CONVOLUTIONAL CODING/VITERBI DECODING OF PCM/FM

T. F. Cox and M. H. Nichols
Naval Weapons Center
China Lake, Calif

SUMMARY

Tests were run with a binary PCM/FM transmission channel using limiter/discriminator detection and (7, 1/2) convolutional coding with Viterbi decoding. In all cases, soft decision decoding showed little or no improvement relative to hard decision, evidently because of the pops in the FM demodulator. Under optimum PCM/FM conditions, namely p-p deviation = 0.7 f_s and IFBW = f_s, the coding improvement at BER = 10^{-6} was about 2.2 dB. The binary symbol rate in the transmission channel is f_s. In the course of the test, it was discovered that with PCM/FM under optimum conditions, channel errors frequently occurred in pairs. This severely reduces the value of parity bit when used with uncoded PCM/FM and also degrades the Viterbi decoder performance by about 1 dB.

INTRODUCTION

Bit error rate (BER) tests were run with a PCM/FM transmission channel in order to determine the gain using convolutional coding with Viterbi decoding. Figure 1 is a block diagram of the test setup. To establish a reference and to verify the performance of the Linkabit hardware, a BER characteristic was run using PCM/PM with ±90 deg deviation. No premodulation filtering was used and the IF bandwidth (IFBW) was four times the binary symbol rate. Good agreement with published data on coder/decoder (CODEC) was obtained.

BER TEST WITH CODED PCM/FM

The tests were repeated with binary PCM/FM with no premodulation filtering using the block diagram of Figure 1 and operating at a symbol rate $f_s = 10^6$/sec. The p-p carrier deviation was 0.7 f_s. Figure 2 shows the test results using IFBW = f_s. The abscissa is E_b/N_o, where E_b is the energy per <u>data</u> bit and N_o is the one-sided spectral noise density in the IF, in order to give a direct indication of the CODEC gain relative to the equivalent PCM/FM link operating at the data bit rate.

Note first that the improvement of soft decision over hard decision is very small. This is evidently due to pop noise in the discriminator. A pop occurs when s(t) + n(t) vector encircles the origin.[(1)] This causes a “step function” increase (or decrease) in the phase that, when differentiated by the discriminator, produces a large pulse (pop) causing a bit error when of opposite polarity. Since the pops are mostly large (not Gaussian distributed), the soft decision is not effective. Also, note that the improvement for hard decision is less by about 1 dB (see Table 1) than predicted from published CODEC data where the bit errors are assumed to be random, i.e., not correlated.

Table 1. Degradation of Hard Decision Performance
for Different IFBW Expressed in dB.

CODEC Pe	IFBW		
	f_s	1.5 f_s	2.4 f_s
10^{-3}	-0.6	. . .	. . .
10^{-4}	-0.8	-0.1	0.1
10^{-5}	-0.9	-0.2	-0.2
10^{-6}	-1.0	-0.25	-0.2
	-1.1	-0.3	-0.2

It was suggested by T. Cox that with narrow band PCM/FM bit errors are correlated. Figure 3a shows an oscillogram of PCM/FM error strobes from the BER tester with sweep synchronized on the error strobes. Note comparative brightness of the first two error strobes. Figure 3b shows the corresponding oscillograms for PCM/PM ±90 deg, with 4 MHz IF bandwidth, in which all error strobes except the first are of equal intensity. To test the bandwidth hypothesis, PCM/FM test data for IFBW = f_s, 1.5 f_s, and 2.4 f_s were obtained; and CODEC degradation is summarized in Table 1 for hard decision, there being little difference between hard and soft decision in all cases. Note that at IFBW = 2.4 f_s, the degradation is negligible and at 1.5 f_s, nearly so.

To quantify the phenomenon in Figure 3, a circuit for detecting double errors was constructed and the frequency of double errors was counted to determine the probability, P_{de}, of occurrence of double errors. Table 2 gives some preliminary results for PCM/FM. The P_e2 column gives the probability of occurrence of double errors if the errors are uncorrelated. Note that at IFBW = 4 MHz = 4 f_b, $P_{de} \cong P_e^2$. Thus, at this IFBW and greater, the errors are uncorrelated.

Table 2. Double Error Probability for PCM/FM Versus IFBW. The Bit detection was F/S, bit rate $f_b = 10_6$/sec.

IFBW, MHz	P_e	P_{de}	P_e^2
1.0	1.05×10^{-3}	5.13×10^{-5}	1.10×10^{-6}
1.0	2.25×10^{-3}	1.47×10^{-4}	5.06×10^{-6}
1.0	5.04×10^{-3}	4.55×10^{-4}	2.54×10^{-5}
1.5	1.07×10^{-3}	9.52×10^{-6}	1.14×10^{-6}
2.4	1.10×10^{-3}	2.22×10^{-6}	1.21×10^{-6}
4.0	1.10×10^{-3}	1.12×10^{-6}	1.21×10^{-6}

Preliminary double error tests were also run with PCM/PM format with phase modulation set for carrier disappearance. The results are summarized in Table 3. Note that for IFBW less than 6 MHz, i.e., less than 6 f_b, - $P_e^2 > P_{de}$. Also note that with IFBW = 2.4 MHz, the inequality is stronger for I/D detection than for F/S. The results in Table 3 are in contrast with those in Table 2 where $P_{de} > P_e^2$ for IFBW < 4 MHz = 4 f_b.

Table 3. Double Error Probability for PCM/PM Versus IFBW. The PM modulation level was set for carrier disappearance with $f_b = 10^6$/sec. The Costas loop bandwidth was 300 Hz and the Costas data bandwidth was 2 MHz.

IFBW, MHz	P_e	P_{de}	P_e^2	Mode
1.0	1.10×10^{-3}	1.00×10^{-7}	1.21×10^{-6}	F/s
1.5	1.04×10^{-3}	4.30×10^{-7}	1.08×10^{-6}	F/s
2.4	1.10×10^{-3}	6.50×10^{-7}	1.21×10^{-6}	F/s
2.4	1.07×10^{-3}	1.62×10^{-7}	1.15×10^{-6}	I/D
6.0	1.07×10^{-3}	9.50×10^{-7}	1.15×10^{-6}	I/D

PARITY BIT USED IN CONNECTION WITH UNCODED PCM/FM

By adding a parity bit to the data words, odd numbers of errors per word can be detected. Even numbers of errors cannot be detected. Let the word length including parity bit be N and assume that the errors are random, i.e., not correlated. Then the probability of a single error per word, $P_w(1)$, is

$$P_w(1) = NP_e\,(1 - P_e)^{N-1} \tag{1}$$

where P_e = bit error probability

Likewise

$$P_w(2) = \frac{N(N - 1)}{2!} P_e^2 (1 - P_e)^{N-2} \tag{2}$$

$$P_w(3) = \frac{N(N - 1)(N - 2)}{3!} P_e^3 (1 - P_e)^{N-3} \tag{3}$$

etc.

Suppose that N = 8 and $P_e = 10^{-3}$. Then

$$P_w(1) \cong 8 \times 10^{-3} \tag{4}$$

$$P_w(2) \cong 2.8 \times 10^{-5} \tag{5}$$

$$P_w(3) \cong 5.6 \times 10^{-8} \tag{6}$$

Thus a parity failure most likely (by a considerable margin) indicates a single error in the word. If a word passes parity, the probability of an undetected even number of errors is approximately 2.8×10^{-5} for this example. Note that this result depends on the assumption that the errors are independent, i.e., not correlated. Thus, in this case, it can be said that with parity, the probability of undetected word error is approximately 2.8×10^{-5}. These numbers illustrate how parity can be used as a data quality indicator and also to flag words in error for attention in the data reduction process.

In order to illustrate the effect of the error correlation found for narrow band PCM/FM, tests were run in which, by a special circuit, using 8-bit words, the number of single, double, and triple bit errors per word were logged and the corresponding probabilities calculated. The results are summarized in Table 4 and compared with the probabilities calculated for uncorrelated errors. It is evident that the correlation results in decreased effectiveness of the parity bit in detecting word errors. For example, when the IFBW = f_b, the probability of undetected word error increases from 1.1×10^{-4} to 1.2×10^{-3}, a factor of 10. With IFBW = 1.5 f_b, the increase is a factor of 4 and with IFBW = 2.4 f_b, the factor is 1.4.

Thus, to use parity effectively with uncoded PCM/FM, the IFBW should be greater than about 2.4 f_b. However, the penalty relative to optimum conditions, i.e., IFBW = f_b, is about 2.5 dB.[2]

If the probability of double errors is known, the probability of two errors per word, $P_w(2)$, can be calculated from the equation:

$$P_w(2) = \left\{ (N\text{-}1)P_{de} + \frac{(N\text{-}1)(N\text{-}2)P_e^2}{2 \quad 2} \right\} (1\text{-}P_e)^{N-2} \qquad (7)$$

Note from Table 2 that P_{de} is a function of P_e as well as of IFBW.

Table 4. Measured and Calculated Probabilities for One, Two, and Three Errors Per 8-Bit Word.

Probability of (.)/Word						
Modulation	P_e	IFBW	One error	Two errors	Three errors	Mode
PCM/FM	2×10^{-3}	1 MHz	1.30×10^{-2} 1.59×10^{-2}	1.20×10^{-3} 1.12×10^{-4}	2.20×10^{-5} 4.50×10^{-7}	Measured Calculated
PCM/FM	2.30×10^{-5}	1 MHz	1.80×10^{-4} 1.84×10^{-4}	3.90×10^{-6} 1.48×10^{-8}		Measured Calculated
PCM/FM	1.98×10^{-3}	1.5 MHz	1.43×10^{-2} 1.56×10^{-2}	4.55×10^{-4} 1.10×10^{-4}	1.40×10^{-5} 4.40×10^{-7}	Measured Calculated
PCM/FM	2.01×10^{-3}	2.4 MHz	1.59×10^{-2} 1.59×10^{-2}	1.58×10^{-4} 1.12×10^{-4}	1.02×10^{-6} 4.50×10^{-7}	Measured Calculated
BPSK	2.05×10^{-3}	4 MHz	1.61×10^{-2} 1.62×10^{-2}	1.11×10^{-4} 1.16×10^{-4}		Measured Calculated

Note: Filter/Sample bit detection was used for all measurements.

DISCUSSION

The improvement of hard decision (7, 1/2) convolutional coding/Viterbi decoding using PCM/FM with IFBW = f_b is about 2 dB at 10^{-5} BER. This compares with 5 dB(3) for BPSK for soft decision (which is not effective for PCM/FM).

Under optimum PCM/FM conditions, correlation between bit errors in PCM/FM causes about 1 dB degradation of the Viterbi hard decision performance relative to that for uncorrelated errors. This degradation is nearly removed when IFBW = 2.4 f_b, but this wider IFBW degrades the PCM/FM channel performance by about 2.5 dB.(2) Interleaving can be used to reduce the effects of correlated errors so that in the limit, 3 dB CODEC improvement could be obtained with optimum PCM/FM.(3) However, with rate 1/2 coding, spectrum occupancy would be doubled.

The correlation between adjacent bit errors increases the probability of undetected word errors when using the parity bit relative to the probability of undetected word errors using the parity bit with uncorrelated bit errors. The IFBW = 2.4 f_b nearly removes the correlation but costs about 2.5 dB on BER performance relative to IFBW = f_b.[(2)] These remarks about parity bit are relative to uncoded PCM/FM.

The causes for the correlation of adjacent errors in PCM/FM and PCM/PM reported in Tables 2 and 3 were not investigated in this work.

EQUIPMENT LIST FOR FIGURE 1

Linkabit CODEC, Model LV 7017A
Microdyne Signal Generator Model 7100SL-(A)
Microdyne Receiver Model 1200MR
EMR BER Test Set Model 721
DSI BSSC Model 7700
DSI BER Test Set Model 7191

REFERENCES

(1) Rice, S. O., 1963. "Noise in FM Receivers," *Proceedings of the Symposium on Time Series Analysis*, p. 395. John Wiley and Sons.

(2) Hayes, J. J., et al, 1968. "Wideband PCM/FM Bit Error Probability Using Discriminator Detection," *ITC 1968 Proceedings*, p. 233.

(3) Simon, M. K. "The Impact of Mismatch on the Performance of Coded Narrow-Band FM With Limiter/ Discriminator Detection," *IEEE Vol COM-31* #1, Jan 1983.

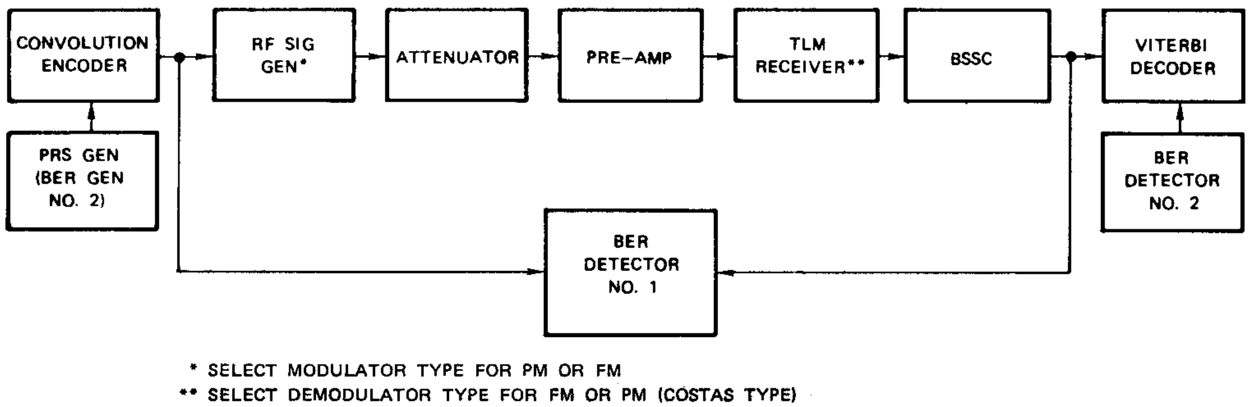

Figure 1. Block Diagram of Test Setup Including Convolutional Coding and Viterbi Decoding Function Blocks.

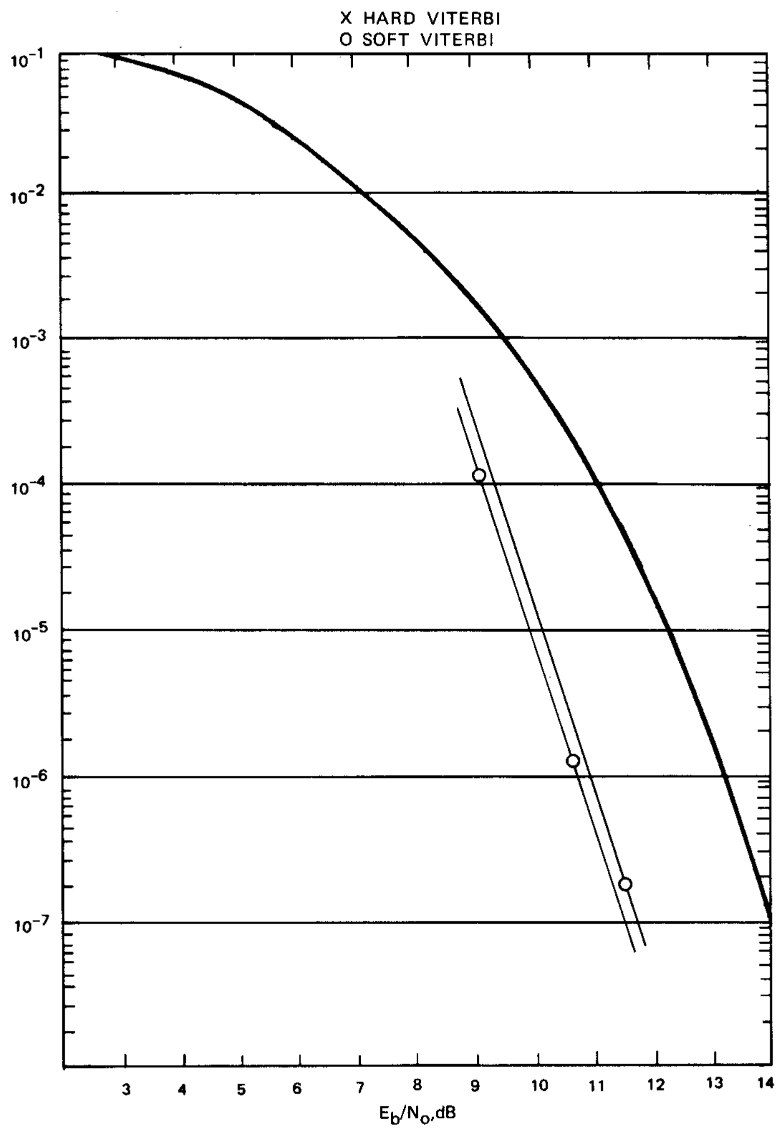

Figure 2. CODEC Performance Relative to Equivalent Uncoded PCM/FM Data Channel.

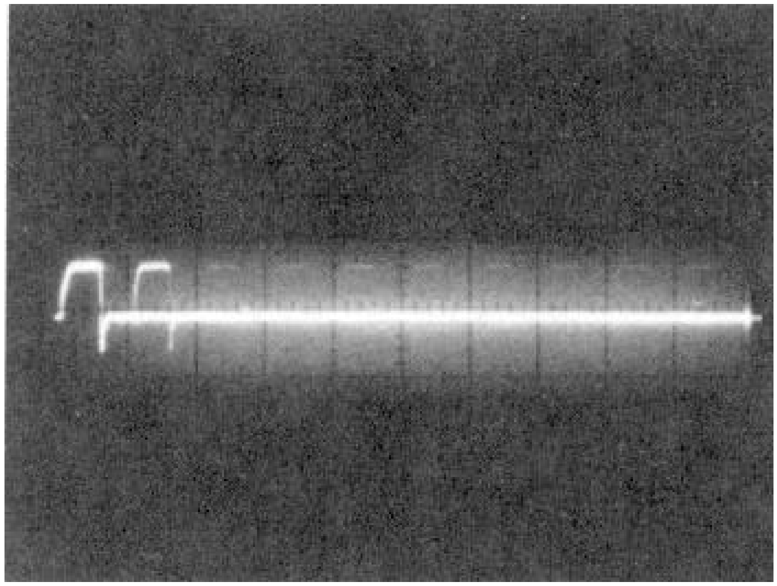

Figure 3a. Oscillogram of Bit Errors for PCM/FM With IFBW = f_b and f_b = 1 Mb/s.

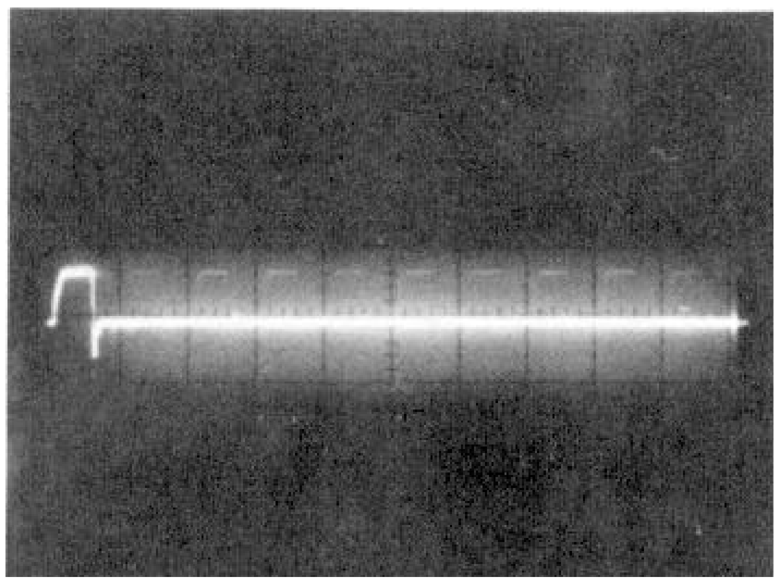

Figure 3b. Oscillogram of Bit Errors for PCM/FM ±90 With IFBW = $4f_b$ and f_b = 1 Mb/s.