Optimum Symbol Synchronization

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Abstract

Although most closed-loop synchronizers employ maximum likelihood estimators for symbol value decisions, in general, their symbol timing estimates are not optimum. It would seem only natural that an optimum timing estimator would choose interval partitions based on maximizing the observed sample signal-to-noise ratio. The symbol synchronizer described below achieves optimum performance when decisions on present symbol values are based on current and previously-received symbol samples. This procedure attempts to reestablish the interval independence criterion, thereby reducing timing estimator variance. The realization presented is motivated by an open-loop maximum a posteriori (MAP) structure analysis.

Introduction

When symbol transition time is known precisely, the maximum signal-to-noise power is obtained utilizing a match filter. It is well known that a filter matched to the transmitted waveform minimizes the probability of error through maximizing the observed sampled signal-to-noise power. By evaluating estimation strategies utilized for symbol synchronization, the concept of interval independence eliminates the necessity to consider the value of the previously received symbols. This important criterion states that the symbol value transmitted in each interval be statistically independent. However, during the symbol recovery process, it is only by making the assumption that interval timing be known a priori that the symbol error performance be divorced from interval time or timing error. This suggests that the optimum symbol recovery process include not only a match filter for symbol value detection, but an optimum interval timing estimation process. How to invoke such an estimation process provides the motivation for this paper.

In the case of a Phase Shift Keyed (PSK) modulated signal, two different symbol types are used to generate the signaling waveform. This waveform in the presence of additive white gaussian noise has a bit-error performance degradation due to timing misalignment given by

$$P_e = pQ\left(\sqrt{\frac{2E}{N_0}}(1-2\xi)\right)+(1-p)Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

where

$$Q(x) = \int_{\mu/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

$$x = \sqrt{\frac{\mu}{\sigma^2}} = \frac{\sqrt{E}}{\sqrt{\frac{N_0}{2}}} = \sqrt{\frac{2E}{N_0}}$$

$$\xi = \frac{\theta}{T}$$

θ = timing misalignment

T = bit period and

p = the probability of a transition

The probability of a bit error in the expression above is interpreted as the probability of error given a transition with timing misalignment θ, plus the probability of error given no transition. Note, when no transition occurs, the probability of error remains unaffected by the timing misalignment parameter θ. The effect of timing misalignment on bit-error probability is illustrated in the figure below with the probability of a transition equal to one-half i.e., p=1/2.

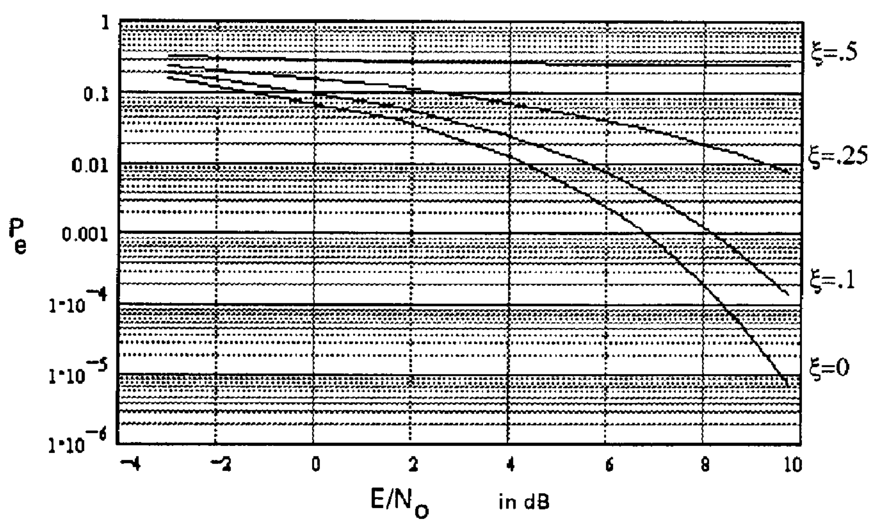

Figure 1. Bit-Error Probability versus E/N_0 (Bit Energy to Noise Density.

The Bayesian Estimation Approach

Given n measurements $z=(z_1,z_2,......,z_n)$, with a conditional a posteriori probability density function $p(\theta \mid z)$, and a cost function $C(\theta(z),\theta)$, the conditional risk incurred by using the estimator $\theta(z)$, when the true parameter value being estimated is θ, is denoted by

$$R(\theta|\mathbf{z}) = \int C(\theta(\mathbf{z}),\theta)p(\theta|\mathbf{z})d\theta$$

By averaging with respect to the a priori probability density function $p(z)$ the conditional risk $R(\theta \mid z)$, the average risk $C(\theta(z))$ per experiment is obtained.

$$C(\theta(\mathbf{z})) = \int R(\theta(\mathbf{z})|\theta)p(\mathbf{z})d\mathbf{z} = \int p(\mathbf{z})\left[\int C(\theta(\mathbf{z}),\theta)p(\theta|\mathbf{z})d\theta\right]d\mathbf{z}$$

An estimator $\theta(z)$ which minimizes the average risk is known as a Bayes estimate. Since the a priori probability density function is positive over the range of value of z, the average risk $C(\theta(z))$ can be minimized by minimizing the conditional risk, $R(\theta \mid z)$. It can be shown that when the cost function assigns a uniformed value for an error magnitude which exceeds some fixed threshold (otherwise it assigns a zero cost) the problem of minimizing the average risk reduces to maximizing the a posteriori probability density function, $p(\theta|z)$. The maximum a posteriori (MAP) estimate of θ is that θ which maximizes $p(\theta|z)$. The derivation of the conditional a priori probability density function $p(z \mid \theta)$ is fairly straight forward. Utilizing Bayes rule permits representing the conditional a posteriori probability as a function of the conditional a priori probability; i.e.,

$$p(\theta|\mathbf{z}) = \frac{p(\mathbf{z}|\theta)p(\theta)}{p(\mathbf{z})}$$

where

$$p(\mathbf{z}) = \int_{\theta} p(\mathbf{z}|\theta)p(\theta)d\theta$$

Note, here the a priori density $p(\theta)$ is a uniformly distributed for $\theta \in (0,T)$. From [1] we find that a PSK modulated waveform immersed in additive white gaussian noise has an a posteriori probability density given by

$$p(\theta|\mathbf{z}) = \frac{\prod_{j=1}^{L} \cosh\left[\frac{2}{N_o}\int_{\theta+(j-1)T}^{\theta+jT} z(t)s(t,\theta)dt\right]}{\int_{\theta[0,T]}\prod_{j=1}^{L} \cosh\left[\frac{2}{N_o}\int_{\theta+(j-1)T}^{\theta+jT} z(t)s(t,\theta)dt\right]d\theta}$$

Here s(t,θ) represents the modulated waveform and z(t) represents the observed process. Since θ is integrated out in the denominator of this equation it only remains that the numerator be maximized; that is,

$$\max_{\theta\in[0,T]} l(\theta) = \prod_{j=1}^{L} \cosh\left[\frac{2}{N_o}\int_{\theta+(j-1)T}^{\theta+jT} z(t)s(t,\theta)dt\right]$$

Since the logarithm of l(θ) is more suitable for implementation and its monotonic behavior guarantees equivalence results, we can now write

$$\max_{\theta\in[0,T]} \Lambda(\theta) = \max_{\theta\in[0,T]} \ln l(\theta) = \sum_{j=1}^{L} \cosh\left[\frac{2}{N_o}\int_{\theta+(j-1)T}^{\theta+jT} z(t)s(t,\theta)dt\right]$$

The basic idea of the MAP implementation shown below is to choose $\theta=\theta_j\in\{\theta_1,\theta_2,.......\theta_m\}$ such that $\Lambda(\theta_j)$ is maximized. The θ which maximizes Λ(θ) is the Bayes estimate. It should also be noted that this implementation employs no feedback. Some authors [2] refer to this realization as an open-loop synchronizer. Here, a summation over L bit times takes place before a decision as to which θ yielded the largest Λ(θ).

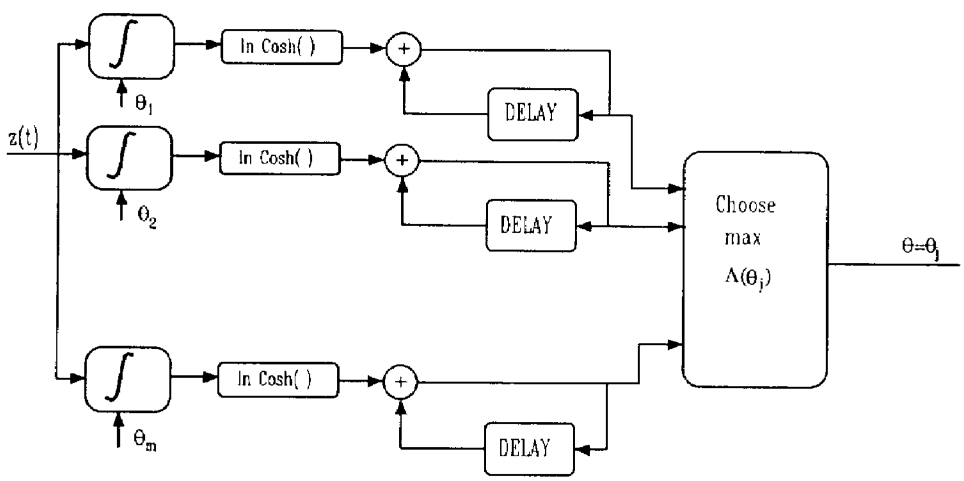

Figure 2. A MAP Estimator Realization (Closed-Loop Synchronization).

Close-Loop Implementation

For burst transmissions, the open-loop implementation will provide the optimum recovered sequence when L is at least as great as the symbol sequence length. Here it is assumed that a means of detecting data presence is incorporated into this realization. However, when dealing with a continuous symbol sequence, there needs to be some means of concatenating L-symbol segments in such a way as to guarantee that the segments will not overlap. Additionally, all symbols contained in the continuous sequence would be mapped into disjointed L-symbol segments.

Even if the initial misalignment parameter θ is known, the timing relationship between the symbol sequence generator and the symbol recovery process would in time drift apart due to the clock stability among the two processes. Thus, the misalignment parameter θ changes with time. This implies some continuous adjustment in timing be employed to prevent skipping or repeating symbols by the recovery process.

The newly-proposed closed-loop realization uses the largest computed power metric, $\Lambda(\theta)$ to provide timing feedback control. When processing baseband non-return-to-zero (NRZ) symbols, a finite impulse response (FIR) filter using constants coefficients performs the realization of a digital integrator. Note, output symbol estimates for each symbol phase $\theta \in \{\theta_1, \theta_2, ...\theta_m\}$ are obtained at the sample rate. For each of the m phases, an associated partitioning of L symbols is obtained. Additionally, with each of the m partitionings of length L, we can associate a metric which represents the power contained in the L symbol sequence. The expression $\Lambda(\theta)$ is termed the power metric, although it provides an indication. The optimum symbol recovery process with closed-loop timing controls are summarized in the paragraph that follows.

After L-symbols (i.e., mL samples) have been processed, an initial timing phase is chosen corresponding to the largest computed $\Lambda(\theta)$. Once m additional samples are obtained, the largest computed metric $\Lambda(\theta)$ is used to determine in which direction the timing phase moves. The timing phase is only allowed to advance or retard one phase increment or remain at the same phase. This means the oldest symbol value from a L-symbol partitioning, corresponding to the largest computed power metric $\Lambda(\theta)$ is generated during phase j=m-1,0, or 1. Additionally, the power metric $\Lambda(\theta)$ computation is based on the current symbol sample and the previous L-l symbols. For each cycle of m samples (i.e., one symbol time), the oldest symbol is deleted from the symbol memory and that symbol's contribution to the power metric $\Lambda(\theta)$ computation is removed .

Summary

We started off by demonstrating that unless the received symbol sequence timing is known precisely, a degradation in bit-error performance would result. By following closely the work of [1], a MAP estimate of the timing misalignment parameter θ could be obtained. Additionally, if we assume this misalignment parameter θ to have a priori probability density $p(\theta)$ which is uniformly distributed for $\theta \in (0,T)$, a

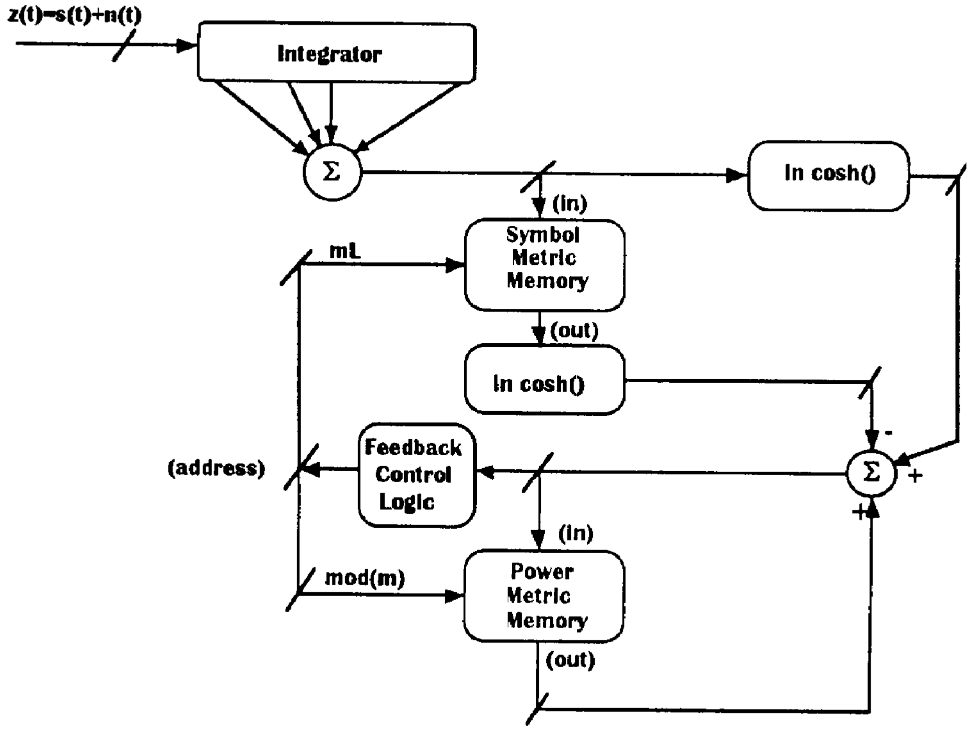

Figure 3. Closed-Loop Implementation of a MAP Estimator.

conditional a posteriori probability could be obtained which when maximized provides an open-loop realization of the optimum symbol synchronizer. The open-loop realization chooses the optimum partitioning of L-symbol samples. Using the computed power metric associated with the L-symbol segment, we can derive a closed-loop realization which continuously computes a new L-symbol segment for each of the m samples; I. e., a new L-symbol segment is computed each symbol time. The oldest symbol value from each L-symbol segment is used as the output.

References

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