GAME-THEORETIC CONTRACT MODELS FOR EQUIPMENT LEASING AND MAINTENANCE SERVICE OUTSOURCING

by

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SIGNED: Maryam Hamidi
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DEDICATION

I dedicate this dissertation to my wonderful family.
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ABSTRACT

There is a major trend that manufacturers sell their services to customers instead of selling their products. These services can be provided through leasing, warranty, or maintenance outsourcing. In this dissertation, we have studied leasing and maintenance outsourcing services from different aspects of reliability-based maintenance, game-theoretic decision making, and inventory and supply chain management. We have studied how different interactions and relationships between the manufacturer and customer in service contracting affect the decisions they make and the profits they gain. The methods used to tackle the related decision-making processes are stochastic modeling, non-convex optimization, game-theoretical framework, and simulation.

For equipment leasing, two non-cooperative game-theoretic models and a cooperative model have been developed to describe the relationships between the manufacturer (lessor) and customer (lessee). Through the lease contracts, the lessor decides on the maintenance policy of the leased equipment, and the lessee decides on the lease period and usage rate. In the non-cooperative simultaneous move scenario, the lessee and the lessor act simultaneously and independently to make their decisions. In the leader-follower non-cooperative contract, the lessor is the leader who specifies the maintenance policy first, and the lessee, as the follower, decides on the lease period and usage rate accordingly. We have next determined the total maximum profit and shown that the Nash and Stackelberg equilibria are different from the total maximum solution. As a result, the players can increase their total profit by cooperation. We have implemented the cooperative solution as an equilibrium through a nonlinear transfer-payment contract. Our results illustrate that cooperation can be regarded as a value-added strategy in establishing such lease contracts. Besides, our numerical results show that although cooperation always increases the total
profit of the players, the magnitude of increase is case specific. When the lease price is low or the revenue is high, the profits in the non-cooperative contracts will be close to the cooperative alternative, while the cooperation may increase the total profit significantly in other cases.

For maintenance outsourcing, we have studied different bargaining scenarios in determining the contract terms. First, we have considered the Nash bargaining solution to compute the bargaining profit of players. Next, we have considered the case where players pose threat against each other in order to increase their own bargaining position. We have determined the optimal threat strategy for each player. Our result shows that although such threatening decreases the efficiency of the contract, it can dramatically increase the profit of the player with a higher bargaining position. We have finally provided a solution to the problem of how the service agent and customer can cooperate and negotiate on the price. We have determined the discounted price as a result of negotiation. Indeed, the discounted price induces the customer to choose the total maximum maintenance policy. Our numerical examples illustrate the feasibility of using such a price-discount contract in maintenance service outsourcing. Moreover, one can see that both the customer and agent can benefit from this price-discount contract.
CHAPTER 1

INTRODUCTION

In this chapter, the research motivation and the contributions of the dissertation are introduced first. Then, the definition and assumptions are introduced, and finally the organization of the dissertation is outlined.

1.1 Motivation

Schindler, GE and other manufacturing companies today generate up to 75% of their sales volume through services, such as leasing, warranty, and maintenance outsourcing. It is increasingly common for companies to use these services offered by manufacturers (Jaturonnatee et al., 2006). Over 80% of American businesses lease at least one of their equipment acquisitions, and nearly 90% say they would choose to lease again (Giddy, 2004). According to Campbell (1995), 35% of north American businesses have considered outsourcing some of their maintenance needs. It is a common practice for hospital equipment, aircraft engine and brakes, mining machinery, and manufacturing processes (Martin, 1997; Lankford and Parsa, 1999; Tarakci et al., 2006). For example, Federal Aviation Administration announced in 2007 that major air carriers outsourced an average of 64% of their maintenance expenses as opposed to 37% in 1996 (McFadden and Worrells, 2012).

One of the major motivations for companies to buy these services is to save the maintenance cost. It is not surprising that maintenance costs can account for 15 to 70% of the expenditures of companies and can even exceed companies’ annual net profits (Ding and Kamaruddin, 2014). Through service contracts, the companies outsource the equipment maintenance, which can help them reduce the operation cost, labor, and spare parts in-
ventory expenses. Besides, it lets companies focus more on their core businesses (Wang, 2010; Kersten et al., 2007).

However, there is a lack of methodological study on designing service contracts agreeable to both manufacturer and customer (Murthy and Jack, 2014). A service contract should be an instruction that is satisfactory to both parties (Martin, 1997). To this end, developing a framework to address the individual perspectives of decision makers is crucial in designing service contracts. This dissertation is focused on leasing and maintenance outsourcing services from different aspects of game-theoretic contracting, reliability-based maintenance, inventory and supply chain management.

1.2 Contributions

In this dissertation, we have developed different service contracts for equipment leasing and maintenance outsourcing. For the equipment leasing case, which is a process by which the owner (lessor) rents a piece of equipment to a user (lessee), the lessee decides on the optimal lease period and usage rate, and the lessor is responsible for developing a maintenance policy for the equipment. We have considered the effects of both age (time) and usage of the leased equipment on its deterioration and salvage value and on the production revenue. Technically, we have defined the payoffs of the lessee and lessor considering a non-homogenous Poisson process for equipment deterioration.

Two non-cooperative game-theoretic models and a cooperative model have been developed to describe the relationships between the two decision makers in the supply chain. For non-cooperative cases, the Nash and Stackelberg equilibria have been obtained respectively. For the cooperative game, the solution targeting on total profit maximization has been derived, and it is shown that this solution can be implemented as an equilibrium using a nonlinear transfer-payment contract. Besides, we have analytically compared the Nash equilibrium, Stackelberg equilibrium, and the total maximum solution to each other, and showed that the lessee and lessor can gain more profit from the cooperative contract.
than from the non-cooperative alternatives. Numerical examples are used to illustrate the different solution methodologies and the value of cooperation.

Next we have addressed maintenance outsourcing problem where the owner of a piece of non-repairable equipment plans on outsourcing preventive and failure replacement services to a service agent. The owner (i.e., customer) and the agent negotiate on the maintenance policy and spare part ordering strategy considering lead-time as well as inventory and shortage costs. We have derived the customer’s and agents long-run profits per unit of time based on the renewal reward theory and formulated the solution using Nash bargaining model. Then, we have considered the case that the players pose threat against each other to increase their profit. The customer can make threat against the agent using the replacement policy, and the agent can threat the customer by spare part availability. Finally, we have considered cooperation through price discount negotiation.

In order to fully characterize this two-person Nash bargaining problem we have characterized the solution to three values: customer’s threat point (max-min profit), agent’s threat point (max-min profit), and the maximum total profit of the customer and agent. Through a four-step procedure, we have shown how the total maximum profit can be obtained if the agent adjusts the service charges. Through numerical studies we have illustrated the effects of the price discount scheme and threats on the individual and total profits of the supply chain. The results of this study show that the manufacturers can increase their own profit and also customers profits by offering proper price discounts to customers at the time of service contracting. Price discounting motivates the customers to change their decisions to more profitable ones for both parties.

### 1.3 Definitions and Assumptions

#### 1.3.1 Reliability-based Maintenance

Generally, the reliability-based maintenance (RBM) or reliability-centered maintenance (RCM) can be classified into different maintenance strategies to address a potential failure
mechanism:

- Corrective maintenance (CM) or run to failure policy- Under this policy, maintenance is performed when the equipment fails. CM repairs or restores the system to its operational condition after the equipment is failed. This is the oldest policy used in industry, but due to unexpected failures this maintenance policy can cause large losses in production, serious damage to the system, person, and environment.

- Preventive maintenance (PM) policy- This policy is used to prevent the failures or to decrease the expected number of failures.

A preventive maintenance policy should achieve an appropriate trade-off between increased reliability (which reduces the failure maintenance cost) and the PM cost. When determining optimal maintenance policy repairable products are treated differently from nonrepairable systems and products. Models used for one type of system is not applicable to the other. A repairable product can be restored to satisfactory condition after failure, while a nonrepairable equipment has to be removed and replaced after failure. An example of a nonrepairable product is lightbulb which can’t be repaired after failure and has to be replaced. Examples of repairable products are car engines and aircrafts.

### 1.3.2 Game theory

One of the most popular approaches for dealing with processes involving two or more decision makers is game theory \cite{Murthy1999, Murthy1995}. Generally each decision maker tries to optimize his own objective function, while he/she wants to reach an agreement or a mutually acceptable solution with the rest of decision makers. The solution to be obtained depends on the interaction and relationship between the decision makers. In the literature of game theory the decision makers are called players, the decision alternatives are called strategies, and the objective functions of the players are called payoff functions. A game-theoretic model can be divided to
two major groups depending on the behavior of the players; non-cooperative game and cooperative game.

Non-cooperative Game

In the non-cooperative game the decision makers decide independently, and there is no interaction or negotiation between the players. This formulation is appropriate when each decision maker considers the other one as a competitor and is not willing to help each other even if that may be beneficial to him. In this case, no communication is needed between the decision makers and each of them follows his own equilibrium strategy independent of the others’. Two major solutions to the non-cooperative games are Nash and Stackelberg equilibria, both of which are based on best response function of the players.

The best response is the strategy which produces the optimum objective function for a player, taking other players’ strategies as given. The best response function in a two-person game can be described as follows. Suppose \( \Pi_1(x_1, x_2) \) and \( \Pi_2(x_1, x_2) \) are the payoff functions of players 1 and 2 respectively, where \( x_1 \in X_1 \) and \( x_2 \in X_2 \) are the chosen strategies of player 1 and 2 over their feasible regions \( X_1 \) and \( X_2 \). We call the strategy of player 1 that maximizes his payoff, given that player 2’s strategy is \( \hat{x}_2 \), the best response of player 1 to \( \hat{x}_2 \) and is denoted by \( x_{1R}^{\ast}(\hat{x}_2) \). The best response function of the second player, \( x_{2R}^{\ast}(x_1) \), as function of \( x_1 \), can be defined in the same manner.

If the players simultaneously and non-cooperatively optimize their payoffs with respect to any given strategy set of the other players, this is called a simultaneous move game, and the solution concept for this game structure is called the Nash equilibrium. In the equilibrium each player makes an optimal decision given the behavior of the other player, and therefore neither player has an incentive to deviate unilaterally from the equilibrium. A Nash equilibrium should satisfy the best response functions of both players. Given this definition of best responses, a pair \((a_1, a_2)\) of actions is a Nash equilibrium if and only if player 1’s action \( a_1 \) is a best response to player 2’s action \( a_2 \), and player 2’s action \( a_2 \) is a best response to player 1’s action \( a_1 \). That is, in order to find a Nash
equilibrium we need to find a pair \((a_1, a_2)\) of actions such that \(a_1\) is a best response to \(a_2\), and vice versa. If we denote player 1’s best response to \(a_2\) by \(x_{1R}^*(a_2)\) and player 2’s best response to \(a_1\) by \(x_{2R}^*(a_1)\) then we can write the condition for a Nash equilibrium more compactly. The pair \((a_1, a_2)\) of actions is a Nash equilibrium if and only if \(a_1 = x_{1R}^*(a_2)\) and \(a_2 = x_{2R}^*(a_1)\). The most useful method of finding Nash equilibrium is to derive the players’ best response functions and then solving the two simultaneous equations:

\[
a_1 = x_{1R}^*(a_2)
\]

\[
a_2 = x_{2R}^*(a_1)
\]

In a Stackelberg game, a player named leader moves first and then the player named follower moves sequentially. In this game the leader commits himself to a particular strategy and announces his choice to the follower, and the follower then chooses his best decision accordingly. The leader optimizes his payoff by specifying his strategies taking into account the expected behavior of the follower. Assuming a rational player, the leader expects the follower to select his best response. Substituting the best response of the follower into the leader’s payoff function transforms leader’s payoff into a function of leader’s decision variables which can be solved using optimization methods.

It should be noted that in the non-cooperative formulations, where the players compete to maximize their individual profits are not efficient since they do not maximize the total profit of the players (Cachon [2003]). It is known that the Nash and Stackelberg equilibria are not Pareto optimal in most games, like in the well-known prisoners dilemma (Matsumoto and Szidarovszky [2016]). The payoffs of the players can improve simultaneously from the equilibrium if they cooperate. When they cooperate in determining their decisions (Matsumoto and Szidarovszky [2016], Giannoccaro and Pontrandolfo [2004], Leng and Parlar [2005]) they will gain a better outcome than the Nash or Stackelberg equilibria (Nagarajan and Sošić [2008], Kim and Ha [2003]).
Cooperative Game

The second concept is applicable when the decision makers cooperate to gain more profit. Here, players try to maximize their overall profit and to choose policies that maximize their overall profit. Cooperation might generate a larger profit for one of the players and a much less for the other. The only way to motivate the other player to agree on the policy is by sharing the profit. Cooperation generally involves distribution of the maximal total profit between the profit. We consider two solutions to allocate the total profit. The first one is Shapley value which is based on the characteristic function definition. The second one is the Nash bargaining solution which is not a characteristic based solution.

Shapley values measure the value of each player in a game based on the characteristic function values. There are four possible subsets (coalitions) for the two-person game: \([C], [A], [C, A]\) and \([\emptyset]\), where \(C\) represents the first player and \(A\) stands for the second player. For each of the coalitions, the characteristic function values can be defined as

\[
\nu([C]) = \max_{x_1, x_2} \Pi_1, \ \nu([A]) = \max_{x_1, x_2} \Pi_2, \ \nu([C, A]) = \max_{x_1, x_2} \Pi_1 + \Pi_2, \ \text{and} \ \nu([\emptyset]) = 0,
\]

where \(\Pi_1\) and \(\Pi_2\) are the payoff functions of players 1 and 2, respectively. The interpretation of the characteristic function value of any coalition is that the players outside the coalition try to punish the members of the coalition by minimizing their overall payoffs while the players inside the coalition try to maximize it. There are two coalitions with the first player as a member: \([C]\) and \([C, A]\). The marginal contributions of the first player to these coalitions are

\[
\begin{align*}
d_C([C]) &= \nu([C]) - \nu([\emptyset]) \\
d_C([C, A]) &= \nu([C, A]) - \nu([A])
\end{align*}
\]

respectively, with equal probability of 0.5. Calculating the expected marginal contributions of the first player yields his Shapley value \(y_C\):

\[
y_C = \frac{\nu([C]) + \nu([C, A]) - \nu([A])}{2}
\]

Similarly, the agents Shapley value \(y_A\) can be determined as:

\[
y_A = \frac{\nu([A]) + \nu([C, A]) - \nu([C])}{2}
\]

Shapley value is a fair and acceptable profit for both players since it considers the contribution of each player in the maximal total profit. One can see that the sum of the Shapley
values equals the maximal total profit. To determine the Shapley values, the characteristic function values should be calculated first, and the Shapley values should be determined accordingly.

The next solution we have considered for profit allocation is the Nash bargaining solution. This solution is based on maximizing the Nash product of the payoffs. The theoretical basis of this concept is the following. A two player conflict game is mathematically defined as a pair \((H, \Pi^*)\), where \(H \subseteq \mathbb{R}^2\) is convex, closed, bounded and comprehensive \((\Pi \leq \Pi' \in H \Rightarrow \Pi \in H)\) payoff space and \(\Pi^* = (\Pi_1^*, \Pi_2^*)\) is the disagreement point which is the payoff that the players achieve if no agreement is reached. The solution is a vector valued function \(\Pi = \psi(H, \Pi^*)\) which satisfies the following axioms:

1. Feasibility: \(\psi(H, \Pi^*) \in H\). That is the players can distribute only existing amount of money.

2. Rationality: \(\psi(H, \Pi^*) \geq \Pi^*\). This means the players cannot accept solutions which give them lower payoff than the disagreement payoff values.

3. Pareto optimality: \(\Pi \in H\) and \(\Pi \geq \psi(H, \Pi^*)\) imply \(\Pi = \psi(H, \Pi^*)\) which shows that if both players can increase their payoffs, then this solution has to be included in the agreement.

4. Independence from unfavorable alternatives: If \(H_1 \in H\) and \(\psi(H, \Pi^*) \in H_1\), \(\psi(H, \Pi^*) = \psi(H_1, \Pi^*)\). This is the same as the well-known property of optimization models.

5. Independence from monotone increasing linear transformations: Let \(\alpha_k > 0, \beta_k (k = 1, 2)\) be constants and \(\Pi' = (\alpha_1 \Pi_1^* + \beta_1, \alpha_2 \Pi_2^* + \beta_2)\), \(H' = \{(\alpha_1 h_1 + \beta_1, \alpha_2 h_2 + \beta_2) | (h_1, h_2) \in H\}\) then \(\psi(H', \Pi') = (\alpha_1 \psi_1 + \beta_1, \alpha_2 \psi_2 + \beta_2)\) which states that changing the unit in which the payoffs are computed cannot change the solution.

6. Symmetry: If \(\Pi_1^* = \Pi_2^*\) at the disagreement vector and \((\Pi_1, \Pi_2) \in H\) if and only if \((\Pi_2, \Pi_1) \in H\), then at the solution \((\psi_1, \psi_2)\) necessarily \(\psi_1 = \psi_2\). This means that
if there is no difference between the players in the disagreement payoff and in the feasible set, then there is no reason to make difference in the solution.

It can be proved that there is a unique solution function $\psi$ that satisfies these six axioms and this solution is the same as $\arg\max(\Pi_1 - \Pi_1^*)(\Pi_2 - \Pi_2^*)$ subjected to $\Pi \geq \Pi^*$, $\Pi \in H$ where $\Pi_1$ and $\Pi_2$ are the payoff functions of players one and two, respectively.

### 1.4 Dissertation Organization

The remainder of the dissertation is organized as follows. In Chapter 2, a literature review on maintenance and game theory is presented. In Chapter 3, the lease contracts are studied. The mathematical formulations for equipment failure intensity, maintenance costs, revenue, residual value, as well as the lessee’s and the lessor’s payoff functions are given in Section 3.2. The non-cooperative solution methodologies are described in Section 3.3, where the best response functions of lessee and lessor are derived, the Nash and Stackelberg equilibria are determined, and the corresponding strategies and payoffs are compared. Section 3.4 analyzes the cooperative game solution and compares it with the non-cooperative solutions. Next, a nonlinear transfer-payment function is introduced to implement the cooperative solution. Section 3.5 presents numerical examples to illustrate the three lease contract models and to investigate their performance. Finally, Section 3.6 concludes this chapter. In Chapter 4, we have studied maintenance outsourcing contracts. Section 4.2 provides a description of the problem using renewal reward theorem and derives the payoff functions of the customer and the agent. Section 4.3 briefly describes how to model negotiation through the Nash bargaining game. In Sections 4.3.1 and 4.3.2, the optimal threat strategies that agent and customer, respectively, can pose against the other player in order to increase his bargaining position are derived. Section 4.3.3 determines the total maximum profit, and the allocation of profit obtained from price negotiation is given in Section 4.3.4. In Section 4.4, we numerically examine the effect of negotia-
tion with and without side payment on the outcome of the contract. Finally, Section 4.5 concludes the chapter and outlines the directions for future research. We conclude the dissertation in Chapter 5 by summarizing the contributions of this research and discussing future research directions.
CHAPTER 2

LITERATURE REVIEW

In this chapter, the literature for maintenance optimization and game theory are discussed.

2.1 Maintenance

Ensuring the operational availability of critical equipment subject to failure is of great importance. In order to facilitate maintenance activities, optimum maintenance policy have been extensively studied (Wang, 2002; Jardine and Tsang, 2013). Maintenance can be categorized in two major groups: corrective maintenance and preventive maintenance. Corrective maintenance (CM) is performed when an equipment is failed. Preventive maintenance (PM) is a maintenance that is performed regularly on an item to decrease the probability of random failure. A basic assumption for performing PM is that the equipment has an increasing failure rate over its life cycle. We will study four different types of PM policies in this section; age-dependent, periodic, failure limit and sequential PM policies.

Age-dependent preventive replacement policy is one of the mostly used models. Under this policy an item is preventively maintained at constant age $T$ or at the time of failure, whichever occurs first (Barlow and Hunter, 1960). Since this maintenance policy is based on the age of the equipment, this model is called age-dependent. It should be noted that both PM actions and CM ones at the time of failure can be minimal, imperfect, or perfect maintenance actions. A review of such models can be seen in Pham and Wang (1996). Nakagawa (1984) extended this policy by performing PM for an item at age $T$ or after $N$ number of failures, whichever occurs first. All the failures are maintained minimally. The decisions to be made are $N$ and $T$. It should be noted that if $N = 1$ this policy is equivalent to classical age-dependent PM policy.
In the periodic PM policy, the equipment is preventively maintained at equal time intervals $T$ and all the failures are also repaired. The PM is performed at time epochs $T, 2T, 3T, ...$. A specific model of this policy is Block replacement policy which can be used for nonrepairable equipments. Under this policy the equipment is replaced at periodic time $T$ and at the time of random failures. Another well-known maintenance policy in this category is periodic replacement with minimal repairs at the time of failure. Under this policy all random failures are minimally repaired and the item is replaced at the time of periodic PM ($T, 2T, 3T, ...$) (Barlow and Hunter, 1960). An extension of the latter model is when the equipment is imperfectly maintained at every $T$ time unit and perfectly maintained after $N$ number of imperfect PM actions (Liu et al., 1995).

Another PM policy is failure limit policy. Under this policy PM is performed when the failure intensity reaches a predetermined threshold. Lie and Chun (1986) models a maintenance policy where the equipment is preventively maintained at a specific threshold while all random failures are minimally repaired. Other extensions of this policy can be found in Suresh and Chaudhuri (1994); Love and Guo (1996); Wang (2002).

The last maintenance policy covered here is sequential PM policy. Under this policy the equipment is maintained preventively at unequal time intervals. As the equipment ages, the length of PM actions decreases. Nakagawa (1986, 1988) considered extended version of this policy where the equipment is replaced at $N^{th}$ imperfect PM actions. The failure distribution of the equipment varies after each PM action.

The preventive maintenance of the leased equipment borne by the lessor is usually specified in the lease contract (Barlow and Hunter, 1960). A number of imperfect maintenance policies for a repairable leased equipment have been proposed in the literature. Jaturonnatee et al. (2006) considered sequential PM policy and derived the optimal number of PM actions to be carried out during the lease period along with time instant and degree of each action. The degree of a PM shows the reduction in the equipment’s failure intensity due to maintenance. Yeh et al. (2009) considered similar problem assuming a fixed degree for all maintenance actions. Pongpech and Murthy (2006) determined the
degree of each action considering a periodic maintenance policy. Yeh and Chen (2006) also considered a periodic PM policy with a fixed degree for all actions. Periodic PM policy can easily be specified in a contract and implemented in practice. Chang and Lo (2011) studied a failure limit policy and assumes that PM actions are carried out when the equipment’s failure intensity reaches a specified level.

A group of studies focuses on minimizing the expected maintenance cost per unit time for a nonrepairable equipment with different assumptions about spare part lead time, inventory and shortage costs. These literature try to optimize both maintenance policy and spare part order policy simultaneously. Osaki (1977) examined the optimal ordering policy when the lead time is constant and no inventory exists. Park and Park (1986) investigated the optimum ordering and maintenance policy in the case of variable lead time assuming the higher cost of failure replacement due to emergency spare part ordering. Thomas and Osaki (1978) generalized ordering policy for constant and variable lead time and also for positive and negative ordering times. It is worth mentioning that most work mentioned that higher costs of failure replacements are incurred by unscheduled or emergency orderings while Armstrong and Atkins (1996) highlighted another reason that an equipment failure causes damage to other parts of the system.

Most of the maintenance policies studied in the literature assume deterioration of equipment is caused by only age (time) and ignore the effect of usage. Most of the real world equipments deteriorate by both time and the usage. Usage can be distance traveled by a car, the output produced by a photocopier, flight hours operated by an aircraft or weight carried by a mining haul truck (Jack et al., 2009). Some studies have considered two-dimensional warranty policies, with one dimension representing time and the other representing usage (Moskowitz and Chun, 1994; Murthy et al., 1995; Kim and Rao, 2000; Chen and Popova, 2002; Iskandar and Murthy, 2003; Iskandar et al., 2005; Jack et al., 2009).
2.2 Game Theory

The classical models of operations management can’t directly be applied on current supply chains since the chains consist of several different companies, and the decisions of different companies affect on the profit of the other companies in the supply chain. Classical models focus generally on one main company in supply chain and determine the optimal decisions such as inventory policy, shipment and production. Recently there has been an increasing interest in applying game theory framework in order to study the interaction between different companies and decision makers in supply chain. Here we study the literature which designed side payment contracts and mechanisms to coordinate the decision makers in a supply chain (Jackson and Pascual 2008; Bier and Haphuriwat 2011; Zhuang et al. 2014). Side payment can be defined as a monetary transfer between players, which is used as an incentive for a particular contract concession (Carter and Ferrin 1995).

Sucky (2006) designed a contract to coordinate one supplier and one buyer for the case that the buyer has a stronger negotiation power than the supplier. It studied two different bargaining problems; one with full information and the second with partial information. For each model the supplier tries to motivate the buyer to order quantity suitable for the supply chain. Cachon and Zipkin (1999) studied both cooperative and competitive interactions for a supply chain where the supplier and retailer try to determine their base stock. Since the back order of the retailer affects on the profit of the supplier, the study designed a side-payment contract from the supplier to the retailer to motivate the retailer to choose the total optimal inventory policy. Caldentey and Wein (2003) considers the case where the supplier decides on production capacity decision and the retailer decides on base-stock policy. Ernst and Powell (1998) designed a contract between a manufacturer and a retailer where the manufacturer tries to affect on the service level decision of the retailer, since the demand of the final customer is a function of the service level of the retailer. Clearly, the demand affects on profit of both manufacturer and retailer and the manufacturer tries
to increase it. Weng (1997) considered the case where the distributor motivates the manufacturer to decrease the price. Since the demand is price sensitive, lower price will induce higher demand, and the manufacturer and the distributor can share the extra profit. Leng and Parlar (2005) provided a review on applications of game theory in supply chain management. Esmaeili et al. (2009) studied a non-cooperative game based on the Stackelberg strategy solution concept and a cooperative game based on total optimal solution for a seller-buyer model. Cachon and Zipkin (1999) investigated the inventory control policy based on Nash and Stackelberg equilibria and compared the result with the cooperative regime. Kim and Ha (2003) developed a buyer-supplier cooperation model. Dong et al. (2004) developed a supply chain competitive network model and derived the Nash equilibrium conditions. Leng and Zhu (2009) reviewed two-person side payment contracts, including price discount, revenue sharing, and sales rebate contracts. Tarakci et al. (2006) discussed three long-term incentive maintenance outsourcing contracts based on different overall-profit distributions. Regarding maintenance service contracts, Murthy and Asgharizadeh (1999) and Ashgarizadeh and Murthy (2000) employed the non-cooperative Stackelberg game theory to model maintenance outsourcing contracts and determined the agent’s optimal pricing strategy and number of customers to service as well as the customers’ optimal contract option.

The game theory literature has overwhelmingly showed cases where decision makers negotiate over different contract terms (Nagarajan and Bassok 2008; Hoda et al. 2009). Gurnani and Shi (2006) and Nagarajan and Sošić (2008) provided reviews of such contracts. Bajari et al. (2009) analyzed a comprehensive data set of building construction contracts and observed that almost half of the contracts were developed through negotiation. The study suggested that more complicated projects were more likely to be negotiated. The negotiation can be modeled using the Nash bargaining solution (Nash 1950). This solution approach is simple and robust (Nagarajan and Bassok 2008). Besides, compared to other alternatives, the experimental bargaining theory indicates stronger empirical evidence to the Nash bargaining theory (Neslin and Greenhalgh 1983; Kagel and
Similar approaches have been considered in distribution channels, franchising arrangements, and inventory control systems (Chen et al., 2001; Nagarajan and Bassok, 2008; Leng and Zhu, 2009; Leng and Parlar, 2010). In negotiation each party can start bargaining by threatening the other player in order to improve his own position and decrease the other player’s position (Anbarci et al., 2002). Threatening is applicable in many interactive settings, such as litigation, international and political relations, and supply chain. This will make the other player more reluctant to risk a conflict in negotiation (Harsanyi, 1977; Myerson, 1991).
CHAPTER 3

NON-COOPERATIVE AND COOPERATIVE GAME-THEORETIC MODELS FOR USAGE-BASED LEASE CONTRACTS

In this chapter, we study game-theoretic models for lease contracts, by which the owner (lessor) rents a piece of equipment to a user (lessee). Two of the major advantages of leasing over purchasing is saving on initial investment and flexibility on equipment upgrading. The companies can use the leased equipment for monthly or annual fees, which are much less than the purchasing price of the equipment. On the other hand, the leased product remains the property of the manufacturer at the end of the service period and can be remanufactured and reused by the manufacturer at the end of period. We study different contracts through which the lessee decides on the optimal lease period and usage rate, and the lessor decides on maintenance policy of the equipment.

3.1 Introduction

Determining the optimal PM policy for the leased equipment is of great importance. Many studies have focused only on effect of age (time) on deterioration of the leased equipment while ignoring the effect of actual usage. Some examples of usage include the number of pages produced by a photocopier, the flight hours operated by aircraft (Jack et al., 2009), and the weight carried by a mining haul truck. In reality also many lease contracts are characterized by both the lease period (time) and the usage. A typical example is an automobile leased for 3 years and up to 10,000 miles per year. In this chapter, we model the failure intensity as a function of both age and usage and determine the optimal periodic PM policy for the lessor (Yeh and Chen, 2006) with a fixed degree for all actions. Such
PM actions not only reduce the number of failures during the lease period but also increase the residual value of the equipment by the time when the lease contract expires. Besides, we consider that the productivity of the leased equipment decreases as it deteriorates, so PM actions also reduce the revenue loss caused by deterioration (Wu et al. 2011).

In order to determine lease contracts agreeable to both lessor and lessee, we study three lease contract models based on non-cooperative and cooperative games. For the non-cooperative games, two scenarios are considered: non-cooperative simultaneous move game and non-cooperative leader-follower game. In the first scenario, the lessee and lessor choose their strategies simultaneously while, in the second scenario, the lessor dominates the lessee by determining his maintenance policy first, and then the lessee chooses the lease period and usage rate. We respectively derive the Nash and Stackelberg equilibria and compare the corresponding strategies and payoffs. Next, for the cooperative game model, we derive the solution targeting on total profit maximization. It is of interest to compare the performance of the non-cooperative contracts and the cooperative alternative. One measure of performance is the difference between the total profit of a non-cooperative contract and that of a cooperative one (which has the maximum total profit). By comparing the total profits, we will demonstrate the conditions under which the non-cooperative lease contracts have a poor performance, and thus the lessee and lessor can gain much more profits by switching to the cooperative alternative. Should the players wish to cooperate we have shown that this solution can be implemented as an equilibrium using a nonlinear transfer-payment contract.

The remainder of this chapter is organized as follows. The mathematical formulations for equipment failure intensity, maintenance costs, revenue, residual value, as well as the lessee’s and the lessor’s payoff functions are given in Section 3.2. The non-cooperative solution methodologies are described in Section 3.3, where the best response functions of lessee and lessor are derived, the Nash and Stackelberg equilibria are determined, and the corresponding strategies and payoffs are compared. Section 3.4 analyzes the cooperative game solution and compares it with the non-cooperative solutions. Next, a nonlinear
transfer-payment function is introduced to implement the cooperative solution. Section 3.5 presents numerical examples to illustrate the three lease contract models and to investigate their performance. Finally, Section 3.6 concludes the chapter.

3.2 Problem Description and Model Formulation

It is assumed that the new equipment to be leased has life cycle $L$, and its maximum usage rate is $r_m$ (Iskandar et al., 2005; Jack et al., 2009) which is the maximum allowable capacity per unit of time (e.g., 20,000 miles per year for a leased car or 240 tons per day for a mining haul truck). The lessee leases the equipment at a usage rate $r \leq r_m$ for a certain period $K$ and pays the lessor an annual payment based on the usage rate. During the lease period, the lessor will perform periodic PM for $N$ times each with degree $\delta$ which to be specified in the contract. The value of $\delta$ is between 0 (perfect maintenance) and 1 (minimum repair). In addition, the lessor will do minimal repair (i.e., corrective maintenance (CM) that does not change the equipment’s failure intensity) to fix any failure that occurs within the lease period. The equipment generates revenue $Y(r, K, N, \delta)$ for the lessee during the lease period, and it has residual value $SV(r, K, N, \delta)$ for the lessor at the end of the lease period.

In this chapter, the lease contracts are modeled in a two-person game-theoretic framework. Specifically, the decision variables of the lessee are $K$ and $r$, and his strategy $x_1 = (r, K)$ belongs to the following strategy set:

$$X_1 = \{(r, K) \mid 0 \leq r \leq r_m, \ 0 \leq K \leq L\} \quad (3.1)$$

where we specify that the lease period $K$ should not exceed the equipment’s life cycle $L$. On the other hand, the lessor’s strategy $x_2 = (N, \delta)$ is within the strategy set $X_2$:

$$X_2 = \{(N, \delta) \mid N \geq 0 \text{ integer, } 0 \leq \delta \leq 1\} \quad (3.2)$$

In addition to the decision strategies, the players’ objective functions (payoff functions) need to be determined. Next, we will formulate the payoff functions of the lessee and the


3.2.1 Failure Intensity Function

The leased equipment deteriorates according to a known failure intensity function $\lambda_0(r, t)$ which is an increasing function of both usage rate and age. To model the failure intensity, we use one-dimensional point process which treats the usage as a function of age (Chen and Popova, 2002; Iskandar and Murthy, 2003; Iskandar et al., 2005). Let $T(t)$ be the equipment’s cumulative operating time up to time $t$ and $Z(t)$ be the total usage. For simplicity, we assume that the equipment’s operating time equals its age, i.e., $T(t) = t$. This is true if all failures in $(0, t)$ are repaired, and the repair time is negligible compared to the operating time (Iskandar et al., 2005). In practice also calendar time is widely used as the operating time of leased equipment (Huang and Yen, 2009). Besides, we consider a constant usage rate $r$ such that $Z(t) = rt$. Based on these assumptions, linear, log-linear, and power intensity functions reduce to:

$$
\lambda_0(t) = \theta_1 Z(t) + \theta_2 T(t) = t(\theta_1 r + \theta_2) \quad (3.3)
$$

$$
\lambda_0(t) = e^{\theta_1 Z(t) + \theta_2 T(t)} = e^{\theta_1 rt + \theta_2 t} \quad (3.4)
$$

$$
\lambda_0(t) = \theta_0 Z(t)^{\theta_1} T(t)^{\theta_2} = \theta_0 r^{\theta_1} t^{\theta_2+\theta_1} \quad (3.5)
$$

with parameters $\theta_i \geq 0$ for $i = 0, 1, 2$ (see Iskandar et al., 2005 for other models). It can be easily seen that model (3.3) is in the form of a Weibull intensity function, $\lambda_0(t) = \alpha \beta (\alpha t) ^{\beta - 1}$, with shape parameter $\beta = 2$ and scale parameter $\alpha = \sqrt{\frac{\theta_1 r + \theta_2}{2}}$ that is a function of usage rate $r$; model (3.4) describes a Gumbel intensity function, $\lambda_0(t) = \frac{e^{\theta_1 rt + \theta_2 t}}{e^{\theta_1 rt + \theta_2 t}}$ with $\mu = \frac{\ln(\theta_1 r + \theta_2)}{\theta_1 r + \theta_2}$ and $\sigma = 1/(\theta_1 r + \theta_2)$, and model (3.5) is also a Weibull intensity function with $\alpha = (\frac{\theta_0 r^{\theta_1}}{\theta_1 + \theta_2 + 1})^{1/(\theta_1 + \theta_2 + 1)}$ and $\beta = \theta_1 + \theta_2 + 1 > 1$. Without loss of generality, model (3.3) is considered in this chapter, and the optimal values of usage rate $r$ and lease period $K$ will be determined and compared based on the three different lease contract models. The other intensity functions can be used in an analogous manner.
3.2.2 Preventive Maintenance and Cost

The lessor will perform periodic PM at time epochs $\tau, 2\tau, ..., N\tau$, where $\tau = \frac{K}{N+1}$ is the constant time interval between two successive PM actions (Pongpech and Murthy, 2006; Yeh and Chen, 2006; Chang and Lo, 2011). The effect of PM can be formulated by age reduction models which assume that PM reduces the age of a maintained item to a younger age (Levitin and Lisnianski, 2000; Kim and Ha, 2003; Kahle, 2007; Bartholomew-Biggs et al., 2009; Yeh et al., 2011). In this chapter, we use Kijima type-I age reduction model (Kijima, 1989) where PM reduces the equipment’s age from $t$ to a virtual age $\nu(t) < t$.

We assume all periodic PM actions have the same degree $0 \leq \delta \leq 1$. As shown in Fig. 3.1, the failure intensity function without PM is $\lambda_0(t)$, and after performing PM, the
failure intensity function is reduced to $\lambda(t)$ where $\lambda(t) < \lambda_0(t)$. According to Kijima type-I model, the virtual age of the equipment immediately after performing PM at time epoch $i\tau$ is $\nu_i = i\delta\tau$ for $i = 0, 1, ..., N$, meaning that PM reduces the equipment’s total age by $(1 - \delta)\tau$ at each time epoch. As a result, the equipment’s virtual age between the $i^{th}$ and $(i + 1)^{th}$ PM actions can be expressed as:

$$\nu(t) = i\delta\tau + t - i\tau, \quad \text{for } i\tau \leq t < (i + 1)\tau$$  \hspace{1cm} (3.6)

Replacing the equipment’s age $t$ in (3.3) by its virtual age $\nu(t)$, the failure intensity function with PM can be expressed as:

$$\lambda(t) = \lambda_0(\nu(t)) = \nu(t)(\theta_1 r + \theta_2) = (i\delta\tau + t - i\tau)(\theta_1 r + \theta_2)$$  \hspace{1cm} (3.7)

It is reasonable to assume that the cost of a PM action is a nonnegative and nondecreasing function of age reduction $(1 - \delta)\tau$. In particular, we consider a linear function for the cost of each PM action, i.e., $C_p = a + b(1 - \delta)\tau$. As a result, the total cost of PM actions during the lease period $K = (N + 1)\tau$ is given by:

$$C_p = Na + \frac{NbK(1 - \delta)}{(N + 1)}$$  \hspace{1cm} (3.8)

where $a, b > 0$ \cite{Pongpech and Murthy, 2006, Jaturonnatee et al., 2006, Yeh et al., 2011}. In the latter part of the chapter, the optimal values of $N$ and $\delta$ for the three different contract lease models will be determined and compared.

### 3.2.3 Corrective Maintenance and Cost

During the lease period, any failure of the leased equipment is minimally repaired by the lessor. Such minimal repairs make the equipment operational but keep the failure intensity the same as that just before the failure. Assuming each minimal repair incurs a fixed cost $C_f$, the expected total repair cost depends on the expected number of failures during the lease period.

Without performing any PM actions, the failure process of the leased equipment is Nonhomogeneous Poisson Process (NHPP) with failure intensity $\lambda_0(t)$ \cite{Jaturonnatee et al.}.
et al., 2006; Chang and Lo, 2011). When PM is considered, the failure process of the equipment in each interval \([i\tau, (i + 1)\tau]\) for \(i = 0, 1, ..., N\) is still an NHPP with intensity function \(\lambda(t) = \lambda_0(\nu(t))\), meaning that failures happen according to the virtual age of the equipment (Kim et al., 2004). As a result, the total expected number of failures with PM over the lease period is:

\[
\sum_{i=0}^{N} \int_{i\tau}^{(i+1)\tau} \lambda_0(\nu(t)) dt = \sum_{i=0}^{N} \int_{i\tau}^{(i+1)\tau} (i\delta\tau + t - i\tau)(\theta_1 r + \theta_2) dt \\
= \frac{K^2(\theta_1 r + \theta_2)(N\delta + 1)}{2(N + 1)} \tag{3.9}
\]

Combining (3.9) with the cost of each CM, \(C_f\), the total cost of CM is given by:

\[
C_m = \frac{C_f K^2(\theta_1 r + \theta_2)(N\delta + 1)}{2(N + 1)} \tag{3.10}
\]

### 3.2.4 Lessee’s Revenue by Using the Equipment

Ideally, a higher usage rate results in higher revenue per unit of time. In reality, however, the efficiency of equipment decreases because of aging and usage. Essentially, deterioration causes revenue loss (Wu et al., 2011) due to reduced speed, poor product quality, and increased operating costs such as fuel consumption (Al-Najjar and Alsyouf, 2004; Marais and Saleh, 2009). Let \(u_m\) be the potential revenue per unit of time that can be generated by the new equipment if its capacity is fully used (i.e., \(u(t = 0; r = r_m)\)) (Wu et al., 2011).

We also assume that the revenue rate at the end of the equipment’s life cycle is zero, i.e., \(u(t = L, r) = 0\). Similar to the Cobb-Douglas production function, the revenue rate can be formulated as the product of an increasing function in \(r\) and a decreasing function in \(t\) as:

\[
u_0(r, t) = \frac{u_m}{r_m} r \left(1 - \frac{t}{L}\right) \tag{3.11}
\]

Because PM makes the equipment younger, it slows down the revenue loss caused by deterioration. By substituting time \(t\) with the equipment’s virtual age \(\nu(t)\) in model (3.11),
the total revenue generated for the entire lease period $K$ can be expressed as:

$$Y(r, K, N, \delta) = \sum_{i=0}^{N} \int_{i\tau}^{(i+1)\tau} u_0(r, \nu(t))dt = \sum_{i=0}^{N} \int_{i\tau}^{(i+1)\tau} \frac{u_m r}{r_m} \left(1 - \frac{i\delta \tau + t - i \tau}{L}\right)dt$$

$$= \frac{u_m r K}{r_m L} \left[L - \frac{K(1 + \delta N)}{2(N + 1)}\right] \quad (3.12)$$

### 3.2.5 Residual Value of the Equipment

The residual value of the leased equipment at the end of the lease period ($K$) is a critical factor in setting lease terms. [Monga and Zuo (2001)](2001) proposed that the equipment’s residual value can be modeled as a function of its failure intensity. [Chang and Lo (2011)](2011) considered the residual value as a linearly decreasing function of time. In this chapter, we consider both the total usage and age (e.g., the residual value of a car largely depends on its mileage and age) in modeling the equipment’s residual value:

$$SV_0(r, t) = \theta_5 - \theta_3 X^2(t) - \theta_4 T^2(t) \quad (3.13)$$

with $\theta_3, \theta_4, \theta_5 \geq 0$. It is worth pointing out that PM increases the equipment’s residual value. By substituting the virtual age $\nu(K)$ into (3.13), the residual value with PM at the end of the lease period $K$ becomes:

$$SV(r, K, N, \delta) = SV_0(r, \nu(K)) = \theta_5 - \frac{(\theta_3 r^2 + \theta_4)(N\delta + 1)^2 K^2}{(N + 1)^2} \quad (3.14)$$

We also assume that the equipment’s purchase price is $SV(0, 0) = V_0$, and at the end of the life cycle $L$ when it is used at the maximum usage rate, it has residual value $SV(r_m, L) = V_L$. As a result, we have $\theta_5 = V_0$ and $\theta_4 = \frac{V_0 - V_L}{L^2} - \theta_3 r_m^2$.

### 3.2.6 Payoff Functions of the Lessee and Lessor

The payoff of the lessee is defined as the total profit he gains by using the equipment. In particular, his total profit, denoted by $\Pi_1(r, K, N, \delta)$, can be calculated considering the revenue generated by the equipment (3.12) and the total price paid for leasing as:

$$\Pi_1(r, K, N, \delta) = \frac{u_m r K}{r_m L} \left[L - \frac{K(N\delta + 1)}{2(N + 1)}\right] - \alpha_0 r^2 K \quad (3.15)$$
On the other hand, the total profit of the lessor is defined as the price the lessee pays for leasing, minus the costs of performing PM (3.8) and CM (3.10), and the equipment depreciation (3.14). As a result, the lessor’s profit can be expressed as:

$$
\Pi_2(r, K, N, \delta) = \alpha_0 r^2 K - Na - \frac{NbK(1 - \delta)}{(N + 1)} - \frac{K^2C_f(\theta_1 r + \theta_2)(N\delta + 1)}{2(N + 1)} - \frac{K^2(\theta_3r^2 + \theta_4)(N\delta + 1)^2}{(N + 1)^2}
$$

(3.16)

### 3.3 Non-cooperative Solutions

We first consider that the lessor and lessee compete in a non-cooperative fashion, and the governing optimization/equilibrium concept underlying non-cooperative behavior is that of Nash. A pair of strategies is a Nash equilibrium if each player maximizes his own payoff function assuming the other player chooses his equilibrium strategy. We next consider Stackelberg equilibrium with lessor as the leader. The Stackelberg version represents a situation where the lessor is dominant in the game. The usual way to find Nash and Stackelberg equilibria is based on the players’ best response functions.

The best response function in a two-person game can be described as follows. Suppose $\Pi_1(x_1, x_2)$ and $\Pi_2(x_1, x_2)$ are the payoff functions of players 1 and 2 when they choose $x_1 \in X_1$ and $x_2 \in X_2$ from their respective strategy sets. We call the strategy of player 1, denoted by $x_{1R}^*(\hat{x}_2)$, which maximizes his payoff given that player 2’s strategy is $\hat{x}_2$, the best response of player 1 to $\hat{x}_2$. The best response function of player 2, $x_{2R}^*(x_1)$, as function of $x_1$, can be defined in the same manner. In this section, the best response functions of the lessee and lessor will be determined (Sections 3.3.1 and 3.3.2), and the Nash and Stackelberg non-cooperative game models will be developed (Sections 3.3.3 and 3.3.4) and compared (Sections 3.3.5).
3.3.1 Lessee’s Best Response Function

The best response function of the lessee can be determined by solving the following problem:

\[
\max_{r,K} \Pi_1(r, K) = \frac{u_mrK}{r_mL} \left[ L - \frac{K(N\delta + 1)}{2(N + 1)} \right] - \alpha_0 r^2 K
\]

s.t. \[0 \leq r \leq r_m\]
\[0 \leq K \leq L\] (3.17)

In the problem formulation, the lessee seeks to maximize his profit based on his strategy set in (3.1), while taking the lessor’s strategies \((N, \delta)\) as given. Setting the first derivative of \(\Pi_1(r, K)\) with respect to \(r\) equal to zero and solving for \(r\) yields:

\[
r^*_1(K) = \frac{u_m}{2\alpha_0 r_m L} \left[ L - \frac{K(N\delta + 1)}{2(N + 1)} \right] \tag{3.18}
\]

Similarly, setting the first derivative of \(\Pi_1(r, K)\) with respect to \(K\) equal to zero and solving for \(K\) yields:

\[
K^*_1(r) = \frac{(N + 1)}{(N\delta + 1)} \left[ L - \frac{\alpha_0 r_m L r}{u_m} \right] \tag{3.19}
\]

Accordingly, solving the problem in (4.6) gives the optimality conditions for the lessor and lessee, as presented in Proposition 1 (see Appendix for the proof).

**Proposition 1.** The lessee’s best response in terms of usage rate and lease period \((r, K)\) is:

i. \((r^*_1(L), L)\) when \(A_1 > 0, D_1 < 0\)

ii. \((r_m, L)\) when \(C_1 > 0, D_1 > 0\)

iii. \((r_m, K^*_1(r_m))\) when \(B_1 > 0, C_1 < 0\)

iv. \((\bar{r}_1, \bar{K}_1)\) when \(A_1 < 0, B_1 < 0\)

where \((\bar{r}_1, \bar{K}_1)\) is the solution to (3.18) - (3.19) as \((\bar{r}_1 = \frac{u_m}{3\alpha_0 r_m}, \bar{K}_1 = \frac{2L(N+1)}{3(N\delta+1)})\), and \(A_1 = 2N - 3N\delta - 1, B_1 = u_m - 3\alpha_0 r^2_m, C_1 = u_m(N - N\delta) - (N + 1)\alpha_0 r^2_m,\) and \(D_1 = u_m(2N - N\delta + 1) - 4(N + 1)\alpha_0 r^2_m.\)
It is worth pointing out that for any given preventive maintenance policy \((N, \delta)\) of the lessor, this proposition gives strategies \((r, K)\) that maximize lessee’s payoff.

### 3.3.2 Lessor’s Best Response Function

The best response function of the lessor can be derived by solving the following problem:

\[
\begin{align*}
\max_{N, \delta} & \quad \alpha_0 r^2 K - Na - \frac{2NbK(1-\delta) + K^2 C_f(\theta_1 r + \theta_2)(N\delta + 1)}{2(N+1)} - \frac{K^2(\theta_3 r^2 + \theta_4)(N\delta + 1)^2}{(N+1)^2} \\
\text{s.t.} & \quad 0 \leq \delta \leq 1 \\
& \quad N = 0, 1, \ldots
\end{align*}
\]

\(3.20\)

In the problem formulation, the lessor seeks to maximize his profit \(\Pi_2\) subject to two sets of constraints in \((3.2)\), while taking the lessee’s strategies \((r, K)\) as given. The first order derivative of the objective function with respect to \(N\) is:

\[
\Pi_2' = -a - \frac{2bK(1-\delta) - K^2 C_f(\theta_1 r + \theta_2)(1-\delta)}{2(N+1)^2} + \frac{2K^2(\theta_3 r^2 + \theta_4)(N\delta + 1)(1-\delta)}{(N+1)^3}
\]

\(3.21\)

Setting this expression equal to zero and solving for \(N\) leads to a cubic equation whose solution \(N_2^*\) can be determined in closed form using the Cardano formula. The unique maximum \(N_2^*\) is given by (see Proposition 2 and the proof in Appendix):

\[
N_2^* = \frac{1}{3a} \left( C_2 + \frac{3aB_2}{C_2} \right) - 1
\]

\(3.22\)

where \(B_2 = \frac{K^2}{2} C_f(\theta_1 r + \theta_2) - bK\), \(C_2 = \sqrt[3]{\frac{54a^3 K^2 (\theta_3 r^2 + \theta_4)}{2} + \sqrt{-27a^2 Q_2}}, \) and \(Q_2 = 4aB_2^3 - 108a^2 K^4 (\theta_3 r^2 + \theta_4)^2\).

**Proposition 2.** \(N_2^*\) is the unique maximum.

Accordingly, the following proposition characterizes the best response function of the lessor (see Appendix for the proof).
Proposition 3. The lessor’s best response in terms of maintenance policy \((N, \delta)\) is \((N_2^*, 0)\).

This proposition gives the strategies \((N, \delta)\) which maximize the lessor’s payoff for any given strategies \((r, K)\) of the lessee. It can be seen that \(N\), the optimal number of PM, depends on the strategies the lessee might pick \((r\) and \(K\)), while \(\delta = 0\) (perfect maintenance) is the lessor’s best response to any lease period and usage rate (dominant strategy). It should be noted that in solving the problem we treated \(N\) as a continuous variable. After solving the problem, if \(N_2^*\) is not an integer, we choose either \(\lfloor N_2^* \rfloor\) (the largest integer not greater than \(N_2^*\)) or \(\lceil N_2^* \rceil\) (the smallest integer not less than \(N_2^*\)), whichever yields a larger payoff value.

3.3.3 The Nash Equilibrium

In this section, we assume that the lessor and lessee simultaneously maximize their profits in a non-cooperative way with respect to any possible strategy of the other player. This is called a simultaneous move game, and the solution concept for this game structure is called the Nash equilibrium. In the equilibrium, each player makes an optimal decision given the behavior of the other player, and therefore neither player has an incentive to deviate unilaterally from the equilibrium. In a Nash equilibrium, the best response functions of both players are satisfied. Accordingly, the following theorem provides the Nash equilibrium and the corresponding conditions (see Appendix for the proof).

Theorem 1. The Nash equilibrium \(x_n^* = (r_n^*, K_n^*, N_n^*, \delta_n^*)\) is given by:

i. \((r_1^*, L, N_3^*, 0)\) if \(D_1 < 0\)

ii. \((r_m^*, L, N_2^*, 0)\) if \(C_1 > 0, D_1 > 0\)

It can be seen that the lease period and degree of each PM action are constant \(K_n^* = L\) and \(\delta_n^* = 0\) (perfect maintenance), but the usage rate \(r_n^*\) and the number of PM \(N_n^*\) are
functions of other strategies. Besides, it should be noted that in case ii., \( N^*_2 \) can be obtained from equation (3.22), and in case i., \( N^*_3 \) can be obtained by simultaneously solving the two first order conditions for the lessor and the lessee in (3.18) and (3.21). This can be calculated by rounding the solution of equation \( M_3(N) = 0 \) where:

\[
M_3(N) = -a(N + 1)^5 - bL(N + 1)^3 + \frac{L^2C_f(N + 1)^2(2N + 1)u_m}{8\alpha_0 r_m} + \frac{L^2C_f\theta_2(N + 1)^3}{2} + \frac{L^2\theta_3 u_m^2(2N + 1)^2}{8\alpha_0^2 r_m^2} + 2L^2\theta_4(N + 1)^2
\]  

(3.23)

In case of multiple solutions, the one with the largest payoff value is selected.

### 3.3.4 The Stackelberg Equilibrium

In a Stackelberg game, the lessor commits himself to a particular maintenance policy and announces his choice to the lessee. The lessee as the follower then chooses the best lease period and usage rate accordingly. The lessor maximizes his profit by specifying the maintenance policy taking into account the expected behavior of the lessee. Assuming a rational player, the lessor expects the lessee to select his best response. Substituting the best response of the lessee (see Proposition 1) into the lessor’s payoff function transforms \( \Pi_2 \) into a nonlinear function of two variables \( N \) and \( \delta \), whose optimal solution is specified in the following theorem (see Appendix for the proof).

**Theorem 2.** The Stackelberg equilibrium \( x^*_s = (r^*_s, K^*_s, N^*_s, \delta^*_s) \) is given by:

i. \((r^*_1, L, N^*_4, 0)\) when \( D_1 < 0 \)

ii. \((r_m, L, N^*_2, 0)\) when \( C_1 > 0, D_1 > 0 \)

iii. \((r_m, K^*_1(r_m), 0, 1)\) when \( B_1 > 0, C_1 < 0, M_1 > 0 \)

iv. \((\bar{r}_1, \bar{K}_1, 0, 1)\) when \( A_1 < 0, B_1 < 0, M_1 > 0 \)
where
\[ M_1 = -\alpha_0 r^2 + b + \frac{A_3 C_f (\theta_1 r + \theta_2)}{2} \]  
(3.24)
and \( A_3 = L - \frac{a_0 r_m L}{u_m} \). It should be noted that in cases \( \text{ii, iii, and iv} \), the equilibria are given directly; but in case \( \text{i} \) equation \( M_4(N) = 0 \) needs to be solved for \( N \), where:
\[
M_4(N) = -a(N + 1)^2 + (N + 1)^3 \left( -b L + \frac{L^2 C_f \theta_2}{2} \right) + (N + 1)^2 (2N + 1) \left( \frac{L^2 C_f u_m r_m + L u_m^2}{8 \alpha_0 r_m^2} \right) \\
+ \frac{L^2 \theta_3 u_m^2 (2N + 1)^2}{8 \alpha_0^2 r_m^2} + (N + 1)^2 \left( 2L^2 \theta_4 - \frac{L^2 C_f \theta_1 u_m}{8 \alpha_0 r_m} \right) - \frac{L^2 \theta_3 u_m^2 (2N + 1)}{8 \alpha_0^2 r_m^2} 
\]  
(3.25)
This is a fifth degree polynomial equation possibly with multiple solutions. In case of multiple solutions, the one with the largest payoff value is selected.

### 3.3.5 Comparison of the Nash and Stackelberg Equilibria

In this section, we compare the two cases of the Nash equilibrium stated in Theorem 1 with the corresponding cases of the Stackelberg equilibrium stated in Theorem 2 (i and ii). One can see that the lease period \( K = L \) (life cycle of the equipment) and the degree of each PM action \( \delta = 0 \) (perfect maintenance) are the same for both games. We need to compare \( N \), the number of PM, and \( r \), the usage rate, as a function of \( N \), but no general results can be proved because of the possible multiple solutions. However, in case of unique solutions it is possible to perform comparison and obtain the following results (see Appendix for the proofs).

**Proposition 4.** One can see that:

i. When \( D_1 < 0 \), the number of PM is higher in the simultaneous game (i.e., \( N^*_3 > N^*_4 \)) if \( M_2(0) < 0 \) and \( \Pi'_2(r^*_1, L, N_A, 0) < 0 \), and otherwise it is smaller.

ii. When \( C_1 > 0 \) and \( D_1 > 0 \), the number of PM in the simultaneous game is the same as in the Stackelberg game (\( N^*_3 \)).
where $\Pi'_{2N}$ is defined in (3.21), and $N_A$ is the finite unique solution to:

$$M_2(N) = 2\alpha_0 r_1^* L - \frac{L^2 C_f \theta_1}{2(N + 1)} - \frac{2L^2 \theta_3 r_1^*}{(N + 1)^2} = 0$$  (3.26)

It is worth pointing out that the same results apply to the comparison of usage rates in both games, as $r_1^*$ is an increasing function of $N$. Next, we compare the payoffs of these games.

**Proposition 5.** One can see that:

i. The lessor always prefers the leader-follower structure rather than the simultaneous move structure.

ii. The lessee prefers the game with a higher number of PM.

The first statement is true because the lessor can get at least the payoff of simultaneous move by choosing the Nash equilibrium. For the second statement, one can see that the lessee’s payoff is increasing in $N$, so the number of PM determines the lessee’s preference (see Proposition 4 regarding the number of PM).

### 3.4 Cooperative Solution

In the previous section, we focused on the results for both the non-cooperative simultaneous move game and the leader-follower game, but the corresponding Nash and Stackelberg equilibria generally do not lead to the maximum total profit for the players. An alternative solution is for the lessor and lessee to act cooperatively in order to increase their total profit (Cachon and Zipkin, 1999; Nagarajan and Sošić, 2008).

#### 3.4.1 Maximum Total Profit

In a cooperative regime, the strategy set of the lessee and lessor is $X = \{ (r, K, N, \delta) \mid 0 \leq r \leq r_m, \ 0 \leq K \leq L, \ N \geq 0 \ \text{integer}, \ 0 \leq \delta \leq 1 \}$. The players choose the set of strategies
\( x^*_c \in X \) that solves \( \max_x \{ \Pi \} = \max_x \{ \Pi_1(x) + \Pi_2(x) \} \) where \( \Pi_1 \) and \( \Pi_2 \) are the payoffs of the lessee and the lessor given in (3.15) and (3.16), respectively. The cooperative solution is obtained by solving the following problem:

\[
\max_{r,K,N,\delta} \Pi = \frac{u_m r K}{r_m L} \left[ L - \frac{K(N\delta + 1)}{2(N + 1)} \right] - Na - \frac{NbK(1 - \delta)}{(N + 1)} - K^2 C_f(\theta_1 r + \theta_2) \frac{(N\delta + 1)}{2(N + 1)} \\
- K^2(\theta_3 r^2 + \theta_4) \frac{(N\delta + 1)^2}{(N + 1)^2}
\]

s.t. \( 0 \leq r \leq r_m \)

\( 0 \leq K \leq L \)

\( N \geq 0 \) integer

\( 0 \leq \delta \leq 1 \)

In the problem formulation, the lessee and lessor seek to jointly maximize the sum of their profits. The following proposition characterizes the optimum solution (see Appendix for the proof).

**Proposition 6.** The solution that maximizes the total profit \( \Pi \) is \( x^*_c = (r^*_c, K^*_c, N^*_c, \delta^*_c) = (r_m, L, N^*, 0) \), where \( N^* = \frac{1}{3a}(C + \frac{3aB}{C}) - 1 \), with \( B = \frac{K^2}{2}(\frac{u_m}{L} + C_f(\theta_1 r + \theta_2)) - bK \), \( C = \sqrt{\frac{54a^2 K^2(\theta_3 r^2 + \theta_4)^3 - 27a^2 Q}{2}} \), and \( Q = 4aB^3 - 108a^2 K^4(\theta_3 r^2 + \theta_4)^2 \).

In this chapter, we call the solution \( x^*_c \) the total maximum solution or the cooperative solution. Again, if \( N^* \) is not an integer, we can use the same technique mentioned after Proposition 3. Besides, the following result regarding \( N^* \) can be obtained (see Appendix for the proof).

**Proposition 7.** \( N^* \) is the unique maximum.

These two propositions indicate that the cooperative solution \( x^*_c \), which maximizes the sum of the lessor’s and the lessee’s profits, is the unique strategy vector, for which the optimum usage rate equals the maximum usage rate \( r^*_c = r_m \), lease period is the equipment’s life cycle \( K^*_c = L \), number of PM is unique \( N^*_c = N^* \), and degree of each maintenance is \( \delta^*_c = 0 \) (perfect maintenance).
3.4.2 Comparison of the Non-cooperative and Cooperative Solutions

By comparing the non-cooperative and cooperative solutions, the following useful managerial insights can be obtained (see Appendix for the proof).

**Proposition 8.** The lessor’s numbers of PM in both the Nash and the Stackelberg equilibria are less than the one in the total maximum solution \( N^* > N_2^* > N_3^* \) and \( N^* > N_2^* > N_4^* \).

**Proposition 9.** The lessee’s usage rates in both the Nash and the Stackelberg equilibria are not higher than the one in the total maximum solution \( r_1^*(N) < r_m \).

These two propositions indicate that the maintenance quality for the equipment under the cooperative contract is better than the ones under both non-cooperative alternatives. Besides, the equipment’s usage rate under the cooperative contract is higher than those under the non-cooperative ones. In other words, competition lowers both the usage rate and the maintenance quality of the leased equipment. It should be noted that the non-cooperative and cooperative schemes result in the same lease period \( K \) and PM degree \( \delta \). Since the Nash and Stackelberg equilibria are different from the total maximum solution, the following result is expected (see Appendix for the proof).

**Proposition 10.** The total profit in the cooperative scheme is higher than the total profits in both the Nash and Stackelberg equilibria.

This proposition is a well-known general result in the literature, which means that the lessor and the lessee can increase their total profit by switching to the cooperative contract and choosing the total maximum solution \( x_c^* \). That is \( \Pi(x_c^*) > \Pi(x_n^*) \) and \( \Pi(x_c^*) > \Pi(x_s^*) \) where \( \Pi \) is the sum of the profits of the players as \( \Pi = \Pi_1 + \Pi_2 \). Next, we define \( \Delta \Pi \) as:

\[
\Delta \Pi = \Pi(x_c^*) - \Pi(x_n^*) = \Pi_1(x_c^*) + \Pi_2(x_c^*) - \Pi_1(x_n^*) - \Pi_2(x_n^*) > 0 \quad (3.28)
\]

We call \( \Delta \Pi \) the joint profit gain, which is the profit the players jointly achieve by moving from the Nash scheme to the cooperative contract. Similarly, the joint profit gain for
switching from Stackelberg equilibria to the total maximum solution can be determined by substituting $x_s^*$ for $x_n^*$ in (3.28).

### 3.4.3 Transfer-payment Contract

In this section, we demonstrate that the cooperative solution can be implemented under transfer-payment (or side-payment) contracts. There are two criteria for a proper side-payment contract to satisfy (Leng and Zhu, 2009). First, selecting the total maximum solution should make each player better off compared to the non-cooperative alternatives. Otherwise, the player who is worse off loses his incentive to cooperate and won’t choose the total maximum solution. Second, to avoid probable deviations of the players, the total maximum solution should be an equilibrium for the side-payment contract (Moses and Seshadri, 2000). To satisfy these two criteria, we propose a transfer-payment from the lessor to lessee with three parameters ($\gamma, \bar{\alpha}_0, \beta$) as:

$$H(x) = \gamma + \bar{\alpha}_0 r^2 K + \frac{\beta}{N + 1} \tag{3.29}$$

The transfer payment $H(x)$ modifies the payoff functions of the lessor and lessee in (3.15) and (3.16), respectively, to:

$$\tilde{\Pi}_1(x) = \Pi_1(x) + H(x)$$

$$\tilde{\Pi}_2(x) = \Pi_2(x) - H(x)$$

The desired value of constant parameter $\gamma$ needs to be such that both the lessor and lessee are better off by cooperation (satisfying the first criterion) and the set of two parameters $(\bar{\alpha}_0, \beta)$ such that the total optimal solution $x_c^* = (r_m, L, N^*, 0)$ is an equilibrium for the modified game with side-payment (satisfying the second criterion).

In order to determine $\gamma$, we assume that the lessor and lessee negotiate over the share of the maximum total profit. We allocate the profit based on Nash Bargaining solution since it’s simple and robust, besides empirical evidence supports this solution compared to other alternatives (Nagarajan and Bassok, 2008). The following result determines the
constant side-payment term \( \gamma \) based on this solution. See [Muthoo (1999)] and [Leng and Zhu (2009)] for proof.

**Proposition 11.** If the players equally distribute the joint profit gain, \( \Delta \Pi \), as suggested by the Nash bargaining solution, then \( \gamma \) is determined as follows:

\[
\gamma = \frac{\sum_{i=1}^{2} (-1)^i [\Pi_i(x^*_i) + (-1)^{i+1} \left( \bar{\alpha}_0 r_m^2 L + \frac{\beta}{N^*+1} \right) - \Pi_i(x^*_n)]}{2}
\]

where \( \Delta \Pi \) is defined in (3.28). This proposition states that when the players negotiate in cooperation, they agree to give each other the Nash profits \( \Pi_1(x^*_n) \) and \( \Pi_2(x^*_n) \), which are the payoffs they will obtain if they do not cooperate at all, and moreover, they equally split the joint profit gain, \( \Delta \Pi \). The constant value \( \gamma \) motivates the players to cooperate and to choose the total maximum solution \( x^*_c \), by ensuring that the lessee’s net profit of \( \bar{\Pi}_1(x^*_c) \) and the lessor’s profit of \( \bar{\Pi}_2(x^*_c) \) are \( \Delta \Pi / 2 \) higher than the non-cooperative profits. Similarly, the value of \( \gamma \) for the Stackelberg game can be determined by substituting \( x^*_s \) for \( x^*_n \).

Next, we determine proper values of \( \bar{\alpha}_0 \) and \( \beta \) such that the total optimal solution \((r_m, L)\) is the best response for the lessee and \((N^*, 0)\) is the best response for the lessor. Based on the two cases of Nash equilibrium in Theorem 1, two side-payment contracts are designed and presented in Propositions 12 and 13 (see Appendix for the proofs).

**Proposition 12.** In the case of \( D_1 < 0 \), a proper side-payment contract is \( H(x) = \gamma + \bar{\alpha}_0 r^2 K + \frac{\mu w L}{2(N+1)} \), where \( \bar{\alpha}_0 \) satisfies relation \( \bar{\alpha}_0 \geq \alpha_0 - \frac{\mu w (2N^*+1)}{4\bar{\alpha}_0 (N^*+1)} \).

Based on the proposition, a proper transfer function which makes the the total maximum solution \((r_m, L)\) the best response for the lessee is \( \bar{\alpha}_0 r^2 K \). One interpretation of this function is that the lessor is reducing the original price function from \( \alpha_0 r^2 K \) to \((\alpha_0 - \bar{\alpha}_0) r^2 K \) to make the total maximum solution the best response for the lessee. The next term in the transfer function, \( \frac{\mu w L}{2(N+1)} \) (or \( \beta = \frac{\mu w L}{2} \)), makes \((N^*, 0)\) the best response of the lessor and can be interpreted as the penalty the lessee charges the lessor for the revenue
loss caused by deterioration of the equipment (see section \ref{sec3.2.4}). This penalty function is decreasing in $N$, since better maintenance quality decreases the loss of revenue. As a result, the transfer function $\bar{\alpha}_0 r^2 K + \frac{u_m L}{2(N+1)}$ makes the total optimal solution a Nash equilibrium for the game (satisfying the second criterion), and the fixed transfer payment $\gamma$ ensures that this contract is beneficial for both parties (satisfying the first criterion).

**Proposition 13.** In the case of $C_1 > 0$ and $D_1 > 0$, the unique proper side-payment contract is $H(x) = \gamma + \frac{u_m L}{2(N+1)}$, where $\gamma \leq 0$.

It can be seen that when $C_1 > 0$ and $D_1 > 0$ (case ii of Theorem 1), the total maximum solution $(r_m, L)$ is already the best response of the lessee so $\bar{\alpha}_0 = 0$. In order to make $N^*$ the best response of the lessor, we use the transfer function $\frac{u_m L}{2(N+1)}$, same as Proposition 12. This is the unique function that makes the first order derivative of the lessor’s payoff function identical to that of the total payoff function. $\gamma \leq 0$ shows that the lessee transfers the value $|\gamma|$ to the lessor to motivate him to cooperate and choose the total maximum solution $N^*$.

It should be noted that in our case, the transfer functions in Propositions 12 and 13 are the same for both simultaneous and Stackelberg games cases i and ii. The reason is that the lessee’s (follower’s) best response $(r_m, L)$ is constant in terms of $N$ and $\delta$, and substituting $(r_m, L)$ in the lessor’s (leader’s) payoff $\bar{\Pi}_2$ doesn’t change the lessor’s best response. A similar approach is needed to determine the transfer function for the cases iii and iv of the leader-follower game. Generally, the side-payment functions of the simultaneous and Stackelberg game are different when the best response of the follower is a function of the leader’s strategies. Another point is that although the choice of transfer payment function $\frac{u_m L}{2(N+1)}$ is unique, the transfer function $\alpha_0 r^2 K$ is not unique. Our proposed transfer function $\bar{\alpha}_0 r^2 K$ is in the form of our original price function $\alpha_0 r^2 K$, which can properly coordinate the players, and has a nice price discount interpretation and implementation. Besides, although the transfer payment $\gamma$ is essential in assuring that the contract is attractive for both lessor and lessee, it has no impact on the total maximum solution and profit or on the
strategies each of the lessor and the lessee will choose. The following three-step process can be used to derive a proper side-payment contract $H(x)$:

1. Derive the Nash equilibrium $x^*_n$ using Theorem 1 and compute the corresponding total profit $\Pi(x^*_n) = \Pi_1(x^*_n) + \Pi_2(x^*_n)$.

2. Derive the total maximum solution $x^*_c$ using Proposition 6 and compute the maximum total profit $\Pi(x^*_c) = \Pi_1(x^*_c) + \Pi_2(x^*_c)$.

3. Calculate the joint profit gain $\Delta \Pi$ and three parameters ($\gamma, \bar{\alpha}_0, \beta$) based on Propositions 11-13.

The same process can be used to derive a proper side-payment contract in leader-follower scenario by considering Stackelberg equilibrium.

### 3.5 Numerical Example

We have shown that cooperation can increase the total profit of the players. In this section, a numerical study is carried out to illustrate how large the value of cooperation can be. To this end, we consider both the simultaneous and Stackelberg games described in Sections 3.3.3 and 3.3.4, and compare their total profits with the maximum total profit.

We consider leasing a vehicle with the following parameters. The initial value of the vehicle is $V_0 = 20,000$, and its failure intensity is the one in (3.7). Its life cycle is $L = 10$ years, and the maximum usage rate of the vehicle is $r_m = 20,000$ miles/year. If the lessee leases the vehicle for one year at the maximum usage rate $r_m = 20,000$, he needs to pay the lessor $2,800 (\alpha_0 = 7 \times 10^{-6})$. The vehicle generates up to $6,000$ annual revenue ($u_m = 6,000/\text{year}$) for the lessee, and the salvage value of the vehicle is $2,000$ after 10 years of usage at the highest annual usage rate ($r_m$).

Fig. 3.2 shows the total profits of the non-cooperative contracts, $\Pi(x^*_n)$ and $\Pi(x^*_c)$, as a percentage of the maximum total profit $\Pi(x^*_c)$ in the cooperative solution. It can be seen that cooperation can increase the players’ total profit, but the magnitude of increase is
case specific. As an example, when \( u_m = $3,600/year \) the total profits under the Nash and Stackelberg games are 51.58\% and 53.69\% of the maximum total profit, respectively, indicating that the value of cooperation is significant; while when \( u_m = $6000/year \) the two quantities are around 99\%, suggesting that cooperation can increase the total profit for only one percent. According to the analytical results of Theorem 1, one can see

![Figure 3.2: Total profits of non-cooperative contracts as a percentage of total profit in cooperative one](image)

that when \( D_1 = u_m(2N^*_n + 1) - 4(N^*_n + 1)\alpha_0r^2_m > 0 \) (i.e., the revenue \( u_m \) is high enough or the price \( \alpha_0 \) is low) the non-cooperative strategy \( x^*_n \) is similar to the cooperative one \( x^*_c \) except for the number of PM, \( N \). In this case, the performance of the non-cooperative solution is nearly as good as the cooperative solution, but when \( D_1 < 0 \) the non-cooperative strategy \( x^*_n \) differs from the cooperative one \( x^*_c \) in both \( r \) and \( N \). In this case, the value of cooperation can be substantial, so engaging the lessor and lessee in price negotiation is encouraged.

Table 3.1 shows the strategies and payoffs in both simultaneous move and Stackelberg non-cooperative contracts and the corresponding strategies and payoffs in the cooperative scheme. When \( u_m = $3,600/year \), the usage rate that the lessee chooses in the simultaneous move game is \( r^*_n = 12,214 \text{ miles/year} \), and the number of PM the lessor chooses is \( N^*_n = 9 \). Next, if the players choose to cooperate the lessee chooses the maximum usage
rate \( r^*_c = 20,000 \text{ miles/year} \), and the lessor chooses \( N^*_c = 19 \). As a result of this contract, the lessee’s profit increases from $10,440 to $17,119.5 and the lessor’s profit from $3,796 to $10,475.5. One can see that the cooperative scheme can result in major performance improvements for the two players in this case. However, when \( u_m = $6000/\text{year} \) (high revenue) the lessee has enough motivation to lease the vehicle at the maximum annual usage rate \( r^*_n = 20,000 \text{ miles} \) in the non-cooperative contract. In this case should the players wish to cooperate, the lessor increases the number of PM from \( N^*_n = 13 \) to \( N^*_c = 22 \). The profit of lessee as a result of switching to the cooperative contract increases from $29,857 to $30,110, and the lessor’s profit increases from $20,667 to $20,920, which are not considered major improvements. It should be noted that the optimal lease periods for all the games in the table are \( K = 10 \text{ years} \), and the corresponding degrees of PM are \( \delta = 0 \).

Table 3.1: Non-cooperative and cooperative strategies and payoffs

<table>
<thead>
<tr>
<th>Max revenue ( u_m($/y) )</th>
<th>Non-cooperative</th>
<th>Cooperative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usage ( r^*_n(\text{miles/y}) )</td>
<td>Usage ( r^*_c(\text{miles/y}) )</td>
<td>Usage ( r^*_n(\text{miles/y}) )</td>
</tr>
<tr>
<td>PM ( N^*_n )</td>
<td>PM ( N^*_c )</td>
<td>PM ( N^*_n )</td>
</tr>
<tr>
<td>Lessee ( \Pi_1($) )</td>
<td>Lessee ( \Pi_2($) )</td>
<td>Lessee ( \Pi_1($) )</td>
</tr>
<tr>
<td>Simultaneous</td>
<td>Stackelberg</td>
<td></td>
</tr>
<tr>
<td>3600</td>
<td>12214</td>
<td>9</td>
</tr>
<tr>
<td>4000</td>
<td>13636</td>
<td>10</td>
</tr>
<tr>
<td>4400</td>
<td>15060</td>
<td>11</td>
</tr>
<tr>
<td>4800</td>
<td>16429</td>
<td>11</td>
</tr>
<tr>
<td>6000</td>
<td>20000</td>
<td>13</td>
</tr>
<tr>
<td>4000</td>
<td>13839</td>
<td>15</td>
</tr>
<tr>
<td>4400</td>
<td>15278</td>
<td>17</td>
</tr>
<tr>
<td>4800</td>
<td>16692</td>
<td>18</td>
</tr>
<tr>
<td>6000</td>
<td>20000</td>
<td>13</td>
</tr>
</tbody>
</table>

Note that when comparing the Nash and Stackelberg games, it is clear that the lessor is always better off with the leader-follower scheme. In this example, the lessee is also better off with the Stackelberg game (which is not a general result based on Proposition 5).

The results given in Table 3.1 are also shown in Fig. 3.3 where the relative increases of usage rate \( r \) in the cooperative contract over the non-cooperative alternatives are defined as \( \frac{r^*_c - r^*_n}{r^*_n} \times 100\% \) and \( \frac{r^*_c - r^*_s}{r^*_s} \times 100\% \), respectively for simultaneous and Stackelberg games, as revenue \( u_m \) increases. In the same manner, the relative increases in the number of PM
and the profits of the lessee and lessor are also presented. It can be seen that when $u_m = $6000/year (high revenue) the relative increase in usage rate is zero (usage rate is 20,000 (miles/year) in both the non-cooperative and cooperative solutions). In this case, the number of PM increases for 69.23% as a result of cooperation, which increases the lessee’s profit only by 0.84% and the lessor’s profit by 1.22%, but when $u_m = $3600/year, cooperation increases the lessee’s profit by 63.97% and the lessor’s profit by 175.96% which is a significant increase in profits.

Figure 3.3: Relative increases in $r$, $N$, $\Pi_1$, and $\Pi_2$ by switching from the non-cooperative contracts to the cooperative one
3.6 Conclusion

In this chapter, we modeled two non-cooperative lease contracts considering Nash and Stackelberg equilibria and a cooperative one aiming at maximizing the total payoff of the players. We showed that the Nash and Stackelberg equilibria are different from the total maximum solution, and thus the players can increase their total profit by cooperation. Implementation of the cooperative solution requires two major criteria. We have shown that these two criteria can be achieved by a nonlinear transfer-payment contract. Our results illustrate that cooperation can be regarded as a value-added strategy in establishing such lease contracts. Besides, our numerical results show that while cooperation always increases the total profit, the magnitude of increase is case specific. When the lease price is low or the revenue is high, the profits in the non-cooperative contracts will be close to the cooperative alternative, while in other cases the cooperation may increase the total profit significantly.

An interesting direction for a future research is to consider cases where there are several players with different payoff parameters. The development of such models will provide insights into the effect of increased competition on lease contracting.
CHAPTER 4

MAINTENANCE OUTSOURCING CONTRACTS BASED ON BARGAINING THEORY

We address a maintenance outsourcing problem where the owner of a piece of critical equipment plans on outsourcing preventive and failure replacement services to a service agent. The equipment owner (i.e., customer) and the agent negotiate on the maintenance policy and spare part ordering strategy. We first provide the Nash bargaining solution to the problem and derive the optimal threats that the customer and agent can pose against each other in contract negotiation. We next show how the players can negotiate the price. In this case a discounted negotiated price can provide an attractive scheme for the customer to choose the maintenance policy that maximizes the total profit. A numerical study illustrates the effects of threats and price discount scheme on the individual and total profits.

4.1 Introduction

In this chapter, we use bargaining game-theoretic approach to design contracts for the cases that the owner of a piece of equipment plans on outsourcing preventive and failure replacement services to a service agent. Here, we call the equipment owner the customer of the agent or generally customer, and we call the service agent, who can be the original equipment manufacturer or a third party service provider, agent. The customer decides on the preventive replacement time of the equipment, and the agent decides on the ordering time for the required spare part. We first formulate the problem using Nash bargaining model. Then, we consider the case that the players pose threat against each other in
bargaining. The customer can make threat against the agent using the replacement policy, and the agent can threat the customer by spare part availability. Finally, we consider cooperation through price discount negotiation.

We can fully characterize this two-person Nash bargaining problem by three numbers: customer’s threat point, agent’s threat point, and the maximum total profit of the customer and agent (Myerson 1991). We first determine the solution to these three values and then determine the Nash bargaining solution for different cases. Through a four-step procedure, we show how the total maximum profit can be obtained if the agent adjusts the service charges. A numerical study shows how the policies and profit allocation alter through the use of threats and price discount.

The remainder of this chapter is organized as follows. Section 4.2 provides a description of the problem and derives the payoff functions of the customer and the agent. Section 4.3 briefly describes how to model negotiation through the Nash bargaining game and determines the allocation of profit obtained from price negotiation. In Section 4.4, we numerically examine the effect of negotiation with and without side payment on the outcome of the contract. Finally, Section 4.5 concludes the chapter and outlines the directions for future research.

### 4.2 Problem Description and Model Formulation

The owner of a piece of equipment makes revenue $R$ per unit time when the equipment is in operation and makes no revenue when it fails. The time to failure of the equipment, denoted by $x$, has a known probability density function (pdf) $f(x)$, cumulative distribution function (cdf) $F(x)$, and reliability function $\bar{F}(x) = 1 - F(x)$. The equipment’s failure rate $\lambda(x) = f(x)/\bar{F}(x)$ is an increasing function of time (i.e., increasing failure rate).

The owner outsources preventive and failure replacement services to a service agent and thus becomes a customer of the service agent. If the agent and the customer come to an agreement, the service agent is responsible for doing preventive replacement at
equipment age $T_R$ and failure replacement whenever the equipment fails based on the contract. For both cases, spare parts are required to fulfill the service, and the equipment is as good as new after replacement. The service agent orders a spare part after time $T_O$ followed by each service, and the lead time $L$ is fixed. The service agent can hold at most one spare part with inventory holding cost of $C_i$ per unit of time. We assume that $T_O + L \leq T_R$ [Armstrong and Atkins, 1996; Thomas and Osaki, 1978] and $T_O \geq 0$ to ensure that the inventory is empty upon the arrival of a new spare part. When a replacement service is requested, there are two possibilities. If a spare part is on-hand, the agent does an immediate replacement; otherwise replacement is delayed until the ordered spare part arrives and the agent has to pay shortage cost $S$ per unit time to the customer to compensate the downtime loss. The agent charges the customer $P_p$ and $P_f$ for each preventive and failure replacement, respectively, where $P_p \leq P_f$. In this chapter, we first assume that the charges are exogenously determined by the market (contract without side payment), and later relax this assumption by making them negotiable between the customer and the agent (contract with side payment). For the agent, the cost $C_f$ for performing each failure replacement is higher than the one $C_p$ for preventive replacement.

It is worth pointing out that Murthy and Yeung (1995) assumed a zero lead time and uniformly distributed repair time, but we consider a fixed lead time and instantaneous repairs.

In establishing the maintenance service contract, decision variables (i.e., terms to be specified in the contract) are the preventive replacement age $T_R$ for the customer and the spare part reordering time $T_O$ for the agent. According to game-theoretic terminology, the two decision variables are called the strategies of the two players. Particularly, the set of simultaneous strategies is given by:

$$A = \{(T_R, T_O) | T_O \geq 0, T_O + L \leq T_R \}. \quad (4.1)$$

The expected profits per unit time can be naturally considered as the payoff functions of the players, which will be derived next.
4.2.1 Customer’s Payoff Function

The customer’s long-run profit per unit of time can be determined based on the renewal reward theorem (Ross, 1970; Murthy and Yeung, 1995), which can be expressed as the expected cycle profit divided by the expected cycle length. We define a service cycle length as the time interval between the installation of a new part and its replacement. Under the assumption that \( T_O + L \leq T_R \), three scenarios may occur in a cycle. The first one is that the equipment fails before the agent receives a spare part, i.e., \( X < T_O + L \). In this case, replacement is delayed until the agent receives the part at \( T_O + L \), so the corresponding cycle length is \( T_O + L \). The customer’s profit in such a cycle is \( RX + S(T_O + L - X) - P_f \), as the agent must compensate the shortage cost to the customer.

The second scenario is that a failure occurs after the arrival of the ordered part while before the preventive replacement, i.e., \( T_O + L < X < T_R \). In this case, the agent replaces the failed part immediately, so the cycle length is \( X \) and the customer’s profit is \( RX - P_f \).

Lastly, when the equipment does not fail before the scheduled preventive replacement time, i.e., \( X > T_R \), the agent performs preventive replacement at \( T_R \), so the cycle length is \( T_R \) and the customer’s profit is \( RT_R - P_p \).

Considering these scenarios, the expected cycle profit for the customer, \( EPC \), can be expressed as:

\[
EPC = \int_{0}^{T_O + L} (Rx + S(T_O + L - x) - P_f) f(x) dx + \int_{T_O + L}^{T_R} (Rx - P_f) f(x) dx + \int_{T_R}^{\infty} (RT_R - P_p) f(x) dx.
\]

After simplification, we have:

\[
EPC = R \int_{0}^{T_R} \tilde{F}(x) dx + S \int_{0}^{T_O + L} F(x) dx - P_f F(T_R) - P_p \tilde{F}(T_R).
\]

On the other hand, the expected cycle length can be expressed as:

\[
ECL = \int_{0}^{T_O + L} (T_O + L) f(x) dx + \int_{T_O + L}^{T_R} x f(x) dx + \int_{T_R}^{\infty} T_R f(x) dx = T_R - \int_{T_O + L}^{T_R} F(x) dx.
\]
As a result, the long-run profit per unit time for the customer is given by:

\[
\Pi_c = \frac{EPC}{ECL} = \frac{R \int_0^{T_R} \bar{F}(x)dx + S \int_{T_O+L}^{T_o+L} F(x)dx - P_f F(T_R) - P_p \bar{F}(T_R)}{T_R - \int_{T_O+L}^{T_o+L} F(x)dx}.
\] (4.2)

### 4.2.2 Agent’s Payoff Function

The expected cycle profit for the agent can also be determined based on the three scenarios: if \( X < T_O + L \), the agent’s profit is \( P_f - C_f - S(T_O + L - X) \); if \( T_O + L < X < T_R \), the profit is \( P_f - C_f - (X - T_O - L)C_i \), and it should be noted that the agent pays the holding cost in such a cycle; if \( X > T_R \), the profit is \( P_p - C_p - (T_R - T_O - L)C_i \). Therefore, the expected cycle profit for the agent, \( EPA \), can be expressed as:

\[
EPA = \int_0^{T_o+L} (P_f - C_f - S(T_O + L - x))f(x)dx \\
+ \int_{T_o+L}^{T_R} (P_f - C_f - (x - T_O - L)C_i)f(x)dx \\
+ \int_{T_R}^{\infty} (P_p - C_p - (T_R - T_O - L)C_i)f(x)dx.
\]

After simplification, we have:

\[
EPA = (P_f - C_f)F(T_R) + (P_p - C_p)\bar{F}(T_R) - S \int_0^{T_o+L} F(x)dx - C_i \int_{T_o+L}^{T_R} \bar{F}(x)dx,
\]

and the agent’s long-run profit per unit time can be expressed as:

\[
\Pi_a = \frac{EPA}{ECL} = \frac{(P_f - C_f)F(T_R) + (P_p - C_p)\bar{F}(T_R) - S \int_0^{T_o+L} F(x)dx - C_i \int_{T_o+L}^{T_R} \bar{F}(x)dx}{T_R - \int_{T_O+L}^{T_o+L} F(x)dx},
\] (4.3)

### 4.3 Nash Bargaining Solution

The decision-making process involves bargaining, where the customer and agent determine their strategies \( T_R \) and \( T_O \) via negotiation. We model the bargaining process by the Nash bargaining model [Nash, 1953]. Nash presented four axioms that all bargaining solutions should satisfy and proved that there is a unique solution that meets these axioms.
In particular, the four axioms are: 1) symmetry meaning that if the players are identical, they receive identical payoffs; 2) feasibility requiring that the players can distribute only existing amount of profit; 3) Pareto optimality showing that if both players can increase their payoffs, this solution has to be included in the agreement; 4) independence from monotone increasing linear transformations stating that changing the unit in which the payoffs are computed cannot change the solution. A thorough explanation of the axioms can be seen in Roth (1979).

In our case, the Nash bargaining solution \((\Pi_B^c, \Pi_B^a)\) is defined as the profits of the customer and agent, which maximize the product of the differences between the payoff functions and fixed disagreement payoffs given by the following optimization problem:

\[
\begin{align*}
\text{max}_{\Pi_c, \Pi_a} & \quad (\Pi_c - \Pi^*_c)(\Pi_a - \Pi^*_a) \\
\text{subject to} & \quad \Pi_c \geq \Pi^*_c, \quad (4.5) \\
& \quad \Pi_a \geq \Pi^*_a, \quad (4.6) \\
& \quad (\Pi_c, \Pi_a) \in H, \quad (4.7)
\end{align*}
\]

where \(\Pi_c\) and \(\Pi_a\) are given by (4.2) and (4.3) respectively, \(\Pi^*_c\) and \(\Pi^*_a\) are the disagreement points, and \(H\) is the payoff set \(H = \{(\Pi_c, \Pi_a) | (T_R, T_O) \in A\}\) where \(A\) is the strategy set defined in (4.1).

It is important to mention that the payoff vector \((\Pi^*_c, \Pi^*_a)\) is defined as the guaranteed payoff obtained by the players in case they disagree to negotiate or cause disagreement in negotiation. To determine the disagreement point, different alternatives can be used. One is to let \((\Pi^*_c, \Pi^*_a) = (0, 0)\), another possibility is an equilibrium point, and a third one is to consider it as a threat point (Myerson, 1991). Here we assume that the customer needs to outsource the maintenance services, and the agent is the only service provider, and on the other hand, the equipment owner is the major customer of the service agent ((0, 0) is not the best choice). So some sort of business has to take place between the customer and the agent and we consider the threat point as the disagreement point for the players (Anbarci et al., 2002). The main purpose of making threat is to increase the cost of
possible conflict to the other player, in order to make him negotiate and agree on mutually satisfactory strategies (Harsanyi, 1977). However, if the customer and agent have the possibility to avoid each other and to have business with others, the disagreement payoffs for both players can be considered as zero. Note that the threat point is predetermined and independent of negotiation (Nagarajan and Bassok, 2008). Constraints (4.5)-(4.6) assert that neither player should get less in the bargaining solution than he could get without negotiation, so the Nash bargaining payoff is at or above this security level for each player. Since the players are rational, they will find it profitable to agree through negotiation and will never choose to disagree or to carry out the threats (Nagarajan and Bassok, 2008).

In this chapter, we assume feasible set of bargaining problem is not empty and derive the threat point of the customer and agent in sections 4.3.1 and 4.3.2, respectively, by the maximin value (Myerson, 1991; Reyniers and Tapiero 1995; Thomas, 1986; Cagalj et al., 2005).

The optimal solution of problem (4.4)-(4.7) is Pareto optimal. That is, there is no other feasible solution which is better than the Nash bargaining solution for one player and not worse for the other player. In other words, it ensures that all other solutions, which make one player better off, make the other player worse off. The form of Pareto optimal set depends on the way the negotiation between the players is conducted (Nash, 1953).

We first consider the case with no side payment (no price discount). In this case, the Pareto frontier, \( \omega \), can be determined by \( \max_{T_R, T_O} (1 - \theta) \Pi_c(T_R, T_O) + \theta \Pi_a(T_R, T_O) \) where \( \theta \in [0, 1] \). At \( \theta = 0 \), the objective function considers only the payoff function of the customer, at \( \theta = 1 \) it considers only the agent, and for values between 0 and 1 it considers some trade-off between the profits. For each constant \( \theta \in [0, 1] \), the optimum strategy, \( (T^\theta_R, T^\theta_O) \), and the corresponding values of \( \Pi_c(T^\theta_R, T^\theta_O) \) and \( \Pi_a(T^\theta_R, T^\theta_O) \) can be calculated, and the entire Pareto frontier can be determined. The Nash Bargaining solution \( (T^B_R, T^B_O) \) selects the unique point from the Pareto frontier, which maximizes the objective function in (4.4) while satisfying constraints (4.5) and (4.6). This can be seen in Fig. 4.1 The corresponding Nash profits of the customer and agent are shown by \( \Pi^B_c = \Pi_c(T^B_R, T^B_O) \)
and $\Pi^B_a = \Pi_a(T^B_R, T^B_O)$. The parameters of the contract are $(T^B_R, T^B_O)$, which determine the profits of the customer and agent. Because of the very different non-algebraic properties of the Nash bargaining solution (Anbarci et al., 2002), it is difficult to derive the solution analytically. Instead, we will look closely into this solution using simulation in the numerical study.

![Nash bargaining solution with no utility transfer](image)

Figure 4.1: Nash bargaining solution with no utility transfer

We next consider the case where the agent uses price discount as a mean to influence the customer’s decision $T_R$. The goal of the agent is to induce the customer to choose the maintenance policy that maximizes the total profit. The Pareto optimal set $\omega$ of the game with side payment (price discount) satisfies $\Pi_c + \Pi_a \leq \Pi^*$ where $\Pi^*$ is the highest possible total profit in the game (see Fig. 4.2). We will derive $\Pi^*$ and the corresponding optimal strategies in section 4.3.3 and then solve problem (4.4)-(4.7) accordingly in section 4.3.4 and determine the corresponding discounted prices.

### 4.3.1 Threat Point for the Agent

To make the agent reluctant to cause conflict in negotiation, the customer can adopt the threat strategy $T^*_{Ro} = \arg \min_{T_R} \Pi_a$ in case of disagreement which causes the greatest damage to the agent. Given the threat of the customer, the agent improves his bargaining
position by choosing strategy $T_{Oa}^\circ = \arg \max_{T_O} \Pi_a(T_{Ra})$, which maximizes his payoff (maximin strategy). Therefore, the threat profit $\Pi_a^\circ$ (maximin profit) is the guaranteed payoff for the agent in the worst case, and the agent won’t agree on any less profit when he negotiates. Technically, the agent’s threat payoff, $\Pi_a^\circ$, and the corresponding optimal strategies (Harsanyi, 1956) can be determined by solving the following problem:

$$
\Pi_a^\circ = \max_{T_O} \min_{T_R} \frac{(P_f - C_f)F(T_R) + (P_p - C_p)\bar{F}(T_R) - S \int_0^{T_O+L} F(x) dx - C_i \int_{T_O+L}^{T_R} \bar{F}(x) dx}{T_R - \int_{T_O+L}^{T_R} F(x) dx}
$$

subject to $0 \leq T_O \leq T_R - L$.

(4.8)

To solve this problem, we consider a two-step optimization process. In the first step, we find the customer’s threat strategy against the agent $T_{Ra}^\circ$; in the second step, we determine the reordering time $T_{Oa}^\circ$ that maximizes the agent’s payoff function for the value of $T_{Ra}^\circ$ determined in the first step.

In particular, the first step solves:

$$
\min_{T_R} \frac{(P_f - C_f)F(T_R) + (P_p - C_p)\bar{F}(T_R) - S \int_0^{T_O+L} F(x) dx - C_i \int_{T_O+L}^{T_R} \bar{F}(x) dx}{T_R - \int_{T_O+L}^{T_R} F(x) dx}
$$

subject to $T_O + L \leq T_R$.

We assume that the profits the agent obtains from failure replacement and preventive replacement are the same (i.e., $P_f - C_f = P_p - C_p$). As a result, the numerator of the derivative of the objective function with respect to $T_R$ divided by $\bar{F}(T_R)$ is:

$$
D(T_O) = -(P_p - C_p) + S \int_0^{T_O+L} F(x) dx - C_i(T_O + L),
$$

which is independent of $T_R$. The following proposition provides the optimum $T_R$ and the corresponding condition (see Appendix for the proof).

**Proposition 14.** For any fixed $T_O$, the value of $T_R$ that minimizes the agent’s payoff function is: $T_{Ra}^\circ = \infty$ if $D(T_O) < 0$, $T_{Ra}^\circ = T_O + L$ if $D(T_O) > 0$, and all $T_R \geq T_O + L$ if $D(T_O) = 0$. 
The next step is to determine the maximum of \( \Pi_a \) with respect to \( T_O \). Due to the tedious computation required to obtain analytical results, we will only consider one parameter set corresponding to the case of \( D(T_O) < 0 \), and the other cases of \( D(T_O) \geq 0 \) can be dealt with similarly. Substituting \( T_{Ra} = \infty \) into \( \Pi_a \), the problem to be solved is:

\[
\max_{T_O} \frac{P_f - C_f - S \int_{0}^{T_O + L} F(x) \, dx + C_i(T_O + L)}{\mu + \int_{0}^{T_O + L} F(x) \, dx} - C_i
\]

subject to \( T_O \geq 0 \),

(4.9)

where \( \mu = \int_{0}^{\infty} xf(x) \, dx \). The numerator of the derivative of the objective function with respect to \( T_O \) is:

\[
G(T_O) = C_i \left( \mu + \int_{0}^{T_O + L} F(x) \, dx \right) - F(T_O + L) \left( P_f - C_f + C_i(T_O + L) + S \mu \right).
\]

(4.10)

The following theorem provides the optimum \( T_O \) and the corresponding conditions (see Appendix for the proof).

**Theorem 3.** The objective function in (4.9) is unimodal and pseudo-concave in \( T_O \). For \( T_{Ra} = \infty \), the value of \( T_O \) that maximizes the agent’s payoff function satisfies \( G(T_{Oa}) = 0 \).

In this case, the following proposition states the threat point (maximin payoff) for the agent, which can be obtained by substituting (4.10) into the objective function in (4.9).

**Proposition 15.** The threat value for the agent is \( \Pi_{a} = -S + C_i \frac{F(T_{Oa})}{F(T_{Oa} + L)} \) if \( D(T_{Oa}) < 0 \), and the corresponding threat strategies are \( T_{Ra} = \infty \) and \( G(T_{Oa}) = 0 \).

Proposition [15] shows that the customer threatens the agent by the threat strategy \( T_{Ra} = \infty \) (do not do preventive replacement), which hurts the agent as much as possible (minimizes agent’s payoff). Given the threat of the customer, the agent improves his bargaining position by choosing strategy \( T_{Oa}^o \) which maximizes his payoff, and the corresponding threat payoff \( \Pi_{a}^o = \Pi_a(T_{Ra}, T_{Oa}) \) is the agent’s security payoff in the bargaining.
4.3.2 Threat Point for the Customer

The agent threatens the customer that if he causes disagreement in negotiation, the agent will implement the threat strategy, \( T^*_Oc = \arg \min_{T_O} \Pi_c \), which hurts the customer as much as possible (minimizes customer’s payoff). The purpose of the agent by making threat against the customer is to make him more reluctant to risk a conflict in negotiation. Given the threat of the agent, the customer improves his bargaining position by choosing strategy \( T^*_Rc = \arg \max_{T_R} \Pi_c(T^*_Oc) \) which maximizes his payoff. The customer’s threat profit, \( \Pi^*_c \), can be determined by solving the following problem:

\[
\Pi^*_c = \max_{T_R} \min_{T_O} \left( \frac{R \int_0^{T_R} \bar{F}(x)dx + S \int_0^{T_O+L} F(x)dx - P_f F(T_R) - P_p \bar{F}(T_R)}{T_R - \int_{T_O+L}^{T_R} F(x)dx} \right)
\]

subject to \( 0 \leq T_O \leq T_R - L \),

where the objective function is the customer’s payoff function (4.2). In order to solve the problem, we consider a two-step optimization approach: in the first step, we find \( T^*_Oc \) as the ordering time that minimizes the customer’s payoff function for a given value of \( T_R \); and next, we determine \( T^*_Rc \), as the optimum preventive replacement time that maximizes the customer’s payoff function for the value of \( T^*_Oc \) determined in the first step (Danskin, 1966). In the first step, the following problem is solved:

\[
\min_{T_O} \left( \frac{R \int_0^{T_R} \bar{F}(x)dx + S \int_0^{T_O+L} F(x)dx - P_f F(T_R) - P_p \bar{F}(T_R)}{T_R - \int_{T_O+L}^{T_R} F(x)dx} \right)
\]

subject to \( 0 \leq T_O \leq T_R - L \).

Clearly, the numerator of the derivative of the objective function with respect to \( T_O \) divided by \( F(T_O + L) \) is independent of \( T_O \):

\[
W(T_R) = (S - R) \int_0^{T_R} \bar{F}(x)dx + P_f F(T_R) + P_p \bar{F}(T_R).
\]

The following proposition provides the optimum \( T_O \) and the corresponding condition (see Appendix for the proof).
Proposition 16. For any fixed $T_R$, the value of $T_O$ that minimizes the customer’s payoff function is $T^o_{Oc} = 0$ when $W(T_R) > 0$, and $T^o_{Oc} = T_R - L$ when $W(T_R) < 0$.

Proposition 16 indicates that the agent can pose threat against the customer by adjusting spare part availability. That is the agent can delay a failure replacement when the customer prefers an instant replacement, or he can do an instant replacement when the customer prefers a delayed one. In other words, the spare part availability can affect the number of failure replacements and the customer’s profit.

The next step for the customer is to choose the action $T^o_{Rc}$ that maximizes the worst-case payoff. We will only consider the case for $W(T_R) > 0$, and the case for $W(T_R) < 0$ can be dealt with similarly. By substituting $T^o_{Oc} = 0$ into $\Pi_c$, the problem for the customer to solve is:

$$\max_{T_R} \frac{R \int_0^{T_R} \bar{F}(x) \, dx + S \int_0^L F(x) \, dx - P_f F(T_R) - P_p \bar{F}(T_R)}{T_R - \int_L^{T_R} F(x) \, dx}$$

subject to $T_R - L \geq 0$.

The numerator of the derivative of the objective function with respect to $T_R$ divided by $\bar{F}(T_R)$ is:

$$B(T_R) = (P_p - P_f) \lambda(T_R)(T_R - \int_L^{T_R} F(x) \, dx) + (R - S) \int_0^L F(x) \, dx + P_f F(T_R) + P_p \bar{F}(T_R).$$

The following theorem gives the optimality conditions (see Appendix for the proof).

Theorem 4. The objective function in (4.13) is unimodal and pseudo-concave in $T_R$. For $T^o_{Oc} = 0$, the value of $T_R$ that maximizes the customer’s payoff function satisfies $B(T^o_{Rc}) = 0$.

The following proposition provides the threat point for the customer, which can be proved by substituting $B(T_R) = 0$ into the objective function in (4.13).

Proposition 17. The threat point for the customer is $\Pi^c = (P_p - P_f) \lambda(T^o_{Rc}) + R$ if $W(T^o_{Rc}) > 0$, and the corresponding threat strategies are $T^o_{Oc} = 0$ and $B(T^o_{Rc}) = 0$. 
Proposition 17 shows that the agent can threaten the customer with the threat strategy $T^o_{Oc} = 0$ (doing failure replacement when the customer prefers a delayed replacement), which minimizes the customer’s payoff. Given the threat of the agent, the customer improves his bargaining position by choosing strategy $T^o_{Rc}$ that maximizes his payoff, and the corresponding threat payoff of the customer $\Pi^c$ is his security payoff in bargaining.

4.3.3 Total Maximum Profit

We next determine the maximum total profit of the customer and agent by solving:

$$\Pi^* = \max_{T_O, T_R} \Pi_c + \Pi_a = \frac{R \int_0^{T_R} \bar{F}(x) \, dx - C_i \int_{T_O+L}^{T_R} \bar{F}(x) \, dx - C_f F(T_R) - C_p \bar{F}(T_R)}{T_R - \int_{T_O+L}^{T_R} F(x) \, dx}$$

subject to $0 \leq T_O \leq T_R - L.$

(4.15)

In this formulation, the customer and agent seek to jointly maximize their total profit. We consider a two-step optimization process to solve the problem: in the first step, we find $T^*_R$ as the optimum preventive replacement age that maximizes the total payoff function for a given value of $T_O$; in the second step, we determine $T^*_O$ as the optimum ordering time that maximizes the total payoff function for the value of $T^*_R$ determined in the first step. One can see that the numerator of the derivative of the objective function with respect to $T_R$ is:

$$K(T_R) = -(C_f - C_p) \lambda(T_R)(T_R - \int_{T_O+L}^{T_R} F(x) \, dx) + C_f F(T_R)$$

$$+ C_p \bar{F}(T_R) + R \int_0^{T_O+L} F(x) \, dx - C_i(T_O + L).$$

The following proposition gives the optimum $T_R$ and the corresponding condition (see Appendix for the proof).

**Proposition 18.** For any fixed $T_O$, the value of $T_R$ that maximizes the total payoff function is $T^*_R = T_O + L$, when $K(T_R = T_O + L) < 0$.

The next step is to determine the maximum of $\Pi_c + \Pi_a$ with respect to $T_O$. After
substituting $T_R^* = T_O + L$ into $\Pi_c + \Pi_a$, problem (4.15) becomes:

$$\max_{T_O} \frac{R \int_{0}^{T_O+L} \bar{F}(x) \, dx - C_f F(T_O + L) - C_p \bar{F}(T_O + L)}{T_O + L}$$

subject to $T_O \geq 0$.  \hspace{1cm} (4.16)

The numerator of the derivative of the objective function with respect to $T_O$ is:

$$H(T_R = T_O + L) = [R \bar{F}(T_R) - (C_f - C_p)f(T_R)]T_R - R \int_{0}^{T_R} \bar{F}(x) \, dx + C_f F(T_R) + C_p \bar{F}(T_R).$$

Here we assume $Rf(T_R) + \bar{f}(T_R)(C_f - C_p) > 0$ to ensure that the objective function in (4.16) is a unimodal and pseudo-concave function in $T_O$. The following proposition addresses the optimum solution (see Appendix for the proof).

**Proposition 19.** The total maximum profit is $\Pi^* = R \bar{F}(T_R^*) - (C_f - C_p)f(T_R^*)$, and the corresponding strategies are $(T_R^*, T_O^*) = (T_O^* + L, H(T_O^*) = 0)$ when $K(T_R^*, T_O^*) < 0$.

### 4.3.4 Price Discount Contract

In this section, we consider the case where the agent introduces price discount as an incentive scheme to influence the customer’s decision $T_R$, and we assume that the appropriate discounted price is determined via bargaining. The agent wants to induce the customer to choose the maintenance policy which maximizes the total profit, and he should adjust the prices such that the expected profit of the customer is identical to the Nash bargaining solution. The bargaining problem to be solved when the players are willing to transfer the side payment is:

$$\max_{\Pi_c, \Pi_a} \left( \Pi_c - \Pi_c^\circ \right) \left( \Pi_a - \Pi_a^\circ \right)$$

subject to $\Pi_c \geq \Pi_c^\circ$, \hspace{1cm} (4.18)

$\Pi_a \geq \Pi_a^\circ$,

$\Pi_c + \Pi_a \leq \Pi^*$,

where $\Pi^*$ is the maximum total profit of the players given in Proposition 19 and $\Pi_c^\circ$ and $\Pi_a^\circ$ are the threat payoffs of the customer and agent given in Propositions 15 and 17.
respectively. The Nash bargaining solution of the game with side payment (4.18) can be described as follows.

**Proposition 20.** The Nash bargaining solution implies that the profits of the customer and agent are:

\[
\bar{\Pi}_c = \Pi_c^\circ + \frac{\Delta \Pi}{2} = \frac{\Pi^* + \Pi_c^\circ - \Pi_a^\circ}{2},
\]

\[
\bar{\Pi}_a = \Pi_a^\circ + \frac{\Delta \Pi}{2} = \frac{\Pi^* + \Pi_a^\circ - \Pi_c^\circ}{2},
\]

(4.19)

respectively, where \(\Delta \Pi = \Pi^* - \Pi^\circ\) is the surplus profit generated by negotiation, and \(\Pi^* = \Pi_c^\circ + \Pi_a^\circ\).

The proof can be seen in [Muthoo (1999)]. Proposition 20 states that when the players bargain over the partition of total profit \(\Pi^*\), they agree to give each other the profits \(\Pi_c^\circ\) and \(\Pi_a^\circ\), which are the guaranteed payoffs they would obtain if they do not bargain at all, and then they split equally the surplus profit \(\Delta \Pi\) [Muthoo (1999)]. This is illustrated in Fig. 4.2 where by splitting equally the surplus profit, the Nash bargaining solution is the midpoint between \(a_1\) and \(a_2\). Note that the sum of the expected profits \(\bar{\Pi}_c\) and \(\bar{\Pi}_a\) is equal to the total maximum profit \(\Pi^*\). It is worth pointing out that the Nash bargaining solution with side payment coincides with the Shapley values that are also based on certain fairness axioms [Shapley (1953)] where the maximin values are known as the characteristic function values of the players [Matsumoto and Szidarovszky (2016), Tauman and Watanabe (2007), Schwarz (2011)].

We next study the use of a price discount to ensure that the net profits of the players will be \(\bar{\Pi}_c\) and \(\bar{\Pi}_a\) in (4.19) as a result of this contract \((T_R^*, T_O^*)\). By setting \(\bar{\Pi}_a = \Pi_a(T_R^*, T_O^*, \bar{P}_p, \bar{P}_f)\) and \(\bar{\Pi}_c = \Pi_c(T_R^*, T_O^*, \bar{P}_p, \bar{P}_f)\), the values of \(\bar{P}_p\) and \(\bar{P}_f\) can be found using the following proposition (see appendix for the proof).

**Proposition 21.** The negotiated preventive and failure replacement prices are given by:

\[
\bar{P}_p = P_p - \tau T_R^*,
\]

(4.20)

\[
\bar{P}_f = P_f - \tau T_R^*,
\]

(4.21)
where $\tau$ specifies the expected pay period side payment:

$$\tau = \Pi^*_a - \bar{\Pi}_a.$$  (4.22)

The discounted prices administer the transfer of payment $\tau \geq 0$ per unit of time from the agent to the customer. We present the following procedure to find the proper discounted prices.

1. Uses Theorem 3 and Proposition 17 to compute threat point $(\Pi_c^\circ, \Pi_a^\circ)$ and derive
   $$\Pi^0 = \Pi_c^\circ + \Pi_a^\circ.$$

2. Use Proposition 19 to compute the global optimal solution $(T^*_R, T^*_O)$ which maximizes the total profit and derive
   $$\Pi^* = \Pi_c^* + \Pi_a^*.$$

3. Use proposition 20 to compute the bargaining profits $\bar{\Pi}_c$ and $\bar{\Pi}_a$ for the customer and agent.

4. Use Proposition 21 to calculate the negotiated preventive and failure maintenance prices, $\bar{P}_p$ and $\bar{P}_f$.

Therefore, if the agent chooses $\bar{P}_p$ and $\bar{P}_f$ such that $\bar{\Pi}_c = \Pi_c(T^*_R, T^*_O, \bar{P}_p, \bar{P}_f)$, the customer will agree on $T^*_R$ for the preventive replacement age. This contract results in the maximum total profit of the customer and the agent.
4.4 Numerical Examples

In this section, numerical examples are presented to illustrate the application of the bargaining model for the customer and agent. We begin with a base case where the equipment’s failure-time distribution is assumed to be Weibull with pdf 

\[ f(x) = \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^\beta}, \]

with scale parameter \( \alpha = 40 \), and shape parameter \( \beta = 3 \). Other parameters assumed for the problem setting are order lead time \( L = 10 \) days, inventory cost \( C_i = $10 \) /day, revenue \( R = $30 \) /day, failure replacement cost \( C_f = $300 \), preventive replacement cost \( C_p = $100 \), price of each failure replacement \( P_f = $600 \), price of each preventive replacement \( P_p = $400 \), and shortage cost \( S = $30 \) /day. We study the sensitivity of the model to the scale parameter \( \alpha \) and revenue \( R \) by considering their different values: \( \alpha \in \{30, 40, 50, 60\} \) and \( R \in \{20, 30, 40, 50\} \).

Tables 4.1 and 4.2 show the results for those cases where the prices of preventive and failure replacement, \( P_p \) and \( P_f \), are exogenous. We numerically obtain the threat strategies \( T_{Oc}^\circ \) (column 3) and \( T_{Ra}^\circ \) (column 5) that the agent and customer pose against each other, and the strategies \( T_{Rc}^\circ \) (column 2) and \( T_{Oa}^\circ \) (column 6) that they choose to improve their individual payoffs if the counterpart chooses his threat strategy. Accordingly, the threat profit of the customer \( \Pi_c^\circ \) and the threat profit of the agent \( \Pi_a^\circ \) are summarized in columns 4 and 7. Next, the bargaining solutions to the problem in (4.4)-(4.7) are calculated, and the results are presented in the last five columns. \( T_R^B \) is the preventive replacement age of the equipment, and \( T_O^B \) is the time the agent has to order the spare part after each replacement. The profits of the customer and agent \( \Pi_c^B = \Pi_c(T_R^B, T_O^B) \) and \( \Pi_a^B = \Pi_a(T_R^B, T_O^B) \) are presented in columns 10 and 11. The total profit of the players \( \Pi^B = \Pi_c^B + \Pi_a^B \) is shown in the last columns.

One can see that:

- The threat profits and the bargaining outcome are highly sensitive to the scale parameter of the equipment’s failure-time distribution. For example, when the scale parameter increases from \( \alpha = 30 \) to 60 (mean time to failure increases from 26.78
days to 53.57), the threat profit of the customer increases from \( \Pi^c = 8.36/\text{day} \) to 19.14, and his bargaining profit increases from \( \Pi^c_B = 9.99/\text{day} \) to \( \Pi^B = 19.97 \), while the threat payoff of the agent decreases from \( \Pi^a = 6.34/\text{day} \) to 0.98, and his bargaining payoff decreases from \( \Pi^a_B = 9.16 \) to 2.97. This shows that a more reliable equipment generates higher bargaining profit for the customer and lower profit for the agent.

- The higher the revenue generated by the equipment, the higher the threat point and the bargaining profit of the customer. We assume that depending on the activity of the customer, the revenue generated by the equipment varies. When revenue increases from \( R = 20/\text{day} \) to \( 50/\text{day} \), the threat profit of the customer increases from \( \Pi^c = 3.73 \) to 33.02, and his bargaining payoff increases from \( \Pi^B = 5.75 \) to 33.48. The threat profit of the agent is not a function of revenue, so his bargaining profit is not sensitive to it.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( T_{Rc} (\text{day}) )</th>
<th>( T_{Oc} (\text{day}) )</th>
<th>( \Pi^c (\text{day}) )</th>
<th>( T_{Ra} (\text{day}) )</th>
<th>( T_{Oa} (\text{day}) )</th>
<th>( \Pi^a (\text{day}) )</th>
<th>( \Pi^c_B (\text{day}) )</th>
<th>( \Pi^B (\text{day}) )</th>
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<td>6.34</td>
<td>24</td>
<td>14</td>
<td>9.99</td>
</tr>
<tr>
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<td>infinite</td>
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<td>3.65</td>
<td>31.9</td>
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<td>14.96</td>
</tr>
<tr>
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<td>52.09</td>
<td>0</td>
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<td>0.98</td>
<td>47.8</td>
<td>37.8</td>
<td>19.97</td>
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<table>
<thead>
<tr>
<th>( R ) ($/\text{day})</th>
<th>( T_{Rc} (\text{day}) )</th>
<th>( T_{Oc} (\text{day}) )</th>
<th>( \Pi^c (\text{day}) )</th>
<th>( T_{Ra} (\text{day}) )</th>
<th>( T_{Oa} (\text{day}) )</th>
<th>( \Pi^B (\text{day}) )</th>
<th>( \Pi^c_B (\text{day}) )</th>
<th>( \Pi^B (\text{day}) )</th>
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<tr>
<td>20</td>
<td>41.65</td>
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<td>3.73</td>
<td>infinite</td>
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<td>3.65</td>
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<td>21.1</td>
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<td>3.65</td>
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One of the assumptions we made is that the players make threat against each other. Here we see how bargaining solution varies if both players have the same bargaining position, i.e., the threat point payoff of each player is zero (no threat). We numerically calculate the bargaining solutions to the problem in (4.4)-(4.7), assuming \((\Pi^c, \Pi^c_o) = (0, 0), \)
and present the bargaining profits \((\Pi^c_n, \Pi^a_n)\) and strategies \((T^c_R, T^a_R)\) in Tables 4.3 and 4.4. The relative increase in the agent’s profit in bargaining contract with threat over contract with no threat is calculated by:

\[
\Delta \Pi^a_n = \frac{\Pi^a_n - \Pi^B_n}{\Pi^B_n} \times 100\%,
\]

and the results are presented in columns 7. The relative increase in profit of the customer and total profit are also determined in the same manner and presented in columns 6 and 8 and Fig. 4.3. The following results can be seen from Tables 4.3 and 4.4.

• The bargaining outcome is highly sensitive to the threat profit of the players. As an example at \(\alpha = 40\), if the threat payoff decreases from \((\Pi^c_e, \Pi^a_e) = (13.73, 3.65)\) (Table 4.1) to \((0, 0)\), the bargaining solution changes from \((\Pi^c_B, \Pi^a_B) = (14.96, 6.09)\) to \((\Pi^c_n, \Pi^a_n) = (10.96, 11.85)\), which is a 26.73% decrease in profit for the customer and a \(\Delta \Pi^a_n = 94.58\%\) increase for the agent. This shows that threatening not to out-source any preventive maintenance is a powerful way for the customer to improve his bargaining position and his bargaining payoff.

• Posing threat influences the bargaining result by shrinking the bargaining feasible region. Fig. 4.4 shows at \(\alpha = 40\), the feasible region for the case of no threat is the entire shaded region in the first quadrant while the feasible region for the case with threat is just a small subset of points greater than the threat point \((\Pi^c_e, \Pi^a_e) = (13.73, 3.65)\). This shows how threat point works as a reserved profit level that no player accepts less than this announced level. In this environment, the players get this reserved profit and bargain over a small subset of profit.

• Comparing bargaining solutions in terms of efficiency reveals that posing threat decreases the efficiency. As an example, at \(\alpha = 40\) the total profit of the players increase \(\Delta \Pi^e = 8.36\%\) if the players pose no threat against each other. The results are revisited in Fig. 4.3.
Figure 4.3: Increase in profits assuming no threat

Figure 4.4: Effects of threat point and price discount on the feasible region and bargaining outcome.
Table 4.3: Results for different scale parameters without threat and price discount

<table>
<thead>
<tr>
<th>α</th>
<th>(T^R) (day)</th>
<th>(T^O) (day)</th>
<th>(\Pi^c) ($/day)</th>
<th>(\Pi^a) ($/day)</th>
<th>(\Delta\Pi^c) (%)</th>
<th>(\Delta\Pi^a) (%)</th>
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</thead>
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Table 4.4: Results for different revenues without threat and price discount

<table>
<thead>
<tr>
<th>(R) ($/day)</th>
<th>(T^R) (day)</th>
<th>(T^O) (day)</th>
<th>(\Pi^c) ($/day)</th>
<th>(\Pi^a) ($/day)</th>
<th>(\Delta\Pi^c) (%)</th>
<th>(\Delta\Pi^a) (%)</th>
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</thead>
<tbody>
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We next consider the case where the players pose threat against each other and the agent offers price discount to the customer in order to induce the customer to choose the maintenance policy which maximizes their total profit. We numerically solve the problem in (4.15) and derive the total optimal preventive replacement age and ordering time, \((T^*_R, T^*_O) = \text{arg max}_{T^R, T^O}(\Pi^c + \Pi^a)\), which are summarized in columns 2 and 3 of Tables 4.5 and 4.6 respectively. By choosing these policies, the payoffs of the customer and agent with exogenous market prices \(P^c\) and \(P^f\) are \(\Pi^*_c = \Pi_c(T^*_R, T^*_O, P^c, P^f)\) and \(\Pi^*_a = \Pi_a(T^*_R, T^*_O, P^c, P^f)\) presented in columns 4 and 5, and their expected payoffs with negotiated prices are \(\hat{\Pi}_c = \Pi_c(T^*_R, T^*_O, \hat{P}_p, \hat{P}_f)\) and \(\hat{\Pi}_a = \Pi_a(T^*_R, T^*_O, \hat{P}_p, \hat{P}_f)\), presented in columns 6 and 7. The side payment values, \(\tau\), are calculated based on (4.22), and the negotiated preventive and failure maintenance prices are presented in columns 9 and 10. The total profit of players \(\hat{\Pi} = \hat{\Pi}_c + \hat{\Pi}_a\) is presented in the last columns. The terms of the contract are \(T^*_R\) and \(T^*_O\) as well as the negotiated prices \(\hat{P}_p\) and \(\hat{P}_f\). The profits of players as a result of this contract are \(\hat{\Pi}_c\) and \(\hat{\Pi}_a\).

One can see that:

- Achieving maximum value of total profit requires price discount, otherwise the customer won’t agree on this contract. For example, at \(\alpha = 50\), if the customer switches from \(T^R = 39.9\) to total maximum maintenance policy \(T^*_R = 24.41\), although the
total profit increases from $\Pi^B = 22.17$ (Table 4.1) to $\bar{\Pi} = 24.14$ (Table 4.5) and the agent’s profit increases from $\Pi^B_a = 4.20$ to $\Pi^*_a = 11.44$, the customer’s profit decreases from $\Pi^B_c = 17.97$ to $\Pi^*_c = 12.71$ when no price discount is involved. The agent increases his profit by imposing the cost of a more expensive contract (due to smaller maintenance interval $T^*_R < T^B_R$) on the customer. The customer paying a higher maintenance cost will expect a price discount, otherwise he won’t agree on this contract.

- In order that the customer agrees on the maintenance policy $T^*_R$, the customer and agent negotiate on maintenance prices. As a result of negotiation, at $\alpha = 50$ the preventive maintenance price decreases from $P_p = 400$ to $\bar{P}_p = 233.40$, and the failure maintenance price decreases from $P_f = 800$ to $\bar{P}_f = 433.40$. With negotiated prices, the expected profit of the customer increases from $\Pi^B_c = 17.97$ to $\bar{\Pi}_c = 19.53$ and the profit of the agent increases to $\bar{\Pi}_a = 4.61 \$/day$. In fact, the price adjustment results in transfer of $\tau = 6.82$ profit per day from the agent to the customer (side payment) which induces the customer to adopt the optimal preventive replacement age, $T^*_R$.

- The price discount contract influences the bargaining outcome by expanding the bargaining feasible region. Fig. 4.4 shows how at $\alpha = 40$ price discounting can expand the feasible region and generate revenue for the customer and agent.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$T^*_R$ (day)</th>
<th>$T^*_O$ (day)</th>
<th>$\Pi^*_c$ ($/day$)</th>
<th>$\Pi^*_a$ ($/day$)</th>
<th>$\Pi_c$ ($/day$)</th>
<th>$\Pi_a$ ($/day$)</th>
<th>$\tau$ ($/day$)</th>
<th>$P_p$ ($)</th>
<th>$P_f$ ($)</th>
<th>$\bar{\Pi}$ ($/day$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>15.99</td>
<td>8.99</td>
<td>3.23</td>
<td>17.67</td>
<td>11.46</td>
<td>9.44</td>
<td>8.23</td>
<td>268.40</td>
<td>468.40</td>
<td>20.9</td>
</tr>
<tr>
<td>50</td>
<td>24.41</td>
<td>14.41</td>
<td><strong>12.71</strong></td>
<td><strong>11.44</strong></td>
<td><strong>19.53</strong></td>
<td><strong>4.61</strong></td>
<td><strong>6.82</strong></td>
<td><strong>233.40</strong></td>
<td><strong>433.40</strong></td>
<td><strong>24.14</strong></td>
</tr>
<tr>
<td>60</td>
<td>28.32</td>
<td>18.32</td>
<td>15.17</td>
<td>9.82</td>
<td>21.57</td>
<td>3.41</td>
<td>6.40</td>
<td>218.61</td>
<td>418.61</td>
<td>24.98</td>
</tr>
</tbody>
</table>

It should be noted that different bargaining solutions induce different payoffs to the two players and therefore different side payments and discounted prices. Note that in this
Table 4.6: Results for different revenues considering threat and price discount

<table>
<thead>
<tr>
<th>R ($/day)</th>
<th>( T_R^* ) (day)</th>
<th>( T_R^o ) (day)</th>
<th>( \Pi_B ) ($/day)</th>
<th>( \Pi_a ) ($/day)</th>
<th>( \Pi_c ) ($/day)</th>
<th>( \tau ) (day)</th>
<th>( P_p ) ($)</th>
<th>( P_f ) ($)</th>
<th>( \Pi ) ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>21.73</td>
<td>11.73</td>
<td>0.22</td>
<td>13.03</td>
<td>6.66</td>
<td>6.58</td>
<td>6.44</td>
<td>260.02</td>
<td>460.02</td>
</tr>
<tr>
<td>50</td>
<td>18.51</td>
<td>8.51</td>
<td>27.37</td>
<td>15.00</td>
<td>35.87</td>
<td>6.49</td>
<td>8.50</td>
<td>242.65</td>
<td>442.65</td>
</tr>
</tbody>
</table>

chapter we study the Nash bargaining solution without comparing the effect of different bargaining solutions on the payoffs (Anbarci et al., 2002).

In order to evaluate the value of price discount contract, we define the relative increase in the agent’s profit in the price discount contract over the general bargaining contract as:

\[
\Delta \Pi_a = \frac{\bar{\Pi}_a - \Pi_a^B}{\Pi_a^B} 100%.
\]

The results are presented in columns 6 of Tables 4.7 and 4.8. The relative increase in profit of the customer and total profit are also determined in the same manner and presented in columns 5 and 7. Furthermore, we define the following two performance measures to show the advantage of price-discount contract over the general bargaining contract:

\[
\Delta T_R = \frac{T_R^* - T_R^B}{T_R^B} 100%,
\]

\[
\Delta P_p = \frac{\bar{P}_p - P_p^B}{P_p^B} 100%,
\]

where \( \Delta T_R \) and \( \Delta P_p \) are the relative changes in preventive replacement age and price, respectively.

Several results are as follows:

- The preventive maintenance age in price-discount contract is shorter than the general bargaining contract, \( \Delta T_R < 0 \). This shows that the price discount contract increases the maintenance quality of equipment.

- The agent should be more willing to offer price discount when the customer generates higher revenue with the equipment. The reason is that the customer can generate a higher surplus profit with the better maintained equipment where the
agent is the owner of half of it. As an example, $\Delta \Pi_a = 0.68\%$ at $R = 20$/day, while $\Delta \Pi_a = 20.34\%$ at $R = 40$/day.

- Price-discount scheme is economically beneficial to both parties. At $\alpha = 50$, the agent induces the customer to decrease the preventive maintenance interval by 38.82\% by decreasing preventive maintenance price by 41.64\% and failure maintenance price by 27.76\%, and this increases the profit of customer by $\Delta \Pi_c = 8.70\%$ and the profit of the agent by $\Delta \Pi_a = 9.88\%$ (see in Fig. 4.5).

- The value of price-discount contract for the agent depends on the context of the problem. For example, at $\alpha = 30$ this contract increases the agent’s profit by $\Delta \Pi_a = 3.05\%$, while for $\alpha = 60$ it increases his profit by $\Delta \Pi = 14.98\%$ (see Fig. 4.5).

Table 4.7: Effects of scale parameter on replacement policy, prices, and profits

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\Delta T$</th>
<th>$\Delta P_p$</th>
<th>$\Delta P_f$</th>
<th>$\Delta \Pi_c$</th>
<th>$\Delta \Pi_a$</th>
<th>$\Delta \Pi$</th>
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</thead>
<tbody>
<tr>
<td>30</td>
<td>-33.57</td>
<td>-32.89</td>
<td>-21.93</td>
<td>14.71</td>
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<td>9.13</td>
</tr>
<tr>
<td>40</td>
<td>-36.30</td>
<td>-37.54</td>
<td>-25.02</td>
<td>10.29</td>
<td>5.41</td>
<td>8.88</td>
</tr>
<tr>
<td>50</td>
<td>-38.82</td>
<td>-41.64</td>
<td>-27.76</td>
<td>8.70</td>
<td>9.88</td>
<td>8.93</td>
</tr>
<tr>
<td>60</td>
<td>-40.75</td>
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<td>-30.23</td>
<td>8.03</td>
<td>14.98</td>
<td>8.93</td>
</tr>
</tbody>
</table>

Table 4.8: Effects of revenue on replacement policy, prices, and profits

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\Delta T$</th>
<th>$\Delta P_p$</th>
<th>$\Delta P_f$</th>
<th>$\Delta \Pi_c$</th>
<th>$\Delta \Pi_a$</th>
<th>$\Delta \Pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
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<td>-34.99</td>
<td>-23.32</td>
<td>15.91</td>
<td>0.68</td>
<td>7.81</td>
</tr>
<tr>
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<td>-36.30</td>
<td>-37.52</td>
<td>-25.01</td>
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<td>5.41</td>
<td>8.88</td>
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<td>-44.85</td>
<td>-39.45</td>
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<td>50</td>
<td>-44.97</td>
<td>-39.33</td>
<td>-26.22</td>
<td>7.15</td>
<td>17.46</td>
<td>8.61</td>
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</table>

4.5 Conclusion and Future Research

In this chapter, we studied a problem that an equipment owner outsources preventive and failure replacement services to a service agent, where the players bargain to determine the terms of contract. We considered the Nash bargaining solution to compute the bargaining profit of players and determined the optimal threat strategies a player can pose against the
other player in order to increase his bargaining position. We next provided solution to the problem of how the service agent can provide price discount incentive to the customer in order to induce him to choose the total maximum maintenance policy. Our numerical examples illustrated the feasibility of using such a price-discount contract in maintenance service outsourcing.

Our result shows that although threatening decreases the efficiency of the contract, it can dramatically increase the profit of the player with a higher bargaining position (the customer in our example). Moreover, one can see that both the customer and agent can benefit if the agent offers a price discount in order to induce the customer to increase the preventive maintenance service level.

A critical assumption in this chapter is that the contract with price discount is based on mutual trust. A contract mechanism can be designed to coordinate the decisions such that the total optimal solution is a Nash equilibrium of the game (Moses and Seshadri, 2000; Ernest and Cohen, 1992). This will eliminate the incentive for the players to deviate. We need a scheme that allows each player to optimize his own profit and still chooses the total optimal solution. This scheme is more complex than the model presented in this chapter. Moreover, the analysis in this chapter is focused on a two-person game in the context
of contract negotiation. An interesting direction for future research is to consider cases where there are several players with different payoff parameters. The development of such models will provide insights into the effect of increased competition on maintenance outsourcing contracts.
In this dissertation, we have shown how different relationships and communication between supply chain members can affect the decisions they could make and the profits they could gain. We have developed service contracts for the manufacturer and customer in different scenarios and relationships and compared the profit of players in different contracts. We have determined the conditions under which each contract is the most profitable. For equipment leasing, we have modeled three contracts, i.e., two non-cooperative lease contracts considering Nash and Stackelberg equilibria, respectively, and a cooperative one aiming at maximizing the total payoff of the players. We have shown that the Nash and Stackelberg equilibria are different from the total maximum solution, and thus the players can increase their total profit via cooperation. Next, we have designed a contract to implement the cooperative solution, which requires two major criteria. We have shown that these two criteria can be achieved by a nonlinear transfer-payment contract. Our results illustrate that cooperation can be regarded as a value-added strategy in establishing such lease contracts. Besides, our numerical results show that while cooperation always increases the total profit, the magnitude of increase is case specific. When the lease price is low or the revenue is high, the profits in the non-cooperative contracts will be close to that of the cooperative alternative, while in other cases the cooperation may increase the total profit significantly.

For maintenance outsourcing, we have studied three different bargaining scenarios in determining the contract terms. We have considered the Nash bargaining solution to compute the bargaining profit of players. We then have considered the case where a player poses threat against the other player to increase his bargaining position, and have determined the optimal threat strategies for each player. Finally, we have provided
a solution to the problem that the agent and customer negotiate on the service price. Our result shows that although threatening decreases the efficiency of the contract, it can dramatically increase the profit of the player with a higher bargaining position. Moreover, one can see that both the customer and agent can benefit if the agent offers a price discount in order to induce the customer to increase the preventive maintenance service level. This can result in full cooperation since it provides an incentive to the customer to choose the total maximum policy.

Besides, we have addressed the maintenance policy in service contracts. To ensure the operational availability of the critical equipment under service, for each contract we have determined the optimal preventive maintenance policy. For the leasing case, we have considered the equipment to be repairable and determined the optimal time and effect for each maintenance. For the maintenance outsourcing contracts, we have considered a piece of non-repairable equipment and determined the optimal replacement time and as well as the optimal spare part ordering policy.

In our future research, we would like to extend our work to develop a general framework for service contracts. One of the modules to be considered under this framework is failure data analysis for products under service. Recently, electronic sensors become a major source for monitoring and reporting real-time information about how, when and under what field environment the equipment is being used. This information can be used to schedule maintenance activities and protect the system from costly in-service failure. Besides, the data can be used to motivate the customer not to use the equipment under conditions that cause serious damage to the equipment. Another interesting research direction is to consider cases where there are many players in the supply chain. The development of such models will provide useful insights into the effects of increased competition and cooperation in supply chain as the demand or supply in service contracts changes.

Such general framework will lead to fundamental technical contributions to the areas of reliability engineering, condition based maintenance, and customized supply chain service contracts, as it integrates disjoint tasks into a more efficient and viable framework.


PROOFS

Proof of Proposition 1 The problem to be solved is $\max_{r,K} \Pi_1(r, K)$. We consider a two-step optimization approach: in the first step, we find $r_{opt}$ as the usage rate that maximizes the payoff function for a given value of $(N, \delta)$; and next, we determine $K_{opt}$, as the optimum lease period that maximizes the payoff function for the value of $r_{opt}$ determined in the first step. After taking the first order derivative with respect to $r$, one can see that:

- If $\frac{u_m}{2\alpha_0 r_m L} \left[ L - \frac{K(N\delta+1)}{2(N+1)} \right] > r_m$, then $r_{opt} = r_m$. After substituting $r_m$ into the objective function and taking the first order derivative with respect to $K$, two cases can be seen: (1) if $0 < \frac{(N+1)}{(N\delta+1)} \left[ L - \frac{\alpha_0 r_m^2}{u_m} \right] < L$ then $K_{opt} = \frac{(N+1)}{(N\delta+1)} \left[ L - \frac{\alpha_0 r_m^2}{u_m} \right]$ (case iii. of Proposition 1), and (2) if $\frac{(N+1)}{(N\delta+1)} \left[ L - \frac{\alpha_0 r_m^2}{u_m} \right] > L$ then $K_{opt} = L$ (case ii.).

- If $0 < \frac{u_m}{2\alpha_0 r_m L} \left[ L - \frac{K(N\delta+1)}{2(N+1)} \right] \leq r_m$, then $r_{opt} = \frac{u_m}{2\alpha_0 r_m L} \left[ L - \frac{K(N\delta+1)}{2(N+1)} \right]$. Taking the first order derivative of the objective function with respect to $K$ and substituting $r_{opt}$ for $r$ two cases can be seen: (1) if $L < \frac{2L(N+1)}{3(N\delta+1)}$ then $K_{opt} = L$ (case i.), and (2) if $L > \frac{2L(N+1)}{3(N\delta+1)}$ then $K_{opt} = \frac{2L(N+1)}{3(N\delta+1)}$ (case iv.).

Table A.1: Optimality conditions for Proposition 1

<table>
<thead>
<tr>
<th>#</th>
<th>$A_1$</th>
<th>$B_1$</th>
<th>$C_1$</th>
<th>$D_1$</th>
<th>Opt</th>
<th>#</th>
<th>$A_1$</th>
<th>$B_1$</th>
<th>$C_1$</th>
<th>$D_1$</th>
<th>Opt</th>
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<tr>
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<td>i.</td>
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<td>+</td>
<td>i.</td>
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<td>+</td>
<td>i.</td>
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<td>-</td>
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<td>-</td>
<td>Imp</td>
</tr>
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<td>Imp</td>
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<td>+</td>
<td>-</td>
<td>-</td>
<td>i.</td>
</tr>
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<td>+</td>
<td>-</td>
<td>i.</td>
<td>15</td>
<td>-</td>
<td>-</td>
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<td>i.</td>
<td>16</td>
<td>-</td>
<td>-</td>
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</table>

Table A.1 gives sixteen cases considering the signs of $A_1$, $B_1$, $C_1$ and $D_1$. The optimal solutions are summarized in columns 6 and 12 (Opt) where the numbers i. to iv. correspond
to Proposition [1] The impossible cases denoted by Imp are impossible cases explained as follows. Observe that $D_1 = C_1 + (N + 1)B_1$, so when $B_1 > 0$ and $C_1 > 0$ we should have $D_1 > 0$ making cases #2 and #8 impossible; if $B_1 < 0$ and $C_1 < 0$, then $D_1 < 0$ making cases #10 and #13 impossible. Another property is $N(1 - \delta)B_1 + A_1\alpha_0 r_m^2 = C_1$, so for $A_1 > 0$ and $B_1 > 0$ we should have $C_1 > 0$ making cases #3, #6, #9, and #15 impossible. It is worth pointing out that the same optimization results can be derived using KKT conditions.

□

Proof of Proposition 2. Clearly, $\Pi'_{2N}$ (3.21) has the same sign as the cubic function:

$$M_9(N) = -a(N + 1)^3 + \left(-bK + \frac{K^2}{2}C_f(\theta_1 r + \theta_2)\right)(1 - \delta)(N + 1) + 2K^2(\theta_3 r^2 + \theta_4)(1 - \delta)(N\delta + 1)$$

and $\lim_{N \to -1} M_9 > 0$ and $\lim_{N \to \infty} M_9 = -\infty$. Therefore, there is at least one solution with $N > -1$. However, it is impossible to have three solutions, because if there are three real roots, then $L_9'(N)$ should have two real roots larger than $-1$. The first derivative of $M_9(N)$ simply shows that this is impossible. As a result, the solution satisfying the first order condition is the unique maximum.

□

Proof of Proposition 3. Taking the first derivative of $\Pi_2$ with respect to $\delta$ yields:

$$\Pi'_{2\delta} = \frac{NbK}{(N + 1)} - \frac{K^2C_f(\theta_1 r + \theta_2)N}{2(N + 1)} - \frac{2K^2(\theta_3 r^2 + \theta_4)(N\delta + 1)N}{(N + 1)^2}$$

Utilizing $\Pi'_{2N}$ (3.21) we have:

$$\Pi'_{2\delta} = \frac{-N(N + 1)}{1 - \delta}(\Pi'_{2N} + a) \quad (A.1)$$

So, at $N_2$ where $\Pi'_{2N} = 0$, we have $\Pi'_{2\delta} < 0$. Therefore, $\Pi_2$ is a decreasing function of $\delta$ for given value of $N_2$, and $(N_2, 0)$ is the optimal solution.

□

Proof of Theorem 1. For the lessee, there are four best response possibilities; i, ii, iii, and iv, as presented in Proposition [1] and for the lessor, only one possibility exists as
addressed in Proposition 3. Considering these cases, there are four overall candidates for the equilibria, for which only two of them satisfy the corresponding conditions. At $\delta = 0$, we have $A_1 > 0$ and $B_1 > 0$, which contradicts $C_1 < 0$ ($C_1 = NB_1 + A_1a_0r_m^2$). Therefore, the corresponding conditions for cases iii and iv in Proposition 1 are not satisfied, and no equilibrium exists in these two cases.

**Proof of Proposition 4** To prove cases iii and iv, notice that substituting $K^* = \frac{A_1(N+1)}{(N\delta+1)}$ into $\Pi_2$ yields the following optimization problem for the lessor:

$$
\max_{\delta,N} \quad \alpha_0r^2A_3(N + 1) - Na - \frac{NbA_3(1 - \delta)}{(N\delta + 1)} - \frac{A_3^2(N + 1)C_f(\theta_1r + \theta_2)}{2(N\delta + 1)} - A_3\theta_3r^2 + \theta_4
$$

Taking the first derivative of the above objective function with respect to $\delta$ yields $M_1$ in (3.24), which is a constant function of $\delta$. When $M_1 > 0$, we have $\delta^*_i = 1$, which means PM is minimal. Due to the tedious derivation involved, we only show the case with $M_1 > 0$, and the case for $M_1 < 0$ can be dealt with in a similar manner.

For case $i$, after substituting $r^*_1$ into $\Pi_2$, the first derivatives of $\Pi_2$ with respect to $N$ and $\delta$ have the same relationship as (A.1). Notice that $r^*_1$, the first derivative of $r^*_1$ with respect to $N$, and $r^*_1$, the first derivative of $r^*_1$ with respect to $\delta$, satisfy $r^*_1 = \frac{-N(N+1)}{(1-\delta)}r^*_1$. Therefore, for any interior optimal $N_3^*$, we have $\delta^*_i = 0$.

**Proof of Theorem 2** To prove cases iii and iv, notice that substituting $K^* = \frac{A_1(N+1)}{(N\delta+1)}$ into $\Pi_2$ yields the following optimization problem for the lessor:

$$
\max_{\delta,N} \quad \alpha_0r^2A_3(N + 1) - Na - \frac{NbA_3(1 - \delta)}{(N\delta + 1)} - \frac{A_3^2(N + 1)C_f(\theta_1r + \theta_2)}{2(N\delta + 1)} - A_3\theta_3r^2 + \theta_4
$$

Taking the first derivative of the above objective function with respect to $\delta$ yields $M_1$ in (3.24), which is a constant function of $\delta$. When $M_1 > 0$, we have $\delta^*_i = 1$, which means PM is minimal. Due to the tedious derivation involved, we only show the case with $M_1 > 0$, and the case for $M_1 < 0$ can be dealt with in a similar manner.

For case $i$, after substituting $r^*_1$ into $\Pi_2$, the first derivatives of $\Pi_2$ with respect to $N$ and $\delta$ have the same relationship as (A.1). Notice that $r^*_1$, the first derivative of $r^*_1$ with respect to $N$, and $r^*_1$, the first derivative of $r^*_1$ with respect to $\delta$, satisfy $r^*_1 = \frac{-N(N+1)}{(1-\delta)}r^*_1$. Therefore, for any interior optimal $N_3^*$, we have $\delta^*_i = 0$.

**Proof of Proposition 4** For cases iii and iv of Theorem 2, no Nash equilibrium exists. In case [2], the two solutions are identical. In case [3], we compare the finite and unique values $N_3^*$ and $N_4^*$. $N_3^*$ is derived by substituting $r^*_1 = \frac{a_m}{2N_1r_m}[\frac{2N+1}{2(N+1)}]$ into $\Pi_2 = 0$ (3.21). This is equivalent to solving $M_5(N) = 0$, where:

$$
M_5(N) = -a - \frac{bL}{(N+1)^2} + \frac{L^2C_f(\theta_1r^*_1 + \theta_2)}{2(N + 1)^2} + \frac{2L^2(\theta_3r^*_1)^2 + \theta_4)}{(N + 1)^3}
$$

$N_4^*$ is derived by substituting $r^*_1$ into $\Pi_2$ and setting its first derivative with respect to $N$ to zero. This is equivalent to solving $M_6(N) = 0$, where:

$$
M_6(N) = -a - \frac{bL}{(N + 1)^2} + \frac{L^2C_f(\theta_1r^*_1 + \theta_2)}{2(N + 1)^2} + \frac{2L^2(\theta_3r^*_1)^2 + \theta_4)}{(N + 1)^3}
$$
which is an increasing function of \( i.e., \pi \leq \pi(\text{leader}) \) is unique, the leader’s profit will not be less than that at Nash equilibrium, \( \pi^N \).

Proof of Proposition 7

Proof of Proposition 5

Proof of Proposition 6

Proof of Proposition 3

\[ \text{Proof of Proposition 6} \]

Taking the first derivative of \( \Pi \) with respect to \( N \) yields:

\[
\pi'_{N} = -a - \frac{bK(1 - \delta)}{(N + 1)^2} + \frac{K^2(1 - \delta)}{2(N + 1)^2} \left[ \frac{u_mr}{r_m L} + \frac{C_f(\theta_1 r + \theta_4)}{r_m L} \right] + \frac{2K^2(\theta_2 r^2 + \theta_4)(N\delta + 1)(1 - \delta)}{(N + 1)^3} \]  

(A.3)

Observe that \( \pi'_{\delta} \) and \( \pi'_{N} \) satisfy \( \pi'_{\delta} = \frac{-\pi'_{N}(N + 1)^3}{K(1 - \delta)} \), which shows that \( \pi \) is a decreasing function of \( \delta \) when \( \pi'_{N} < 0 \). As a result, we have \( \delta^* = 0 \). The first order derivatives of \( \pi \) with respect to \( K \) and \( r \), \( \pi'_{K} \) and \( \pi'_{r} \), yield the following relationship:

\[ r\pi'_{r} = 2K\pi'_{K} + C_f K^2(N + 1)(\theta_1 r + 2\theta_2) + 4K^2\theta_4 + 2N\theta_4(N + 1)K + \frac{K^2 u_m r(N + 1)}{r_m L} \]

which shows that \( \pi'_{r} > 0 \) (as \( \pi'_{K} > 0 \)) and \( x^* = (r_m, L, N^*, 0) \) is the total maximum solution.

Proof of Proposition 7

Substituting \( \delta = 0 \) into (A.3), \( \pi'_{N} = 0 \) will have the same solution as \( M_7(N) = 0 \), where

\[ M_7(N) = -a(N + 1)^3 + \left( -bK + \frac{K^2}{2} C_f(\theta_1 r + \theta_2) + \frac{K^2 u_m r}{2r_m L} \right) (N + 1) + 2K^2(\theta_2 r^2 + \theta_4) \]  

(A.4)
Following the same manner as proof of proposition \ref{prop:1}, one can see that there is only one real positive root. \hfill \Box

**Proof of Proposition \ref{prop:8}** After substituting $\delta = 0$, $\Pi'_{2N} = 0$ in (3.21) has the same solution as $M_8(N) = 0$ where:

$$M_8(N) = -a(N + 1)^3 + \left(-bK + \frac{K^2}{2}C_f(\theta_1 r + \theta_2)\right)(N + 1) + 2K^2(\theta_3 r^2 + \theta_4) \quad (A.5)$$

One can see that $\lim_{N \to -1} M_7 = \lim_{N \to -1} M_8 > 0$ and $\lim_{N \to \infty} M_7 = \lim_{N \to \infty} M_8 = -\infty$, and each of the equations has one real root as proved before. Besides, it can be seen that for fixed $r$ and $K$, the two cubic functions in (A.4) and (A.5) have the same coefficients except for the second terms where the coefficient of $(N + 1)$ in (A.4) has a larger value. As a result, we have $M_7(N) > M_8(N)$, and $N^* > N_2^*$. Next, we prove that $N_2^* > N_3^*$. Notice that $M_8(N)$ is an increasing function of $r$, so substituting $r_m$ with $r^* < r_m$ yields $M_8(N, r^*_m) > M_8(N, r^*_1)$. As a result, we have $M_7(N, r^*_m) > M_8(N, r^*_m) > M_8(N, r^*_1)$, so $N_3^* < N_2^* < N^*$. Moreover, observe that (A.2) is an increasing function of $r$ under the conditions of the problem, so $N_4^* < N_2^* < N^*$.

\hfill \Box

**Proof of Proposition \ref{prop:10}** From Proposition \ref{prop:8} we see that the Nash and Stackelberg equilibria are different from the total maximum solution, so their total payoff is less than the maximum total payoff in the cooperative scheme. \hfill \Box

**Proof of Proposition \ref{prop:11}** We need to determine the parameter $\gamma$ such that $\Pi_1(x^*_n) = \Pi_1(x^*_m) + \Delta \Pi/2$. That is $\Pi_1(x^*_n) + H(x^*_n) = \Pi_1(x^*_m) + \Delta \Pi/2$, where $\Delta \Pi = \Pi_1(x^*_n) + \Pi_2(x^*_n) - \Pi_1(x^*_m) - \Pi_2(x^*_m)$ and $H(x) = \gamma + \bar{\alpha}_0 r^2 K + \frac{\beta}{N+1}$. Solving for $\gamma$ proves the results. \hfill \Box

**Proof of Proposition \ref{prop:12}** It can be seen by case $i$ of Theorem \ref{thm:1} that when $D_1 < 0$ i.e., $\alpha_0 > \frac{u_m(2N^*+1)}{4r_m^2(N^*+1)}$, then the total maximum solution $(r_m, L)$ and $N^*$ are not the best response of lessee and lessor, respectively. Besides, it can be seen by case $ii$ that when $C_1 \geq 0$, i.e., $\alpha_0 \leq \frac{u_mN^*}{r_m(N^*+1)}$, and $D_1 \geq 0$, i.e., $\alpha_0 \leq \frac{u_m(2N^*+1)}{4r_m^2(N^*+1)}$, then $(r_m, L)$ is the best response of the lessee. Observe that $\frac{u_mN^*}{r_m(N^*+1)} \geq \frac{u_m(2N^*+1)}{4r_m^2(N^*+1)}$, so $\alpha_0 \leq \frac{u_m(2N^*+1)}{4r_m^2(N^*+1)}$ is the necessary and sufficient
condition for the lessee to choose the total maximum solution. So we introduce a transfer-payment function term as \(\bar{\alpha}_0 r^2 K\) with \(\bar{\alpha}_0\) satisfying the relation \(\alpha_0 - \bar{\alpha}_0 \leq \frac{u_m(2N^*+1)}{4L(N^*+1)}\) to make \((r_m, L)\) the best response of the lessee. Next, we show that \(\frac{L u_m}{2(N+1)}\) is a proper transfer function, which makes \(N^*\) the best response of lessor. The first order partial derivatives of the lessor’s payoff function and total payoff function are given in (3.21) and (A.3) respectively. A comparison shows that adding the term \(K_2 u_m r (1 - \delta) \frac{2}{r_m} L (N^* + 1)\) to (3.21) makes the two functions equal, so we modify the payoff functions by \(\beta = \frac{K_2 u_m r (1 - \delta)}{2L} |x = (r_m, L, 0)\) to make the first order derivatives equal. The same proof as Proposition 2 shows that \((N^*, 0)\) is the unique maximum for the lessor’s payoff, \(\bar{\Pi}_2\). □

Proof of Proposition 13 It can be seen by Theorem 1 that when \(C_1 \geq 0\) and \(D_1 \geq 0\), then \((r_m, L)\) is already the best response of the lessee, so \(\bar{\alpha}_0 = 0\). With the same proof as Proposition 12, it can be seen that \(\frac{L u_m}{2(N+1)}\) is a proper transfer function, which makes \(N^*\) the best response of lessor. Next, we show that \(\gamma \leq 0\). It can be seen by (3.30) that \(\gamma = \frac{\Pi_1(x^*_a) - \Pi_1(x^*_c) + \Pi_2(x^*_c) - \Pi_2(x^*_a) - \frac{L u_m}{2(N+1)}}{2}\). The sum of the first two terms is negative, since \(\Pi_1\) is increasing in \(N\), and \(N^* > N^*_a\). The sum of the terms three and four is also negative since \(N^*_a\) is the optimal solution of \(\Pi_2\), so \(\Pi_2(x^*_a) > \Pi_2(x^*_c)\). Finally, \(- \frac{u_m L}{(N^*+1)} < 0\), and the proof is complete. □

Proof of Proposition 14 If \(D(T_O) < 0\) then \(\Pi_a\) is a decreasing function, and its minimum is at \(T_{Ra}^* = \infty\). If \(D(T_O) > 0\), then \(\Pi_a\) is an increasing function of \(T_R\) and achieves the minimum at \(T_{Ra}^* = T_0 + L\). This completes the proof. □

Proof of Theorem 3 One can see that:

\[
\frac{\partial G(T_O)}{\partial T_O} = -f(T_O + L)(P_f - C_f) + C_i(T_O + L) + S \mu < 0
\]

as \(P_f > C_f\) and \(f(T_O + L) > 0\). Therefore, \(G(T_O)\) has at most one zero point, and if it exists it must be a maximum, otherwise, the function is monotonic. In other words, the objective function is unimodal and pseudo-concave in \(T_O\). If there is a strategy \(T_{Oa}^*\) at
which $G(T_{Oa}^\circ) = 0$, this strategy must be the unique maximum of (4.9). This completes the proof. □

**Proof of Proposition 16** Notice that $S \leq R$, so depending on the sign of $W$, there are two possibilities for the optimum strategy of $T_{Oc}^\circ$ for a given value of $T_R$. For any fixed $T_R$, if $W(T_R) > 0$, $\Pi_c$ is increasing in $T_O$. According to the constraint $0 \leq T_O \leq T_R - L$, minimum of $\Pi_c$ is at $T_{Oc}^\circ = 0$. If $W \leq 0$, $\Pi_c$ is decreasing in $T_O$, so one minimum is at $T_{Oc}^\circ = T_R - L$. This completes the proof. □

**Proof of Theorem 4** The derivative of $B(T_R)$ with respect to $T_R$ is:
\[
\frac{\partial B(T_R)}{\partial T_R} = \lambda'(T_R)(P_p - P_f)[T_R - \int_{T_o}^{T_R} F(x) dx] < 0
\]
because $P_p < P_f$, $\lambda(T_R)$ is an increasing function of $T_R$ (i.e., $\lambda'(T_R) > 0$), and $T_R = \int_0^{T_R} 1 dx > \int_0^{T_R} F(x) dx > \int_{T_o}^{T_R} F(x) dx$. Therefore, $B(T_R)$ is a decreasing function of $T_R$. Because there is a unique $T_{Re}^\circ$ strategy such that $B(T_{Re}^\circ) = 0$, where the derivative of (4.13) is zero as well, this strategy gives the unique maximum of (4.13). □

**Proof of Proposition 18** One can see that:
\[
\frac{\partial K(T_R)}{\partial T_R} = -\lambda'(T_R)(C_f - C_p)(T_R - \int_{T_o+L}^{T_R} F(x) dx) < 0,
\] (A.6) since $C_f > C_p$ and the equipment has an increasing failure rate function ($\lambda'(T_R) > 0$). This means that $K(T_R)$ is a decreasing function in $T_R$ where $T_R \geq T_o + L$. If $K(T_R = T_o + L) < 0$ then $K(T_R)$ is always negative, and so the objective function (4.15) is a decreasing function of $T_R$ and achieves the maximum at $T_{Ro}^* = T_o + L$. □

**Proof of Proposition 19** Assuming $Rf(T_R) + f'(T_R)(C_f - C_p) > 0$, there is a unique $T_{O}^*$ strategy such that $H(T_{O}^*) = 0$, where the derivative of (4.16) is zero as well, and this strategy gives the unique maximum of (4.16). Substituting $H(T_R) = 0$ in the objective function in (4.16) yields the value of $\Pi^*$ stated in the Proposition 19. □
Proof of Proposition 21: Substituting $T_R^* = T_O^* + L$ into agent’s payoff function (4.3) and assuming $P_p - C_p = P_f - C_f$, we have:

$$\bar{\Pi}_a = \left( \bar{P}_p - C_p \right) - S \int_0^{T_R} F(x)dx$$

Equating $\tau = \Pi_a^* - \bar{\Pi}_a$ and solving for $\bar{P}_p$ lead to the adjusted price.

□