PCM/FM+FM/FM Design Parameters for Telemetering Systems

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Abstract

In a PCM/FM+FM/FM system, the PCM data is added to the subcarriers at baseband and the composite signal is modulated onto the carrier. When the subcarrier messages are demodulated, part of the PCM signal's spectrum falls within the bandwidth of the subcarrier bandpass filters. This causes interference with the subcarrier messages, particularly those of the lower subcarrier frequencies. When designing a PCM/FM+FM/FM system, one is concerned with the placement of the subcarrier frequencies and the interference suffered by the subcarriers due to the PCM signal. This paper develops a relationship between the lowest frequency subcarrier, PCM bit rate and the resulting interference. The design procedure allows a bit rate or lowest frequency subcarrier to be selected for a specified interference ratio. The expression of the ratio is a complex integral which is reduced to a simple equation involving the system parameters.

Introduction

The PCM/FM+FM/FM system involves a bit stream of nonreturn to zero format (Figure 1). The system sums an input bit stream with n modulated subcarriers, then

FM modulates the sum onto the carrier. After carrier demodulation on the receiver end, the PCM/FM+FM/FM signal's power spectrum is similar to the one in Figure 2.

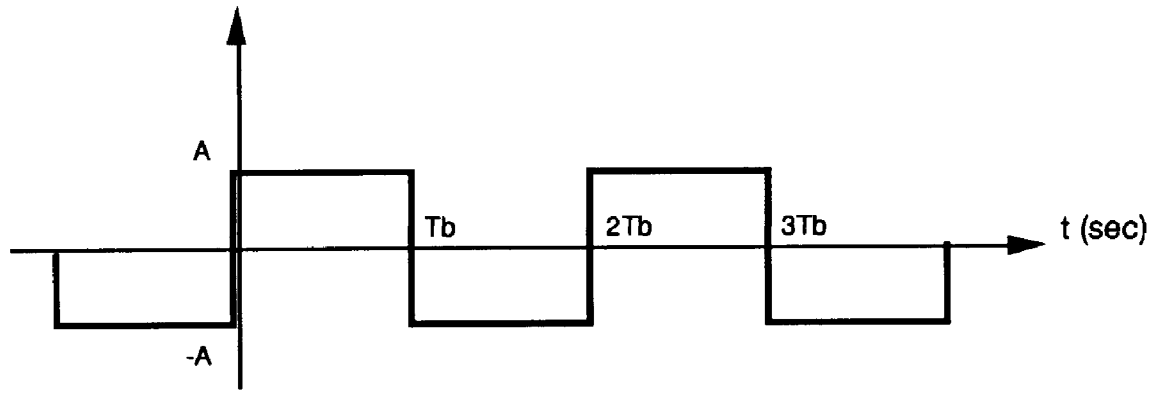

Figure 1. PCM Bit Stream, NRZ-L Format

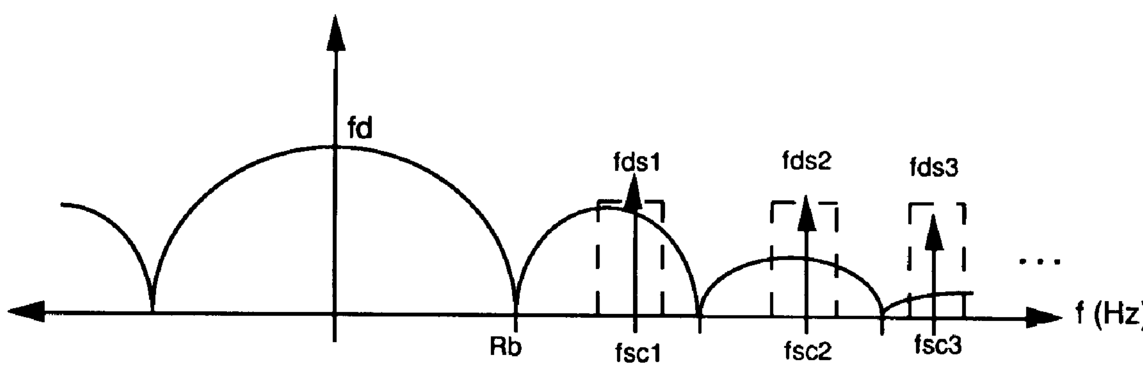

Figure 2. Power Spectral Density after Carrier Demodulation

The n subcarrier messages are filtered by n bandpass filters, where the nth filter is set to pass the nth modulated subcarrier prior to demodulation. The dashed lines in Figure 2 show that the bandpass filters also pass a portion of the bit stream's power spectrum. The presence of the bit stream's power will affect the subcarrier message, particularly the lowest frequency subcarrier, but the designer of the PCM/FM+FM/FM system may choose a bit rate and subcarrier placement that lessens the interference.

Design Procedure

This design procedure assumes a random distribution in the bit stream. Also, the lowest frequency subcarrier message is the most affected. This procedure will adjust the interference of this message. The other subcarrier messages' interference will be as good as or better than the design interference, assuming the subcarrier bandpass filters are IRIG constant bandwidth.

The bit stream will interfere with the lowest subcarrier message because a portion of the PCM spectrum falls in the bandwidth of the message's bandpass filter. The interference power in watts is

$$P_{pcm} = \frac{2f_d^2T_b^2}{T_b}\int_{f_1}^{f_2} Sinc^2(fT_b)|H_{pm}(f)|^2|H_{bp}(f)|^2 df \tag{1}$$

f_d--peak deviation of bit stream by carrier
T_b--period of bit stream
$Sinc^2(fT_b)$--power spectral density of the PCM signal before filtering
$H_{pm}(f)$--frequency response of bit stream's premodulation filter
$H_{bp}(f)$--frequency response of nth bandpass filter
f_2-f_1--bandwidth of first bandpass filter ($f_1 < f_{sc1} < f_2$)

Note that this expression of the power is the integration of the power spectral density of the bit stream, after it is filtered, modulated and demodulated, over the bandwidth of the nth subcarrier bandpass filter. It is multiplied by 2 to capture both sides of the spectral density.

The power of the message on the lowest frequency subcarrier is

$$P_{sc1} = \frac{f_{ds1}^2}{2} \tag{2}$$

where f_{ds1} is the peak deviation of the message by the lowest frequency subcarrier. The ratio of these powers is the interference power to carrier power ratio which will affect the demodulation of the subcarrier. This ratio, in dB, is

$$R = 10\log\left[\frac{P_{pcm}}{P_{sc1}}\right] \text{dB}$$

$$= 10\log\left[\left[\frac{2}{f_{ds1}^2}\right](2f_d^2T_b)\int_{f_1}^{f_2} Sinc^2(fT_b)|H_{pm}(f)|^2|H_{bp}(f)|^2 df\right] \text{dB.} \tag{3}$$

The design procedure uses this ratio expressed as a sum of terms that are now developed. Recalling that $T_b = 1 / R_b$ and grouping the peak deviations together gives

$$R=10\log\left[\frac{2(2f_d^2)}{f_{ds1}^2 R_b}\right] + 10\log\left[\int_{f_1}^{f_2} Sinc^2(fT_b)|H_{pm}(f)|^2|H_{bp}(f)|^2 df\right] \tag{4}$$

$$= 20\log\left[\frac{2f_d}{f_{ds1}\sqrt{R_b}}\right] + 10\log\left[\int_{f_1}^{f_2} Sinc^2(fT_b)|H_{pm}(f)|^2|H_{bp}(f)|^2 df\right] \tag{5}$$

Design Approximation

The second term of Equation 5 is of interest. If the placement of f_{sc1} corresponds with a part of the spectrum of the bit stream that does not vary greatly with frequency and if the bandwidth of the subcarrier bandpass filter is narrow compared to the PCM bit rate, the integration of $f_d Sinc^2(fT_b)$ over the bandwidth of the filter will be approximately equal to $f_d Sinc^2(fT_b)$ evaluated at $f=f_{sc1}$ and multiplied by the filter's bandwidth (See Figure 3).

$$f_d^2 \int_{f_1}^{f_2} Sinc^2(fT_b)df = f_d^2 Sinc^2(f_{sc1}T_b)(f_2-f_1) \tag{6}$$

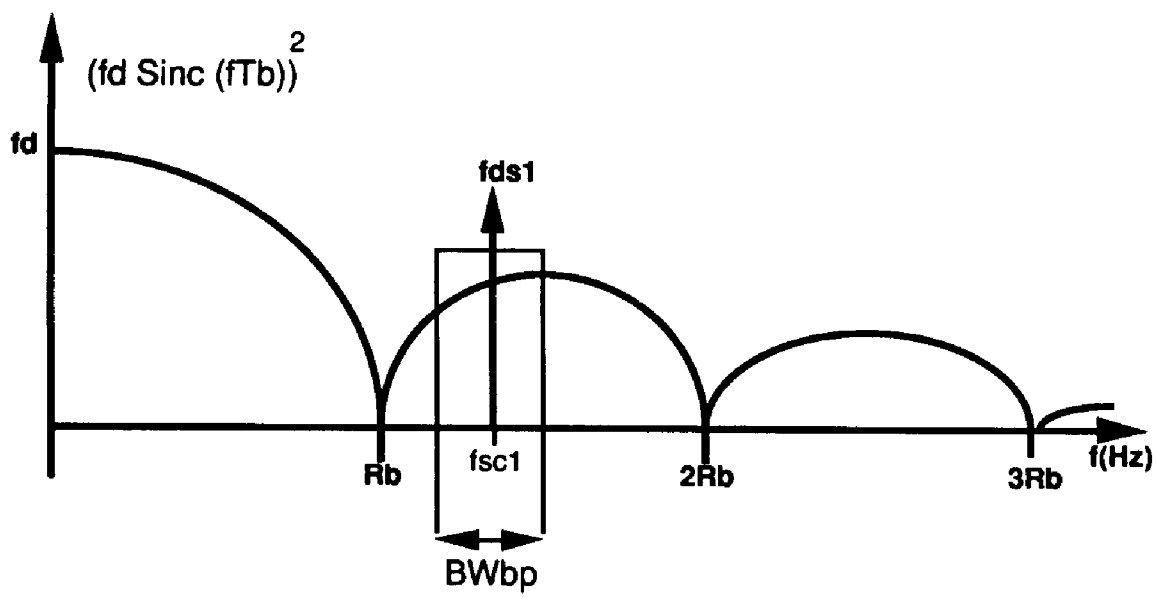

Figure 3. Approximation of Integration over Filter Bandwidth BW_{bp}

This approximation of the integration over the message signal's bandpass filter bandwidth (f_2-f_1) is valid for f_{sc1} and its bandwidth chosen anywhere outside ± 6% of the zero crossing of the bit stream. It has been determined by numerical integration of (6) that this result is within 1 dB of the actual value if the filter bandwidth is outside of the region marked in Figure 4.

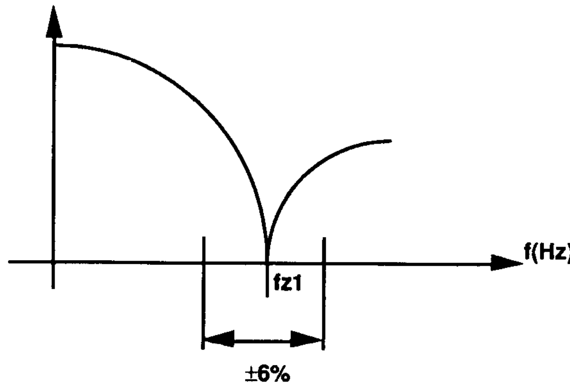

Figure 4. Region over which approximation is not valid (f_{z1}=Rb)

The same approach simplifies the integration of the premodulation filter ($H_{pm}(f)$) and the subcarrier bandpass filter ($H_{bp}(f)$). The once complicated second term of (5) is now

$$\int_{f_1}^{f_2} Sinc^2(fT_b)|H_{pm}(f)|^2|H_{bp}(f)|^2 df$$

$$= Sinc^2(f_{sc1}T_b)\ |H_{pm}(f_{sc1})|^2\ |H_{bp}(f_{sc1})|^2(f_2-f_1). \quad (6)$$

Then the interference power ratio can be expressed as

$$R = 20\log\left[\frac{2f_d}{f_{ds1}\sqrt{R_b}}\right] + 20\log[Sinc(f_{sc1}T_b)H_{pm}(f_{sc1})H_{bp}(f_{sc1})] + 10\log[f_2 - f_1]$$

$$=20\log\left[\frac{2f_d}{f_{ds1}\sqrt{R_b}}\right] + 20\log[Sinc(f_{sc1}T_b)] + 20\log[H_{pm}(f_{sc1})]$$

$$+ 20\log[H_{bp}(f_{sc1})] + 10\log[[f_2 - f_1].\ dB \quad (7)$$

The design procedure is to use Equation 7 to determine the interference power ratio for a given bit rate. If R is too large, the bit rate may be reduced. If R is not significant, the bit rate may be increased as desired.

Design Example

Consider the following specifications for a PCM/FM+FM/FM system.

$A = 1$ volt
$f_d = .41\ R_b$
$f_{ds1} = 17.065x10^3$
$f_{sc1} = 56kHz$

The premodulation filter is a six-pole Bessel filter with $\omega_o=2\pi(0.7)R_b$. The lowest frequency subcarrier's bandpass filter will be considered ideal ($|H_{bp}(f)|=1$) with a fixed bandwidth of ± 2kHz centered on f_{sc1}. It is desired to have an interference power ratio of -35 dB. The rate of the bit stream is the variable in the design. Choosing R_b = 40kHz puts the lowest frequency subcarrier in the first sidelobe of the bit stream power spectrum (see Figure 5a), and 56kHz is outside the ±6% of the bit stream zero crossing at 40kHz. Then the approximation is valid, and the interference power ratio (Equation 7) is

$$R=20\log\left[\frac{32800}{3.413\times10^6}\right] + 20\log[\mathrm{Sinc}(f_{sc1}T_b)]$$

$$+ 20\log[H_{pm}(f_{sc1})] + 20\log[1] + 10\log[[4\times10^3] \text{ dB}$$

$$= -40.3452 + -13.301 + -14.132 + 36.021 \text{ dB}$$

$$= -31.758 \text{ dB.}$$

This bit rate is too high to meet the desired R. Lowering the bit rate to 35kHz keeps the subcarrier in the first sidelobe and the approximation is still valid (see Figure 5b). The ratio is now

$$R = -40.925 - 14.461 - 18.858 + 36.021 \text{ dB}$$

$$= -38.2238 \text{ dB}$$

which is better than the required R of -35 dB. Figure 5 is a sketch of the power spectrum of the bit stream with the premodulation filter taken into account. It shows the placement of the subcarrier relative to the zero crossings of the spectrum with the two bit rates in effect.

Conclusions

The relationships between the bit rate, subcarrier frequencies, and interference power ratio were developed. A complex integral necessary to calculate the interference of the bit stream with respect to a subcarrier channel was simplified by an approximation. This approximation and the developed relationships were used to develop a simplified design process that allows the PCM/FM+FM/FM system to be designed with the interference below a specified level.

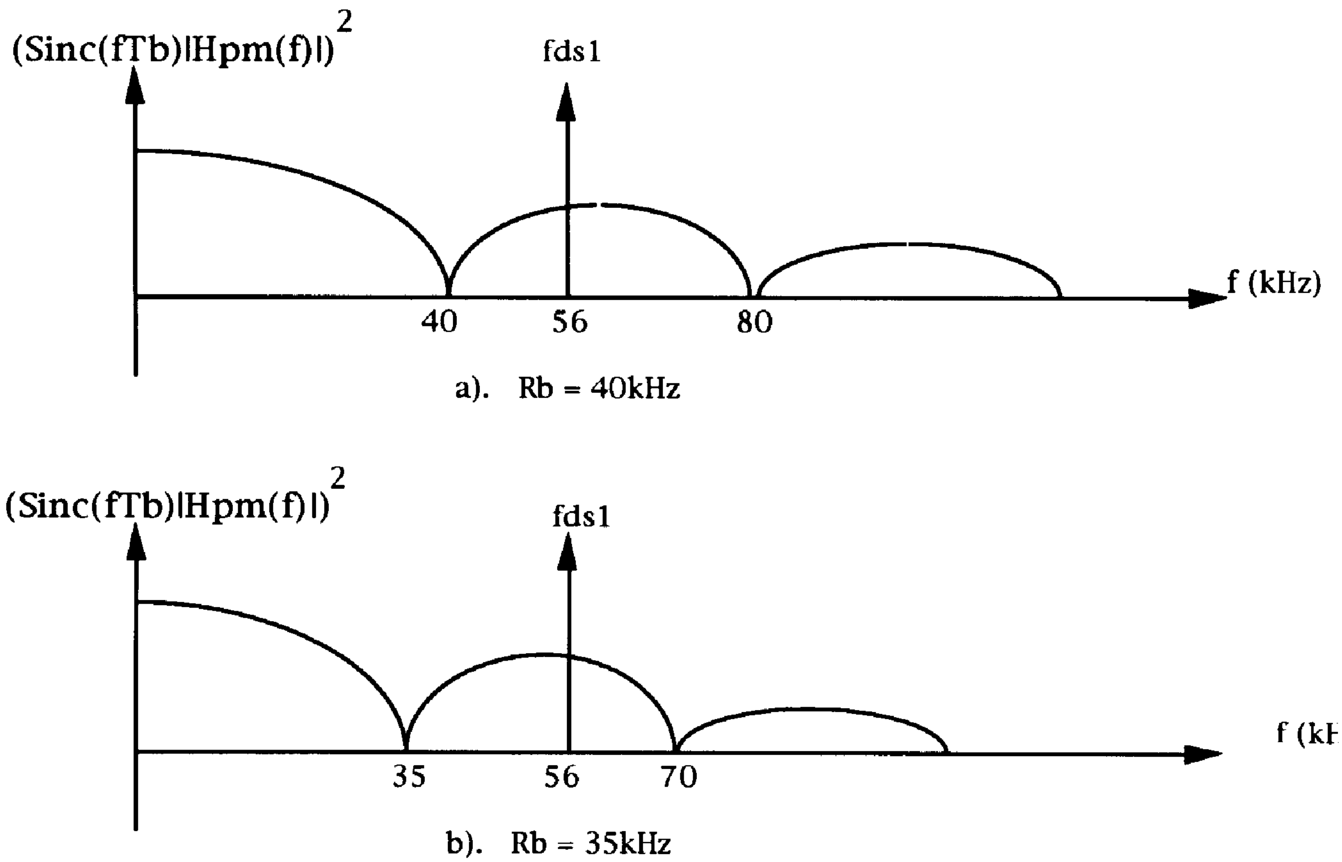

Figure 5. Comparison of the effect of different bit rates on the interference power ratio.

References

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