APPLICATION OF DITHER TO LOW RESOLUTION QUANTIZATION SYSTEMS

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ABSTRACT

A significant problem in the processing chain of a low resolution quantization system is the Analog to Digital converter quantization error. The classical model of quantization treats the error generated as a random additive process that is independent of the input and uniformly distributed. This model is valid for complex or random input signals that are large relative to a least significant bit. But the model fails catastrophically for small, simple signals applied to high resolution quantization systems, and in addition, the model fails for simple signals applied to low resolution quantization systems, i.e. one to 6 bits resolution. This paper will discuss a means of correcting this problem by the application of dither. Two methods of dither will be discussed as well as a real-life implementation of the techniques.

KEYWORDS: A/D conversion, dither, random number generators.

PROBLEMS WITH LOW RESOLUTION QUANTIZATION SYSTEMS

In a low resolution quantization system a major source of spectral related problems in the processing chain is the Analog to Digital (A/D) converter quantization error. The classical model for a quantization system, or A/D converter, is shown in figure 1. The classical model of quantization treats the error generated as a random additive process that is independent of the input and uniformly distributed. This model of quantization error is valid for complex (quasi-random) input signals that are large relative to a least significant bit (LSB). But the model fails catastrophically for small, simple signals applied to high-resolution quantization systems.[1] In addition, the model fails for simple signals applied to low resolution quantization systems.[2] The result of this failure can be seen in figure 2.

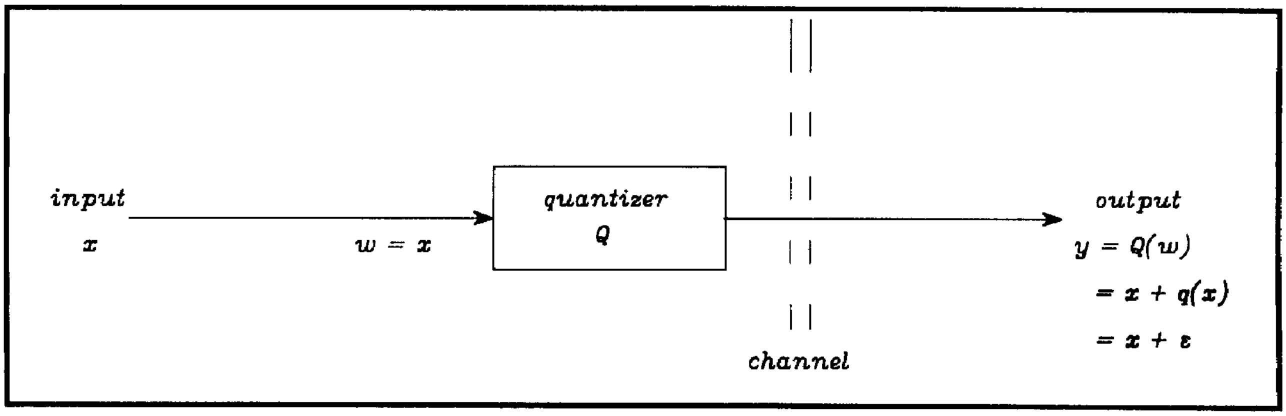

Figure 1. Classical quantizing system. Shown are input x, quantizer input w, error signal , , and system output y.

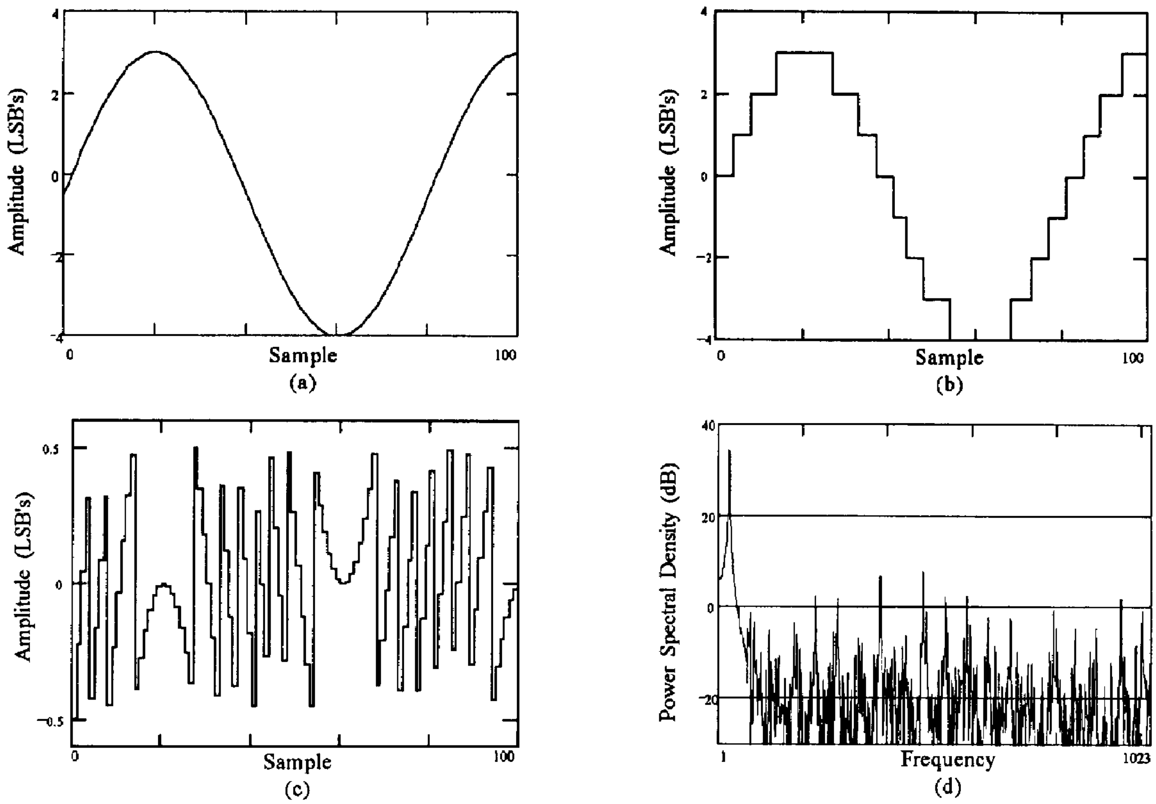

Figure 2. Simulation of a 3-bit quantization system showing, (a) an input signal, (b) the output signal, (c) the resulting quantization error, and (d) the power spectrum of the output.

CORRECTING THE PROBLEM

NONSUBTRACTIVE DITHER

A random, rectangular probability density function (pdf), dither signal with 1-LSB peak-to-peak amplitude is generated. The dither signal is statistically independent of the input and, in an analog approach at its application, is summed with the input signal prior to quantization (A/D conversion). The result is a quantizing system whose total error is uniformly distributed. Figure 3 shows the model for a nonsubtractively dithered quantization system.

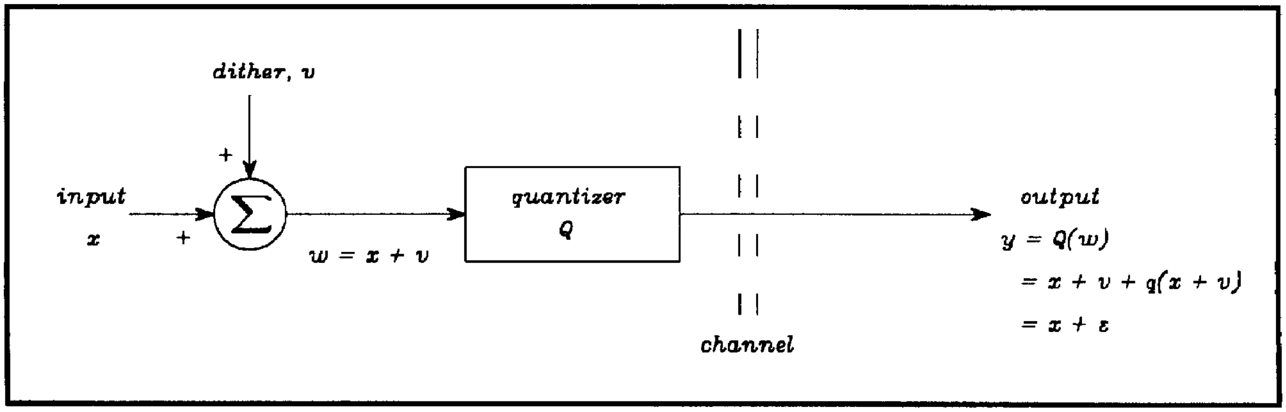

Figure 3. Nonsubtractive dither quantization system. Shown are input x, quantizer input w, dither signal v, error signal ,, and system output y.

Dither randomizes the quantization error signal. This results in a spreading of the spectral energy of the quantization dependent error peaks over a large range of frequencies. The effect of spreading the peak energy signals is a white noise floor, typically 6 to 9 dB lower than the previous highest quantization error component.

An explanation of the effect of dithering is as follows. Assume you have a 2-bit A/D converter, with a zero to 4 volt input range. The voltage range of zero to 1 volt (0.5 volt nominal) equals a binary 00, 1 to 2 volts (1.5 volts nominal) equals 01, 2 to 3 volts (2.5 volts nominal) equals 10, and 3 to 4 volts (3.5 volts nominal) equals 11. Now apply a 1.7-V DC signal without dither. The digital output would be 01, which corresponds to a 11.8 percent error. Now apply the same 1.7 -V DC with a summed random dither signal that varies between ±0.5 volts (±½-LSB or 1 -LSB peak-to-peak). Over a large number of samples, 20 percent of the time the output of the A/D converter would be 10 or 2.5-V, and 80 percent of the time the output would be 01 or 1.5-V. Calculating 20 percent of the difference between 1.5 and 2.5 results in 0.2. Sum 0.2 with the nominal of 1.5 volts and the result is 1.7-V, which is a much more accurate representation of the input signal.

Important notes from the above example are that the random dither signal is assumed to be statistically independent of the input, the dither signal varies at each sample point, and the above example case may be extended to an AC input signal, because DC is a special case of AC. And one final important point, the dither may be added after the A/D conversion, in the digital domain, with comparable results, as long as the dither signal matches the requirements listed above. This allows for a relatively uncomplicated implementation of dither to a low resolution quantization system.

A description of a digitally dithered quantization system follows. Assume the quantization system is required to process information to 3 bits of resolution. A method of implementing this system would be to use a commercial 6-bit A/D converter and use only the three MSB's. This implementation then allows the remaining 3, unused, LSB's to be used for other processing, such as dither. In other words, the 3-bit quantizing system now has a range of 0 to 63 quantization steps, instead of only 0 to 7, and the ±½-LSB count (1-LSB peak-to-peak) dither signal may now be represented by a binary number sequence which ranges from 3 to -4. The two's complement number, representing the random dither signal, can be summed digitally, with the 6-bit quantization result, prior to requantization to 3-bits, thus randomizing the quantization error. The effect of dithering with this method may be seen in figure 4.

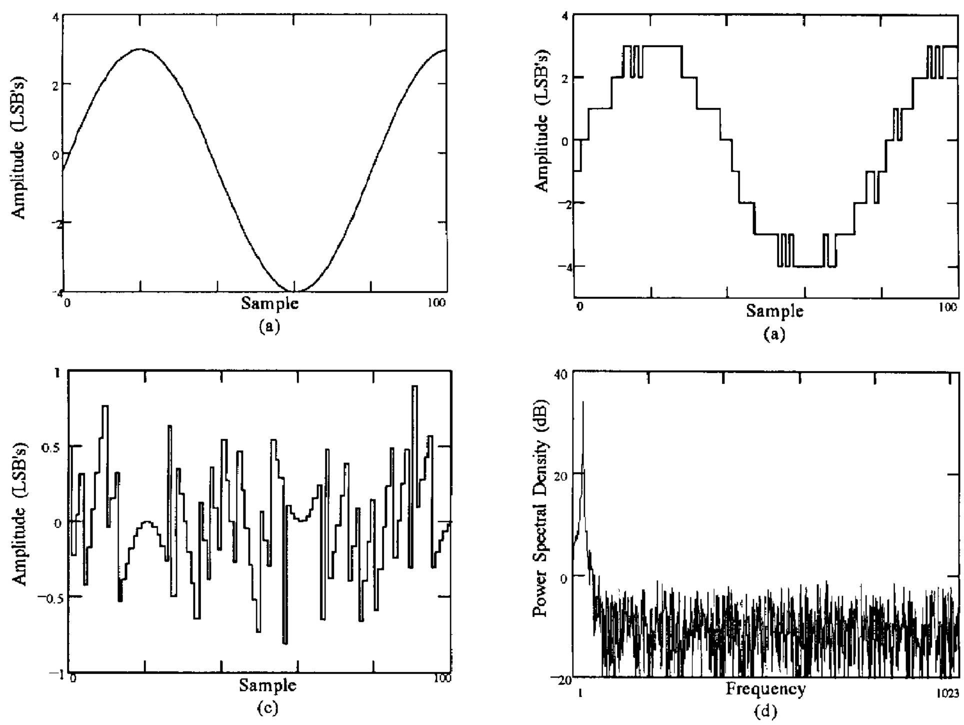

Figure 4. Simulation of a nonsubtractive dithered 3-bit quantization system showing, (a) an input signal, (b) the output signal, (c) the quantization error, and (d) the power spectrum of the output. Note that the peak quantization error is larger, but the peak noise floor in the power spectrum is lower.

Digital dither can be applied to quantization systems with other word sizes. For example, the above describe system can be extended to a one bit quantization system. For the one bit system the dither signal is represented by binary numbers ranging from 15 to -16. The dither number is summed with the 6-bit quantization result and then requantized (truncated) to 1-bit resolution.

It is important to note that digital dither cannot be added to any arbitrary quantized signal. The quantized signal must fulfill the following requirements: (1) The original quantized signal must have a greater resolution than the desired resolution (i.e. greater word size). If dither were added to a quantized signal with zero information in the lower-bit locations, the dither would only increase the apparent noise floor and would not have the desired effect of randomizing the quantization error. (2) The original quantized signal must have a relatively clean noise floor. If the noise floor of the quantized signal is random in nature and is greater than or equal to 1-LSB peak-to-peak, the inherent noise of the system will tend to pre-dither the signal and the application of the dither signal will only increase the resultant noise floor.

RANDOM NUMBER GENERATORS

One problem associated with any dithering scheme is how to generate a statistically independent random number sequence. Maximum length sequences (MLS) using shift registers are inappropriate because the output of the generator has a binary pdf (0 or 1). Although it is possible to use multiple taps from the shift register string, to arrive at a more distributed pdf, it can be shown that each successive sample is strongly correlated.[3] Therefore, the sequence of states is not spectrally white, as is required for proper dithering. A possible, as yet untested, method of eliminating the correlation between successive samples, is to clock the MLS generator at a higher clock rate of N times the required sample rate (where N is greater than or equal to the required dither word size) and then sample the output of the shift register taps every N clocks. This may eliminate the sample to sample correlation, if a suitable clock is available in the system.

A second method of solving this problem is the use of a linear congruential pseudorandom number generator. Linear congruential pseudorandom number generators utilize residue arithmetic to calculate an M length sequence of random numbers, which are then scaled to the required value for optimum dither level (i.e. ±½-LSB). Traditionally, the sequences have been computed with a digital signal processor as part of the signal processing chain. But most applications requiring dither do not have the volume or power allocation for such a processor, therefore a different technique has been devised.

The new technique involves the use of a computer program that calculates the properly scaled, linear congruential pseudorandom number dither sequence and then writes the data sequentially to an erasable programmable read only memory (EPROM) programmer compatible file. The file is then used to program an EPROM of sufficient word length and sequence depth. The data is place sequentially in the EPROM and only a simple counter is required to sequence through the random dither data.

A block diagram of a system using this implementation of digital dither is shown in figure 5.

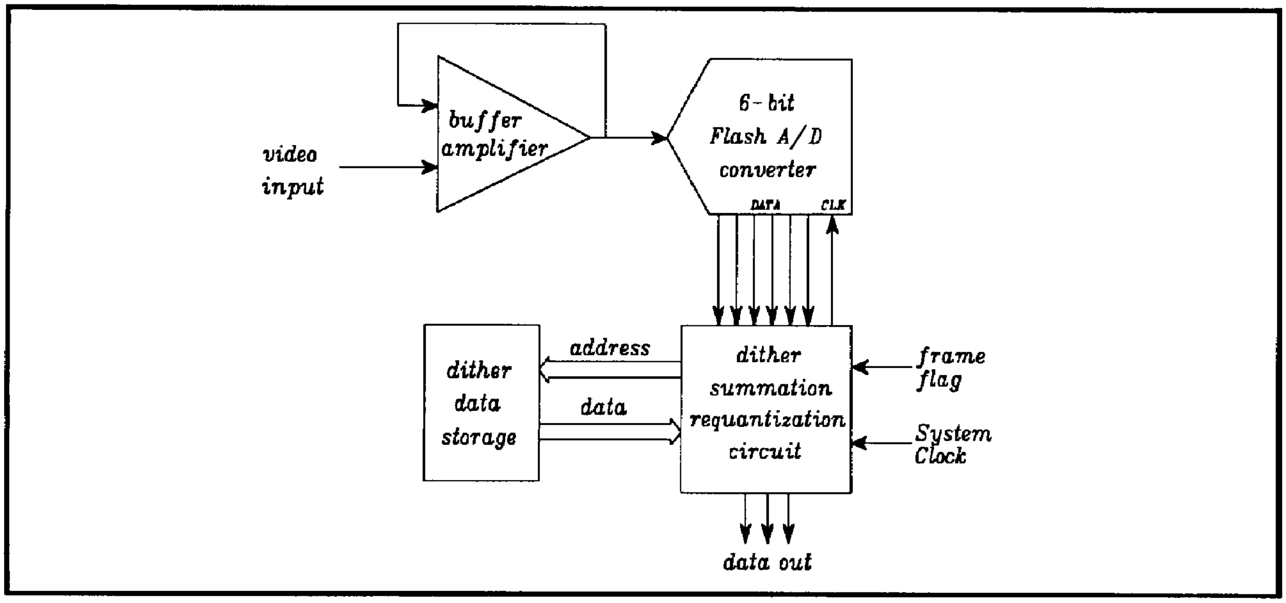

Figure 5. Block diagram of a digital dither system.

SUBTRACTIVE DITHER

A further improvement to a low resolution quantization system, especially as it applies to telemetry is the implementation of subtractive dither. Subtractive dither subtracts the same dither signal that was added to the nonsubtractive quantization system, detailed above, after the data is unpacked at a receiving location. The result is a further reduction of the noise floor, typically another 3 to 4 dB. Figure 6 shows the model for a subtractively dithered quantization system.

The problem in applying subtractive dither to a telemetry system, is that a method must be found to synchronize the additive dither system (at the transmitting location) with the subtractive dither system (at the receiving location). The technique used to implement the nonsubtractive digital dither can also be used to implement subtractive dither. To review the digital dithering method, the value being applied to the dither summation circuit is selected by advancing a counter whose output is connected to the

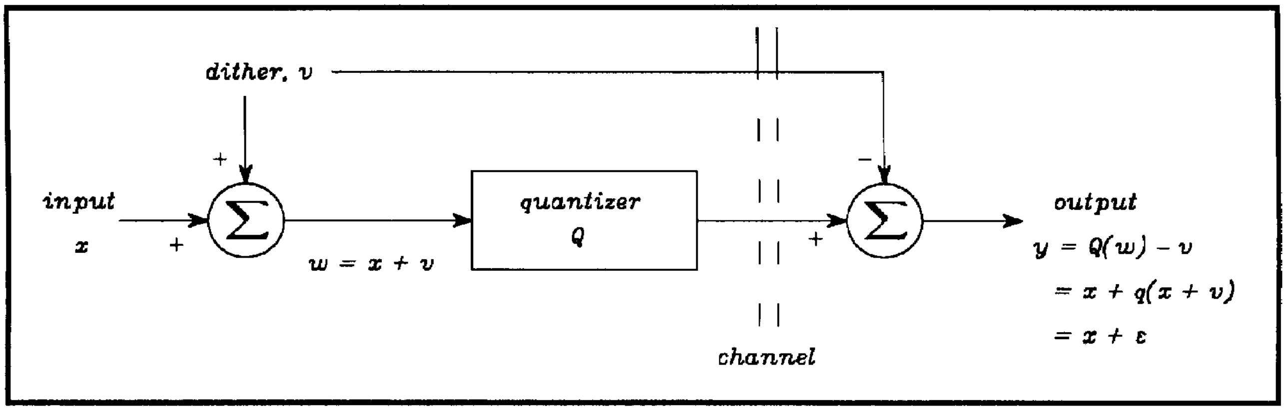

Figure 6. Subtractive dither quantization system. Shown are input x quantizer input w, dither signal v, error signal , , and system output y.

address lines of an EPROM. An analogous system is used in the subtraction system, with a corresponding EPROM containing the subtraction values for the dither signal. Its counter is reset at a coordinated, preselected time, thus synchronizing the two systems.

The preselected synchronization time may be arrived at by several different methods. For example; with most PCM telemetry systems the data is broken up into frames whose beginning or end is indicated by a frame synchronization word. The dither counter may be reset with the frame synchronization word indication in the telemeter. At the receiving station, a decommutation system can derive the exact time of the reset pulse by using the timing of the frame synchronization word. The dither subtraction circuit is then reset and the correct dither value may then be subtracted from the quantized data.

IMPLEMENTATION

A nonsubtractive dithering system has been designed and integrated into the AN/DKT-71 Telemeter, which is used for the Standard Missile Blocks II, III, IIIA, and IV.[4] The AN/DKT-71 Telemeter processes video Doppler information in a selectable resolution of either 3-bits or 1-bits. The video Doppler information is decommutated with a special purpose video Doppler decoder and the result is viewable on a spectrum analyzer.

CONCLUSION

The goals of this paper were to introduce the concept of dithering to the telemetry community and to present a cost-effective, uncomplicated approach to improve the spectral quality of quantization systems without altering the accuracy of the data. The nonsubtractive dither modifications are transparent to, and compatible with existing support equipment. Additional improvement is possible with the implementation of subtractive dither. Very little redesign of most telemetry applications is required to implement any of the techniques outlined in this paper.

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