Probability of Bit Error on a Standard IRIG Telemetry Channel Using the Aeronautical Fading Channel Model

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Abstract

This paper analyzes the probability of bit error for PCM-FM over a standard IRIG channel subject to multipath interference modeled by the aeronautical fading channel. The aeronautical channel model assumes a mobile transmitter and a stationary receiver and specifies the correlation of the fading component. This model describes fading which is typical of that encountered at military test ranges.

An expression for the bit error rate on the fading channel with a delay line demodulator is derived and compared with the error rate for the Gaussian channel. The increase in bit error rate over that of the Gaussian channel is determined along with the power penalty caused by the fading. In addition, the effects of several channel parameters on the probability of bit error are determined.

Key Words

Multipath fading, Delay line demodulator, Probability of bit error, Aeronautical channel.

1 Introduction

The presence of multipath fading on telemetry channels is a problem at test ranges. Because the multipath interference causes burst errors at the receiver, especially at the end of test flights when the vehicle is low on the horizon, multipath fading can cause serious degradation of overall system performance.

The effects of multipath interference have been reported by many authors including [1], [2], [3] , and [4]. [1] and [2] studied the characteristics of the aeronautical channel model without considering a delay line demodulator at the receiver, while [3] and [4] studied the performance of delay line demodulators in the presence of multipath for

MSK communications over the satellite mobile and land mobile channels, but they did not consider the aeronautical channel model.

This paper will analize the performance of the delay line demodulator when used on a standard IRIG telemetry channel subject to fading characterized by the aeronautical fading channel model. Such a demodulator is typical of those used in the telemetry receivers at the test ranges [7]. The effects on the error probability of channel parameters such as fading bandwidth and the ratio of power of the direct path to that of the indirect path (known as the propagation SNR or K-factor) are studied.

The paper proceeds as follows: In section II the channel and system models are described. In section III the development of the expression for the probability of error is presented. The development follows that of [4] and [6]. In section IV the results of the derivation are presented. Plots of the error probability vs. SNR are given and the effects of several channel parameters are shown. In section V conclusions are drawn.

2 System Overview

On the telemetry channels at the test ranges a mobile transmitter communicates with a stationary receiver. The received signal is the sum of a direct ray and a delayed, Doppler shifted indirect ray as illustrated in Figure 1. Such a channel can be modeled with the aeronautical channel model [2]. The indirect ray is the sum of a large number of independent random reflections and hence the magnitude of the reflected ray, $\xi(t)$, is assumed to be a Gaussian random process. The received signal therefore has an envelope which has a Rician distribution [4]. A distinguishing characteristic of the aeronautical channel model is that the autocorrelation of $\xi(t)$ is assumed to have a Gaussian form [1], [3] (see Eq. (3-5)). The receiver used in the derivation models the receivers typically used at military test ranges. The receiver model is shown in Fig. 2. An IF bandwidth equal to the bit rate is assumed [5]. The FM demodulator is a delay line demodulator. The optimum delay for the delay line is dependent on the frequency

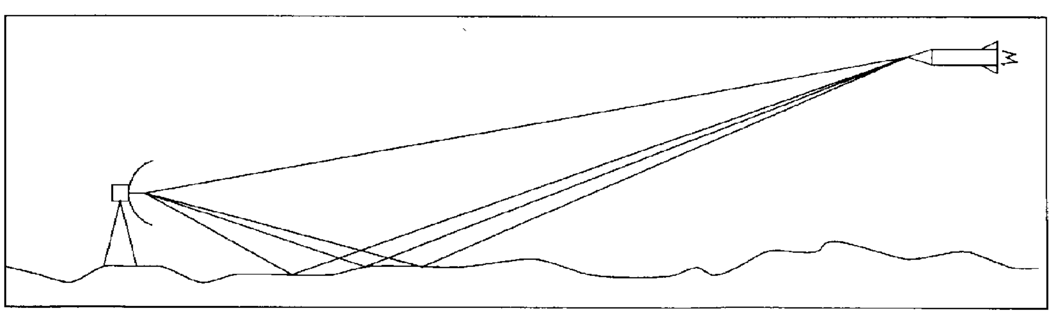
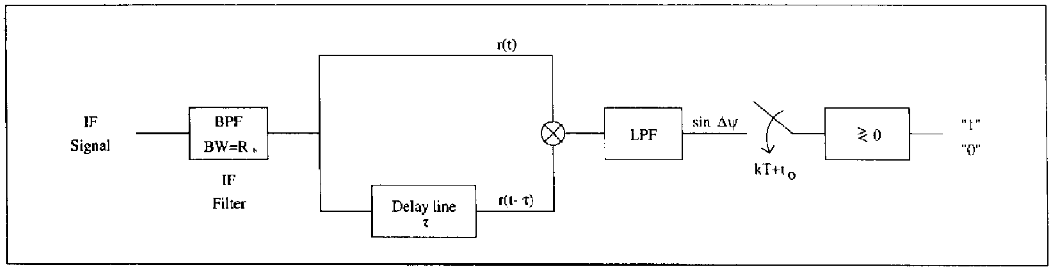


Figure 1: Geometry of the Aeronautical Channel Model.

of the modulator and on the bit rate [6]. A peak-to-peak frequency deviation of h = .7R_b (where R_b is the bit rate) is commonly used [5] and will be assumed in this paper. The delay τ which optimizes the performance of the demodulator for this value of h is .7T where T is the bit time. This value of τ will be used in the numerical results that follow.



igure 2: Delay Line Demodulator.

3 Development

In PCM/FM the transmitted signal can be written as

$$x(t) = \cos(2\pi\ f_c t + 2\ (t)) \tag{3-1}$$

where

$$\theta(t) = \frac{\pi h}{T} \int_{-\infty}^{t} a(t)dt \tag{3-2}$$

and a(t) is the binary data sequence. The received signal after the IF filter is the sum of the direct component, the indirect component, and the receiver noise and can be written as [4]

$$r(t) = \sqrt{2P_s}S(t) + \sqrt{P_d}S(t - t_d)\xi(t)\exp(j2\pi f_D t) + n(t) \tag{3-3}$$

where P_s and P_d are the average powers in the direct and indirect components respectively. f_D is the relative Doppler frequency, t_d is the time delay of the indirect component relative to the direct component, and

$$S(t) = x(t) * h(t) = A_s(t)\ \exp(jN_s(t)). \tag{3-4}$$

h(t) is the low pass equivalent impulse response of the IF filter. The bandwidth of >(t)exp(j2πf_Dt) is assumed to be much less than the bandwidth of the IF filter.

The process ξ(t) is a zero mean, normalized Gaussian random process with autocorrelation [3]

$$R_\xi(\tau) = \gamma_\xi(\tau) = \exp\left(-[\pi B_D \tau]^2\right) \tag{3-5}$$

where B_D is the fading bandwidth. The additive noise n(t) is zero mean, Gaussian noise with autocorrelation

$$R_n(\tau) = N_o \int_{-\infty}^{\infty} |H(f)|^2 \exp(j2\pi f\tau)df \tag{3-6}$$

and normalized autocorrelation

$$\gamma_n(\tau) = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \exp(j2\pi f\tau)df}{\int_{-\infty}^{\infty} |H(f)|^2 df} \tag{3-7}$$

where H(f) is the Fourier transform of h(t).

The resulting signal-to-noise ratio is

$$\text{SNR} = \frac{P_s + P_d}{P_n} \tag{3-8}$$

and the propagation SNR or K-factor is defined as

$$K = \frac{P_s}{P_d}. \tag{3-9}$$

Equation (3-3) can be written as

$$r(t) = R(t)\cos(2\pi f_c t + N(t) + O(t)) \tag{3-10}$$

where

$$\phi(t) = \tan^{-1}\left(\frac{\int_{-\infty}^{t} h(t-\tau)\sin\theta(\tau)d\tau}{\int_{-\infty}^{t} h(t-\tau)\cos\theta(\tau)d\tau}\right) \tag{3-11}$$

$$\eta(t) = \tan^{-1}\left(\frac{n_s(t) + \frac{\xi(t)}{\sqrt{N_0}}\sin V}{\sqrt{2\alpha(t)} + n_c(t) + \frac{\xi(t)}{\sqrt{N_0}}\cos V}\right) \tag{3-12}$$

$$R(t) = \sqrt{\left(\sqrt{2\alpha(t)} + n_c(t) + \frac{\xi(t)}{\sqrt{N_0}}\cos V\right)^2 + \left(n_s(t) + \frac{\xi(t)}{\sqrt{N_0}}\sin V\right)^2} \tag{3-13}$$

$$\alpha(t) = \frac{E_b}{N_0}\left(\frac{a^2(t)}{T\int_{-\infty}^{\infty} |H(f)|^2 df}\right) \tag{3-14}$$

$$a^2(t) = \left(\int_{-\infty}^{t} h(t-\tau)\cos\theta(\tau)d\tau\right)^2 + \left(\int_{-\infty}^{t} h(t-\tau)\sin\theta(\tau)d\tau\right)^2 \tag{3-15}$$

$$V = 2\pi f_D t + 2\pi(f_c + f_D)t_d + \theta(t_d). \tag{3-16}$$

In the above equations, $n_c(t)$ and $n_s(t)$ are the in-phase and quadrature components of the narrow-band noise and have zero mean and unit variance, $\alpha(t)$ is the instantaneous SNR and $E_b = P_s T$ is the energy contained in one bit.

The phase difference ΔR between r(t) and r(t - J) is sampled at time $t_k = kT + t_0$ and the decision variable of the delay line demodulator is [6]

$$\sin \Delta R = \sin(\Delta N - \Delta O) \tag{3-17}$$

where

$$\Delta N = N(t_k) - N(t_k - J) \tag{3 18}$$

and

$$\Delta O = O(t_k) - O(t_k - J). \tag{3- 19}$$

If sin ΔR is positive then the demodulator puts out a 1, otherwise it puts out a 0. If it is assumed that a 1 was sent, then the probability of error is

$$P(E|1) = Pr(-\pi < \Delta R < 0). \tag{3-20}$$

This probability was determined in the absence of fading to be [8]

$$Pr(-\pi < \Delta\psi < 0) = \begin{cases} F(0) - F(-\pi) + 1 & 0 < \Phi_s < \pi \\ F(0) - F(-\pi) & \text{else} \end{cases} \tag{3-21}$$

where $\Phi_s = N s(t_k) - N s(t_k - J\text{-})$ is the phase difference between the signal at time t_k and $t_k - \tau$ in the absence of noise and

$$F(\phi) = \frac{1}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp\left[-WE(\phi, \Phi_s, \theta)\right] I(\phi, \Phi_s, \theta) d\theta \tag{3-22}$$

where

$$E(\phi, \Phi_s, \theta) = \frac{\rho_+ - \rho_- \sin\theta - \cos(\Phi_s - \phi)\cos\theta}{1 - A_\gamma \cos(\phi_\gamma - \phi)\cos\theta} \tag{3-23}$$

$$I(\phi, \Phi_s, \theta) = \frac{\sin(\Phi_s - \phi)}{\rho_+ - \rho_- \sin\theta - \cos(\Phi_s - \phi)\cos\theta} - \frac{A_\gamma \sin(\phi_\gamma - \phi)}{1 - A_\gamma \cos(\phi_\gamma - \phi)\cos\theta} \tag{3-24}$$

$$\rho_\pm = \frac{0.5[\rho(t_k) \pm \rho(t_k - \tau)]}{[\rho(t_k)\rho(t_k - \tau)]^{\frac{1}{2}}} \tag{3-25}$$

$$W = [\rho(t_k)\rho(t_k - \tau)]^{\frac{1}{2}} \tag{3-26}$$

$$\rho(t) = \frac{P_s A_s^2(t)}{P_n} \tag{3-27}$$

$$\gamma = \gamma_n \tag{3-28}$$

where $A_{(}$ and $N_{(}$ are the envelope and phase of the autocorrelation γ, [4] showed that the above result can be used for the multipath case if the sum of the receiver noise and the multipath interference is treated as one Gaussian random process μ(t). The signal to noise ratio p and the noise autocorrelation γ above must then be changed to the following:

$$\begin{aligned} \rho(t) &= \frac{P_s A_s^2(t)}{P_\mu(t)} = \frac{\mathrm{SNR} K A_s^2(t)}{\mathrm{SNR} K A_s^2(t-t_d)+1-K} \\ \gamma &= \gamma_\mu(\tau, t_k) = \frac{R_d(\tau, t_k) + P_n \gamma_n(\tau)}{[P_\mu(t_k) P_\mu(t_k-\tau)]^{\frac{1}{2}}} \end{aligned} \tag{3-29}$$

$$= \frac{\mathrm{SNR} A_s(t_k-t_d) A_s(t_k-t_d-\tau)\gamma_\xi(\tau)\exp[j(2\pi f_D \tau + \Phi_{sd})] + (1+K)\gamma_n(\tau)}{[\mathrm{SNR} A_s^2(t_k-t_d)+1+K]^{\frac{1}{2}}[\mathrm{SNR} A_s^2(t_k-t_d-\tau)+1+K]^{\frac{1}{2}}} \tag{3-30}$$

where

$$\Phi_{sd} = N_s(t_k - t_d) - N_s(t_k - t_d - J). \tag{3-31}$$

In order to compute the average probability of bit error it is necessary to take into account the effects of intersymbol interference (ISI) caused by the IF filter. As in [6] it will be assumed here that only the adjacent bits interfere with the desired bit. The average probability of bit error is then

$$\begin{aligned} PE &= \frac{1}{4} Pr(-\pi \le \Delta\psi < 0|010) + \frac{1}{4} Pr(-\pi \le \Delta\psi < 0|011) \\ &+ \frac{1}{4} Pr(-\pi \le \Delta\psi < 0|110) + \frac{1}{4} Pr(-\pi \le \Delta\psi < 0|111). \end{aligned} \tag{3-32}$$

4 NumericalResults

In this section the average probability of error is plotted for a third order Butterworth IF filter. A relative delay t_d of .025T was assumed for all plots. The optimum delay for the delay line demodulator and the optimum sampling time were also assumed.

Figure 3 shows the effect that different values of K have on PE. As seen in the figure, the propagation SNR is an important parameter in determining the probability of bit error. As K decreases (ie. as the relative power in the indirect component increases) PE increases significantly. On the other hand, when K = ∞ no fading is present and the resulting probability of bit error is that of the Gaussian channel.

Figures 4 and 5 show how the fading bandwidth, B_D, and the relative Doppler shift, $f_D T$, effect PE. As each of these parameters increases the effect of a floor in PE becomes evident. Because of this floor the probability of bit error cannot be made arbitrarily low by increasing the SNR when fading is present.

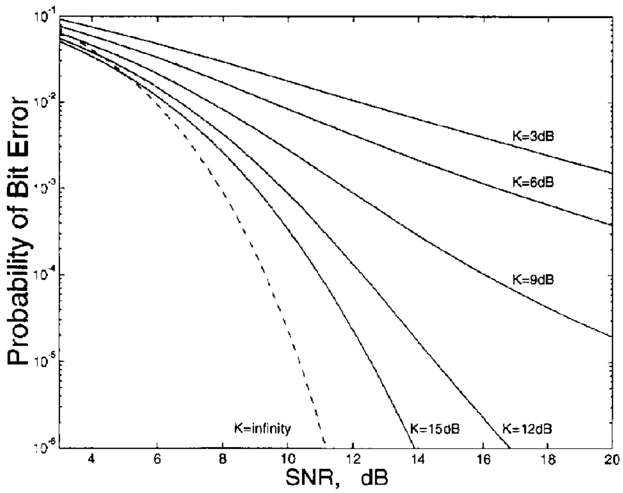


Figure 3: The effect of the propagation SNR on PE (fading bandwidth $B_DT = .01$, $f_DT = 0$).

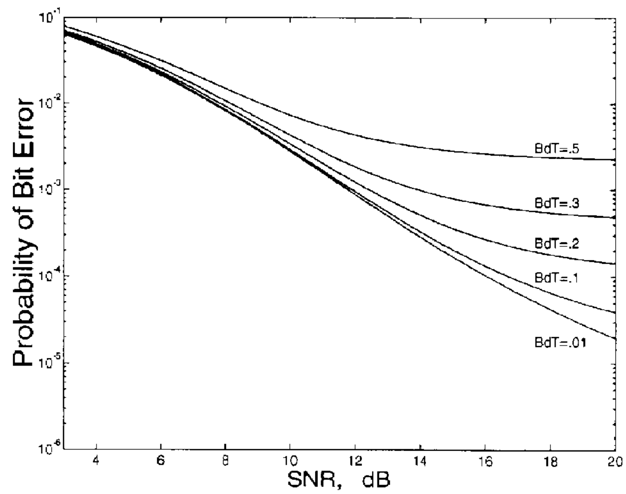


Figure 4: The effect of normalized fading bandwidth B_DT on PE ($K = 9\text{dB}$, $f_DT = 0$).

Figure 6 shows the combined effects that the three parameters K, B_DT, and f_DT have on PE. The figure contains plots representing various levels of fading including the unfaded case (K = ∞). Note that even for relatively light fading (K = 9dB, $B_DT = .1$, $f_DT = .01$) there is about a 6dB penalty at PE= 10^{-6} compared to the unfaded Gaussian channel. For severe fading (K = 3dB, $B_DT = .5$, $f_DT = .2$) PE is between .1 and .2 or all SNR shown.

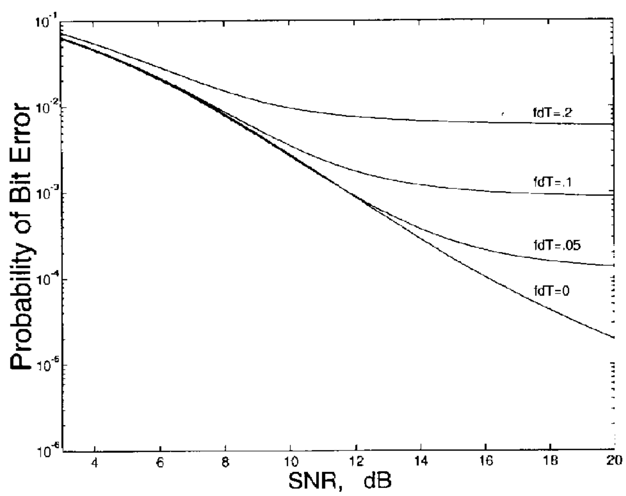


Figure 5: The effect of the relative Doppler shift f_DT on PE ($K = 9\text{dB}$, $B_DT = .01$).

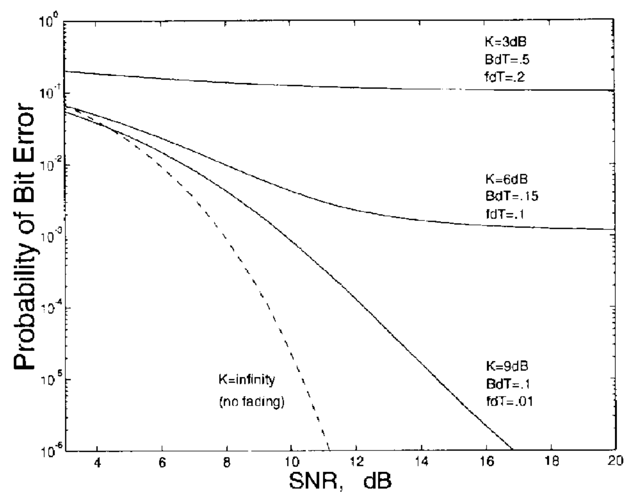


Figure 6: PE for various levels of fading from no fading to severe fading.

5 Conclusion

The above plots show that multipath fading can significantly degrade system performance for telemetry channels at test ranges. The floor in the probability of bit error when K is small or when $B_D T$ or $f_D T$ are large suggests the need for additional measures to counteract the effects of multipath fading. Techniques such as space, frequency, or time diversity might be used in telemetry systems to alleviate the reduction in performance caused by multipath interference.

References

[1] P. A. Bello, "Aeronautical channel characterization," IEEE Trans. Commun., vol. COM-21, pp. 548-563, May 1973.

[2] John H. Painter, Someshwar C. Gupta, and Lewis R. Wilson, "Multipath Modeling for Aeronautical Communications," IEEE Trans, Commun., vol. COM-21, pp. 658-662, May 1973.

[3] Lloyd J. Mason, "Error Probability Evaluation for Systems Employing Differential Detection in a Rician Fast Fading Environment and Gaussian Noise," IEEE Trans. Commun., vol. COM-35, pp. 39-46, Jan. 1987.

[4] Israel Korn, "Error Probability of Digital Modulation in Satellite Mobile, Land Mobile, and Gaussian Channels with Narrow-Band Receiver Filter," IEEE Trans, Commun., vol. COM-40, pp. 697-707, Apr. 1992.

[5] E. L. Law and D. R. Hurt, Telemetry Applications Handbook, Pacific Missile Test Center, Point Mugu, CA Technical Publication TP000044, Sept. 1987.

[6] K. J. P. Fonseka and N. Ekanayake, "Differential Detection of Narrow-Band Binary FM," IEEE Trans. Commun., vol. COM-33, pp. 725-729, July 1985.

[7] Warren Price, Microdyne, Private communication, March 24, 1994.

[8] R. F. Pawula, S. O. Rice, and J. H. Roberts, "Distribution of the Phase Angle Between Two Vectors Perturbed by Gaussian Noise," IEEE Trans. Commun., vol. COM-30, pp. 1828-1841, Aug. 1982.