

ERROR PERFORMANCE OF SINGLE PULSE PPK SYSTEM WITH MAXIMUM VALUE TYPE DECISION MODEL

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ABSTRACT

The error performance of single pulse PPK system with maximum value type decision model is discussed in this paper. The word error probability in the case when noises exist is analysed and the energy necessary for transmitting each information bit is calculated. The detection characteristics of this system are made clear by comparing it with a threshold type decision model and ordinary systems. It is shown that the maximum value type of decision scheme is always superior to the threshold type. It can reduce the word error probability of PPK system further or make the energy efficiency of PPK system higher under the condition of given tolerable error probability.

INTRODUCTION

In 1980, a digitized 2^M -ary pulse communication system (being called PCM-PPK telemetry system) was presented by Li You-Ping(1). A feasible decision device model based on using "time checked" synchronization was introduced. It was shown that the energy efficiency of single pulse PPK system is higher than that of ordinary binary systems. The decision model adopted in that paper was threshold type scheme. However its optimum decision threshold depends on signal-to-noise ratio (see appendix), and any deviation of decision threshold from the optimum value will have some effects on the performance of system(2).

A decision scheme without threshold – the maximum value type decision model is discussed in this paper. There is no problem of selecting optimum threshold in this scheme. The analyses of word error probability and energy efficiency for this scheme show that using maximum value decision can make the word error probability of PPK system lower for a certain signal-to-noise ratio, or the energy efficiency will be higher for a given error probability. The value β of the maximum value type scheme is 2 dB smaller than that of the threshold type, and 4 dB smaller than that of the binary PSK system roughly.

MAXIMUM VALUE DECISION SCHEME

The decision scheme presented in (1) can be simplified as Fig.1, where synchronous timing circuits are omitted. When a certain sample U_k exceeds the threshold level U_o the PPK/PCM converter (a counter in fact) will send a PCM data according to its sampling order number. This scheme can be called a threshold type decision scheme, or be abbreviated to PPK(TT). The selection of threshold value U_o depends on the length M of PCM data word, and the signal-to-noise ratio r . The optimum values α_{opt} of relative threshold $\alpha = U_o / A$ (i.e. the ratio of the decision threshold value U_o to the peak voltage A of IF signal) and the word error probability P_{ew} under α_{opt} are listed in table A-1 and A-2 in appendix.

The block diagram of maximum value type decision scheme, abbreviated to PPK (MV), is shown in Fig.2a. In this scheme the samples U_j are not directly compared with threshold level U_o . The N samples are compared each other in proper order, the largest one of them is considered as the sample of signal pulse, and the sampling order number of this sample is converted to corresponding PCM data by PPK/PCM converter.

The engineering realization of maximum value selector is not very complex, it may be constructed by ingeniously combining sample-hold circuits, comparator and digital circuits (Fig.2b). As long as the velocity permitting, it may consist of digital circuits only.

WORD ERROR PROBABILITY AND ENERGY EFFICIENCY

From the principle of PPK(MV), we know that, so long as the $N-1$ noise samples in space positions do not exceed the sample of signal with noise in signal position, the PPK signal can be decided correctly. Otherwise the word error will appear.

It is well known that the output voltage U_n of a envelope detector is subject to Reyleigh distribution when the signal is absent

$$p(U_n) = \frac{U_n}{\sigma^2} \exp(-U_n^2/2\sigma^2), \quad (1)$$

where σ^2 is the mean square value of noise voltage. Hence the probability of that noise voltage exceeds the mixture of signal and noise U_{s+N} is

$$\begin{aligned}
P_1 &= P(U_N > U_{s+N}) = \int_{U_{s+N}}^{\infty} p(U_N) dU_N \\
&= \int_{U_{s+N}}^{\infty} \frac{U_N}{\sigma^2} e^{-U_N^2/2\sigma^2} dU_N = e^{-U_{s+N}^2/2\sigma^2},
\end{aligned} \tag{2}$$

the mixture of signal and noise U_{s+N} is subject to the Rice distribution

$$p(U_{s+N}) = \frac{U_{s+N}}{\sigma^2} I_0 \left\{ \frac{AU_{s+N}}{\sigma^2} \right\} \exp \left(-\frac{U_{s+N}^2 + A^2}{2\sigma^2} \right), \tag{3}$$

where A – peak voltage of IF signal,
 I_0 – Bessel function of zero order.

The value of U_{s+N} may be vary in the range of $0 \sim \infty$, therefore the average probability of that the noise U_N is larger than the mixture U_{s+N} is

$$\begin{aligned}
P_2 &= \int_0^{\infty} P_1 p(U_{s+N}) dU_{s+N} = \int_0^{\infty} \frac{U_{s+N}}{\sigma^2} I_0 \left\{ \frac{AU_{s+N}}{\sigma^2} \right\} e^{-\frac{2U_{s+N}^2 + A^2}{2\sigma^2}} dU_{s+N} \\
&= \frac{1}{2} \exp \left(-\frac{A^2}{4\sigma^2} \right) \cdot Q \left(\frac{A}{\sqrt{2}\sigma}, 0 \right),
\end{aligned} \tag{4}$$

where

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} t I_0(\alpha t) e^{-(t^2 + \alpha^2)/2} dt \tag{5}$$

is so-called Marcum Q function(3), it has a property as following

$$Q(\alpha, 0) = \int_0^{\infty} t I_0(\alpha t) e^{-(t^2 + \alpha^2)/2} dt = 1. \tag{6}$$

so that formula (4) can be further simplified to

$$P_2 = \frac{1}{2} \exp \left(-\frac{A^2}{4\sigma^2} \right). \tag{7}$$

The probability of that the noise voltage does not exceed the mixture of signal and noise must be $1 - P_2$. The probability of that all the $N-1$ samples in space positions do not exceed the mixture sample in signal position, i.e. the probability of correct decision P_c is

$$P_c = (1 - P_2)^{N-1} \quad (8)$$

Therefore the word error probability P_{ew}

$$\begin{aligned} P_{ew} &= 1 - P_c = 1 - (1 - P_2)^{N-1} = 1 - \left(1 - \frac{1}{2} \exp\left(-\frac{A^2}{4\sigma^2}\right)\right)^{N-1} \\ &= 1 - \left(1 - \frac{1}{2} \exp\left(-\frac{\gamma}{2}\right)\right)^{2^M - 1}, \end{aligned} \quad (9)$$

where

$$\gamma = \frac{A^2}{2\sigma^2} = \frac{P_s}{P_n} \quad (10)$$

is the ratio of pulse signal power to noise power, namely pulse signal-to-noise ratio of the system.

In order to evaluate the energy efficiency (the paid energy cost) the coefficient β proposed by R.W. Sander is used

$$\beta \triangleq E_b/N_o \quad (11)$$

Which means the ratio of the carrier energy needed for transmitting each bit information to noise power density spectrum. Because

$$E_b = P_s \cdot \tau / M, \quad (12)$$

$$N_o = P_n / \Delta f, \quad (13)$$

$$\beta = \frac{E_b}{N_o} = \frac{P_s}{P_n} \frac{\Delta f \cdot \tau}{M}, \quad (14)$$

where P_s – Pulse power of recieved signal
 τ – pulse width
 P_n – noise power
 Δf – system bandwidth

As a result of the consideration that the matched filters have been applied in decision models, we have $\Delta f \cdot \tau = 1$ and $P_s/P_n = \gamma$, so that

$$\beta = \frac{E_b}{N_o} = \frac{\gamma}{M} \quad (15)$$

Substituting(9) into (15) it is obtained that

$$\beta = -\frac{2}{M} \ln \left\{ 2 \cdot (1 - 2^{M-1} \sqrt{1 - P_{ew}}) \right\}. \quad (16)$$

Let P_{ew} be equal to $10^{-2} \sim 10^{-8}$, the values β of PPK(MV) systems for $M=5 \sim 8$ are given in table 1.

Table 1 The Values β of PPK(MV) Systems

β^M P_{ew}	8	7	6	5
10^{-2}	2.3621	2.5004	2.6834	2.9365
10^{-3}	2.9388	3.1595	3.4524	3.8592
10^{-4}	3.5146	3.8175	4.2201	4.7805
10^{-5}	4.0903	4.4754	4.9876	5.7015

β^M P_{ew}	8	7	6	5
10^{-6}	4.6660	5.1334	5.7552	6.6226
10^{-7}	5.2429	5.7939	6.5242	7.5443
10^{-8}	5.8842	6.4827	7.3091	8.4678

COMPARISON

The results of comparing the error performance of PPK(MV) systems with that of the PPK(TT) for $M = 5 \sim 8$ are shown in Fig , 3, where the word error probability of PPK(MV) is calculated by (16) and that of PPK(TT) is under optimum threshold values.

It can be seen in this figure that the PPK(MV) is always superior to the PPK(TT) for the same given tolerable error probability. The main reason for this fact is that some of the slots preceding the signal slot may have exceeded threshold and caused word error even if the signal slot has the maximum sampling value. But such error can not happen in PPK(MV).

On an average, the value β of PPK(MV) is about 2 dB smaller than that of PPK(TT) for the same error probability.

In PPK systems, the information is transmitted word by word. In order to compare it with binary systems, we can convert the bit error probability of binary System P_{eb} to P_{ew} as follows

$$P_{ew} = 1 - (1 - P_{eb})^M. \quad (17)$$

And the P_{ew} of the various binary systems are in turn given as follows

$$P_{ew} = 1 - \left(1 - \frac{1}{2} e^{-\beta/2}\right)^M \quad \text{FSK(Noncoherent)} \quad (18)$$

$$P_{ew} = 1 - \left[1 - \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{\gamma}{2}} \right) \right]^M \text{ FSK (Coherent)} \quad (19)$$

$$P_{ew} = 1 - \left(1 - \frac{1}{2} e^{-\gamma} \right)^M \text{ DPSK} \quad (20)$$

$$P_{ew} = 1 - \left[1 - \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma}) \right]^M \text{ PSK} \quad (21)$$

The word error probability P_{ew} of various binary and these of the two PPK Systems are plotted in Fig.4, where the X axis is the normalized signal-to-noise ratio of each bit γ_b (dB). For binary systems only one bit is carried by each symbol, so

$$r_b = \frac{r}{1} = \Gamma = \frac{P_s}{P_N} = \beta_{\text{binary}} \quad (22)$$

And for 2^M -ary PPK systems, M bits are carried by each symbol, therefore

$$r_b = \frac{r}{M} = \frac{P_s}{P_N} \cdot \frac{1}{M} = \beta_{\text{PPK}} \quad (23)$$

If we insist on using the bit error probability P_{eb} to compare (4), by the way, the relation between P_{ew} and P_{eb} derived from (4) is not correct, the P_{ew} can be converted to the equivalent P_{eb} the relations between P_{ew} and P_{eb} in PPK systems are satisfied following conditions (3)(5)(6).

$$\frac{1}{M} P_{ew} \leq P_{eb} \leq \frac{1}{2} P_{ew} \quad (24)$$

Comparing the upper bound of P_{eb} with binary systems, Fig.5 shows that the performance of PPK systems are also better than that of binary systems within the scope of the engineering practice.

It must be pointed out that the improvement of error performance or energy efficiency of PPK system is obtained at the expense of bandwidth. The bandwidth usage factor of PPK system is very low, the bandwidth demand for PPK system is $2^M / M$ times larger than that of the PSK system under the same information rate. It is only applied to that place where the bandwidth is very rich and the power is limited severely.

CONCLUSION

After analyses of the error probability for PFK(MV) and comparing it with PPK (TT) and binary systems, the following results are obtained:

- 1) The error performance of PPK systems are better than the ordinary binary systems under the same value β , no matter whether P_{eb} or P_{ew} is used as the comparison standard.
- 2) The performance of PPK(MV) is always superior to PPK(TT). The use of the maximum value decision model will bring more advantages of the PPK system into play.

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APPENDIX P_{ew} and α_{opt} of PPK(TT)

Two kinds of errors are existed in PPK(TT), one is false-alarm (probability is P_w), the another is miss (probability is P_L). Both are related to the decision threshold U_o :

$$P_w = P(u_N > U_o) = \int_{U_o}^{\infty} p(u_N) du_N = \exp\left(-\frac{U_o^2}{2\sigma^2}\right), \quad (A-1)$$

$$\begin{aligned} P_L &= P(u_{s+N} < U_o) = \int_0^{U_o} p(u_{s+N}) du_{s+N} = 1 - Q\left(\frac{A}{\sigma}, \frac{U_o}{\sigma}\right) \\ &\simeq \frac{1}{2} \operatorname{erfc}\left(\frac{A-U_o}{\sqrt{2}\sigma}\right) \simeq \frac{1}{2} \frac{1}{\sqrt{\pi} \left(\frac{A-U_o}{\sqrt{2}\sigma}\right)} \exp\left[-\frac{(A-U_o)^2}{2\sigma^2}\right]. \end{aligned} \quad (A-2)$$

Asume that the PPK signal is occurred with equal probability at N positions, then the average value of error probability caused by false-alarm and miss are

$$P_I = \frac{1}{N} \sum_{k=1}^N [1 - (1 - P_w)^{k-1}] \simeq \frac{N-1}{2} P_w, \quad (A-3)$$

$$P_{II} = \frac{1}{N} \sum_{k=1}^N (1 - P_w)^{k-1} \cdot P_L \simeq \left(1 - \frac{N-1}{2} P_w\right) \cdot P_L, \quad (A-4)$$

so the word error probability is

$$\begin{aligned} P_{ew} &= P_I + P_{II} = \frac{N-1}{2} P_w + \left(1 - \frac{N-1}{2} P_w\right) P_L \\ &\simeq \frac{2^M - 1}{2} \exp(-\alpha^2 r) + \frac{1}{2} \frac{1}{(1-\alpha)\sqrt{\pi r}} \exp\left[-r(1-\alpha)^2\right], \end{aligned} \quad (A-5)$$

where

$$r = \frac{A^2}{2\sigma^2} \text{ is the signal-to-noise power ratio}$$

$$\alpha = U_0/A \text{ is the relative decision threshold value}$$

The optimum decision threshold values α_{opt} and the minimum values of word error probability P_{ew} under the α_{opt} with $r = 10 \sim 24$ dB are given in the following tables A-1 and A-2.

Table A-1 The Optimum Decision Threshold Values

α_{opt} M	$r = M\beta$ dB	10	12	14	16	18	20	22	24
4		0.680	0.626	0.585	0.556	0.537	0.525	0.516	0.511
5		0.717	0.649	0.599	0.565	0.543	0.528	0.519	0.512
6		0.750	0.672	0.614	0.575	0.549	0.532	0.521	0.514
7		0.780	0.694	0.628	0.584	0.554	0.535	0.523	0.515
8		0.809	0.716	0.642	0.592	0.560	0.539	0.525	0.516

Table A-2 The Minimum Values of P_{ew}

P_{ew} M	$r = M\beta$ dB	10	12	14	16	18	20	22	24
4		0.174	3.57×10^{-2}	3.18×10^{-3}	7.32×10^{-5}	1.96×10^{-7}	1.74×10^{-11}	7.01×10^{-18}	5.44×10^{-28}
5		0.232	4.82×10^{-2}	4.36×10^{-3}	1.02×10^{-4}	2.76×10^{-7}	2.47×10^{-11}	1.00×10^{-17}	7.75×10^{-28}
6		0.305	6.38×10^{-2}	5.89×10^{-3}	1.40×10^{-4}	3.84×10^{-7}	3.46×10^{-11}	1.41×10^{-17}	1.10×10^{-27}
7		0.395	8.32×10^{-2}	7.85×10^{-3}	1.90×10^{-4}	5.29×10^{-7}	4.82×10^{-11}	1.97×10^{-17}	1.54×10^{-27}
8		0.508	1.07×10^{-1}	1.04×10^{-2}	2.56×10^{-4}	7.25×10^{-7}	6.68×10^{-11}	2.75×10^{-17}	2.17×10^{-27}

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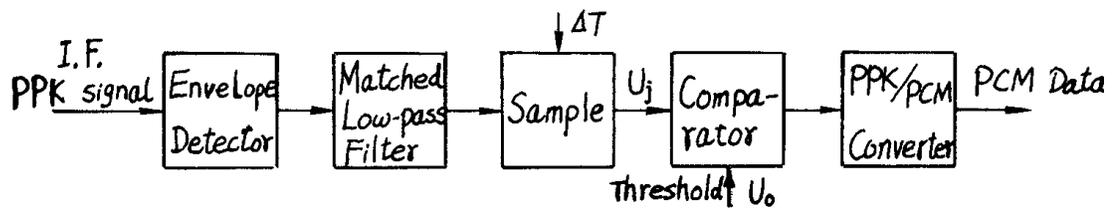
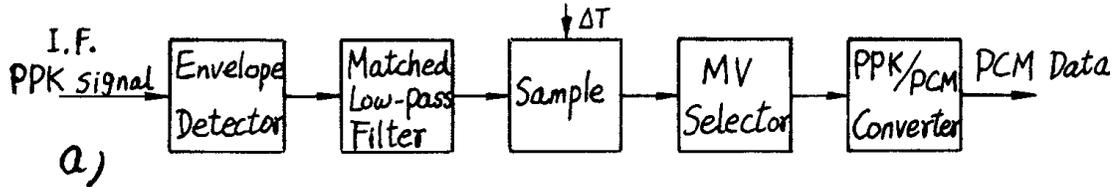
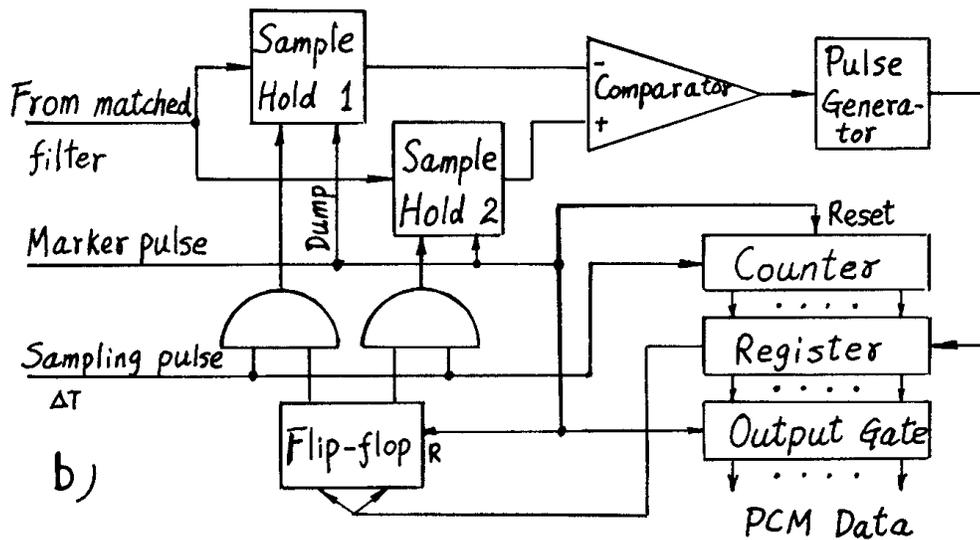


Figure 1 Block diagram of threshold decision



a)



b)

Figure 2 Scheme of maximum value decision
a) block diagram b) A realization of MV decision

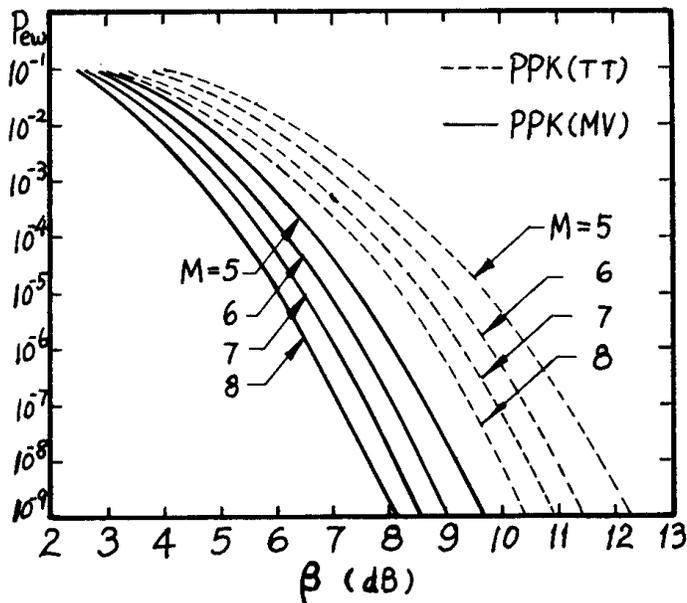


Figure 3 Word error probability for two decision schemes

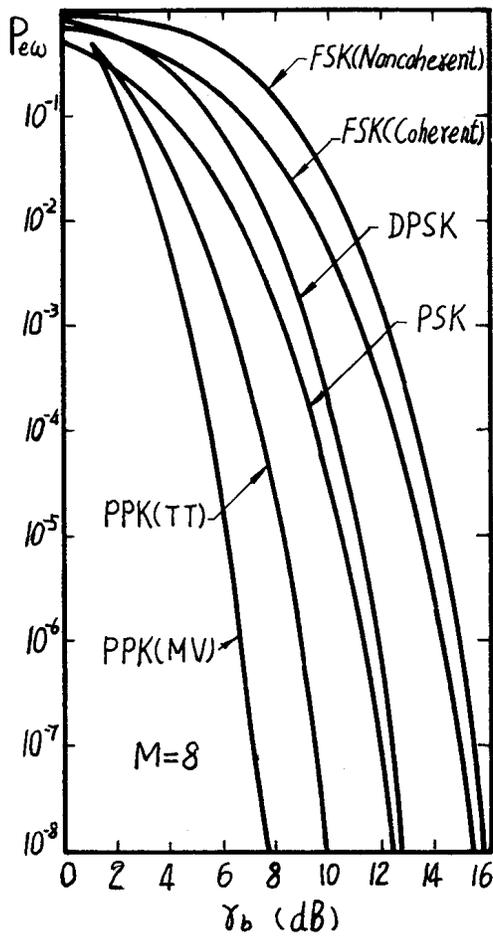


Figure 4 Word error probability P_{ew} for PPK systems and binary systems

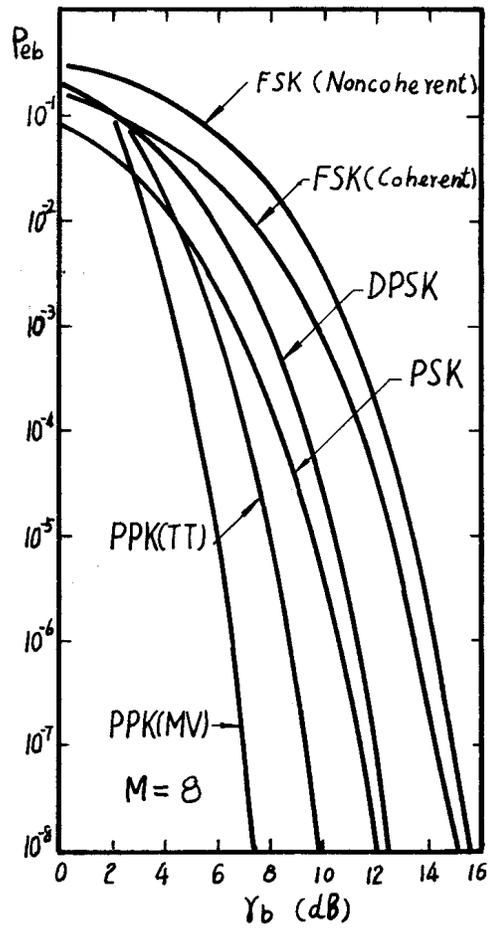


Figure 5 Bit error probability P_{eb} for PPK systems and binary systems