DESIGN PARAMETERS FOR A FM/FM SYSTEM

Frank Carden
New Mexico State University

ABSTRACT

Design parameters for a FM/FM telemetry system are determined in terms of the IRIG specifications for proportional bandwidth channels. Three mathematical models used by designers of the above processes are extended and compared. That is, FM multi-tone models are used to establish the relationship between frequency deviations, modulation indices, signal-to-noise and IF bandwidth for the IRIG channels. Since spectral efficiency and signal quality are of major importance, a goal of the design is to have a minimum IF bandwidth, while fixing as large as possible the values of the modulation indices for the subcarriers modulating the carrier in order to achieve as large as needed output signal-to-noise ratio.

INTRODUCTION

Since the noise spectral density after the carrier demodulator is parabolic, it is necessary to vary system parameters such that the signal quality in each individual channel is the same. In these channels the frequency deviation of the subcarrier oscillator by the signal is a constant percentage of the frequency of the subcarrier and the deviation ratio is nominally set to 5. Since the only remaining unspecified parameters are the frequency deviation of the carrier by the individual subcarriers they must be used in order to achieve equal signal to noise ratios in all the channels. The frequency deviations by the individual subcarriers on the carrier is related to maximum specified peak frequency deviation of the carrier. The FM/FM spectrum is complex and is basically multi-tone. Three mathematical models used by designers of the above processes are extended and compared. That is, FM multi-tone models are used to establish the relationship between frequency deviations, modulation indices, signal-to-noise ratios and IF bandwidth for the IRIG channels. These models are then used to determine parameters such that specifications are met.

The output signal-to-noise ratio, numerically, in the i th subcarrier channels is given by

$$[S/N]oi = [3/4]^{1/2}[Bc/fmi]^{1/2} \; [fdci/fsi][fdsi/fmi] \; [S/N]_C \qquad [1]$$

$$= [3/4]^{1/2}[Bc/fmi]^{1/2} \; Dci \; Dsi \; [S/N]_C \qquad [2]$$

where

Bc= Carrier IF Bandwidth
fmi= maximum information frequency in the i th channel,
fdci=deviation of the carrier by the i th subcarrier,
fsi=frequency of the i th subcarrier,
fdsi=deviation of the i th subcarrier by the message,
Dci=modulation index of the carrier and the i th subcarrier,
Dsi=modulation index of the i th subcarrier and the message.
$[S/N]_C$= carrier-to-noise ratio in the carrier IF

A bandwidth equation, known as Carson's rule, which predicts the necessary IF bandwidth is given by

$$Bc=2(fd+fm), \qquad [3]$$

where
fd=peak frequency deviation of the carrier by the message
fm=maximum frequency of the message.

For single tone modulation and modulation indices greater than one it is known that the Bc predicted by this equation contains all but one percent of the sideband power. However for multi-tone modulation it has been found that a better prediction for the necessary bandwidth is given by

$$Bc=2(fdn+fsh) \qquad [4]$$

where

fdn=RMS of the deviations of the carrier by each subcarrier

$$fdn=[\,fdc1^2+fdc2^2+\;+\;+\;fdcn^2]^{1/2}. \qquad [5]$$

fsh= highest frequency subcarrier.

It is instructive to look at the maximum peak deviation, say fdp, the carrier could have if all the individual peak subcarrier deviations occurred at the same time although this is an extremely low probability event.

This peak value is given by

$$fdp = fdc1 + fdc2 + fdc3 + + + fdcn \qquad [6]$$

This worst case deviation will be determined and compared for the three design procedures.

It is also instructive to look at the spectrum of fm/f m for the simple case of only two subcarriers. For two-tone fm [1] the carrier spectrum is given by

$$c(t) = A_c \Sigma \Sigma J_n(\beta_1) J_m(\beta_2) \mathrm{Cos}(\omega_c + n\omega_1 + m\omega_2), \qquad [7]$$

where

β_1=modulation index of the first tone
β_2=modulation index of the second tone
and n and n are summed from −infinity to +infinity.

A simple but illustrative case occurs for small modulation indices and when $\omega_1 >> \omega_2$. The resulting spectrum is shown in figure 1. The higher frequency establishes sidebands as in single tone modulation. The lower frequency creates sidebands around the carrier and the higher frequency sidebands with the sum and difference frequencies. Each higher frequency sideband appears to be a tone with fm modulation. The net result is that the bandwidth of the two tone fm signal is determined primarily by the higher frequency. Although this is for a simple case of only two tones, it will be seen from the models for the general fm/fm case that the higher subcarriers basically set the required bandwidth.

DESIGN PROCEDURES

I. The first design procedure [2] considered, uses equation [1] and specifies the desired signal-to-noise ratio of the individual subcarrier channels, then solves for the deviation of the carrier by each subcarrier in order to achieve the desired signal-to-noise ratio. In order to use equation [1] using this procedure, the IF bandwidth must be established by some rule of thumb. Rearranging equation [1] such that fdci, the deviation of the carrier by the i^{th} subcarrier is given explicitly,

$$fdci = [4/3]^{1/2} [S/N]oi [S/N]c [1/Bc]^{1/2} (fmi)^{3/2} fsi(1/fdsi). \qquad [8]$$

Setting the carrier-to-noise to 12 db and inserting factors for filter attenuation which results in an effective subcarrier channel output signal-to-noise ratio of 40 db (ibid) gives

$$fdci=40\,(fmi)^{3/2}\,fsi\,(1/fdsi)(1/Bc)^{1/2} \quad [9]$$

Equation [9] (ibid) will be used to design an fm/fm system by computing fdci for fifteen proportional bandwidth subcarriers from fsi=93KHz down to 1.3kHz. Bc is set equal to 500 KHz, one of the available IF bandwidths. On the first iteration, fdci for the highest subcarrier is found to be 40KHz which will not utilize the chosen IF bandwidth therefore a factor of 2 is inserted into equation [9] giving

$$fdci=2(40)\,(fmi)^{3/2}\,fsi\,(1/fdsi)(1/Bc)^{1/2} \quad [10]$$

Using equation [10] to compute the deviation of the carrier by the 93KHz subcarrier gives

$$fdc93=2(40)(1.395)^{3/2}\,(93)(1/6.975)(500)^{1/2} \quad [11]$$
$$=80.2\text{KHz}.$$

Table 1 lists the deviation of the carrier by the fifteen individual subcarriers computed from equation [10]. The second column lists the deviation calculated from equation [10], while the third column list the deviation of the carrier by the subcarriers modified such that all subcarriers will deviate the carrier by at least 10% of the anticipated total deviation.

The rule of thumb used to insert the factor of two into equation [9] leading to equation [10] is that the RMS deviation of the total signal will be 1.3 times the deviation due to the highest subcarrier and that this deviation due to the total signal should be about 1/6 of the IF bandwidth (ibid).

Using equation [4] to compute the fdn or the norm for this case and using the deviations before being modified for the 10% requirement gives

$$fdn(exact)=\ 105.76\ \text{KHz} \quad [12]$$

Computing fdn after increasing the deviation of the lower subcarriers gives

$$fdn(mod)=109.9 \quad [13]$$

Computing fdn for only the highest five subcarriers gives

$$fdn(high5)=105.17 \quad [14]$$

The predicted IF bandwidth based upon the RMS of the individual deviations is given by equation [3] and is

$$Bc=2(fdn+fsh)=2(109.9+93)=405.8KHz. \quad [15]$$

The increase in required bandwidth is marginal after increasing the deviations of the lower frequency subcarriers. Further, using only the highest five subcarriers to predict the bandwidth is a good first cut and supports the multitone model developed for only two tones that suggests the bandwidth is predominantly determined by the higher frequency tones.

The peak carrier deviation, fdp, is

fdp=310 KHz.

II. The second design procedure also employs equation [1] but does not assume an IF bandwidth [3]. Solving equation [1] for fdci gives

$$fdci= [4/3]^{1/2}[S/N]oi[S/N]c[1/Bc]^{1/2}\,(fmi)^{3/2}\,fsi(1/fdsi). \quad [16]$$

Bc is factored out of equation [16] since it is unknown and a preliminary fdcpi is solved for each subcarrier modulating the carrier. This preliminary deviation is given by

$$fdcpi=[4/3]^{1/2}[S/N]oi[S/N]c\,(fmi)^{3/2}\,fsi(1/fdsi). \quad [17]$$

Using equation [5] and letting fdcl be the deviation of the carrier by the highest frequency subcarrier and fdcn be the deviation by the lowest and normalizing with respect to fdcl gives

$$fdn=fdc1[\ 1+(fdc2/fdc1)^2+\ (fdc2/fdc)^2++++(fdcn/fdc1)^2]^{1/2} \quad [18]$$

$$=fdc1[1+A_2{}^2+\ A_3{}^2++++A_n{}^2] \quad [19]$$

$$=fdc1\ [A] \quad [20]$$

where

$$A_i=fdci/fdc1 \quad [21]$$

Note the relationship between fdci and fdcpi is

$$fdci = fdcpi/(Bc)^{1/2} \quad [22]$$

or

$$A_i = fdci/fdc1 = fdcpi/fdcp1 \quad [23]$$

That is by substituting the preliminary fdcpi into the expression for A and noting the Bc term divides out in each term which allows each A_i and hence A to be calculated knowing only the fdcpi's. Further, this allows the calculation of Bc in terms of fdcl. Using equation [3] and [22] gives

$$Bc = 2(fdc1\,A + fs1) \quad [24]$$

Using equation [23] and specifying [S/N]c, fml, fdsl, and fsl allows fdcl to be solved for. Since each A_i is known, multiplying each by fdcl gives fdci the necessary deviation of the carrier by the i^{th} subcarrier to achieve the specified and uniform signal-to-noise out. Also knowing fdcl allows the required IF bandwidth to be calculate using equation [23].

A design process for the fifteen PBW used in process I was completed with a specified [S/N]c=12db and [S/N]oi = 46 db. The calculated deviations of the carrier by the individual subcarriers are shown in table 1 column 4.

The RMS deviation and fdp are given by

$$fdn = 88 \text{ kHz.} \quad [25]$$

$$fdp = 260\text{KHz} \quad [26]$$

III. The third method [4] does not use equa [1]. In proportional bandwidth fm/fm the peak subcarrier frequency deviation, fdsi by the ith subcarrier is set as a constant percentage of the subcarrier frequency f_i. The subcarrier deviation ratio, D_{si}, is also set as a constant. Therefore, the message bandwidth, f_m, capability is proportional f_i. That is

$$fdsi = Pf_i \quad \text{(For the IRIG 7.5 \% channels, P=.075)} \quad [27]$$

$$Dsi = \text{constant (nominally 5 for IRIG channels)} \quad [28]$$

$$f_{mi} = (P/D_{si})f_i \quad \text{(for the 7.5\% IRIG } P/D_m = .075/5 = .015) \quad [29]$$

In order for the signal-to-noise ratios to be equal for all of the subcarrier channels

$$f^2_{dci} = Cf_i^3. \qquad [30]$$

The peak carrier deviation is equal to or less than the sum of the individual subcarrier deviations and is given by

$$fdp \le \Sigma\, fdsi \qquad [31]$$

The equality only holds whenever all the subcarriers are in alignment, an event of low probability. Further constraints and relations are,

$$fsi = fsi-1 + \frac{B_i + B_{i-1}}{2}, \qquad [32]$$

$$B_i = 2(fdsi + f_{mi}). \qquad [33]$$

where

Bi=bandpass filter bandwidth in the ith subcarrier channel.

In order to determine the deviation of the carrier by the individual subcarriers, equations [27], [28], [29], [30], [31], [32], and [33] are solved giving

$$fdci = fdp[(.83+P)/(.83-P)]^{3(i-1)/2} \; \frac{\dfrac{(.83+P)^{3/2}}{(.83-P)^{3/2}} - 1}{\dfrac{(.83+P)^{3N/2}}{(.83-P)^{3N/2}} - 1} \qquad [34]$$

In the IRIG proportional bandwidth channels P=.075 or .15 or .3.

Using equation [34] for the .075 channels gives values as shown in Table 1 for N=15. Column 5 shows fdci in terms of a specified fdp.

Column 5 was generated assuming the carrier peak frequency deviation was specified. Observing both fdn and fdp from procedure I and II it is seen that their relationship is approximately fdp = 3fdn.

This procedure does not specify Bc therefore assume Bc = 500Khz. Using the rule of thumb that fdn = Bc/6 and fdp = $3f_{dn}$ gives fdp = (500/6) 3 = 250KHz. Using this value of fdp, column 6 shows the resulting fdci's. The actual calculated fdn and fdp from the deviations are fdn = 96 KHz (assumed 83) and fdp = 253K Hz (assumed 250).

Summary

All three procedures produce similar subcarrier deviations of the carrier. Estimates of fdn are comparable. Peak deviation of the carrier exceeds the RMS deviation by a factor of 3 on the first two procedures and is so constrained in the latter.

In all procedures the deviation by the lower frequency subcarriers should be increased to 10% of fdn which will have a very small effect on Bc. All three are sound approaches, but the first two seem to furnish more insight into the system parameters.

References

[1] Giacoletta, L.J., "Generalized Theory of Multitone Amplitude and Frequency Modulation," Proc. IRE, Vol. 35, July 1947.

[2] Law, E.L. and D.R. Hust, "Telemetry Applications Handbook", Tech. Pub. TP000044, Sept. 1987, Pacific Missile Test Center, Pt Mugu, CA 93042

[3] Rosen, Charles, "System Transmission Parameters Design for Threshold Performance", McCrocom Corp., 1983, ITC Proceedings, Vol. XIX.

[4] Downing, J.J. (1964) "Modulation Systems and Noise", Prentice-Hall, Englewood Cliffs, N.J.

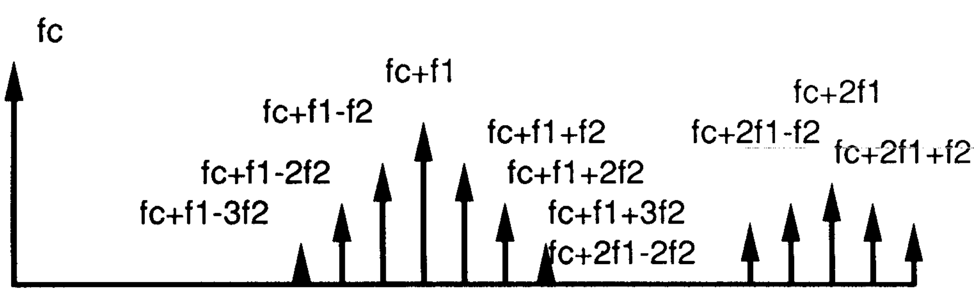

Figure 1
Two Tone FM

Table 1 (KHz)

fsi	fdci	fdci(m)	(fdci)	(fraction of total fdp)	fdci
	I		II	III	
1	2	3	4	5	6
93	80.2	80.2	66.80	(.242)	64.5
70	52.4	52.4	43.62	(.184)	46.0
52.5	34.0	34.0	28.37	(.140)	35.2
40.	22.6	22.6	18.84	(.107)	26.7
30.	14.7	14.7	12.23	(.081)	20.2
22	9.22	10.6	7.68	(.062)	15.5
14.5	4.96	10.6	4.13	(.047)	11.8
10.5	3.06	10.6	2.55	(.036)	9.0
7.35	1.78	10.6	1.48	(.027)	6.8
5.4	1.12	10.6	.934	(.020)	5.25
3.9	.695	10.6	.576	(.015)	4.0
3.0	.465	10.6	.387	(.012)	3.0
2.3	.314	10.6	.267	(.009)	2.3
1.7	.196	10.6	.163	(.007)	1.77
1.3	.134	10.6	.110	(.005)	1.35