A TELEMETRY SYSTEM BASED ON HAAR FUNCTIONS

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ABSTRACT

In this paper a telemetry system based on Hear functions is described. According to the principles of orthogonal multiplexing, the key point is that noninterference between channels can be guaranteed if the subcarrier waveforms are chosen from an orthogonal set [1]. The Haar functions form an orthogonal and orthonormal system of periodic square weaves. The Haar functions can be used as subcarriers for a telemetry system. Finally, an experimental 4-channel model has been constructed to demostrate the principle.

INTRODUCTION

The basic problem of a multiplexing system is how to transmit many information signals over a single wire or radio link and recover them without interference between channels. This is really an interference problem, since to the output detector for any one channel all the other channel signals represent potential interference. Each of these potential interference signal usually is comparable in level to the desired signal.
Non-interference between channels can be guaranteed in a multiplex system if the selected set of subcarriers are mutually orthogonal, meaning that the product of any two subcarrier waveforms integrates to zero over some characteristic interval T. Any two subcarriers P(n,t) and P(m,t) are orthogonal if they satisfy the relationship:

$$\frac{1}{T}\int_{0}^{T} P(n,t)\,P(m,t)\,dt = 0.$$

An optimum multiplex system is shown in Figure 1. In communication systems, frequency division multiplex, time division multiplex and code division multiplex are usually used nowadays. Their common property is the utilization of orthogonal functions. The difference is that the orthogonal function sets used are different. For example, since and cosine functions are used as subcarriers in frequency division multiplex, block pulses are used as subcarriers in time division multiplex, Walsh functions are used as subcarriers in code division multiplex. It is proved that all orthogonal function sets may in principle be used for multiplexing. In practical applications, there are some factors which should be considered in choosing waveforms, such as ease of generation, detectability, and flexibility in engineering.

THE HAAR FUNCTION

The Haar function set forms a complete set of orthogonal rectangular functions similar in several respects to the Walsh function. Haar functions were introduced by the Hungarian methematician Alfred Haar in 1910 [2]. The amplitude values of the Haar functions assume a limited set of values, 0, ±1, ± 2, ±2, ±2 2, ±4 etc. If we consider the time base to be defined as $0 \le t \le 1$, then the Haar function { har (n, k, t) } may be written as follows:

$$\mathrm{har}(0,0,t) = 1, \qquad \text{for } 0 \leqslant t \leqslant 1$$

$$\mathrm{har}(0,1,t) = \begin{cases} +1, & \text{for } 0 \leqslant t < \tfrac{1}{2} \\ -1, & \text{for } \tfrac{1}{2} \leqslant t \leqslant 1 \end{cases}$$

$$\mathrm{har}(1,1,t) = \begin{cases} \sqrt{2}, & \text{for } 0 \leqslant t < \tfrac{1}{4} \\ -\sqrt{2}, & \text{for } \tfrac{1}{4} \leqslant t < \tfrac{1}{2} \\ 0, & \text{for } \tfrac{1}{2} \leqslant t \leqslant 1 \end{cases}$$

$$\mathrm{har}(1,2,t) = \begin{cases} 0, & \text{for } 0 \leqslant t < \tfrac{1}{2} \\ \sqrt{2}, & \text{for } \tfrac{1}{2} \leqslant t < \tfrac{3}{4} \\ -\sqrt{2}, & \text{for } \tfrac{3}{4} \leqslant t \leqslant 1 \end{cases}$$

$$\cdots \quad \cdots \qquad \qquad \cdots$$

$$\mathrm{har}(n,k,t) = \begin{cases} \sqrt{2^n}, & \text{for } \dfrac{2k-2}{2^{n+1}} \leqslant t < \dfrac{2k-1}{2^{n+1}} \\ -\sqrt{2^n}, & \text{for } \dfrac{2k-1}{2^{n+1}} \leqslant t < \dfrac{2k}{2^{n+1}} \\ & n = 0, 1, 2, \ldots\ldots \\ & k = 1, 2, \ldots, 2^n \\ 0, & \text{elsewhere} \end{cases}$$

then the Haar functions can be referred to by order, k, and degree, n, as well as time, t. The degree, n, then denotes a subset having the same number of zero crossings in a given width, $1/2^n$, thus providing a form of comparison with frequency and sequency terminology. The order, k, gives the position of the function within this subset. All members of the subset with the same degree are obtained by shifting the first member along the time axis by an amount proportional to its order.

The first eight Haar functions are shown in Figure 2. The first two functions are identical to wal (0, t) and wal (1,t). The next function, har(1, 1, t) is simply har(0, 1, t) squeezed into the left-hand half of the time base and modified in amplitude to $\pm\sqrt{2}$. The next function har (1, 2, t) is identical but squeezed into the right-hand half of the time base. Subsequent pairs of functions are similarly squeezed and shifted having amplitudes ±1 multiplied by power of $\sqrt{2}$. in general all members of the same function subset (such as har(1, 1, t) and har (1, 2, t) or har (2, 1, t), har (2, 2, t), har (2, 3, t) and har (2, 4, t), etc.) are obtained by a lateral shift of the first member along the time axis by an amount proportional to its length.

Within the interval $0 \le t \le 1$, for same n, if $j > k \ge 1$,

then $$\int_0^1 \text{har}(n, j, t)\ \text{har}(n, k, t)\ dt = 0 \ ;$$

if $m > n$,

$$\int_0^1 \text{har}(m, k, t)\ \text{har}(n, k, t)\ dt$$

$$= \lambda \int_{\frac{2k-2}{2^{m+1}}}^{\frac{2k}{2^{m+1}}} \text{har}(m, k, t)\ dt = \lambda \left[\frac{2^m}{2^{m+1}} - \frac{2^m}{2^{m+1}} \right] = 0$$

where λ is a constant.

$$\int_0^1 \text{har}(0, 0, t)\ \text{har}(n, k, t)\, dt = \int_0^1 \text{har}(n, k, t)\, dt = 0$$

$$\int_0^1 \left[\text{har}(n, k, t) \right]^2 dt = 2(\sqrt{2^n}\ )^2 \frac{1}{2^{n+1}} = 1$$

Therefore the Haar function are an orthogonal and orthonormal system.

SYSTEM DESIGN

The diagram of the Haar telemetry system is identical with the Figure 1. We put emphasis on the multiplier and orthogonal waveform generator.

There are three basic types of multipliers. The first multiplies two voltages that can assume two values only, say +1 V and -1 V. This type of multiplier is implemented by logic circuits and usually referred to as exclusive OR-gate or half adder. The second type multiplies a voltage V_1 having arbitrary values with a voltage V_2 that can assume a few values only. The third basic type of multipier multiplies two arbitrary voltages. In principle, this type can be implemented by Hall effect multipliers, field effect transistors and logarithmic elements. We are interested in the second type multipliers. Figure 3 shows an example of this type [3]. Voltage V_2 assumes the values +1 or -1 only. The output voltage equals either $+V_1$, or $-V_1$, where V_1 may have any value within the voltage range of the operational amplifier A. The circuit works as follows: The non-inverting input terminal (+) of the amplifier is grounded, if the field effect tramsistor FET is fully conducting, V_3 must equal $-V_1$ to bring the inverting input terminal (-) also to ground potential. Let FET be non-conducting. The non-inverting terminal is then at V_1 and the inverting terminal must also be at V_1. This requires V_3 to equal V_1.

For Haar functions V_2 may have the values, 0, +V, -V. So the second type of multiplier must be modified. The circuit is shown in Figure 4 When both of two switches, S_1, S_2 are open, Let $n = (R_1' + R_1'')/R_4$, Amplifier gain is G. then $V_3 = \frac{n \cdot G}{n\,(G+1)+1}\, V_1$

When S_1 is closed, S_2 is open,

$$V_3 = -\frac{G}{n\,(G+1)\,+1}\, V_1 \quad .$$

When both, S_1, S_2 are closed,

$V_3 = 0$.

Let $n = 1$, $G >> 1$, then

$V_3 = V_1$, for, S_1 and S_2 are open.

$V_3 = -V_1$, for, S_1 is closed, S_2 is open.

$V_3 = 0$, for, S_1 and S_2 are closed.

The three-value multiplier is implemented by control S_1 and S_2. The circuit is shown in Figure 4.

The orthogonal wave-form generator in fact is a Haar function generator. According to the properties of block pulse functions and Rademacher functions, from the Figure 5 and 6, we have

$$har(n, k, t) = \sqrt{2^n} \; blo(2^n, k, t) \; R(n+1,$$

It shows that the Haar functions may be obtained by the produt of block pulse functions and Rademacher functions. In practical case, the key point is how to control the switches S_1 and S_2 by use of the waveforms of har(n, k, t). For simplicity, the digital circuit is used for generating two corresponding waveforms to control the switches S_1 and S_2. The modified Haar functions are used as control waveforms. They are shown in Figure 7. The har(n, k, t)' are the control waveforms for S_1, and the har(n, k, t)" are the control waveforms for S_2.
Haar functions have different coefficient. However, the three-value multiplier is only acted as 0, +1 and -1. This problem is properly solved in adder. A model with four channels has been constructed to demostrate the principle of Haar system. Experimental results show that the scheme is reasonable.

CONCLUSION

The Haar functions are orthogonal. According to the principles of orthogonal multiplexing, it is possible to form a telemetry system based on Haar functions. System design has been completed. A laboratory model with four channels has been built. Experimental results are given to justify the design consideration.

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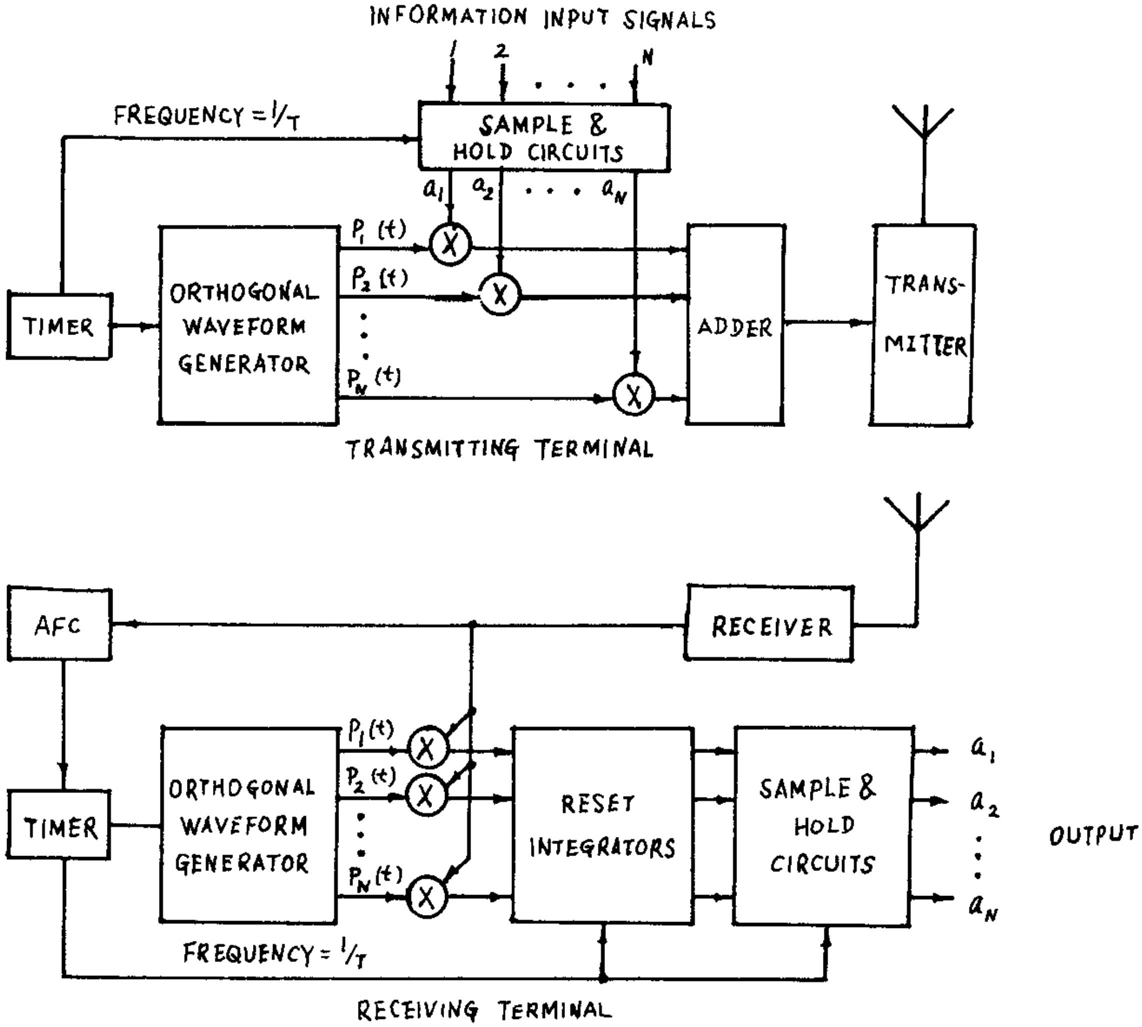

Figure 1. Optimum Multiplex System

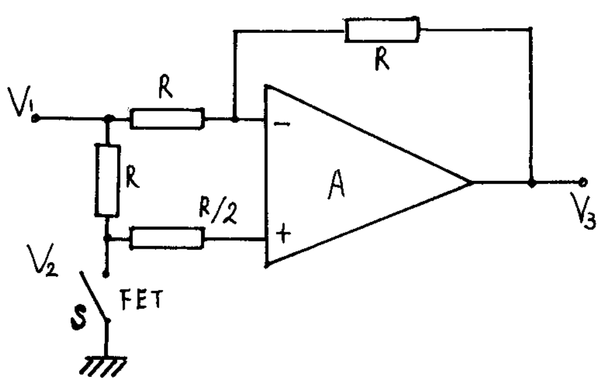

Figure 3. Two-value Multiplier

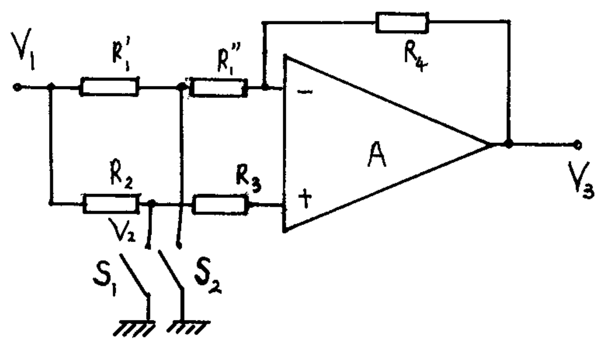

Figure 4. Three-value Multiplier

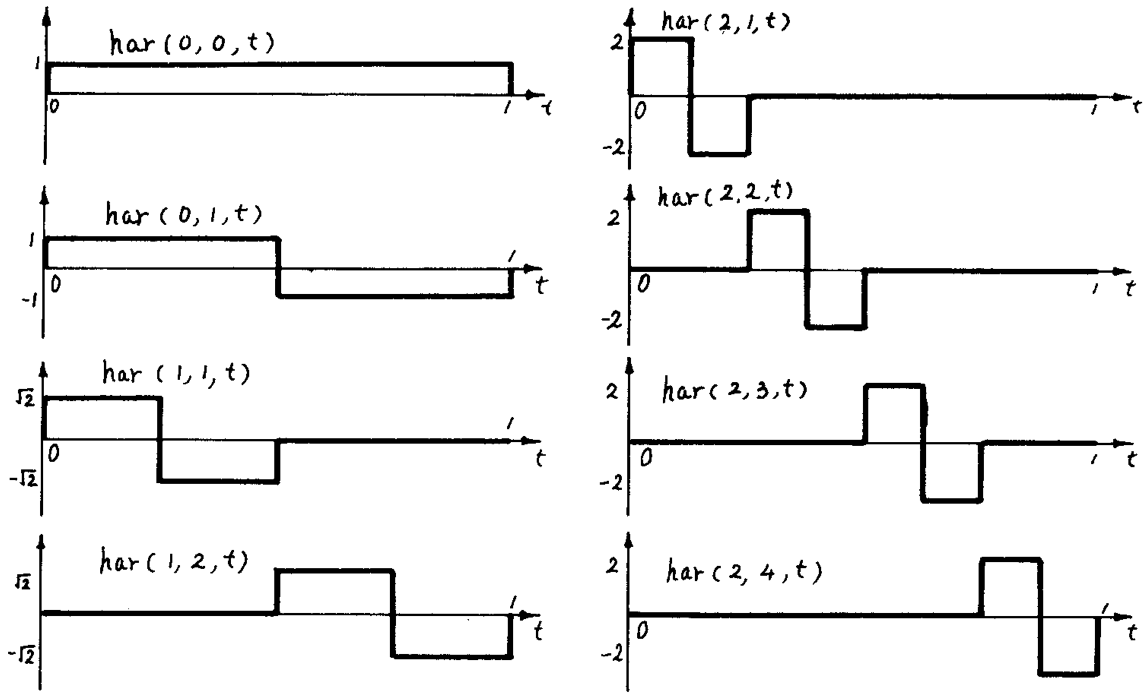

Figure 2. The first eight Haar functions

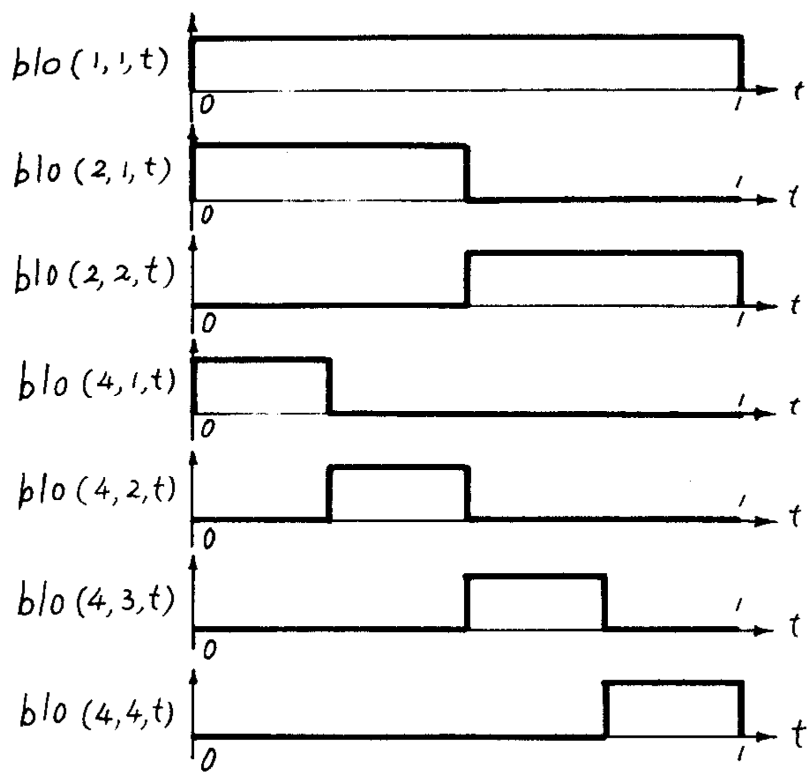

Figure 5. The block pulse functions

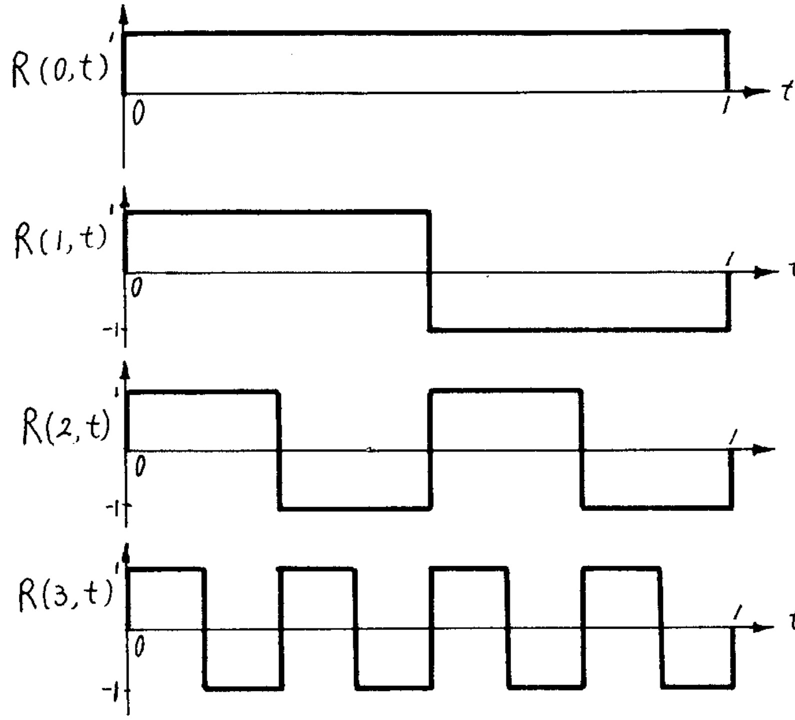

Figure 6. The first four Rademacher functions

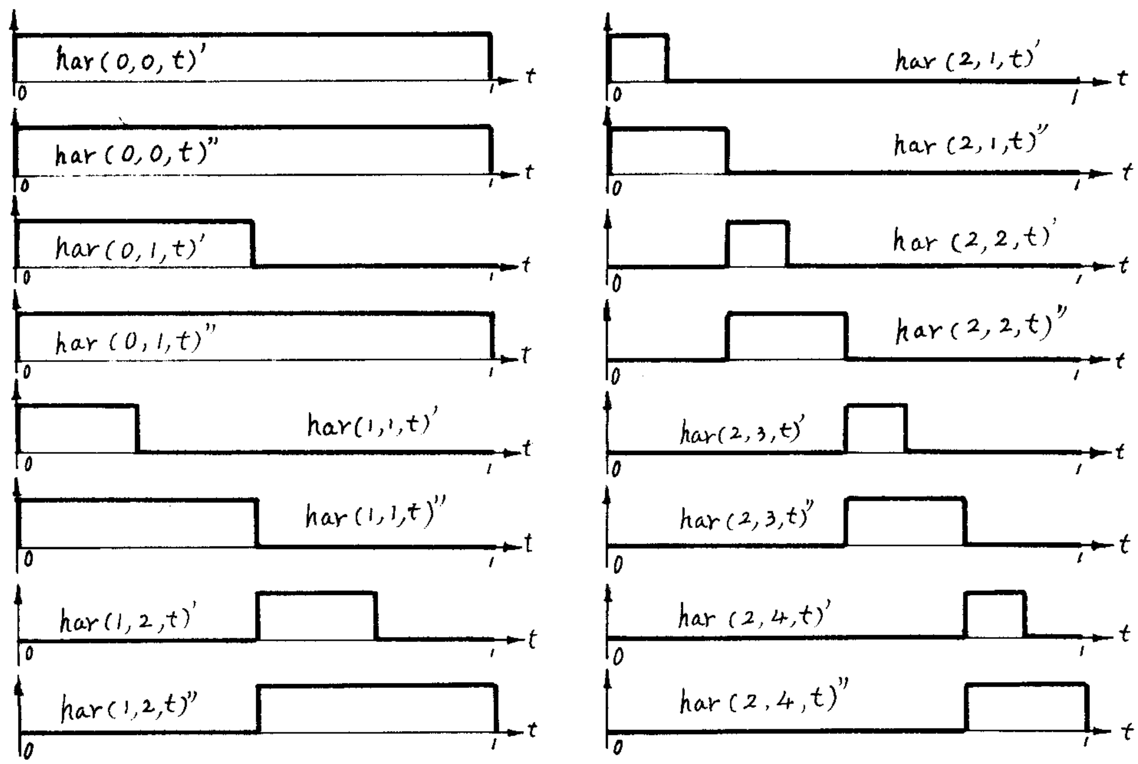

Figure 7. The modified Haar functions