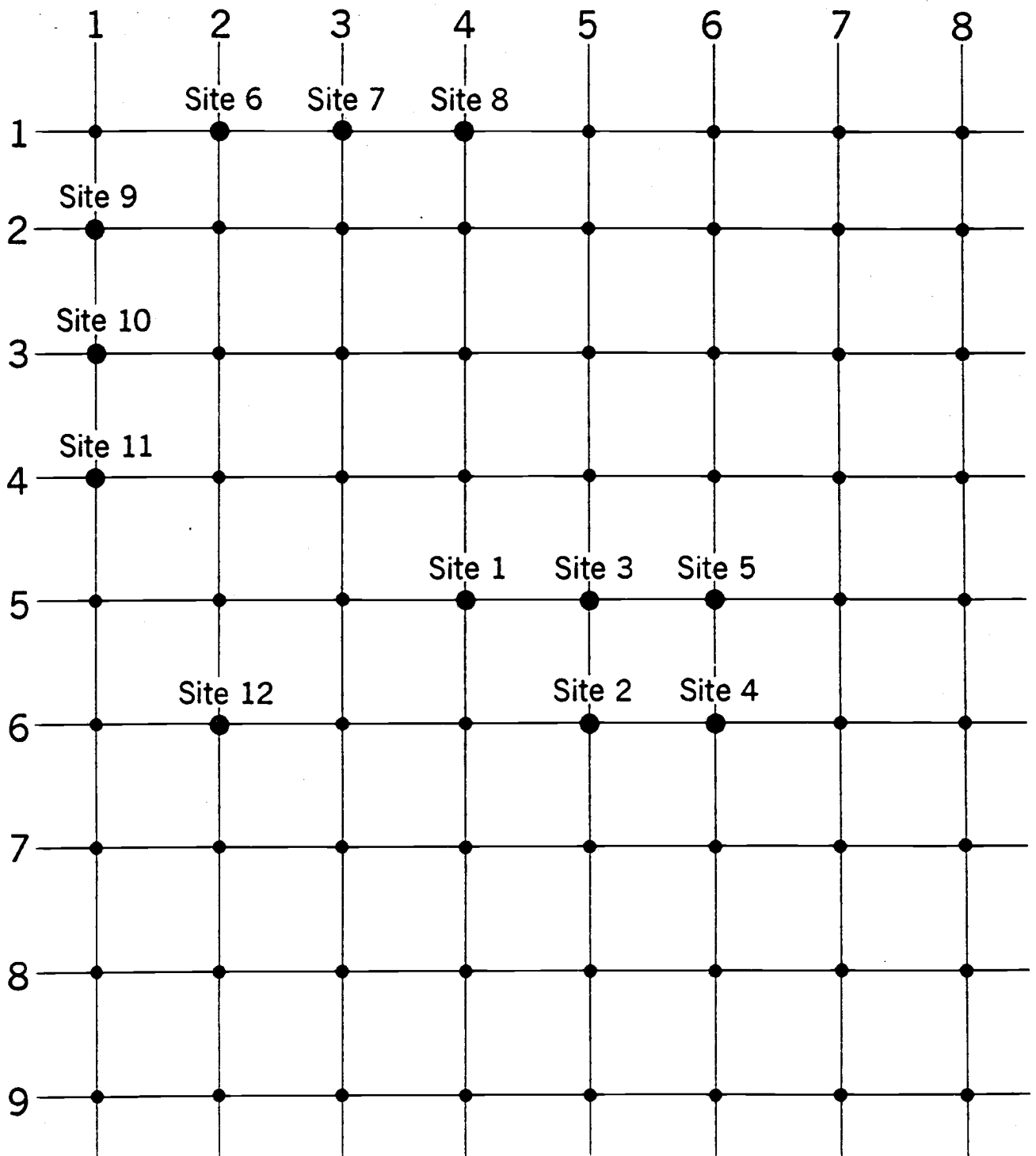


Figure 10. Lattice-centered, finite-difference grid showing the well site locations.

values of temperature, initial pressure, transmissive quality and storativity at each node point of the grid system, and then by solving a subsequent set of difference equations that result from discretizing equation (18) in space and time. The response functions are approximated by

$$R(k,j,i) \approx G(x_k, x_j, (i-1)\Delta t) \Delta t \quad (106)$$

Twelve potential sites are considered for development, five for extraction wells and seven for injection wells. Each site is located at a node point (Figure 10). Typical response functions are given in Table 4, which lists the R values for site 1 over a 20 year design horizon. Up to 10 wells may be located within a site area represented by the node point. A single well is restricted to a withdrawal rate or injection rate no greater than 70,000 g/s. The wells are assumed to be uniformly distributed over the representative nodal area when more than one well is developed at a site. The response functions are calculated for sites not wells, thus the pressure drop or gain calculated at a node is the average pressure drop or gain induced by all the wells within that site.

If more than one well is present at a site withdrawal or injection rates are equally distributed among them. Note that the $Q_E(j,i)$'s or $Q_R(j,i)$'s are restricted to a value no greater than 700,000 g/s (i.e., product of the maximum number of wells at a site, 10 and the restricted withdrawal or injection rate per well, 70,000 g/s).

Table 4
 Response functions for site 1^a
 [1/cm-s]

Year (n)	R(1,1,n)	R(1,2,n)	R(1,3,n)	R(1,4,n)	R(1,5,n)
1	184.221	88.0797	107.9536	69.3484	76.6602
2	67.3156	63.1493	64.6223	60.5776	61.6929
3	58.8358	58.0890	58.3932	57.6123	57.8825
4	56.9865	56.7217	56.8383	56.5903	56.6886
5	56.1180	56.0089	56.0605	55.9638	56.0069
6	55.4962	55.4495	55.4729	55.4325	55.4528
7	54.9679	54.9477	54.9581	54.9411	54.9507
8	54.4902	54.4816	54.4859	54.4793	54.4837
9	54.0473	54.0440	54.0454	54.0434	54.0452
10	53.6319	53.6309	53.6308	53.6309	53.6315
11	53.2392	53.2392	53.2385	53.2394	53.2394
12	52.8660	52.8665	52.8655	52.8668	52.8664
13	52.5100	52.5106	52.5095	52.5109	52.5104
14	52.1691	52.1698	52.1686	52.1701	52.1696
15	51.8418	51.8426	51.8414	51.8428	51.8423
16	51.5269	51.5276	51.5265	51.5279	51.5274
17	51.2231	51.2227	51.2227	51.2241	51.2236
18	50.9296	50.9303	50.9292	50.9306	50.9301
19	50.6455	50.6462	50.6452	50.6465	50.6460
20	50.3701	50.3708	50.3698	50.3710	50.3706

^a Only response functions for extraction wells are listed.

The Power Plant

Table 5 lists operating pressures, enthalpies, entropies and efficiencies for the hypothetical power plant. The key variables in the table are those lying in the pressure column. These are saturation pressure values. The other variables, with the exception of the efficiencies, are provided from steam tables once the saturation pressure values are given. Table 6 lists the thermodynamic properties for the 5 potential extraction well sites outlined in the reservoir model section. Utilizing the values specified in Tables 5 and 6, equations (31) through (39) are used in the power plant model to calculate the material-energy balance for each site. Tables 7 and 8 give the results of these calculations. For reservoir temperatures ranging from 210°C to 230°C the steam fraction leaving the primary separator-flasher ranges from .047 to .092. Below the temperature of 165°C the steam fraction leaving the primary separator-flasher is zero. Thus wells at sites below 165°C can only be used to power the low pressure turbine system. The intensive work for the high pressure turbine system ranges for $.161 \times 10^{-4}$ MW/g to $.315 \times 10^{-4}$ MW/g while the intrinsic work for the low pressure turbine system ranges from $.265 \times 10^{-4}$ MW/g to $.323 \times 10^{-4}$ MW/g.

The temperature of the spent fluid leaving the dump condensor is 85°C. It is injected into reservoir sites with temperatures of 85°C. As may be seen from Figure 10, these sites are generally on the perimeter of the field.

Table 5
Initial conditions for power plant (exogenous variables)

	Pressure $\frac{\text{dyne}}{\text{cm}^2}$	Liquid enthalpy $\frac{\text{erg}}{\text{g}}$	Vapor enthalpy $\frac{\text{erg}}{\text{g}}$	Vapor entropy $\frac{\text{erg}}{\text{g-C}^\circ}$	Liquid entropy $\frac{\text{erg}}{\text{g-C}^\circ}$	Vapor entropy $\frac{\text{erg}}{\text{g-C}^\circ}$
After primary separator-flasher	$1.256 \times 10^7 (P_1)$	8.053×10^9	2.786×10^{10}	2.231×10^7	6.512×10^7	
Into low pressure turbine	$3.495 \times 10^6 (P_2)$	5.818×10^9	2.548×10^{10}	1.722×10^7	6.945×10^7	
Into back compressor	$2.026 \times 10^5 (P_3)$	2.514×10^9	2.457×10^{10}	8.320×10^6	7.908×10^7	
At atmospheric	$1.013 \times 10^6 (P_a)$	4.174×10^9	2.506×10^{10}	1.303×10^7	7.359×10^7	
Out of dump condenser	—	3.000×10^9	—	—	—	—
Efficiencies (mechanical ↔ electrical)						
		high pressure turbine		.85		
		low pressure turbine		.65		
		back compressor		.85		

Table 6

Site thermodynamic properties*

Site	Temperature C°	Enthalpy $\frac{\text{erg}}{\text{g}}$	Initial pressure $\frac{\text{dyne}}{\text{cm}^2}$	Saturation pressure $\frac{\text{dyne}}{\text{cm}^2}$
1	210	8.989×10^9	1.33×10^8	1.81×10^7
2	210	8.989×10^9	1.38×10^8	1.81×10^7
3	230	9.884×10^9	1.35×10^8	2.60×10^7
4	210	8.989×10^9	1.40×10^8	1.81×10^7
5	210	8.989×10^9	1.38×10^8	1.81×10^7
6	85	*	1.18×10^8	*
7	85	*	1.20×10^8	*
8	85	*	1.23×10^8	*
9	85	*	1.18×10^8	*
10	85	*	1.20×10^8	*
11	85	*	1.23×10^8	*
12	85	*	1.32×10^8	*

* The first five wells are extraction wells the rest are injection wells.

Table 7

Material - energy balance for sites 1, 2, 4 and 5

	IN		OUT	
	STEAM	LIQUID	STEAM	LIQUID
Separators - Flashers (Intensive Values -- g/g)				
Primary Flasher	0.0	1.0	0.047232	0.952769
Secondary Flasher	0.0	0.952769	0.108275	0.844494
Flasher to Dump Condenser	0.039050	0.008182	0.039050	0.008182
Separator to L.P. Turbine	0.0	0.852675	0.067124	0.785551
Turbines (Intensive Values -- g/g)				
High Pressure	0.047232	0.0	0.008182	0.039050
Low Pressure	0.147325	0.0	0.135293	0.012032
Condenser - Compressors (Intensive Values -- g/g)				
Back Compressor	0.135293	0.012032	0.146729	0.000596
Dump Condenser	0.213853	0.786147	0.0	1.0

Cooling Water Requirements (Intrinsic Values -- g/g)

Low Pressure Turbine Condenser = 0.573851
 Dump Condenser = 8.083158

Intensive Work (Megawatt/g)

High Pressure Turbine = 0.1609×10^{-4}
 Low Pressure Turbine = 0.2653×10^{-4}

Table 8

Material - energy balance for site 3

	IN		OUT	
	STEAM	LIQUID	STEAM	LIQUID
Separators - Flashers (Intensive Values -- g/g)				
Primary Flasher	0.0	1.0	0.092460	0.907540
Secondary Flasher	0.0	0.907540	0.103135	0.804405
Flasher to Dump Condenser	0.076444	0.016016	0.076444	0.016016
Separator to L.P. Turbine	0.0	0.820422	0.064585	0.755836
Turbines (Intensive Values -- g/g)				
High Pressure	0.092460	0.0	0.016016	0.076444
Low Pressure	0.179578	0.0	0.164913	0.014666
Condenser - Compressors (Intensive Values -- g/g)				
Back Compressor	0.164913	0.014666	0.178852	0.000726
Dump Condenser	0.243437	0.756563	0.0	1.0

Cooling Water Requirements (Intrinsic Values -- g/g)

Low Pressure Turbine Condenser = 0.699484
 Dump Condenser = 8.968708

Intensive Work (Megawatt/g)

High Pressure Turbine = 0.3150×10^{-4}
 Low Pressure Turbine = 0.3234×10^{-4}

The Pricing Structure

All expansion and construction occurs in five year intervals. Extraction wells and injection wells have a ten year life and five year life, respectively while the transmission system and power plant have twenty year lives. The design period is twenty years. The power plant has a load factor of 85 percent based on a 365 day year. The capital costs are as follows.

1. Capital cost for development of extraction wells -- For a 20 year design period, lease acquisition and exploration costs are \$1,500,000. Extraction well cost and dry hole costs are assumed to be \$400,000 and \$300,000, respectively. Twenty percent of the wells drilled are dry holes, and 20 percent standby well capacity is required. Thus for a 5 year construction interval, a_{11} is 540,000 \$/well and b_1 is 375,000 \$, giving

$$CC_{EW}(\ell) = 5.40 \times 10^5 N_{DWE}(\ell) + 3.75 \times 10^5 \quad (107)$$

2. Capital cost for injection wells -- Injection well costs are assumed to be \$400,000 and a 20 percent standby well capacity is required. Lease acquisition and exploration costs apply only to extraction wells. Thus a_{21} is 480,000 \$/well and b_2 is zero, giving

$$CC_{RW}(\ell) = 4.8 \times 10^5 N_{DWR}(\ell) \quad (108)$$

3. Capital cost for transmission system -- Well-related transmission equipment costs are \$40,000 per well and \$30,000 per well for

extraction and injection, respectively. For the assumed field configuration, (Figure 10) capital costs of transmission piping are set at \$6.11 per gram per second of the total design withdrawal rates and \$4.00 per gram per second of the total design injection rates. Thus a_{31} , a_{32} , a_{33} and a_{34} are 40,000 \$/well, 30,000 \$/well, 6.11 \$/g/s and 4.00 \$/g/s respectively, and b_3 is zero, giving

$$CC_{TS}(\ell) = 4.00 \times 10^4 N_{DWE}(\ell) + 3.00 \times 10^4 N_{DWR}(\ell) + 6.11 Q_{DE}(\ell) + 4.00 Q_{DR}(\ell) \quad (109)$$

4. Capital cost for power plant (excluding separator-flasher costs) -- Table 9 list cost functions for crane, turbine-generators, electrical equipment, buildings and installations, switch yards, piping, condensers, instrumentation, cooling systems and miscellaneous developed from the Battelle report (Huber and others, 1975). The costs are combined to give the following values (for a five year construction interval). The a_{41} and a_{42} are 46,200 \$/mw and 42.50 \$/g/s, respectively and b_4 is 835,000 \$, giving

$$CC_{pp}(\ell) = .462 \times 10^5 N_{DMW}(\ell) + 4.25 \times 10^2 S_{DT}(\ell) + 8.35 \times 10^5 \quad (110)$$

5. Capital Cost of the separator-flasher system -- Table 9 also lists the cost function for the separator-flasher system. Thus a_{51} is 3.53 \$/g/s and b_5 is zero, giving

$$CC_{SF}(\ell) = 3.53 S_{DF}(\ell) \quad (111)$$

Table 9

Individual costs resulting in capital costs for power plant

Crane:	$.53 \times 10^3 N_{DMW}(\ell) + 0.160 \times 10^6$
Turbine- generators:	$.50 \times 10^4 N_{DMW}(\ell) + 0.250 \times 10^2 S_{DT}(\ell) + .133 \times 10^7$
Electrical equipment:	$.117 \times 10^5 N_{DMW}(\ell) + 0.783 \times 10^5$
Building and installation:	$.270 \times 10^5 N_{DMW}(\ell) + 0.255 \times 10^6$
Switch yard:	$6.36 N_{DMW}(\ell) + 0.431 \times 10^6$
Piping:	$3.09 S_{DT}(\ell) + 0.200 \times 10^4$
Condensers:	$4.84 S_{DT}(\ell) + 0.370 \times 10^6$
Instrumentation:	$4.76 S_{DT}(\ell)$
Cooling system:	$4.84 S_{DT}(\ell) + 0.690 \times 10^6$
Separator- flashers:	$3.53 S_{DF}(\ell)$
Miscellaneous:	$.196 \times 10^4 N_{DMW}(\ell) + 0.252 \times 10^5$

6. Engineering design and administration cost -- There are capital charges of 10 percent for engineering and design, and 8 percent for administration and indirect costs tacked on to the total capital cost for the power plant. Thus $\alpha = .18$ and

$$\begin{aligned} CC_{DA}(\ell) &= .18(CC_{PP}(\ell) + CC_{SF}(\ell)) \\ &= .832 \times 10^4 N_{DMW}(\ell) + .765 \times 10^1 S_{DT}(\ell) + .635 S_{DF}(\ell) \\ &\quad + 1.504 \times 10^5 \end{aligned} \quad (112)$$

7. The total capital cost for the power plant is

$$\begin{aligned} CC_{PS}(\ell) &= (1+\alpha)(CC_{PP}(\ell) + CC_{SF}(\ell)) \\ &= .546 \times 10^5 N_{DMW}(\ell) + .502 \times 10^2 S_{DT}(\ell) \\ &\quad + .417 \times 10^1 S_{DF}(\ell) + 9.850 \times 10^5 \end{aligned} \quad (113)$$

8. Capital costs for reservoir development

$$\begin{aligned} CC_{RS}(\ell) &= CC_{EW}(\ell) + CC_{RW}(\ell) \\ &= 5.40 \times 10^5 N_{DWE}(\ell) + 4.80 \times 10^5 N_{DWR}(\ell) + 3.75 \times 10^5 \end{aligned} \quad (114)$$

9. Total capital costs

$$\begin{aligned} CC_T(\ell) &= CC_{RS}(\ell) + CC_{TS}(\ell) + CC_{PS}(\ell) \\ &= 5.80 \times 10^5 N_{DWE}(\ell) + 5.10 \times 10^5 N_{DWR}(\ell) + 6.11 Q_{DE}(\ell) \\ &\quad + 4.00 Q_{DR}(\ell) + .546 \times 10^5 N_{DMW}(\ell) + .502 \times 10^2 S_{DT}(\ell) \\ &\quad + .417 \times 10^1 S_{DF}(\ell) + 1.36 \times 10^6 \end{aligned} \quad (115)$$

or in terms of the dependent variables, using equation (68) through (74).

$$\begin{aligned}
 CC_T(\ell) = & \sum_{j=1}^{M_E} (.580 \times 10^6 + 6.11 U_E + .502 \times 10^2 \{X_1^T(j) + X_2^T(j)\} U_E \\
 & + .417 \times 10^1 \{X_1^F(j) + X_2^F(j) + X_3^F(j) + X_4^F(j)\} U_E) \delta_{DWE}(j, \ell) \\
 & + \sum_{j=1}^{M_R} (.510 \times 10^6 + 4.00 U_R) \delta_{DWR}(j, \ell) \\
 & + .546 \times 10^5 T_H \delta_{DTH}(\ell) + .546 \times 10^5 T_L \delta_{DTH}(\ell) \\
 & + 1.36 \times 10^6 \tag{116}
 \end{aligned}$$

and the values of the parameters are given in Tables 10 and 11.

The annual fixed and variable costs are as follows.

1. Annual fixed and variable costs for development of extraction wells -- Extraction wells are assumed to have a ten year life. Well maintenance and associated equipment replacement costs are \$6,000 per well per year (\$7,200 per well per year when standby capacity wells are included). Abandonment costs are \$1,000 per well (\$100 per well per year). Depreciation costs are 10 percent of extraction well costs and 5 percent of lease acquisition and exploration costs. Thus a_{61} is 61,300 \$/well and b_6 is 75,000 \$, giving

$$CV_{EW}(i) = .613 \times 10^5 T_{WE}(i) + .750 \times 10^5 \tag{117}$$

2. Annual fixed and variable cost for development of injection wells -- Injection wells are assumed to have a five year life. Well

Table 10

Steam fractions, intensive work and intensive cooling water requirements (from Tables 7 and 8)

Steam fractions (g/g)	Site 1	Site 2	Site 3	Site 4	Site 5
T _{X1}	0.0472	0.0472	0.0925	0.0472	0.0472
T _{X2}	0.1473	0.1473	0.1796	0.1473	0.1473
F _{X1}	0.0472	0.0472	0.0925	0.0472	0.0472
F _{X2}	0.1083	0.1083	0.1031	0.1083	0.1083
F _{X3}	0.0390	0.0390	0.0764	0.0390	0.0390
F _{X4}	0.0671	0.0671	0.0646	0.0671	0.0671
Intensive Work (megawatt/g)					
W _{e1}	0.1609×10^{-4}	0.1609×10^{-4}	0.3150×10^{-4}	0.1609×10^{-4}	0.1609×10^{-4}
W _{e2}	0.2653×10^{-4}	0.2653×10^{-4}	0.3234×10^{-4}	0.2653×10^{-4}	0.2653×10^{-4}
Intensive Cooling Water Requirements (g/g)					
Y _{c1}	0.5738	0.5738	0.6995	0.5738	0.5738
Y _{c2}	8.0832	8.0832	8.9687	8.0832	8.0832

Table 11

Miscellaneous parameter values for hypothetical geothermal system

parameter	Value
U_E	70,000 g/s
U_R	70,000 g/s
T_H	2MW
T_L	5MW
α_{CW}	.04
M_E	5
M_R	7
α	.18
U_{max}	1000
B_{NWE}	10
B_{NWR}	10
B_{NTH}	25
B_{NTL}	25
p_{max}	2.02×10^7 dy/cm
M	12
N	20
L	4
I_t	5

maintenance and associated equipment replacement costs are \$10,000 per well per year (\$12,000 per well per year when stand by capacity wells are included). Abandonment costs are \$250 per well (\$50 per well per year). Depreciation costs are 10 percent of injection well costs only. The injection pressure is $.202 \times 10^8$ dy/cm² and the density at 85°C is .968 g/cm³. Thus the ratio

$$\frac{P_{\max}}{\rho_T} = 5.797 \times 10^{-7} \frac{\text{kw-hr}}{\text{g}} \quad (118)$$

The a_{71} is 60,000 \$/well. For a power plant with a load factor of 85 percent based on a 365 day year and an energy cost of 0.05 \$/kw-hr, a_{72} is 1,340,000 E-s/kw-h-y and b_7 is zero, giving

$$CV_{RW}(i) = 6.00 \times 10^4 T_{WR}(i) + 7.77 \times 10^{-1} Q_{TR}(i) \quad (119)$$

3. The annual fixed and variable cost for the transmission system -- The transmission system is assumed to have a life of 20 years. The annual maintenance and replacement costs for the transmission system are 5 and 9 percent, respectively, of the transmission system capital cost. The annual property tax and insurance, and the depreciation costs are 2.62 percent and 5 percent, respectively, of the transmission system capital cost. Thus a_{81} , a_{82} , a_{83} and a_{84} are 8,650 \$/well, 6,490 \$/well, 1.322 \$/g/s and .865 \$/g/s, respectively, and b_8 is zero, giving

$$CV_{TS}(i) = .865 \times 10^4 T_{WE}(i) + .649 \times 10^4 T_{WR}(i) + 1.322 Q_{TE}(i) \\ + .865 Q_{TR}(i) \quad (120)$$

4. The annual fixed and variable costs for power plant operations -- The cost of operation has an annual fixed cost of \$6,870 per year and a variable cost of \$852 per megawatt. Thus a_{g1} is $.852 \times 10^3$ \$/MW and $b_g = .687 \times 10^4$ \$, giving

$$CV(i) = .852 \times 10^3 D_{MW}(i) + .687 \times 10^4 \quad (121)$$

For a power plant with a 20 year life, the maintenance cost, interim capital replacement costs, and depreciation are 0.4, 0.35 and 0.6 percent, respectively of the total power plant capital costs, $CC(\lambda)$. Thus a_{101} , a_{102} , a_{103} and b_{10} are .0563 \$/g/s, .678 \$/g/s, 737 \$/MW and 53,200 \$, respectively, giving

$$CV_{MID}(i) = .0563 S_{AF}(i) + .678 S_{AT}(i) + .737 \times 10^3 D_{MW}(i) + .532 \times 10^5 \quad (122)$$

The evaporative rate and blowdown expenditure rate aggregate to a 4 percent loss of cooling water per year. Cooling water make up is purchased at 2.433×10^{-8} \$/g (30 \$/acre-ft). Thus a_{111} is $.652 \frac{\$/g}{y/s}$ and b_{11} is zero, giving

$$CV_{CW}(i) = .652 Q_{CW}(i) \quad (123)$$

5. Annual fixed and variable cost for reservoir development

$$\begin{aligned} CV_{RS}(i) &= CV_{EW}(i) + CV_{RW}(i) \\ &= .613 \times 10^5 T_{WE}(i) + .600 \times 10^5 T_{WR}(i) + .777 Q_{TR}(i) \\ &\quad + .750 \times 10^5 \end{aligned} \quad (124)$$

6. Annual fixed and variable cost for power plant --

$$\begin{aligned}
CV_{PS}(i) &= CV_{OP}(i) + CV_{MID}(i) + CV_{CW}(i) \\
&= .159 \times 10^4 D_{MW}(i) + .0563 S_{AF}(i) + .678 S_{AT}(i) + .652 Q_{CW}(i) \\
&\quad + .601 \times 10^5 \qquad \qquad \qquad (125)
\end{aligned}$$

7. Total fixed annual and variable costs

$$\begin{aligned}
CV_T(i) &= CV_{RS}(i) + CV_{TS}(i) + CV_{PS}(i) \\
&= .699 \times 10^5 T_{WE}(i) + .665 \times 10^5 T_{WR}(i) + 1.642 Q_{TR}(i) \\
&\quad + 1.322 Q_{TE}(i) + .159 \times 10^4 D_{MW}(i) + .0563 S_{AF}(i) \\
&\quad + .679 S_{AT}(i) + .652 Q_{CW}(i) + .135 \times 10^5 \qquad \qquad (126)
\end{aligned}$$

or in terms of the dependent variables using equations (75) through (84) ,

$$\begin{aligned}
CV_T(i) &= \sum_{j=1}^{M_E} (1.322 + .0563 \{X_1^F(j) + X_2^F(j) + X_3^F(j) + X_4^F(j)\}) \\
&\quad + .678 \{X_1^T(j) + X_2^T(j)\} + .652 \alpha_{CW} \{\gamma_{c1}(j) + \gamma_{c2}(j)\} Q_E(j, i) \\
&\quad + \sum_{j=1}^{M_R} 1.642 Q_R(j, i) + .159 \times 10^4 T_H \delta_{TH}(i) + .159 \times 10^4 T_L \delta_{TL}(i) \\
&\quad + \sum_{j=1}^{M_E} .699 \times 10^5 \delta_{WE}(j, i) + \sum_{j=1}^{M_R} .665 \times 10^5 \delta_{WR}(j, i) \\
&\quad + .135 \times 10^6 \qquad \qquad \qquad (127)
\end{aligned}$$

and the values of the parameters are given in Tables 10 and 11.

The power plant receives \$0.05 for a kilowatt-hour of electricity. For a load factor of 85 percent for 365 day operating year the revenue obtained for the i^{th} year is ($a_{121} = 3.723 \times 10^5$ \$/MW-y)

$$R_V(i) = 3.723 \times 10^5 A_{MW}(i) \quad (128)$$

or in terms of the independent variables,

$$R_V(i) = 3.723 \times 10^5 \sum_{j=1}^{M_E} (W_{e1}(j) + W_{e2}(j) Q_E(j,i)) \quad (129)$$

where the W_{e1} 's and W_{e2} 's are given in Table 10.

Management Model Results

Values for the parameters in the constraint sets are found in Tables 6, 10 and 11. For the reservoir system developed without injection, the value of θ is one. If there is a 10 percent loss of extracted fluid to evaporation and all the remaining fluid is reinjected, θ is .10. Unrestricted reinjection requires replacing the equality constraint given by equation (88) by the inequality constraint

$$(1-\theta) \sum_{k=1}^{M_E} Q_E(k,n) - \sum_{\sigma=1}^{M_R} Q_R(\sigma,n) \geq 0 \quad (130)$$

The greater than equal sign allows unrestricted reinjected of extracted fluid (i.e., no more fluid can be reinjected than is extracted, however not all available fluid need be reinjected).

The objective function (equation (85)) and the constraint sets (equations (87)-(105) and possibly (130)) with their appropriate

parameter values developed in the previous sections or given by the tables, form mixed integer programs that are solved using an IBM software package, MPSX-MIP (IBM, 1971).¹ The results of these solutions are reported in the following sections.

A. No reinjection

When none of the extracted fluid is reinjected into the reservoir ($\theta = 1.0$) the following development is projected:

1. All wells are constructed in the first construction interval; there are 4 wells at site 3; no other sites were developed.
2. All turbine systems are constructed in the first construction interval; there are 4 high pressure turbines and 2 low pressure turbines.
3. Over the 20 year design period power production decreases for each construction interval (Figure 11), varying through 16.5, 5.8, 2.4 and 1.3 megawatts.
4. Over the 20 year design period mass flow from wells decreases from 258,681 g/s to 20,578 g/s at site 3:
5. Over the 20 year design period the integrated reservoir-power plant system makes a profit of \$19,210,044 for an average annual profit of \$960,502.

The single phase pressure constraint (equation (99)) affects the withdrawals from the wells at site 3, reducing the quantity of hot

¹ The use of brand name in this report is for identification purposes only and does not imply endorsement by the U.S. Geological Survey or the University of Arizona.

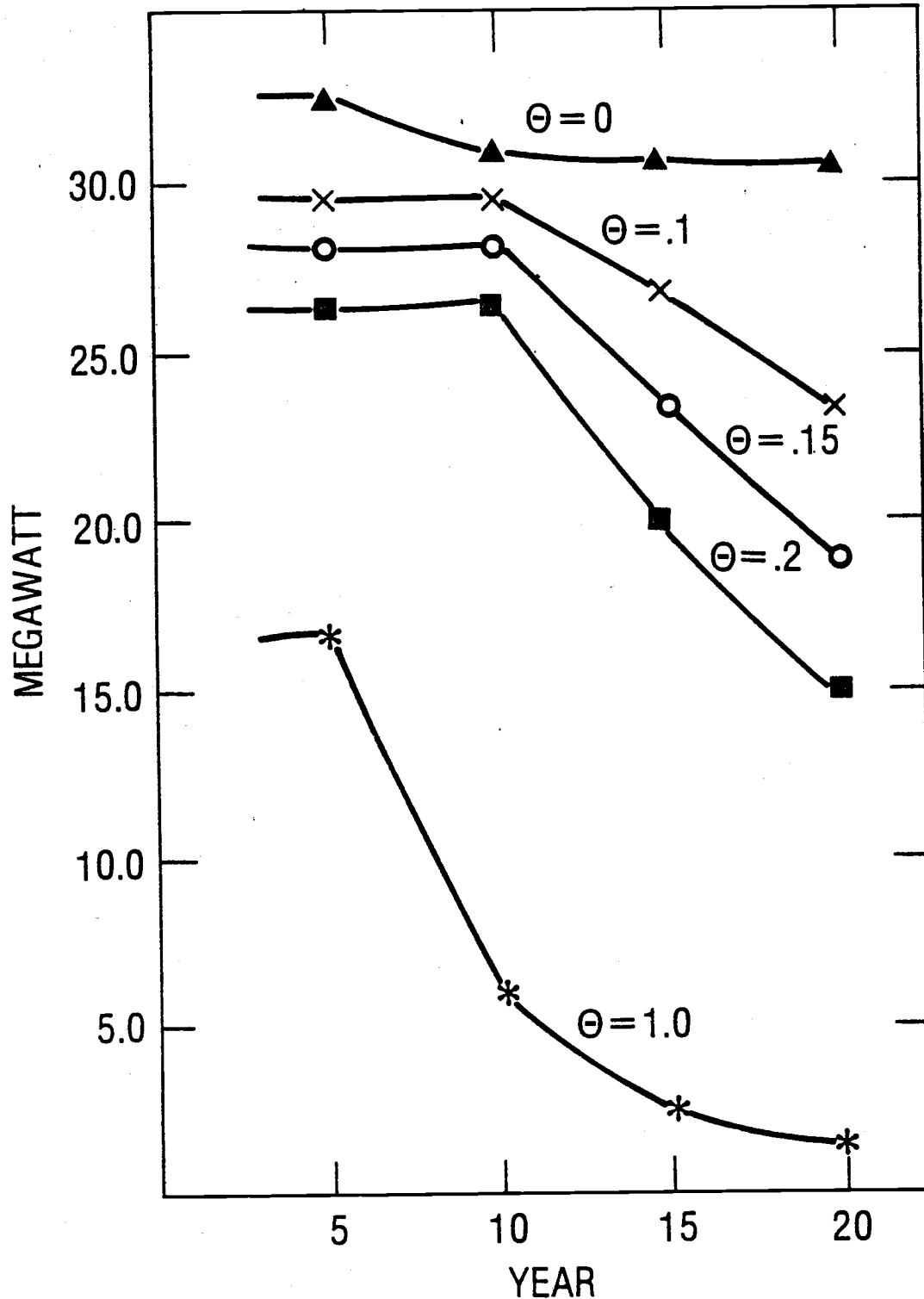


Figure 11. Power production for no reinjection and unrestricted reinjection of available fluid under various consumptive use losses. The θ is the fraction of extracted water lost to leakage or consumptive use. The value $\theta = 1.0$ implies no reinjection of fluid.

water that can be exploited for each construction interval. The leakage from the confining bed is not sufficient to replenish fluid lost from storage.

B. Complete reinjection of available water

Three cases are considered when all available water is reinjected into the reservoir: consumptive use and evaporation losses of 10, 15 and 20 percent ($\theta=.1, .15$ and $.20$). The number of high pressure turbines, low pressure turbines, extraction wells and injection wells for each construction interval; the total power production for each construction interval; and the mass flow from extraction well sites and to injection well sites for each time period are reported in Tables 12, 13 and 14 for the 10, 15 and 20 percent losses, respectively.

1. For a 20 percent loss to consumptive use and evaporation, power production rises sharply from 19.8 megawatts to 29.1 megawatts and then declines to 21.7 megawatts and finally to 16.3 megawatts over the four construction intervals (Figure 12). The temporal variation in power production is due to either the injection pressure constraints (equation (104)) or the single phase pressure constraints (equation (99)). In the first and second construction intervals the injection pressure constraints are binding and the power production increases until the pressure effects from extraction reach the injection wells and allow more fluid to be injected, before the maximum pressure limit is achieved. In the third and fourth construction intervals the single phase constraint becomes binding at site 3 and extractions are reduced to prevent

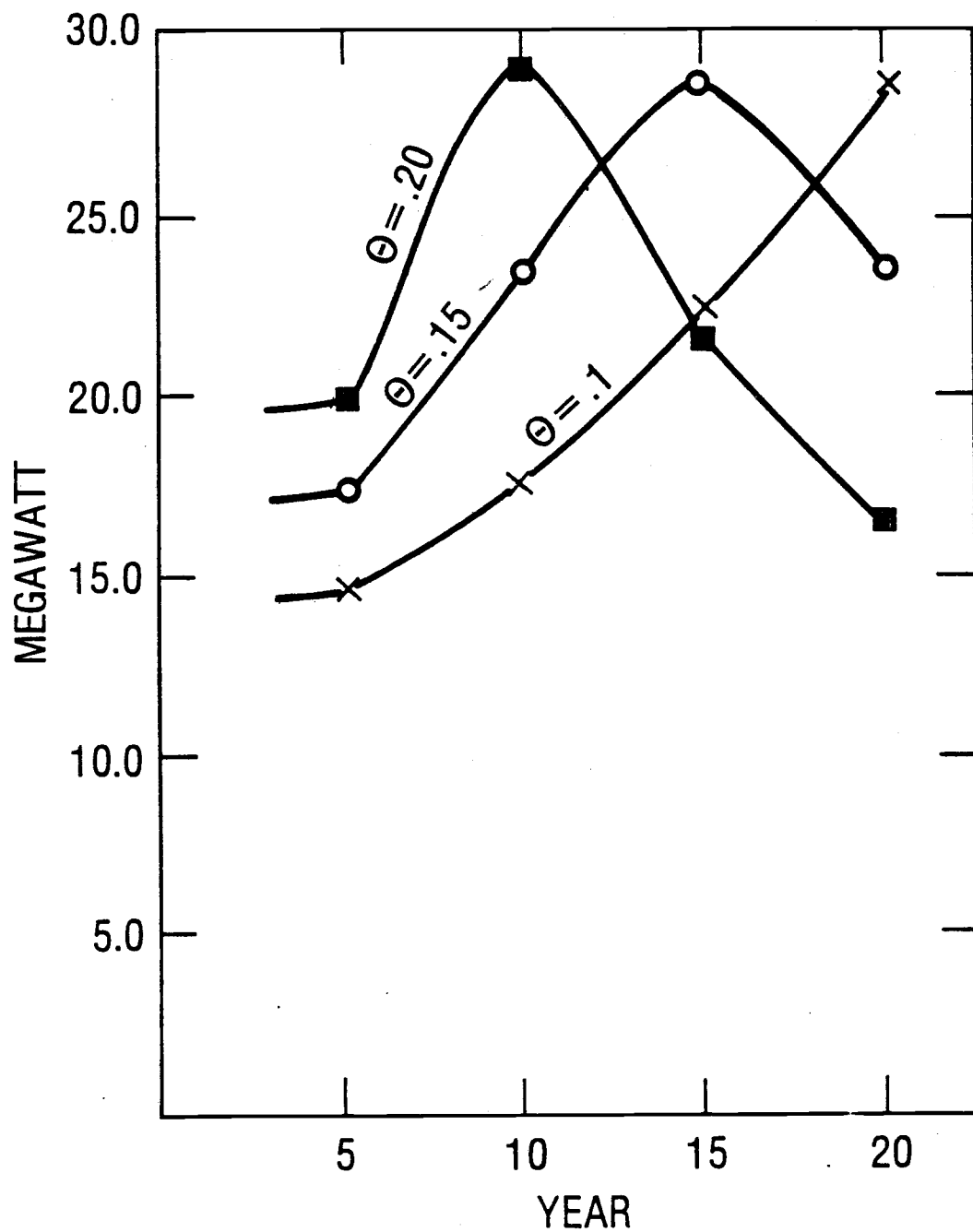


Figure 12. Power production for complete reinjection of available fluid under various consumptive use losses. The θ is the fraction of extracted water lost to evaporation and consumptive use.

Const. Int.	Number of Turbines		Number of Wells at each site										Power Prod. MW	Time Period	Rates of Fluid Extraction (ext.) and Injection											
	Low Press.	High Press.	Ext. 3	Injection						Extract (g/s) 3	Injection (g/s)															
				6	7	8	9	10	11		12	6			7	8	9	10	11	12						
1	2	4	4	1	1	1	1	1	1	1	1	2	230970	1195	18348	54715	0	16230	31624	85762						
				4								2	230970	8262	18348	46376	8915	16230	31624	78118						
												3	230970	8930	18348	46131	8915	16230	28855	80463						
												4	230970	5165	18348	49681	3273	16230	31624	83552						
												5	230970	0	18348	54715	0	15660	31624	87526						
2	2	5	4	1	1	1	1	1	1	1	2	271717	11437	21676	53610	11493	19334	33761	93235							
												6	271717	11437	21676	54715	5700	19334	33761	97923						
												7	271717	11437	21676	54715	1011	19334	33761	102612						
												8	271717	7773	21676	54715	0	19334	33761	107287						
												9	271717	7773	21676	54715	0	19334	33761	107287						
												10	271717	7773	21676	54715	0	19334	33761	107287						
3	3	6	6	1	1	2	1	1	1	1	2	355571	14636	28907	70757	15522	25591	44536	120064							
												11	355571	12581	28907	72126	12561	24694	44709	125266						
												12	355571	14241	28907	75039	0	25591	44709	131257						
												13	355571	8242	28907	75039	0	25591	44709	137256						
												14	355571	8242	28907	75039	0	25591	44709	137256						
												15	355571	8242	28907	75039	0	25591	44709	137256						
4	3	8	7	1	1	2	1	1	1	1	3	450927	18829	36213	89392	19350	32309	56474	153268							
												16	450927	16284	36213	89392	16043	31710	56474	159718						
												17	450927	18829	36213	87854	18055	32309	56474	156100						
												18	450927	12380	36213	89392	19350	32309	56474	159718						
												19	450927	18829	36213	89392	19350	32309	56474	159718						
												20	450927	18829	36213	89392	19350	32309	56474	159718						

Table 12. Complete reinjection of available water.
 $\theta = 0.10$.

Total profits: \$54,738,055
Annual Average Profit: \$ 2,736,903

Const. Int.	Number of Turbines		Number of Wells at Each Site												Power Prod. (MW)	Time Period	Rates of Fluid Extraction and Injection											
	Low Press.	High Press.	Extract			Injection						Extraction (g/s)					Injection (g/s)											
			3	4	6	7	8	9	10	11	12	3	4	6			7	8	9	10	11	12						
1	2	5	4	0	1	1	1	1	1	1	1	1	1	2	17.1	1	268490	0	0	20450	57738	9263	18441	32619	89706			
																2	268490	0	11925	20450	48376	11498	32619	85397				
																3	268490	0	1649	20450	53235	11498	32619	90324				
																4	268490	0	0	20450	60329	0	32619	97136				
																5	268490	0	0	20450	60329	0	32619	104813				
2	3	6	6	0	1	1	1	1	1	1	1	1	2	23.3	6	365263	0	15326	27823	67240	15563	25514	43394	115614				
															7	365263	0	13594	27823	67240	12850	25514	43394	120060				
															8	365263	0	15326	27823	67240	15563	25514	43394	115614				
															9	365263	0	15326	27823	67240	15563	25514	43394	115614				
															10	365263	0	15326	27823	67240	15563	25514	43394	115614				
3	3	7	6	1	1	1	1	2	1	1	1	1	3	28.9	11	412192	61335	15326	27823	101240	15563	25514	43394	173638				
															12	412192	61335	15326	27823	101240	15563	25514	43394	173638				
															13	412192	61335	15326	27823	101240	15563	25514	43394	173638				
															14	412192	61335	15326	27823	101240	15563	25514	43394	173638				
															15	412192	61335	15326	27823	101240	15563	25514	43394	173638				
4	3	7	6	1	1	1	1	2	1	1	1	1	3	23.2	16	363399	0	0	27823	101240	15563	25514	43394	95356				
															17	363399	0	0	27823	101240	0	0	43394	136433				
															18	363399	0	0	27823	101240	0	0	43394	136433				
															19	363399	0	0	0	101240	0	0	34011	173638				
															20	322448	61335	0	7944	101240	0	0	43394	173638				

Total Profits: \$54,541,462

Annual Average Profits: \$2,727,073

Table 13: Complete reinjection of available water.
 $\theta = 0.15$.

the reservoir fluid from changing phase. A marginal increase of 2.02×10^5 dy/cm² (1 psi) in the maximum injection pressure limit for well site 12 would increase profits by \$6,200, \$52,000 or \$2,200 if implemented in time period (year) 1, 2 or 6, respectively. A marginal increase in the maximum injection pressure limit in any other time period would reduce profits. Because the injection wells are not operating against maximum pressure there is an excess injection capacity. In time periods 1 and 6 sites 8 and 12 are at maximum pressure limit but the other injection wells have excess capacity. In time period 2 all injection wells are operating at maximum injection pressure limits and there is no excess capacity.

2. For a 15 percent loss to consumptive use and evaporation, power production rises from 17.1 megawatts to 23.3 megawatts, to 28.9 megawatts, and then drops to 23.2 over the four construction intervals (Figure 12). For time periods 1 through 6, injection well sites 8 and 12 are operating at maximum injection pressure limits; in time periods 2 and 6 all injection wells are at the limits; in time period 7 well sites 6 and 9 are at the limits; and in time period 11 well site 12, is at the limit. Hence, in the first and second construction intervals, the injection pressure constraints are binding. In time period 11 a well is developed at site 4 and is used continuously through time period 15. The development at site 4 produces water at lower temperature than at site 3 (210°C vs. 230°C); however it prevents site 3 from reaching the

single phase limit until time period 15. Single phase lower limit pressures are also reached at site 3 in time periods 19 and 20. A marginal increase of 2.02×10^5 dy/cm² in the maximum injection pressure limit at site 12 in time periods 1 through 6 increases profits by \$13,000, \$53,000, \$1,900, \$2,000 and \$41,000, respectively. The greatest profit increases occur in those time periods when all injection wells are operating against the injection pressure limit.

3. For a 10 percent loss to consumptive use and evaporation, power production rises continuously from 14.7 megawatts through 17.3 and 22.7 megawatts to 28.8 megawatts over the four construction intervals and 22.7 megawatts to 28.8 megawatts over the four construction intervals (Figure 12). The single phase pressure constraint is never binding, and thus the pressure constraint is binding in every construction interval. For example, wells at site 12 are against the upper limit during time periods 1 through 9, 11 through 15 and 16 through 18. Marginal increases in the injection pressure limit produce profit behaviors similar to those seen in the 20 and 15 percent consumptive use and evaporative loss cases. The maximum profit increase occurs if 2.02×10^5 dy/cm² addition injection pressure capability is added to the wells at site 12 during the sixth time period (\$56,400).

It is interesting to note that when all available extracted waters are required to be reinjected into the reservoir, the greatest power production occurs for a system with the greatest water loss to consumptive use or evaporation. Thus a stringent requirement to reinject all spent fluid back into the same reservoir leads to either a reduction of

power production or no incentive to reduce steam leakage from wells, transmission system or power plant.

C. Unrestricted reinjection of available fluid

Unrestricted reinjection implies that all the available water in any time period need not be reinjected into the reservoir. As was done with complete reinjection, three cases are considered: the consumptive use and evaporative losses of 10, 15 and 20 percent. Results are summarized in tables 15 through 17.

1. For a 20 percent loss to consumptive use and evaporation, power production remains constant for the first two construction intervals at 26.2 megawatts, then drops to 19.7 megawatts in the third interval, and finally drops to 14.9 megawatts in the fourth interval (Figure 11). In the first construction interval the injection pressure constraints are binding at reinjection sites 8, 11 and 12 for time period 1 and at all reinjection sites for time periods 2 and 3. The injection pressure constraint is not binding for any time period in the second construction interval; however the single-phase pressure constraint is binding at site 3 in time period 10; the last time period of the second construction interval. The single-phase pressure constraint is also binding at site 3 for the last time periods (15 and 20) of the third and fourth construction intervals. In time periods 1, 2 and 3 there are 35,243 g/s, 29,667 g/s, and 539 g/s, respectively,

Const. Int.	Number of Turbines		Number of Wells at Each Site												Power Prod. (MW)	Time Period	Rates of Fluid Extraction and Injection																																																			
	Low Press.	High Press.	Extract			6			7			8					9			10			11			12																																										
			3	4	12	3	4	12	3	4	12	3	4	12			3	4	12	3	4	12	3	4	12	3	4	12	Light																																							
1.	3	8	7	0	1	1	2	1	1	1	1	3	29.7	1	464931	0	16082	31484	63890	16085	29027	46482	112794	102594	2	464931	0	13434	29007	71962	13443	25756	46755	126908	91172	3	464931	0	14354	32381	81711	14360	28606	51740	142349	52938	4	464931	0	15368	34698	88446	15373	30571	55295	153584	25102	5	464931	0	16082	36297	93098	16085	31930	57787	161620	5538
2	3	8	7	0	1	1	2	1	1	1	3	29.7	6	464931	0	16082	36297	93098	16085	31930	57787	167158	0	7	464931	0	16082	36297	93098	16085	31930	57787	167158	0	8	464931	0	16082	36297	93098	16085	31930	57787	167158	0	9	464931	0	16082	36297	93098	16085	31930	57787	167158	0	10	464931	0	16082	36297	93098	16085	31930	57787	167158	0	
3	3	8	7	2	1	1	2	1	1	1	3	26.8	11	419178	0	0	36297	93098	16085	31930	57787	142062	0	12	419178	0	0	36297	93098	0	31930	57787	158148	0	13	419178	0	0	22920	57787	167158	0	14	401356	26693	0	30904	57787	167158	0	15	367091	78014	14325	31930	57787	167158	0										
4	3	8	7	2	1	1	2	1	1	1	3	23.1	16	361972	0	16082	36297	93098	16085	31930	57787	74494	0	17	361972	0	0	36297	93098	0	0	57787	138592	0	18	361972	0	0	0	57787	167158	0	19	337916	36030	0	0	57787	167158	0	20	309885	78014	0	0	57787	167158	0										

Total Profits: \$71,046,469
Annual Average Profits: \$3,552,323

Table 15. Unrestricted recharge of available water.
 $\theta = 0.10$.

Const. Int.	Number of Turbines		Number of Wells at Each Site												Power Prod. (MW)	Time Period	Rates of Fluid Extraction and Injection											
	Low Press.	High Press.	Extract 3	4	6	7	8	9	10	11	12	Extraction (g/s)					Injection (g/s)											
												3	4	6			7	8	9	10	11	12	Light					
1	3	7	7	0	1	1	2	1	1	1	2	1	1	1	2	28.0	1	438872	0	14571	31427	61994	14575	28973	45177	109260	67064	
																	2		0	13021	27600	68740	13031	24578	44611	121254	60126	
																	3		0	13644	30756	77674	13650	27165	49143	135354	25655	
																	4		0	14571	32887	83873	14575	28973	52409	145674	79	
																	5		0	14571	32887	83873	14575	28973	52409	145753	0	
2	3	7	7	0	1	1	2	1	1	1	2	1	1	1	2	28.0	6	438872	0	14571	32887	83873	14575	28973	52409	145753	0	
																	7		0	14571	32887	83873	14575	28973	52409	145753	0	
																	8		0	14571	32887	83873	14575	28973	52409	145753	0	
																	9		0	14571	32887	83873	14575	28973	52409	145753	0	
																	10		0	14571	32887	83873	14575	28973	52409	145753	0	
3	3	7	7	1	1	1	2	1	1	1	2	1	1	1	2	23.4	11	366322	0	0	32887	83873	0	28973	52409	98656	0	
																	12		0	0	32887	83873	0	28973	52409	113231	0	
																	13		0	0	32887	83873	0	0	52409	142204	0	
																	14		366322	0	29338	83873	0	0	52409	145753	0	
																	15		323488	64155	32887	83873	0	14574	52409	145753	0	
4	3	7	7	1	1	1	2	1	1	1	2	1	1	1	2	18.8	16	294588	0	14571	32887	83873	0	28973	52409	37686	0	
																	17		0	0	32887	83873	0	0	52409	81230	0	
																	18		294588	0	0	32887	83873	0	0	0	133639	0
																	19		284026	15819	0	0	0	0	25242	145753	0	
																	20		251754	64155	0	0	0	0	38897	145753	0	

Total Profits: \$64,694,897

Average Annual Profits: \$32,347,445

Table 16. Unrestricted recharge of available water.
θ = 0.15.

Const. Int.	Number of Turbines		Number of Wells at each site										Power Prod. MW	Time Period	Extract (g/s) 3	Rates of Fluid Extraction (ext.) and Injection									
	Low Press.	High Press.	Ext. 3	Injection												6	7	8	9	10	11	12			
				6	7	8	9	10	11	12	6	7											8	9	10
1	3	7	6	1	1	2	1	1	1	1	1	1	2	12889	29010	61481	12894	25619	45630	105696					
			6	1	1	2	1	1	1	1	1	1	2	12842	26508	65353	12870	23621	42363	115238					
														12889	29010	73338	12894	25618	46351	127823					
														12889	29010	73338	12894	25618	46351	128362					
														12889	29010	73338	12894	25618	46351	128362					
2	3	7	6	1	1	2	1	1	1	1	1	2	12889	29010	73338	12894	25618	46351	128362						
			6	1	1	2	1	1	1	1	1	2	12889	29010	73338	12894	25618	46351	128362						
														12889	29010	73338	12894	25618	46351	128362					
														12889	29010	73338	12894	25618	46351	128362					
3	3	7	6	1	1	2	1	1	1	1	1	2	0	29010	73338	12894	25618	46351	59142						
			6	1	1	2	1	1	1	1	1	2	0	29010	73338	73338	0	25618	46351	72037					
													0	29010	73338	73338	0	0	46351	97655					
													0	0	0	73338	0	0	44654	128362					
													0	0	0	73338	0	0	44654	128362					
4	3	7	6	1	1	2	1	1	1	1	1	2	12889	29010	73338	0	25618	45649	0						
			6	1	1	2	1	1	1	1	1	2	0	29010	73338	73338	0	0	46351	37806					
													0	0	0	73338	0	0	0	113167					
													0	0	0	58143	0	0	0	128362					
													0	0	0	58143	0	0	0	128362					

Table 17. Unrestricted recharge of available water.
 $\theta = 0.20$.

Total Profits: \$58,745,663
Annual Average Profit: \$ 2,937,283

of available fluid to be disposed of by means other than reinjection into the reservoir. For all other time periods the amount injected is exactly equal the amount extracted minus the 20 percent consumptive use and evaporative losses.

2. For a 15 percent loss to consumptive use and evaporation, power production remains constant for two construction intervals at 28 megawatts, then drops to 23.4 megawatts in the third interval and to 18.8 megawatts in the fourth interval (Figure 12). Injection pressure constraints are binding in only the first construction interval (all injection sites for time periods 2 through 4 and sites 7, 8, 11 and 12 for time period 1). The single phase pressure constraint is binding again at site 3 at the end of construction interval 2 (time period 10) and remains for interval 3 (time periods 10, 14 and 15) and interval 4 (time periods 19 and 20). In the first construction interval, time periods 1, 2, 3 and 4 there are 67,064 g/s, 60,126 g/s, 25,655 g/s and 79 g/s, respectively, of available fluid to be disposed of by means other than reinjection. Site 4 is developed for extraction in the third and fourth construction intervals.

3. Power production behaves in the same fashion for 10 percent loss of fluid to consumptive use and evaporation as it did for 20 and 15 percent losses. Power production remains constant at 29.7 megawatts for the first two construction intervals, then drops to 26.8 in third interval and 23.1 in the fourth interval (Figure 11). The injection pressure constraint is binding only in the first construction

interval (all injection sites for time periods 2 through 5, and at all sites except 9 in time period 1). The single phase constraint is binding during the third interval (time periods 10, 14 and 15) and the fourth interval (time periods 18, 19 and 20). Site 4 is again developed for extraction during the third and fourth intervals (time periods 14, 15, 19 and 20). Alternative disposal requirements are 102,594 g/s, 91,172 g/s, 52,938 g/s, 25,102 g/s and 5,538 g/s in time periods 1 through 5, respectively.

Unlike complete reinjection, unrestricted reinjection leads to greater power production with reduction of water losses and thus promotes greater operating efficiency. Unrestricted reinjection produces marginal increases in profit similar to those for complete reinjection when a marginal increase in the maximum injection pressure is allowed.

For unrestricted recharge of available water after a 10 percent loss of fluid, over 460,000 g/s of water at 85°C is reinjected for the 6th through the 10th time periods. This is the maximum rate of injection for any of the cases considered. Large injections of lower temperature fluid could reduce the temperature distribution throughout the reservoir, thus violating the constant temperature assumption. To determine temperature changes produced by the extraction and injection values for the 10 percent loss case (unrestricted recharge) tests were made using a temperature-pressure single phase model

(Mercer, Pinder and Donaldson, 1975). It was found that maximum temperature changes were less than 2°C. Hence the constant temperature assumption, was not violated to a degree to warrant modeling of temperature variation.

The results of the example for no reinjection indicate that all well field and power plant construction occur during the first construction interval. Furthermore, the results predict that if staging does occur, it is in response to the maximum injection constraint. This is contrary to historical developments that have occurred in actual exploitation of geothermal reservoirs. At Wairakei, New Zealand, at The Geysers, California and at Larderello, Italy construction and power production have been staged over time. At Wairakei, for example, production from 1950 to 1963 occurred in two stages. During the first stage (1956 to 1960), a 69 MW power plant was built and supported by 28 wells. In the second stage (1960 to 1963) the power plant was expanded to 112 MW that required the addition of 34 new wells (Haldane and Armstead, 1962). At Wairakei, spent waters are discharged to a nearby river. At the Geyser, although there is reinjection, there is no evidence that the fluid is placed in the extraction zones.

In practice staging may occur in response to three factors: 1) capital or market availability, 2) technological capability, and/or 3) uncertainty as to actual geothermal field conditions.

Capital or Market Availability

Development of a geothermal reservoir and the construction of a power plant and its supportive transmission system are extremely costly. Developers may not have sufficient capital available to initiate all construction within one construction interval. Under this condition it becomes necessary to add capital constraints to the original constraint set given by equations (86) through (105).

These constraints take the form,

$$CC_T(\ell) R(\ell, r) + \sum_{i=I_t}^{I_t \cdot \ell} \frac{CV_T(i)}{(1+r)^i} \leq C_B(\ell) R(\ell, r) \quad (131)$$

where $C_B(\ell)$ is the total cost that can be incurred during the ℓ^{th} construction interval. If a market for the full electric power potential does not exist during the initial or subsequent construction intervals, the power plant may also be staged to respond to an unexpected market growth. To account for the limited market, power availability constraints should be added to the constraint set.

These constraints take the form,

$$\sum_{j=1}^{M_E} \{W_{e1}(j) + W_{e2}(j)\} Q(j, i) \leq W_T(i) \quad i=1, \dots, N \quad (132)$$

where $W_T(i)$ limit to power availability in the i^{th} time period.

If, during the time horizon, the price of electricity should rise, then the power plant may be staged in response to this rise. In this

situation, the revenue term in the objective function should be modified.

Technological Capability

Geothermal power plants are still experimental in nature. Technology with regard to utilization of lower temperature fluids, mixtures of hot waters and steam, or corrosive fluids, may improve during the life of the plant. For example, in the early stages of the development at Wairakei the steam portion of the hot fluid that had flashed in the well bore was separated from the hot water (Haldane and Armstead, 1962). The water was dumped and only the steam portion transported to the power plant. Later, under extremely controlled conditions to prevent boiling, hot water was also transported and utilized in the power plant. Although it is not possible to account explicitly for future technological capabilities within the management model, a limited measure of their effects may be accounted for in a number of ways.

Turbine efficiencies may be increased over the construction intervals to reflect improved turbine technology. Operating pressures for turbines may be changed to reflect improved transmission system technology. The design capacities for wells may be changed to reflect improved well completion and stimulation technology. Costs of production may also be changed to reflect technology changes. The

costs of extraction of fluid, transmission and power generation may decrease with improvements in technology. Subsequent stages of power production may utilize an indirect method of power production or may require an evaluation of different power production methods. Under these circumstances alternative power plant models and their associated cost functions may be developed and tested in the management model.

Uncertainty as to Actual Geothermal Field Conditions

The economic utilization of a geothermal reservoir is in part dependent on the fluid temperatures and the recharge properties of the reservoir; and on the porous media having sufficient volume, porosity and permeability to yield adequate quantities of fluid. In many cases it is impossible to determine the existing field properties prior to development and production. Thus, limited power production and fluid exploitation occurs in the early time intervals in order to ascertain field conditions.

The management model can be extremely useful in handling uncertainties in field conditions. During the initial stages of geothermal power plant sizing, various hypotheses for reservoir properties can be tested in the model to determine their effect on the sizing decision. Later, as the uncertainty in these properties decrease, the model can be used to determine expansion.

ASSUMPTIONS, EXTENSIONS AND CONCLUSIONS

Three assumptions concerning the reservoir model provide the linear form of the equation of flow (equation (4)): the reservoir contains only pure water; the reservoir is liquid dominated; and the spatial distribution of temperature for the reservoir is known and invariant with time. If any of these assumptions are invalid for application to a geothermal reservoir, the flow equation may become coupled with an additional equation relating changes in concentration, changes in phase, or changes in temperature due to withdrawals. This coupling is generally nonlinear.

If concentrations of dissolved solids are greater than 2 percent by weight, the thermodynamic equations (equation (1) and (2) may require modification such that the viscosity and density become dependent on concentration. If concentrations are assumed to be uniform throughout the reservoir, withdrawals will not induce changes in concentrations and equation (4) is still a valid approximation. However, if the concentrations are nonhomogeneous, withdrawals will induce changes in the concentrations, and the flow equation is coupled via the velocity field, to an equation describing the rate of change of concentrations (for example, see INTERCOMP, 1976).

If fluid in the reservoir undergoes a change from single phase, hot water to two phase steam and water, fluid temperatures are lowered,

and heat is extracted from the porous media. When withdrawals induce changes in phase or if prior to development a reservoir is two phase, the equation of flow is coupled with an energy equation requiring that pressure as well as an additional dependent variable such as enthalpy, be determined (Faust and Mercer, 1979). Changes in enthalpy and pressure will be dependent on the velocity field produced by withdrawals, resulting in nonlinear coupling.

If the reservoir remains single-phase, hot water, but temperatures vary with time, such as would be the case if cooled, spent fluid is reinjected into the hotter portion of the reservoir rather than the cooler or if large amounts of leakage from a confining layer at different temperatures occurs over a long design horizon, the equation of flow is again coupled with an energy equation (Mercer, Pinder, Donaldson, 1975). Because there are no changes in phase, temperature may be used as the additional dependent variable to pressure. The energy equation and flow equation are again coupled nonlinearly because the changes in temperatures are dependent on the velocity field produced by withdrawals. It should also be noted that in situations where temperature changes with time, thermodynamic properties such as density and viscosity will also change with time because they are temperature dependent.

Substantial modification to the management model is required if any of the three original assumptions are deemed improper for application to a field problem. The nonlinearity of the coupled equations

prohibits the use of the linear response functions relating changes of pressure to withdrawals. To solve nonlinear problems the constraint set given by equation (87) is replaced by the actual set of finite-difference or finite-element approximations of the partial differential equations. Such a technique is called "embedding." Because the finite-difference or finite-element equations have usually been linearized in some fashion, a global optimization of profits over the entire design period is impossible. Only local optimization of profits over a single time period can be achieved. There is no guarantee that the time sequence of the local optimized profits and their associated decision variables provide a global optimum. However, by applying the linearizing assumptions as a first step, and then proceeding to embedding techniques, some control can be exercised over the time sequence of local optima and a satisfactory set of decision variables can be determined over the design horizon.

Two other assumptions utilized in the reservoir model; porous media and constant porosity, may also affect the applicability of the reservoir model. Some geothermal reservoirs depend on secondary permeability, such as fractures, to allow sufficient flow to wells. In many of these cases, if the reservoir is considered on a regional scale, a porous media model is adequate to simulate the pressure and temperature responses to exploitation. For field situations where this is not true, a model that incorporates flow in fractures is required.

One major problem that arises in the actual exploitation of geothermal energy is land subsidence. If land subsidence is expected to occur, the assumption of constant porosity is may be in valid and a subsidence model should be incorporated into the management model.

The power plant model relies on the pure water assumption in order to use standard steam tables. There are steam property tables available for fluids containing dissolved solids (Haas, 1975a, 1975b). The properties from these tables may be used in the power plant model without changing its structure. However, it is likely that an indirect method of power generation should be used if high concentrations of dissolved solids are present, and should be incorporated into the management model.

No attempt is made to optimize the various operating pressures hypothesized for the power plant. Furthermore, it is assumed that these pressures occurred at saturation. Although explicit optimization of operating pressures is not possible with the existing structure of the management model, various operating pressures can be tested, and an indication of suitable suites of pressures can be determined. Treatment of nonequilibrium conditions such as nonsaturation is beyond present modeling capabilities.

One possible alternative to the direct method for production of electric power from geothermal energy is the method of total flow. For this method a mixture of steam and hot water is fed through a

series of nozzles to drive turbines. Such a method would be highly applicable to reservoirs containing fluid at high pressure (such as geopressed reservoirs), and would utilize the mechanical energy of the fluid. If mechanical energy is an important consideration, the objective function of the management model becomes quadratic because the mechanical power is proportional to the product of the pressure, $p(k,n)$ and the rate of withdrawals from the wells, $Q_E(k,n)$. The cost functions would all change to incorporate the different technology, particularly the costs for separator-flashers that will probably not be needed. However, the basic form of the objective function would be unaltered with the exception of the quadratic mechanical power term. It should be noted that the resulting mixed integer, quadratic program is exceedingly difficult to solve without access to extremely large computers.

The existing power plant model can also be modified to incorporate heat losses due to fluid transmissions. Such a modification would require considerably more detail as to the nature of the transmission system than has been described in this report. It would be necessary to know the length of the transmission system and characteristics of the pipes.

It should be reiterated that the cost functions used in the management model may require modification before actual application to a field problem. The only restriction that must be adhered to is

that the cost functions be linear. Nonlinear cost functions must be approximated by linear functions over their ranges of applicability. In certain cases nonlinear cost functions can be approximated by more than one linear segment. The resulting objective function and constraint set would form a mixed integer, separable program, which can be solved by the MPSX-MIP solution package but at greater cost.

The management model, described herein, can be used to determine individual well spacing. However, the grid spacing of the finite-difference or finite-element technique used to solve equation (18) must be refined so that there is only one well per node. For large field problems this will increase substantially the number of node points and the number of response functions.

The management model assumes a single firm owns and operates the well field, the transmission system and the power plant. One obvious simplification, is to modify the management model to determine optimal development of a geothermal reservoir for the sale of hot water. This is done simply by dropping those terms in the objective function and constraint set related to the power plant and adding a linear revenue term for sale of hot water or adding a constraint set for specified demands for hot water.

Only a very limited sensitivity analysis has been conducted by the authors because of the hypothetical nature of the reservoir and power plant. In an actual application to a field problem, the value

of sensitivity analysis cannot be discounted in any manner. One of the great powers of programming techniques is the simplicity at which post optimal analysis can be performed. The post optimal analysis can be used to determine the range of parameter values for which decision variables would remain unchanged.

In this report a methodology has been developed and demonstrated that provides a beginning for analyzing the decision process inherent in managing an integrated geothermal reservoir-power plant system.

Because this report represents a preliminary effort, many assumptions were made to simplify the analysis, and although many of these assumptions have been discussed in detail and some of their ramifications elucidated, a far more detailed effort is likely to be needed before the methodology is applied to actual field conditions. Such an effort is now underway.

→ How about Geysers? in N. Calif?

APPENDIX A

The two-phase, three dimensional heat transport equations in terms of pressure and enthalpy are (Faust and Mercer, 1979, p. 26).

$$\begin{aligned}
 & \frac{\partial}{\partial t} (\phi \rho(\hat{x}, t)) - \nabla \cdot \left[\frac{k(\hat{x}) k_{rs}(\hat{x}) \rho_s(\hat{x}, t)}{\mu_s(\hat{x}, t)} \cdot (\nabla p(\hat{x}, t) - \rho_s(\hat{x}, t) g \nabla D(\hat{x})) \right] \\
 & - \nabla \cdot \left[\frac{k(\hat{x}) k_{rw}(\hat{x}) \rho_w(\hat{x}, t)}{\mu_w(\hat{x}, t)} \cdot (\nabla p(\hat{x}, t) - \rho_w(\hat{x}, t) g \nabla D(\hat{x})) \right] \\
 & - q_w^1 - q_s^1 = 0 \tag{A1}
 \end{aligned}$$

and

$$\begin{aligned}
 & \frac{\partial}{\partial t} [\phi \rho(\hat{x}, t) h(\hat{x}, t) + (1-\phi) \rho_r(\hat{x}) h_r(\hat{x}, t)] \\
 & = \nabla \cdot \left[\frac{k(\hat{x}) k_{rs}(\hat{x}) \rho_s(\hat{x}, t) h_s(\hat{x}, t)}{\mu_s(\hat{x}, t)} \cdot (\nabla p(\hat{x}, t) - \rho_s(\hat{x}, t) g \nabla D(\hat{x})) \right] \\
 & - \nabla \cdot \left[\frac{k(\hat{x}) k_{rw}(\hat{x}) \rho_w(\hat{x}, t) h_w(\hat{x}, t)}{\mu_w(\hat{x}, t)} \cdot (\nabla p(\hat{x}, t) - \rho_w(\hat{x}, t) g \nabla D(\hat{x})) \right] \\
 & - q_w^1 h_s^1 - q_w^1 h_w^1 - \nabla \cdot (k_m(\hat{x}) \nabla T(\hat{x}, t)) \\
 & - \left[\frac{\partial}{\partial t} (\phi p(\hat{x}, t)) + (v_s(\hat{x}, t) + v_w(\hat{x}, t)) \cdot \nabla p(\hat{x}, t) \right] \tag{A2}
 \end{aligned}$$

where

- ϕ = porosity (dimensionless)
- $\rho(\hat{x}, t)$ = mixture density $[ML^{-3}]$
- $k(\hat{x})$ = permeability tensor $[L^2]$
- $k_{rs}(\hat{x})$ = relative permeability for steam (dimensionless)
- $k_{rw}(\hat{x})$ = relative permeability for water (dimensionless)
- $\rho_s(\hat{x}, t)$ = density of steam $[ML^{-3}]$
- $\rho_w(\hat{x}, t)$ = density of water $[ML^{-3}]$
- $\mu_s(\hat{x}, t)$ = viscosity of steam $[ML^{-1}t^{-1}]$
- $\mu_w(\hat{x}, t)$ = viscosity of water $[ML^{-1}t^{-1}]$
- $p(\hat{x}, t)$ = pressure $[ML^{-1}t^{-2}]$
- g = gravitation constant $[Lt^{-2}]$
- $D(\hat{x})$ = depth $[L]$
- q_s^1 = steam mass source term $[Mt^{-1}]$
- q_w^1 = water mass source term $[Mt^{-1}]$
- $h(\hat{x}, t)$ = mixture enthalpy $[L^2t^{-2}]$
- $\rho_r(\hat{x})$ = rock density $[ML^{-3}]$
- $h_r(\hat{x}, t)$ = rock enthalpy $[L^2t^{-2}]$
- $h_s(\hat{x}, t)$ = saturated steam enthalpy $[L^2t^{-2}]$
- $h_w(\hat{x}, t)$ = saturated water enthalpy $[L^2t^{-2}]$
- $T(\hat{x}, t)$ = temperature $[T]$
- $k_m(\hat{x})$ = medium thermal conductivity
- $v_s(\hat{x}, t)$ = steam velocity $[Lt^{-1}]$
- $v_w(\hat{x}, t)$ = water velocity $[Lt^{-1}]$
- $h_s^1(\hat{x}, t)$ = saturated steam enthalpy at source $[L^2t^{-2}]$
- $h_w^1(\hat{x}, t)$ = saturated water enthalpy at source $[L^2t^{-2}]$
- \hat{x} = point (x, y) $[L]$

If the reservoir is single phase, water, then the relative permeability for steam is zero and that for water is one. The water saturation is one. If the last term of equation (A2) is assumed negligible and

$$h(\hat{x}, t) \approx C_v T(\hat{x}, t) \quad (A3)$$

where C_v is the specific heat then equations (A1) and (A2) reduce to,

$$\frac{\partial}{\partial t} (\phi \rho(\hat{x}, t)) - \nabla \cdot \left[\frac{k(\hat{x}) \rho(\hat{x}, t)}{\mu(\hat{x}, t)} \cdot (\nabla p(\hat{x}, t) - \rho(\hat{x}, t) g \nabla D(\hat{x})) \right] - q^1 = 0 \quad (A4)$$

and

$$\begin{aligned} & \frac{\partial}{\partial t} \{ [\phi \rho(\hat{x}, t) C_v + (1-\phi) \rho_r(\hat{x}) C_{vr}] T(\hat{x}, t) \} \\ & - \cdot \left[\frac{k(\hat{x}) \rho(\hat{x}, t) C_v T(\hat{x}, t)}{\mu(\hat{x}, t)} \cdot (\nabla p(\hat{x}, t) - \rho(\hat{x}, t) g \nabla D(\hat{x})) \right] \\ & - \nabla \cdot [k_m(\hat{x}) \nabla T(\hat{x}, t)] - q^1 C_v^1 T^1 = 0 \quad (A5) \end{aligned}$$

Note that C_{vr} is the specific heat of rock.

If temperature is invariant with time, then $\frac{\partial T}{\partial t}(\hat{x}, t) = 0$ and only the steady state or initial temperature distribution need be

determined. For temperature invariance, equation (A5) vanishes and only the solution of equation (A4) is required. Equation (A4) may be integrated in the vertical direction resulting in an equation in terms of areal dimensions only

$$b \frac{\partial}{\partial t} (\phi \rho(\hat{x}, t)) - \nabla \cdot \left(\frac{b k(\hat{x}) \rho(\hat{x}, t)}{\mu(\hat{x})} \cdot \nabla p(\hat{x}, t) \right) - Q^1 = 0 \quad (A6)$$

where ∇ is now defined over the two horizontal dimensions, x and y , b is the thickness and Q^1 is the mass flux for sources and sinks. The porosity is assumed constant with respect to time

$$b \frac{\partial}{\partial t} (\phi \rho) = b \phi \frac{\partial \rho}{\partial t} \quad (A7)$$

and the following equation of state is assumed

$$\rho(\hat{x}, t) = \rho_0(\hat{x}) + \beta \rho_0(\hat{x}) (p(\hat{x}, t) - p_0(\hat{x})) \quad (A8)$$

hence

$$b \phi \frac{\partial \rho}{\partial t}(\hat{x}, t) = b \phi \beta \rho_0(\hat{x}) \frac{\partial p}{\partial t}(\hat{x}, t) \quad (A9)$$

For a pressure change less than that which would produce two phase flow

$$\nabla \cdot \left(\frac{b k(\hat{x}) \rho(\hat{x}, t)}{\mu(\hat{x})} \cdot \nabla p(\hat{x}, t) \right) \cong \nabla \cdot \left(\frac{b k(\hat{x}) \rho_0(\hat{x})}{\mu(\hat{x})} \nabla p(\hat{x}, t) \right) \quad (A10)$$

Therefore equation (A6) becomes

$$\nabla \cdot (m(\hat{x}) \nabla \cdot p(\hat{x}, t)) = a(\hat{x}) \frac{\partial p(\hat{x}, t)}{\partial t} + Q^i \quad (\text{A11})$$

where

$$a(\hat{x}) = b\phi\beta\rho_o(\hat{x}) \quad (\text{A12})$$

and

$$m(\hat{x}) = \frac{b\rho_o(\hat{x}) k(\hat{x})}{\mu(\hat{x})} \quad (\text{A13})$$

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