FM/FM PREEMPHASIS SCHEDULE DESIGN FOR CONCURRENT ALL CHANNEL DROPOUT AND MINIMUM TRANSMISSION BANDWIDTH

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Abstract

This work is concerned with the design of the FM/FM preemphasis schedule such that all channel dropout occurs concurrently in that threshold is reached at the same signal-to-noise ratio at the same time in all the channels. That is, threshold occurs in the carrier IF at the same time as threshold occurs in all the subcarrier bandpass filters. This design assures that this is the minimum transmission bandwidth achievable with no channel in threshold.

The procedure developed is a systematic approach to the design of the preemphasis schedule that is readily converted to an algorithmic code. When the code is implemented on a computer, the preemphasis schedule may be modified and studied for various anticipated operational parameters such as carrier-to-noise ratio at the receiver and the D_{si}'s. The impact on the output signal-to-noise ratios by the specified D_{si}'s can also be evaluated. A design examples is given.

Introduction

In an FM/FM system the minimum transmission bandwidth achievable occurs whenever the preemphasis schedule is designed such that threshold occurs concurrently in the IF bandpass filter and in all the subcarrier bandpass filters. In order to achieve this goal the relationship between the carrier-to-noise ratio in the IF bandpass filter and the signal-to-noise ratios in all the subcarrier bandpass filters is established. A preemphasis schedule design procedure is developed to design for concurrent all channel dropout in terms of a specified carrier-to-noise ratio and the D_{si}'s, the deviations of the subcarriers with respect to the signals. This design procedure determines the minimum transmission bandwidth achievable for operation above threshold in all channels. The design is general in that different D_{si}'s may be specified for each subcarrier channel. Specifically, the output signal-to-noise ratios in all channels are computed for this design procedure and are shown to be a function of the specified D_{si}'s and cannot be independently specified. This is an important relationship since it shows that designing for all channel drop out and specifying small D_{si}'s will produce unacceptable low output signal-to-noise ratios in the subcarrier channels with small D_{si}'s.

Design Equations Development

In an FM/FM system, the IF bandpass filter, carrier demodulator and the subcarrier bandpass filter comprise a bandpass-to-bandpass FM demodulation module. For this case the signal-to-noise in the subcarrier bandpass filter is given by [1],

$$[S/N]_{o\,BP} = \frac{1}{4}\left[\frac{f_{dc}^2}{f_s^2}\right]\left[\frac{A^2}{h Bi}\right]. \qquad (1)$$

Where

f_s = frequency of the single subcarrier,
f_{dc} = peak deviation of the carrier by the subcarrier,
A = carrier power,
B_i = bandwidth of the subcarrier bandpass filter,
h = one sided power spectral density of the white noise applied to the IF filter.

Observing that the first bracket term is equation (1) is also the mod index, D_c, of the carrier with respect to the single subcarrier, equation (1) may be rewritten as

$$[S/N]_{o\,BP} = \frac{1}{4} D_c^2 \frac{A^2}{h B_i}\ . \qquad (2)$$

Multiplying equation (2) by B_c, the receiver IF bandwidth, rearranging and inserting the subscript i for the ith subcarrier channel gives

$$\left[\frac{S}{N}\right]_{OBPi} = \left[\frac{1}{2}\frac{f_{dci}^2}{f_{si}^2}\frac{B_c}{B_i}\right]\left[\frac{A^2}{2h B_c}\right]. \qquad (3)$$

The term in the second bracket is the carrier-to-noise ratio in the carrier IF. Therefore, equation (3) may be written as

$$\left[\frac{S}{N}\right]_{OBPi} = \left[\frac{1}{2}\frac{f_{dci}^2}{f_{si}^2}\frac{B_c}{B_i}\right]\left[\frac{S}{N}\right]_c \qquad (4)$$

The carrier-to-noise ratio in the carrier IF, by link design, is set at a minimum of 12 dB where threshold occurs. If the first term is made larger than one, then the signal-to-noise in the subcarrier bandpass filter will be above 12 dB and the onset of threshold will occur later in the subcarrier bandpass filter than in the carrier IF which is desirable if there is transmission bandwidth available. However, if the system is being designed for minimum bandwidth utilization, it is necessary for the two thresholds to occur at the same time, and this will occur if the first bracket term is set equal to one. Equating the first bracket term to one, rearranging and noting that $B_i = 2(f_{dsi}+f_{mi})$ [1] gives

$$f_{dci}^2 B_c = 4 f_{si}^2 (f_{dsi} + f_{mi}). \tag{5}$$

The problem in using equation (5) as the starting point in a design procedure is that it is one equation in two unknowns, f_{dci} and B_c, and B_c is a function of the f_{dci}'s.

The way around this dilemma is to develop an equation relating the two unknowns and solve for one in terms of the other. For multi-tone modulation such as FM/FM it has been found that a good prediction for the necessary carrier IF bandwidth is given by [2],

$$B_c = 2(f_{dn} + f_{sh}) \tag{6}$$

where,

f_{dn} = RMS of the deviations of the carrier by each subcarrier,
f_{sh} = highest frequency subcarrier.

That is,

$$f_{dn} = \sqrt{f_{dc1}^2 + f_{dc2}^2 +++ f_{dcn}^2} . \tag{7}$$

Let

f_{dc1} = the deviation of the carrier by the highest frequency subcarrier,
f_{dc2} = the deviation of the carrier by the next highest frequency subcarrier,
*
f_{dcn} = the deviation of the carrier by the lowest frequency subcarrier.

Let

f_{s1} = highest frequency subcarrier,
f_{s2} = next highest frequency subcarrier,
*
*
f_{sn} = lowest frequency subcarrier.

Normalizing equation (7) with respect to f_{dc1}, gives

$$f_{dn} = f_{dc1} \sqrt{1 + \left(\frac{f_{dc2}}{f_{dc1}}\right)^2 + \left(\frac{f_{dc3}}{f_{dc1}}\right)^2 +++ \left(\frac{f_{dcn}}{f_{dc1}}\right)^2} \qquad (8)$$

Define an intermediate parameter A, [1,2] :

$A_1 = 1$
$A_2 = A_3$
$A_3 = f_{dc3} / f_{dc1}$
*
*
*
$A_n = f_{dcn} / f_{dc1}$. (9)

Using the definitions of equation (9) in equation (8) gives

$$f_{dn} = f_{dc1} \sqrt{A_1^2 + A_2^2 + A_3^2 +++ A_n^2} \qquad (10)$$

Defining A_p such that

$$A_p = \sqrt{A_1^2 + A_2^2 + A_3^2 +++ A_n^2} \; . \qquad (11)$$

Then

$$f_{dn} = f_{dc1} A_p . \qquad (12)$$

The A_i's and hence A_p are in terms of the carrier deviations which are unknown. However, the A_i's may be expressed in terms of known parameters by forming the ratio f_{dci}/f_{dc1} using the equation for the FM/FM output signal-to-noise ratio after solving for f_{dci}. That is, the output signal-to-noise ratio for the i^{th} channel is given by

$$[S/N]_{oi} = \sqrt{3/4}\ \frac{f_{dsi}}{f_{mi}}\ \frac{f_{dci}}{f_{si}}\ \sqrt{\frac{B_c}{f_{mi}}}\ [S/N]_c, \qquad (13)$$

Solving for f_{dci} gives

$$f_{dci} = \sqrt{\frac{4}{3}}\ \frac{f_{si}}{f_{dsi}}\ \sqrt{f_{mi}^3}\ \frac{1}{\sqrt{B_c}}\ \frac{[S/N]_{oi}}{[S/N]_c} \qquad (14)$$

Taking the ratio of f_{dci} to f_{dc1} gives

$$A_i = \frac{f_{dci}}{f_{dc1}} = \frac{\sqrt{4/3}}{\sqrt{4/3}}\ \frac{\dfrac{[S/N]_{oi}}{[S/N]_c}}{\dfrac{[S/N]_{01}}{[S/N]_c}}\ \frac{\dfrac{f_{si}}{f_{dsi}}}{\dfrac{f_{s1}}{f_{ds1}}}\ \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{ml}^3}}. \qquad (15)$$

If the output signal-to-noise ratios in all the channels are to be the same, then equation (15) may be expressed as

$$A_i = \frac{f_{si}}{f_{s1}}\ \frac{f_{ds1}}{f_{dsi}}\ \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{ml}^3}}. \qquad (16)$$

Equation (16) is an important equation since it allows the A_i's to be calculated in terms of known parameters. Equation (12) gives f_{dn}, the norm of the total carrier deviation, in terms of f_{dc1} and A_p which is known. Substituting equation (12) into equation (6) gives

$$B_c = 2[\ f_{dn} + \ f_{s1}] = 2[\ f_{dc1}\ A_p + \ f_{s1}]. \qquad (17)$$

Although f_{dc1} is not known, equation (17) gives B_c in terms f_{dc1} and may be substituted into equation (14), reducing this equation into one equation in one unknown. Making this substitution, rearranging and normalizing gives

$$f_{dc1}^3 + \frac{f_{s1}}{A_p} f_{dc1}^2 - 2\frac{f_{s1}^2(f_{ds1} + f_{ml})}{A_p} = 0. \qquad (18)$$

Equation (18) is a cubic equation in one unknown and may be solved for f_{dc1}. Further, since $f_{dci}=f_{dc1}A_i$, all the f_{dci}'s may be solved. This is the preemphasis schedule.

The design procedure, once a group of subcarriers are selected, is to calculate the A_i's and A_p. In using equation (16) to solve for the A_i's it is necessary to specify the D_{si}'s. Equation (18) is used to solve for f_{dc1} and then the other f_{dci}'s are solved for completing the preemphasis schedule design for concurrent all channel dropout. B_c cannot be reduced further without having the receiver operate in threshold in the subcarrier bandpass filters if the carrier IF is operating at 12 dB.

Impact of the D_{ci}'s

In designing for all channel drop out, regardless of the set of subcarriers chosen, whether using constant or proportional bandwidth subcarriers the signal-to-noise output will be 38.5 dB if the D_{si}'s equal five and the carrier-to-noise ratio is 12 dB. This can be seen by solving equation (5) for B_c and noting $D_{ci}=f_{dci}/f_{si}$ which gives

$$B_c = \frac{4(f_{dsi} + f_{mi})}{D_{ci}^2}. \tag{19}$$

Substituting equation (19) into equation (13) results in

$$[S/N]_{oi} = \frac{3}{4}\, D_{si}^2 \quad D_{ci}^2 \, \frac{4(f_{dsi} + f_{mi})}{D_{ci}^2} \, \frac{1}{f_{mi}} \, [S/N]_C$$

$$[S/N]_{oi} = 3\, D_{si}^2 \, (D_{si} + 1) \quad [S/N]_C . \tag{20}$$

Setting $D_{si} = 5$ and taking 10 log of the result will give 38.5 dB. Equation (20) shows that whenever the first bracket term of equation (4) is set equal to one or 0 dB for the concurrent threshold design procedure, the output signal-to-noise is a function of only D_{si} and the input carrier-to-noise ratio. For example, setting $D_{si} = 1$ gives a $[S/N]_{oi}$ = 16.77 dB, which is an unsatisfactory result. Prudence must be exercised whenever an all channel drop out design is initiated with the D_{si}'s less than five.

Example

A design example will be worked using the five constant bandwidth subcarriers shown in column 1 of table 1. D_{si}'s are set to 5. The computed A_i's are shown in column 5 and the preemphasis schedule is given in column 6.

Table 1

Ch	f_{si} (kHz)	f_{dsi} (kHz)	f_{mi} (kHz)	A_i	f_{dci} (kHz)
111E	896	32	6.4	1.0	221
95E	768	32	6.4	.857	189
79E	640	32	6.4	.714	158
63E	512	32	6.4	.571	126
47E	384	32	6.4	.482	95

All the D_{ci}'s = .246.

The output signal-to-noise in all the channels is computed to be

$$[S/N]_{oi} = 38.53 \text{ dB.}$$

The required bandwidth, B_c and f_{dn} is

$$f_{dn} = 367 \text{ kHz,}$$
$$B_c = 2.5 \text{ Mhz.}$$

Unequal D_{si}'s

Whenever the D_{si}'s are specified to not be equal, it can be shown [1] that the A_i's are given by

$$A_i = \frac{D_{si}}{D_{s1}} \sqrt{\frac{1+D_{si}}{1+D_{s1}}} \; \frac{f_{si}}{f_{s1}} \; \frac{f_{ds1}}{f_{dsi}} \; \frac{\sqrt{f_{mi}^3}}{\sqrt{f_{m1}^3}} . \qquad (21)$$

Further it should be noted that each channel signal-to-noise ratio will be different depending upon the different D_{si}'s.

Summary

Minimum Bandwidth and Concurrent All Channel Drop out Design Procedure

1) Select the group of subcarriers compatible with the sensors.
2) Specify the D_{si}'s for the individual subcarrier channels .
3) Calculate the A_i's.
4) Calculate A_p.
5) Solve for f_{dc1} using the cubic equation (19).
6) Calculate the remaining f_{dci}'s.

7) Calculate the output signal-to-noise ratio in the channels.
8) Calculate the required bandwidth, B_c from equation (14).

[1] Carden, F.F., Telemetry System Design , Boston, MA, Artech House Inc., 1995.
[2] Rosen, C., "System transmission parameters design for threshold performance," Proceedings of the International Telemetering Conference, Vol. XIX, pp. 145-182, 1988.