THE EFFECT OF PREMODULATION FILTERS ON BIT ERROR RATE PERFORMANCE

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This paper examines the effect of different types of premodulation filters on the time waveshapes of PCM signals. Using a simplified model of this effect, an expression for the Probability of Error in the presence of Gausian Noise is derived and compared for systems with and without premodulation filtering. A simple single bit decision feedback detector is designed and an evaluation made of its usefulness in improving bit error rate performance using different filters in the presence of different amounts of noise.

WHY USE A PREMOD FILTER?

Premodulation filtering is used to reduce unwanted frequencies that result when a carrier is modulated by a square-sided PCM signal. By prefiltering the data, the modulating signal is rounded thereby reducing the energy generated outside the passband of the channel. This prefiltering produces “intersymbol interference” where the energy of previous bits affects future bits upon detection. Not only is the signal energy reduced, but the degree of reduction is highly dependent on the bit pattern.

The configuration shown in fig. 1A was used to examine the effect of prefiltering on different bit patterns. The resulting waveforms with and without a premodulation filter are shown. To a good approximation, the waveforms demonstrate that only adjacent bits interfere with each other. There are essentially two magnitudes to the peaks. If the adjacent bits were similar, the magnitude of the signal is what it would have been without prefiltering. If the adjacent bits are different, there is a reduced threshold for the bit decision.

WHAT IS THE ERROR RATE?

An NRZ-L PCM signal is detected by sampling the filtered waveform at its peaks using a synchronous clock usually extracted from the signal. This peak value is compared to a threshold to determine if it was a “1” or a “0”. If Gaussian noise is superimposed on the signal at its peak, then an error is expected whenever the noise makes it appear that the signal was was on the opposite side of the threshold. The probability of error with a given threshold is the shaded area under the curve in fig. 2. This probability is expressed in the famous definite integral:

$$P_E(V_T) = \frac{-1}{\sigma\sqrt{2\pi}} \int_{V_T}^{\infty} e^{\frac{v^2}{2\sigma^2}}\, dV = \tfrac{1}{2}\ \mathrm{Erfc}\left(\frac{V_T}{\sigma\sqrt{2}}\right)$$

Where V_T is the threshold level and σ is the r.m.s. noise level. Hence $V_T/\sigma\sqrt{2}$ is the signal to noise ration.

When the signal is prefiltered, half the peaks are reduced. This is enumerated in fig. 3, where it has been assumed that a “1” and a “0” are equally likely. Let v be the average threshold and let ΔV be the magnitude that the actual peaks are above and below this threshold. Clearly ΔV depends on the nature of prefiltering. For the moment it is assumed that it has been characterized. An expression for the average probability of error for the prefiltered signal is obtained by considering the probability for each threshold multiplied by its frequency of occurrence and summing. This is shown in fig. 3.

WHY BIT DECISION FEEDBACK?

The nature of the complementary error function is that it obeys the following inequality:

$\mathrm{Erfc}(X + \Delta X) + \mathrm{Erfc}(X - \Delta X) > 2\,\mathrm{Erfc}(X)$; $v\ \forall\ \Delta X > 0$

This suggests that it would have been better had all the peaks been reduced to the average rather than some above and others below. This is the motivation of bit decision feedback in which the previous bit decision is used to change the threshold level. Specificaltly consider the following algorithm: If the previous bit was a “1” then the threshold is shifted to +ΔV. If the previous bit was a “0” then the threshold is shifted to -ΔV. What is the resulting probability of error? To answer this question, it is necessary to enumerate all possible combinations of two bits and see what the resulting thresholds are in each case. Then the probability of error is determined for each threshold and weighted by its relative occurrence. This is done in fig. 4. Note that the expected threshold differs depending on whether the previous bit was detected correctly. Since the future probability depends on the past, this suggests a recurrence relation. Let P_B be the probability of an error in detecting a bit. Then $(1-P_B)$ is the probability that there was no error in detecting the bit. These definitions produce the results shown in fig. 4.

IS THE PERFORMANCE IMPROVEMENT WORTH IT?

It is instructive to quantify the theoretical probability of error for three cases assuming an apriori signal to noise ratio. These results are shown in fig. 5. Fig. 6 tabulates the change in peak signal level using various Bessel and Butterworth filters to prefilter the signal.

As can be seen, the simple bit decision feedback can dramatically improve performance. It must be cautioned, however, that only the error due to changes in signal amplitude was considered here. The issue of how prefiltering affects the extraction of sync and hence the clock are beyond the scope of this paper. The above analysis assumed the existence of a perfect clock enabling the sampling of the bits at the peaks. Note also that while performance was greatly enhanced by using bit decision feedback, it is still an order of magnitude from the non-prefiltered error rate.

WHAT IF BIT DECISION FEEDBACK IS USED ON NON-PREFILTERED SIGNALS?

It is instructive to consider the effect of bit decision feedback on non-prefiltered signals. To enumerate the possible thresholds and their frequency of occurrence, note that regardless of whether the previous bit was detected correctly, the threshold will be either $V + \Delta V$ or $V - \Delta V$. This is exactly the same result we would obtain had the signal been prefiltered in a manner to obtain that ΔV without bit decision feedback having been used. Thus it can be concluded that shifting the threshold too much is no more detrimental than not shifting it enough. The optimal threshold shift is one matched to the characteristic of the premodulation filter as shown in the table in fig. 6.

A more useful result is obtained by the use of a more general inequality:

$$\mathrm{Erfc}\,(X + E) + \mathrm{Erfc}\,(X - \epsilon) > \mathrm{Erfc}\,(X + \delta) + \mathrm{Erfc}(X - \delta)\ ;\ \forall |\epsilon| > |\delta|$$

This statement implies that any change in the threshold that makes the magnitude of the peaks appear closer to their average magnitude reduces the theoretical error rate. Specifically any threshold shift less than twice the optimum (fig. 6.) will improve bit error rate performance of prefiltered data.

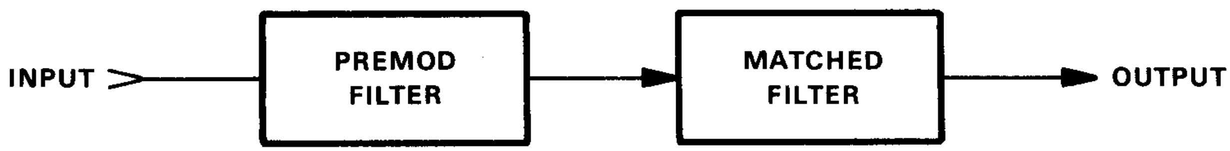


Figure 1A. Test Configuration

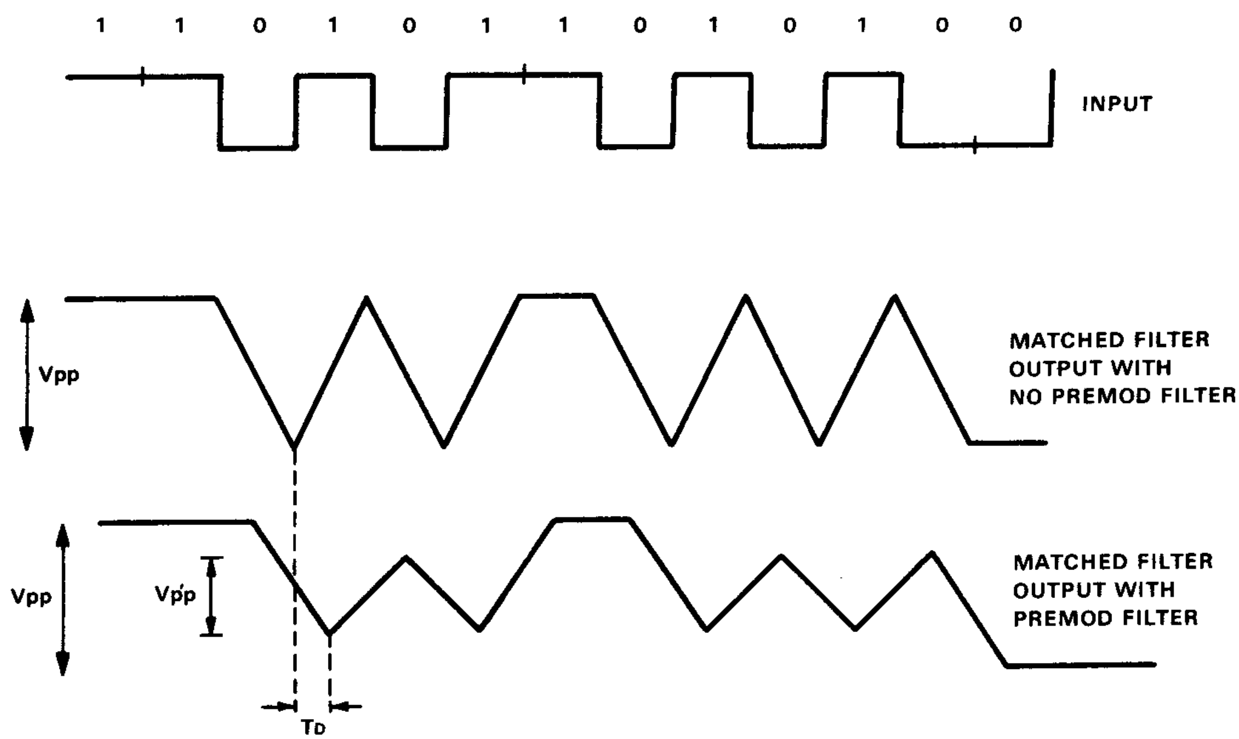
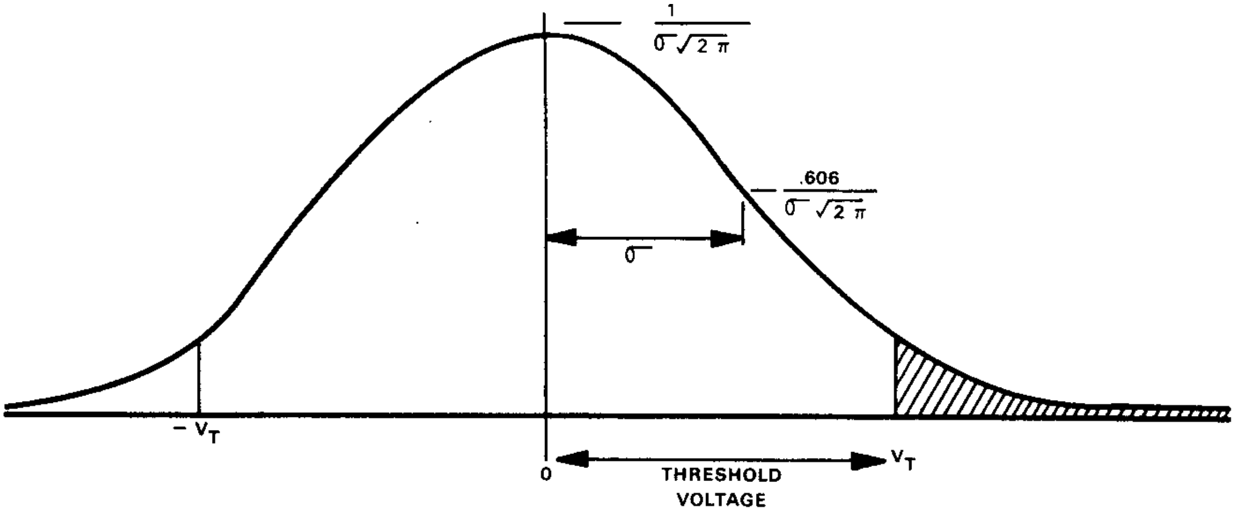


Figure 1B.



$$P_E(V_T) = \frac{-1}{\sigma\sqrt{2\pi}} \int_{V_T}^{\infty} e^{\frac{V^2}{2\sigma^2}} \, dV$$

LET $u^2 = \frac{V^2}{2\sigma^2} \Rightarrow dv = \sqrt{2}\,\sigma\, du$

$$\Rightarrow P_E(V_T) = \frac{-1}{\sqrt{\pi}} \int_{\frac{V_T}{\sigma\sqrt{2}}}^{\infty} e^{-\mu^2} \, du = \tfrac{1}{2}\, \mathrm{Erfc}\left(\frac{V_T}{\sigma\sqrt{2}}\right)$$

WHERE (WITH RESPECT TO FIG 1B):

$V_T = \tfrac{1}{2}\, Vpp$

$$\text{AND } \mathrm{Erfc}(x) = 1 - \mathrm{Erf}(x) \cong \frac{e^{-x^2}}{x\sqrt{\pi}} \left[1 - \frac{1}{2x^2} + \cdots + (-1)^n \frac{(1 \cdot 3 \cdot 5 \cdot \ldots (2n-1))}{(2x^2)^n}\right]$$

(HERE $n < \infty$ BECAUSE THE SERIES DOES NOT CONVERGE)

Figure 2.
Calculating the probability of error in making a bit decision with gaussian noise added to signal.

REFERRING TO FIG 1B :

$$\text{LET } V = \tfrac{1}{2}\left(\frac{V_{pp} + V_{p'p}}{2}\right) \quad \text{AND} \quad \Delta V = \tfrac{1}{2}\left(\frac{V_{pp} - V_{p'p}}{2}\right)$$

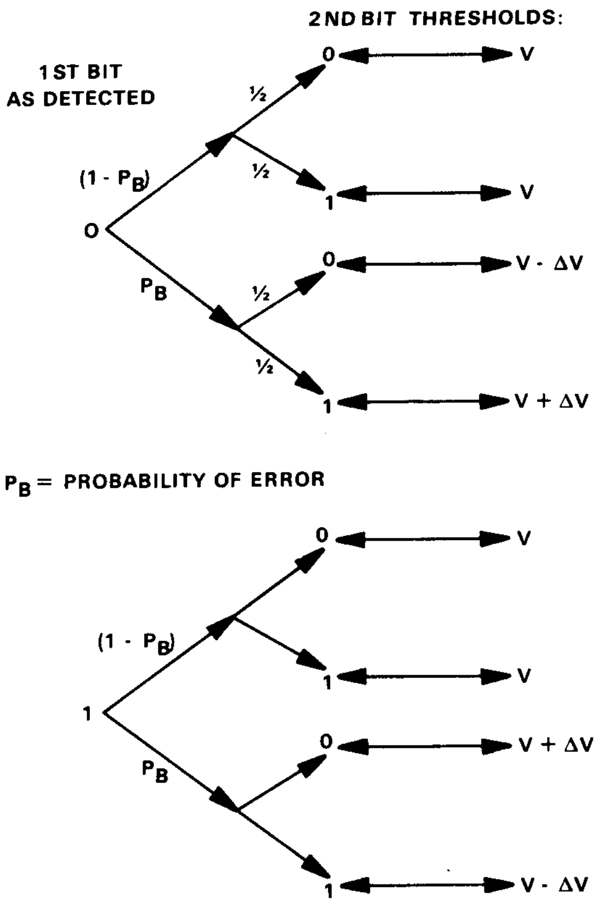
2-BIT PATTERNS	00 11	01 10
RESULTING THRESHOLD MAGNITUDE	$V + \Delta V$	$V - \Delta V$

THEORETICAL ERROR PROBABILITY FOR PREFILTERED SIGNAL

$$P_F = \tfrac{1}{2}\, P_E(V + \Delta V) + \tfrac{1}{2}\, P_E(V - \Delta V)$$

$$\therefore\ P_F = \tfrac{1}{4}\,\mathrm{Erfc}\left(\frac{V + \Delta V}{\sigma\sqrt{2}}\right) + \tfrac{1}{4}\,\mathrm{Erfc}\left(\frac{V - \Delta V}{\sigma\sqrt{2}}\right)$$

Figure 3.
Enumeration of 2-bit patterns and resulting threshold voltages assuming all two bit patterns occur with equal probability.



IF PREVIOUS BIT IS DETECTED CORRECTLY, PROBABILITY OF ERROR FOR DETECTING NEXT BIT IS $P_E(V)$

IF PREVIOUS BIT IS DETECTED INCORRECTLY, PROBABILITY OF ERROR IS

$$\tfrac{1}{2} P_E(V + \Delta V) + \tfrac{1}{2} P_E(V - \Delta V) = P_F(V)$$

$$\therefore \quad P_B = (1 - P_B)\, P_E(V) + P_B\, P_F(V)$$

$$\Rightarrow P_B = \frac{P_E(V)}{1 + P_E(V) - P_F(V)}$$

$$\approx P_E(V)$$

SINCE $P_E(V) - P_F(V) \ll 1$

UNDER NORMAL CIRCUMSTANCES

Figure 4.
Enumerating thresholds for prefiltered signal using bit decision feedback.

$$V_T = 1\,V; \quad \frac{V_T}{\sigma\sqrt{2}} = 4; \quad V + \Delta V = V_T; \quad \frac{\Delta V}{V_T} = 0.1; \quad \frac{\Delta V}{\sigma\sqrt{2}} = 0.4$$

$$P_E(V_T) = \tfrac{1}{2}\,\mathrm{Erfc}(4) \cong 7.7 \times 10^{-9}$$

$$P_F(V) = \tfrac{1}{4}\,\mathrm{Erfc}(4) + \tfrac{1}{4}\,\mathrm{Erfc}(3.2) \cong 1.5 \times 10^{-6}$$

$$P_B(V) = \tfrac{1}{2}\,\mathrm{Erfc}(3.6) \cong 1.8 \times 10^{-7}$$

Figure 5.
Calculation of P_E, P_F, and P_B for a 2Vp-p signal with a 4:1 signal to noise ratio and $\Delta V/V$ of 0.1.

	BESSEL	BUTTERWORTH
- 3dB @	AV/V	AV/ V
0.5 BIT RATE	0.2	0.25
0.75 BIT RATE	0.125	0.2

Figure 6.
Approximate values for ΔV/V for Besse] and Butterworth filters of orders 2-6 pole. Referring to Figure 1B, V = 1/4 (Vpp + Vpp') and ΔV = 1/4 (Vpp-Vpp').