RF SPECTRAL CHARACTERISTICS OF RANDOM PCM/FM AND PSK SIGNALS

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ABSTRACT
The telemetry radio frequency (RF) spectrum is rapidly becoming more crowded. Therefore, telemetry system engineers and frequency managers must become more knowledgeable about the RF spectral characteristics of telemetry signals. This paper presents methods to calculate the expected RF spectrum of random non-return-to-zero (NRZ) pulse code modulation (PCM)/frequency modulation (FM) and phase shift key (PSK) signals. The discussion includes the effects of bit rate, peak deviation, premodulation filtering, and spectrum analyzer resolution bandwidth. The methods are easily implemented using a personal computer and a spreadsheet program with graphics capability. Calculated spectra agree well with measured spectra. Equations are presented for accurately estimating the peak deviation and unmodulated carrier power of a random NRZ PCM/FM signal from the measured RF spectrum. Adjacent channel interference is also calculated. Key words: radio frequency spectral occupancy, pulse code modulation, frequency modulation, phase shift keying, premodulation filtering, adjacent channel interference.

PCM/FM RF SPECTRA
The calculated PCM/FM spectra are based on a paper by M. G. Pelchat [1]. Pelchat presents an exact expression for the power spectrum of an unfiltered, random PCM/FM signal. The power spectrum (dBc) can be calculated using (spectrum analyzer bandwidth added to Pelchat's expression):

$$S(f) = 10\log\left(\frac{4B_{SA}}{f_B}\left(\frac{D}{\pi(D^2-X^2)}\right)^2 \frac{(\cos\pi D-\cos\pi X)^2}{1-2\cos\pi D\cos\pi X+\cos^2\pi D}\right), \quad |\cos\pi D| < 1 \qquad (1)$$

where:

$S(f)$	=	power spectrum (dBc) at frequency f
B_{SA}	=	spectrum analyzer resolution bandwidth
f_B	=	bit rate
D	=	$2\Delta f/f_B$
X	=	$2(f-f_c)/f_B$

Δf = peak deviation
f_c = carrier frequency.

Figure 1 shows the spectrum calculated using equation (1) and the measured spectrum for the following parameters: bit rate = 800 kb/s, peak deviation = 280 kHz, and spectrum analyzer resolution bandwidth (RBW) = 30 kHz. The agreement is excellent. The measured values were increased by 1.7 dB to correct for the fact that Hewlett-Packard spectrum analyzers do not accurately measure power when many spectral components are present in the resolution bandwidth. The correction factor may be different for other spectrum analyzers. The measured spectra in this paper are "average" spectra. The averaging was accomplished by using a video bandwidth which was much narrower than the resolution bandwidth. I believe that average spectra provide a more accurate measure of interference than "peak" spectra (measured using peak hold feature) for random NRZ signals. I selected 201 points and an RF bandwidth of six times the bit rate for the spreadsheet calculations in this paper. This number of points gives a good representation of the actual spectrum. Using less points results in a less accurate spectrum while using more points resulted in slow response (I used an 8 MHZ AT). It is also desirable to insert a very small frequency offset in $f-f_c$ (e.g., 0.001 Hz) to minimize the probability of dividing by zero. Figure 2 shows the agreement for a peak deviation of 560 kHz. The agreement is again excellent. The optimum deviation for PCM/FM is approximately 0.35 times the bit rate. For this deviation, more than 90% of the total power is contained in a bandwidth equal to the bit rate. The example with a peak deviation of 0.7 times the bit rate was included only to demonstrate that this equation works well for more than one peak deviation. A peak deviation of 0.7 times the bit rate is not a recommended peak deviation. The nulls in the RF spectrum occur when $\cos \pi D = \cos \pi X$, $(X \neq D)$. The nulls occur at offsets from the carrier frequency of $\pm(nf_B \pm \Delta f)$, that is, integer multiples of the bit rate away from the peak frequency deviation frequencies. This fact can be used to accurately estimate the peak deviation ($\Delta f < f_B$) when the bit rate is known using

$$\Delta f = f_B - \frac{\textit{null spacing}}{2} \tag{2}$$

The null spacing is the frequency difference between the first nulls (one on each side of center frequency). The null spacing is 1040 kHz in figure 1 and 480 kHz in figure 2.

Equation (1) shows that S(f) is inversely proportional to the bit rate and directly proportional to the RBW (for any given values of X and D). Figure 3 presents the effect of increasing the bit rate from 800 kb/s to 2400 kb/s (RBW = 30 kHz). The power spectral density changed by ten times the log of 800/2400 or -4.8 dB. Decreasing the resolution bandwidth from 30 kHz to 3 kHz will change the displayed power spectrum by ten times the log of 3/30 or -10 dB.

PCM/FM RF SPECTRA WITH PREMODULATION FILTERING

The sidebands in figures 1 to 3 decrease at 12 dB/octave. The rolloff rate can be increased by using a premodulation filter. This filtering minimizes the interference with adjacent channels. General, exact expressions do not appear to exist for PCM/FM RF spectra with premodulation filtering. The basic problem is that frequency modulation is a complex, non-linear operation. Watt, Zurick, and Coon [2] discuss the calculation of spectral sidebands for PCM/FM with premodulation filtering. Their basic approach is to shift the low pass filter zero frequency to the peak deviation limits of the modulated signal. This method can be easily implemented on a spreadsheet. A premodulation filter bandwidth of 0.7 times the bit rate does not have a significant impact on PCM/FM bit error rate performance [3]. Figure 4 presents the measured and calculated spectra for a 4-pole constant delay (CD), also known as linear phase, premodulation filter with -3 dB bandwidth of 560 kHz. Figure 5 displays the measured and calculated spectra for a 1-pole resistor-capacitor (RC) filter with -3 dB bandwidth of 540 kHz. The agreement between measured and calculated spectra is reasonably good for the 4-pole CD and 1-pole filters. The calculated spectra are slightly wider than the measured spectra. The calculated spectra have slightly less power near center frequency than the measured spectra. The best match for this 4-pole CD filter occurs with a frequency shift of 200 kHz. Figure 6 displays the measured and calculated frequency responses for the 4-pole CD filter. Figure 7 shows the measured and calculated spectra for a 4-pole constant amplitude (CA) filter. The calculated spectrum is slightly narrower than the measured spectrum for this CA filter. Figure 8 shows that the RF spectra are similar for both a 4-pole CD and a 4-pole CA premodulation filter (same -3 dB bandwidths).

The power gain as a function of frequency can be calculated for a 4-pole CD filter using the following equation [4]:

$$|H(f)|^2 = \frac{11025}{z^8 + 10z^6 + 135z^4 + 1575z^2 + 11025} \tag{3}$$

where:

$z = 2.114f/f_{-3}$
f_{-3} = desired -3 dB frequency

The power gain as a function of frequency for an n-pole CA filter can be calculated using:

$$|H(f)|^2 = \frac{1}{1 + \left(\frac{f}{f_{-3}}\right)^{2n}} \tag{4}$$

The Telemetry Standards[5] (Inter-Range Instrumentation Group (IRIG) Standard 106-86) states "In any 3 kHz bandwidth outside bandwidth A' (A' = Authorized bandwidth + 1.0 MHZ), the minimum required attenuation for all emissions is 60 dB below the transmitted power, except that it shall not be necessary to attenuate below a level of -25 dBm". Textbooks typically calculate the 99% fractional power containment bandwidth[6] (the frequency band where 0.5 % of the power is below the lower band limit and 0.5% is above the higher band limit). Many modern spectrum analyzers can easily find the bandwidth that contains N% of the total power. It is more difficult to find the -60 dBc points when an unmodulated carrier is not available. Equation (1) can be used to estimate the power in dBc at any point in the spectrum. Setting X=0 yields the power (dBc) at the center frequency:

$$S(f_c) = 10\log\left(\frac{B_{SA} f_B}{\pi^2 (\Delta f)^2}\right) \qquad (5)$$

Equation (5) is plotted in figure 9 for RBWs of 3 and 30 kHz. Equation (5) can be used to accurately estimate the signal power at the spectrum analyzer input by adding $|S(f_c)|$ plus any spectrum analyzer correction factor to the power measured at the center frequency.

The -60 dBc (3 kHz RBW), 99%, 99.9%, and 99.99% bandwidths were measured for several different combinations of bit rate, peak deviation, and premodulation filtering. These values are listed in table 1. The -60 dBc and 99.99% bandwidths are approximately equal for the bit rates, peak deviations, and premodulation filters presented in table 1 (excluding the cases with no premodulation filter). The -60 dBc bandwidth (MHZ) with a 4-pole CD premodulation filter (-3 dB at $0.7f_B$), peak deviation=$0.35f_B$, and bit rates between 0.8 Mb/s and 20 Mb/s can be estimated using $(2.45-0.025f_B)f_B$. The -60 dBc bandwidth for a bit rate of 1 Mb/s is slightly greater than 2.4 MHZ and the -60 dBc bandwidth for a bit rate of 10 Mb/s is approximately 22 MHZ. The 99% power bandwidth is slightly less than $1.2f_B$ for these parameters. Figure 10 plots the -60 dBc bandwidths (RBW=3 kHz) versus bit rate for a 4-pole CD and a 1-pole RC filter with -3 dB points at $0.7f_B$. Figure 10 also shows the A' bandwidths for IRIG 1, 3, 5, and 10 MHZ authorized bandwidths.

Table 1. NRZ PCM/FM Measured Bandwidths.

Bit Rate kb/s	Peak Dev kHz	Premod Filter kHz Type	-60 dBc kHz / f_B	99% kHz / f_B	99.9% kHz / f_B	99.99% kHz / f_B
400	140	None	2450 6.13	716 1.79	1280 3.13	2430 6.08
800	200	None	3360 4.2	960 1.2	2000 2.5	3900 4.88
800	280	None	4030 5.04	1430 1.79	2480 3.1	4830 6.04
800	320	None	4030 5.04	1500 1.88	2860 3.58	4920 6.15
800	560	None	5650 7.06	2100 2.63	4020 5.03	7080 8.85
1600	560	None	6750 4.22	2800 1.75	4850 3.03	9310 5.82
2400	840	None	8570 3.57	4200 1.75	7170 2.99	12200 5.08
400	140	280 5-pole CD	1060 2.65	480 1.2	820 2.05	980 2.45
800	280	560 5-pole CD	2030 2.54	930 1.16	1630 2.04	1930 2.41
800	280	800 4-pole CD	2400 3.0	1070 1.34	1740 2.18	2290 2.86
800	280	560 4-pole CD	2020 2.53	930 1.16	1620 2.03	1920 2.4
800	280	400 4-pole CD	1870 2.34	890 1.11	1460 1.83	1790 2.24
800	280	560 4-pole CA	2060 2.58	930 1.16	1620 2.03	1950 2.44
800	280	540 1-pole RC	2500 3.13	920 1.15	1680 2.1	2490 3.11
800	320	560 4-pole CD	2090 2.61	1180 1.48	1680 2.1	1990 2.49
2400	840	1680 4-pole CA	5760 2.4	2850 1.19	4950 2.06	6020 2.51

PSK RF SPECTRA

The term phase shift keying implies that the information is contained in the relative phase of the transmitted signal. Two methods of generating a binary PSK signal are multiplying a sine wave by ± 1 (also known as amplitude shift keying) and linearly changing the phase between 0 and 180 degrees (also known as phase modulation). The two methods are equivalent with no filtering but are not the same with premodulation filtering. Classical PSK signals are usually generated by multiplying the carrier signal by the baseband modulation signal. This method is equivalent to translating the baseband signal to the carrier frequency (two-sided spectra). The power spectrum with no filtering can be calculated using:

$$S(f) = 10\log\left(\frac{B_{SA}}{f_B}\,\frac{\sin^2\left(\frac{\pi X}{2}\right)}{\left(\frac{\pi X}{2}\right)^2}\right) \qquad (6)$$

Figure II shows the measured and calculated spectra for an unfiltered, random 800 kb/s PSK signal. The agreement is again excellent. An unfiltered PCM/FM signal is also shown in figure 11. The PSK sidebands roll off at 6 dB/octave while the FM sidebands roll off at 12 dB/octave. The 99% power bandwidth [7] for unfiltered PSK is $19.3f_B$. The -60 dBc bandwidth (3 kHz RBW) for unfiltered 800 kb/s PSK is approximately 30 MHZ. The RF spectra for PSK with premodulation filtering can be calculated by multiplying the equation (6) values by the premodulation filter gain (no frequency shift is necessary because the carrier frequency is not being changed). Figure 12 presents the measured and calculated spectra for the 560 kHz 4-pole CD filter. The differences are mainly caused by the extra attenuation of the actual filter between 800 and 2000 kHz and the lower attenuation of the actual filter above 2000 kHz. The measured bandwidths for this PSK signal were: -60 dBc 2.69 MHZ, 99% 1.19 MHZ, 99.9% 2.12 MHZ, 99.99% 2.7 MHZ. These bandwidths are approximately 30% larger than the bandwidths of PCM/FM with the same premodulation filter. The filtered PSK signal does not have a constant envelope (see figure 13). If a filtered PSK signal is amplified using a nonlinear amplifier, the sidebands are restored to approximately the level with no filtering [8].

Offset quadrature phase shift keying (OQPSK) is a modulation technique in which alternate bits modulate the carrier (I) and the carrier phase shifted by 90 degrees (Q). Both the I and Q components contain one-half of the total power. Since the I and Q components do not change state at the same time, the amplitude of only one component goes to zero at any time when a premodulation filter is used. Therefore, the envelope of the filtered signal only decreases to approximately 70% of the amplitude with no premodulation filter (see figure 13). The sidebands are not fully restored when a limiting amplifier is used with filtered OQPSK [8] . Tests using a saturated preamplifier showed a restoration of OQPSK sideband levels of approximately one-half the attenuation expressed in decibels. That is, when the filter caused a sideband reduction of 30 dB, the sideband was only attenuated by approximately 15 dB at the output of the saturated amplifier. The spectrum for OQPSK can be calculated using equation (6) with the bit rate cut in half. Minimum shift keying (MSK) can be viewed as either OQPSK with sinusoidal weighting or as PCM/FM with a peak deviation of 0.25 times the bit rate. The spectra of unfiltered OQPSK and MSK are shown in figure 14. The spectrum of OQPSK with rate ½ coding is the same as the spectrum of unencoded PSK (with linear amplification).

ADJACENT CHANNEL INTERFERENCE

The results of the previous sections can be combined with information about the receiver predetection filters to predict adjacent channel interference expressed as desired signal to interference (S/I) ratios. Figure 15 displays a typical 1 MHZ receiver filter and the receiver intermediate frequency (IF) outputs for a signal centered at the receiver center frequency and a filtered 800 kb/s PCM/FM signal shifted by +1 MHZ. The calculated and measured S/I values were 9.9 and 10 dB respectively. Figure 16 shows the calculated receiver IF output for a signal at center frequency and a filtered PCM/FM signal shifted by +2 MHZ but with 30 dB more power at the receiver input. The calculated and measured values were 14.3 and 18.4 dB respectively. The 4 dB difference was caused by the higher sideband levels of the calculated spectrum. The calculated S/I value with a 1-pole RC premodulation filter was 10.5 dB and the calculated S/I value with no premodulation filter was only 3.6 dB (+2 MHZ, +30 dB). Figure 17 presents the calculated receiver IF output for a signal at center frequency and a filtered 800 kb/s PSK signal shifted by + 1 MHZ. The calculated and measured S/I values were both 10.5 dB. Figure 18 shows the calculated receiver IF output for a signal at center frequency and a filtered PSK signal shifted by +2 MHZ but with 30 dB more power at the receiver input. The calculated and measured values were 8.6 and 9.7 dB respectively. The calculated S/I with no premodulation filter was -11.7 dB (+2 MHZ, +30 dB). The premodulation filter for figures 15 to 18 was a 4-pole CD with -3 dB frequency of 560 kHz.

CONCLUSIONS

1. The RF spectra of unfiltered PCM/FM and PSK signals can be predicted very accurately for various bit rates, peak deviations, and resolution bandwidths using a personal computer and a spreadsheet.

2. The RF spectra with premodulation filtering can be predicted with acceptable accuracy for PCM/FM and PSK.

3. The peak deviation can be accurately estimated when the bit rate and the frequency separation between the first power nulls are known.

4. The unmodulated carrier power of a PCM/FM signal can be accurately estimated when the bit rate, peak deviation, and spectrum analyzer resolution bandwidth are known.

5. The ratio of the -60 dBc bandwidth to the bit rate decreases as the bit rate increases (peak deviation and premodulation filter held at constant fraction of bit rate).

6. The spectral sidebands are lower for PCM/FM than for PSK with the same linear phase premodulation filter.

7. The adjacent channel interference is lower for PCM/FM than for PSK with the same linear phase premodulation filter.

ACKNOWLEDGEMENT

I would like to thank all of the people who have called me with questions about spectral occupancy of digital telemetry signals. These calls aroused my interest in this subject. I would also like to thank the Pacific Missile Test Center for providing Independent Development Program funding for this effort. I would like to give a special thanks to Ted Waddell for his many helpful comments.

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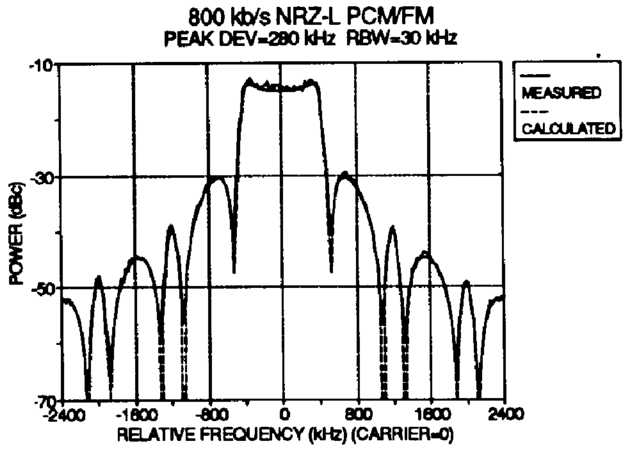

Figure 1 - PCM/FM with $\Delta f=0.35f_B$.

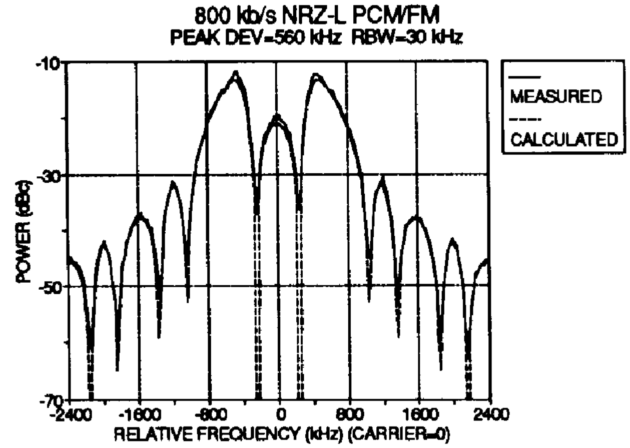

Figure 2 - PCM/FM with $\Delta f=0.7f_B$.

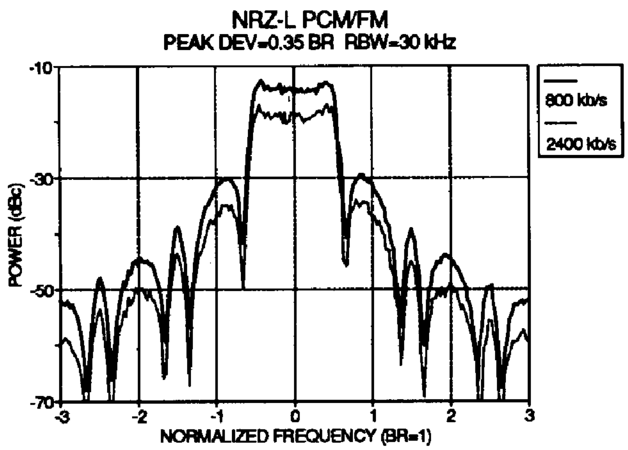

Figure 3 - Spectra for Two Bit Rates.

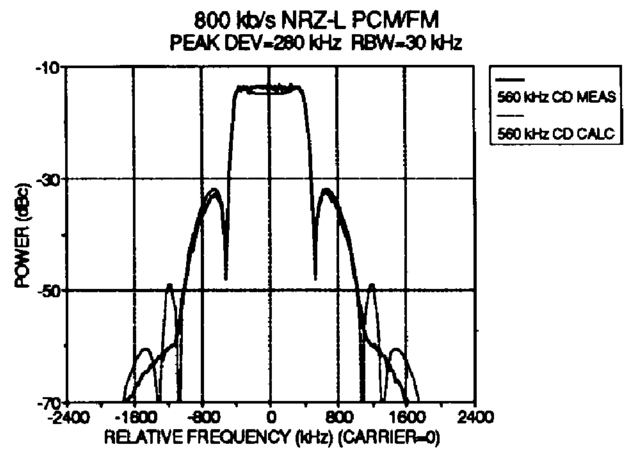

Figure 4 - PCM/FM with CD Filter.

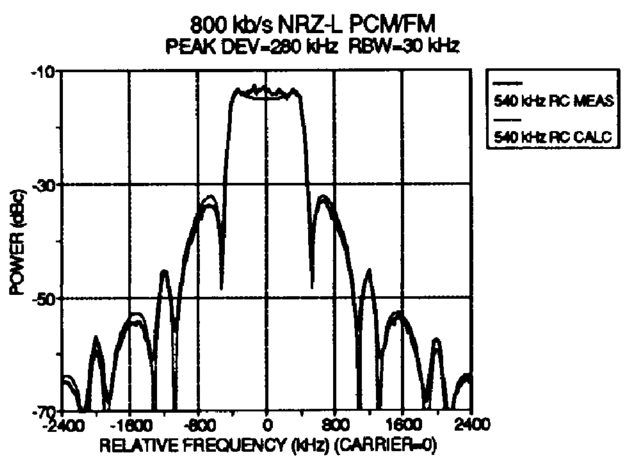

Figure 5 - PCM/FM with RC Filter.

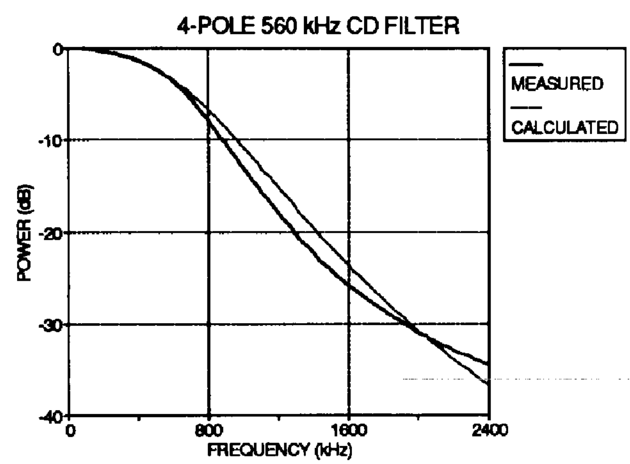

Figure 6 - CD Filter Response.

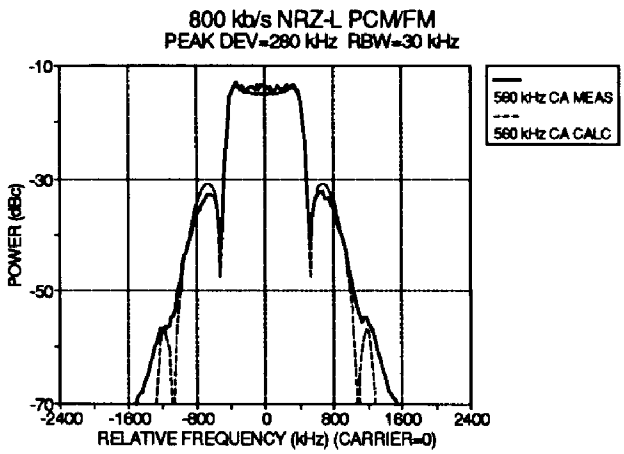

Figure 7 - PCM/FM with CA Filter.

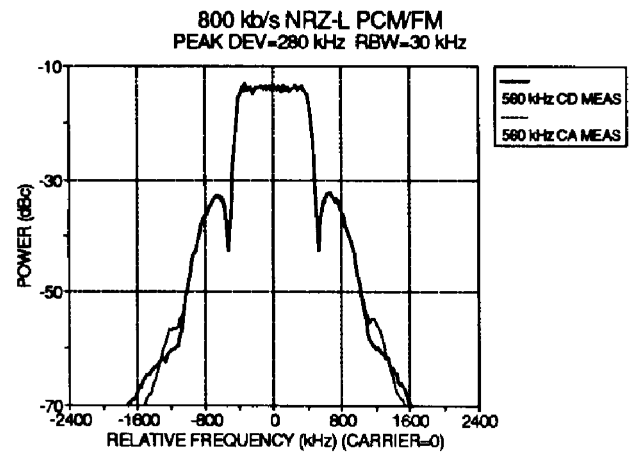

Figure 8 - PCM/FM with CD and CA Filters.

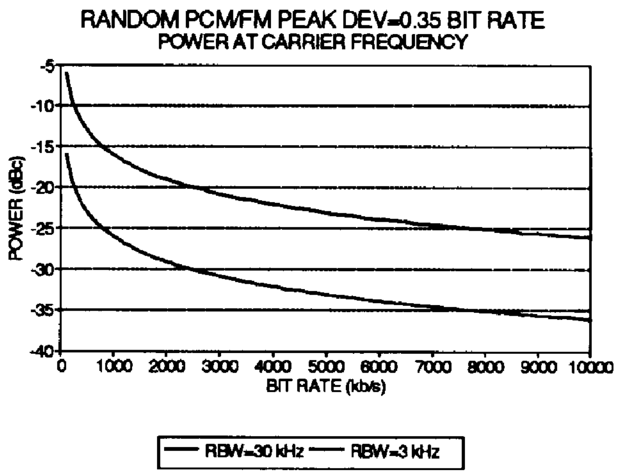

Figure 9 - PCM/FM Center Frequency Power.

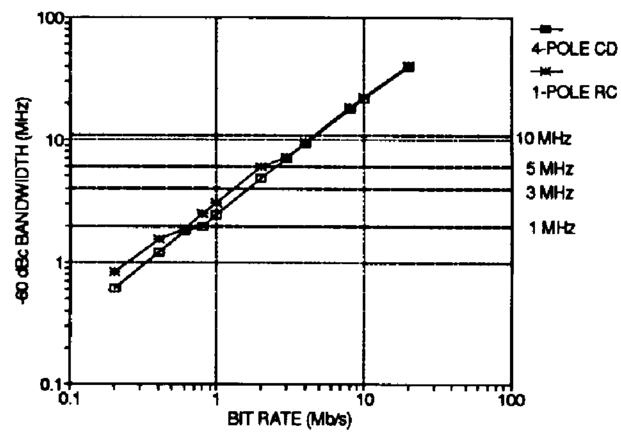

Figure 10 - PCM/FM -60 dBc Bandwidth.

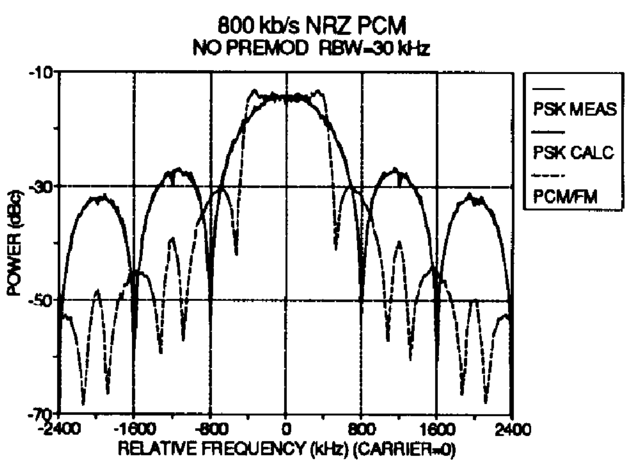

Figure 11 - PSK and PCM/FM Spectra.

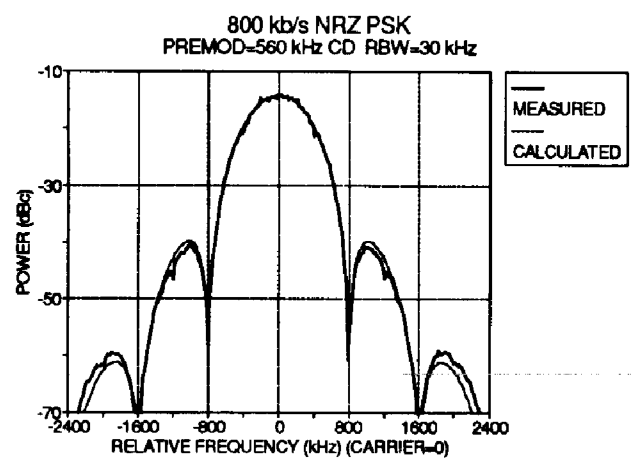

Figure 12 - PSK with CD Filter.

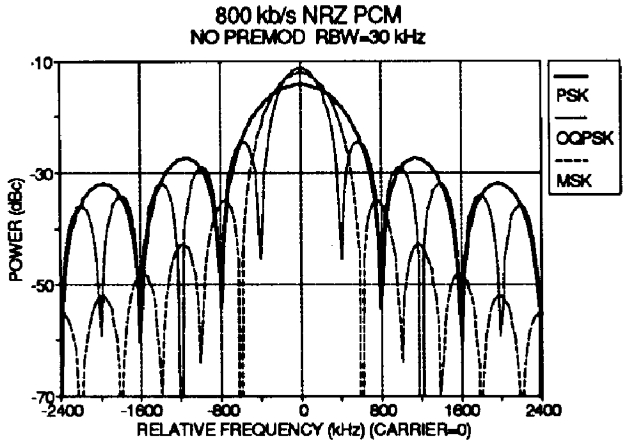

Figure 13 - MSK and OQPSK Spectra.

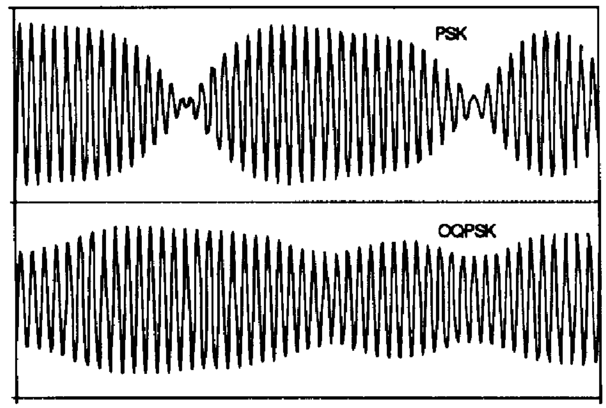

Figure 14 - PSK and OQPSK Envelopes.

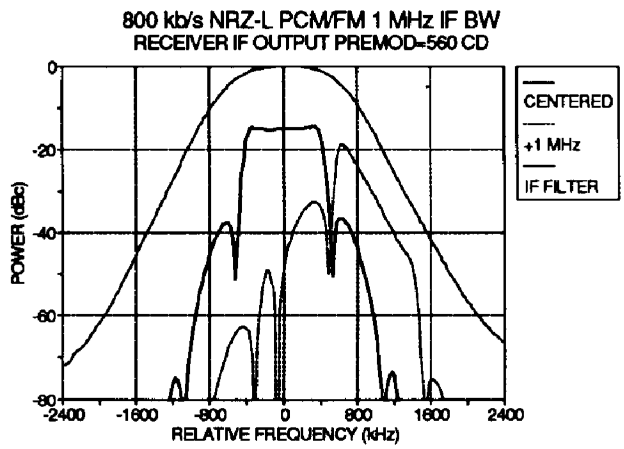

Figure 15 - Adjacent Channel (FM, +1 MHz).

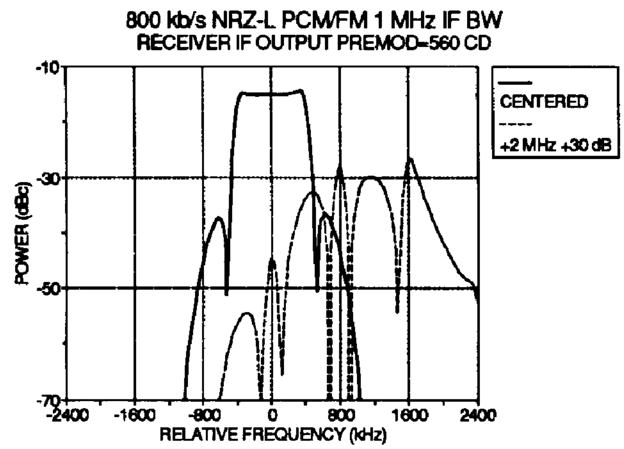

Figure 16 - Adjacent Channel (FM, +2 MHz).

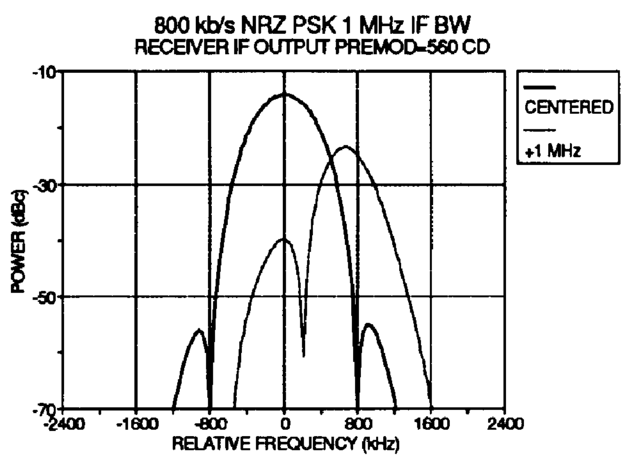

Figure 17 - Adjacent Channel (PSK, +1 MHz).

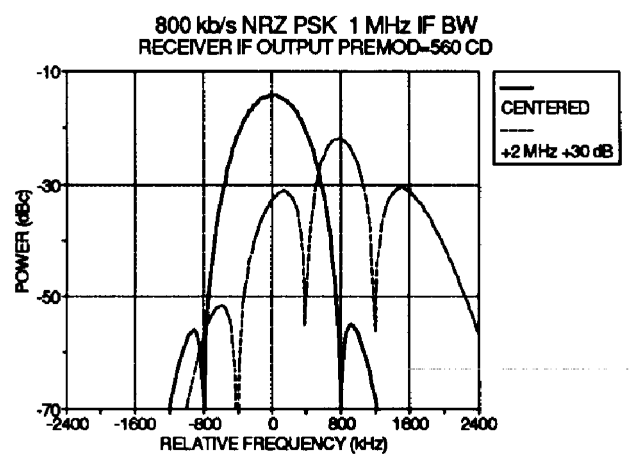

Figure 18 - Adjacent Channel (PSK, +2 MHz).