A NEW ORTHOGONAL MULTIPLEX SYSTEM

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ABSTRACT

The basis of mathematics which can form a telemetering system is orthogonal functions. Three kinds of orthogonal functions are used up to now. First of them is sine and cosine functions. Second one is block pulse functions. The third one is walsh functions. Their corresponding systems are FDM, TDM and SDM.

There are also other orthogonal sets which can form telemetering system, such as Legendre polynomials and Hermite polynomials. Hewever. They are too complex for engineering practice.

Except these functions mentioned above, is there any other orthogonal functions which is suitable for engineering practice? In this paper we presented a new type of orthogonal functions.

Its construction is similar to Walsh functions. The amplitudes of the functions are +1, -1 and 0. In the sence that they close the gap between walsh functions and block functions, it is called Bride functions. The definition and properties are discussed in more detail here. The construction of system is also similar to that of SDM.

INTRODUCTION

The principles of orthogonal multiplexing has been first introduced by A.H. Ballord.(1) The key point of the theory is that non-interference between channels can be guaranteed if the subcarrier waveforms are chosen from an orthogonal set. The orthogonal property requires that all pairs of waveforms have an average product of zero. Thus if $P_n(t)$ and $P_m(t)$ are two subcarrier waveforms with a common repetition period T, they are orthogonal if:

$$\frac{1}{T}\int_0^T P_n(t) \quad P_m(t)\ dt=\begin{cases} 0 & n=m \\ 1 & n=m \end{cases}$$

Equation (1) also implies that each wavef orm has a mean square value of unity. This property is desirable for multiplex subcarriers since it menas that all waveforms have equal power and will be affected equally by random noise.

Sine and cosine subcarriers has been used for a long time in telemetry as well as block pulse subcarriers. They are the simplest orthogonal sets, and non-orerlapping in frequcency or in time. So they are first recognized and utilized in engineering. sine and cosine are used as subcariers in frequency division multiplexing. Block pulses are used as subcarriers in time division multplexing. A sequency division multiplex system using walsh functions has been recently introduced. [(2)] The basis of mathmatics which can form a telemetry system must be orthogonal function.

A MODEL OF ORTHOGONAL MULTIPLEXING

An optimum multiplex system has been proposed in 1962. Its diagram is shown in figure 1. It will be sumarized here only briefly. The operation of the system is as follows:

This system uses identical wareform generators at both terminals to produce the set of orthogonal subcarriers $P_1(t)$, $P_2(t)$, ... $P_N(t)$. Automatic synchronization is provided so that bath generators operate at the same repetition rate of $1/_T$CPS. once during a period T, the information input signals are sampled at the transmitter to obtain the set of amplitudes a_1, a_2, ... q_N. Each subcarrier is then amplifude-modulated by multiplication and the results summed to obtain a composite signal of the form.

$$E(t) = \sum_{1}^{N} a_n P_n(t) \qquad (2)$$

The composite signal is transmitted. At the receiver, the recovered composite signal is multiplied by each subcarrier, the products are integrated over each period T, and the final integrated values are sampled to produce outpnt signals of the form:

$$a_m = \frac{1}{T} \int_{0}^{T} E(t)\, P_m(t)\, dt \qquad (3)$$

where m=l, 2, ... N.

This result follows immediately if the subcarriers $P_m(t)$ are orthogonal. Although E(t) contains N subcarriers, all products integrate to zero except for the product of the desired subcarrier times itself. This product integrates to unity, so that the result is the amplitude of the desired signal.

LEGENDER POLYNOMIALS

According to the principles of orthogonal multiplexing, A set of polynomial waveforms, Legendre polynomials, used as subcarriers. It is said that an experimental system has been constructed to demonstrate the principle.[3]

The first five polynomial functions which satisfy the orthogonality condition within a time duration T are:

$$\begin{aligned} P_0(X) &= 1 \\ P_1(X) &= X \\ P_2(X) &= \frac{1}{2}(3X^2-1) \\ P_3(X) &= \frac{1}{2}(5X^3-3X) \\ P_4(X) &= \frac{1}{8}(35X^4-30X^2+3) \end{aligned} \qquad (4)$$

where X is a normalized independent variable defined as:

$$X = \frac{2}{T}\left(t - \frac{T}{2}\right) \qquad \text{for} \qquad 0 \leqslant t \leqslant T$$

Figure 2 illustrates the form of the first five orthogonal polynomials.

Although it is a new method of multiplex, it is too complex for engineering practice.

MODIFIED HERMITE POLYNOMIALS

An orthogonal multiplexed communications system using modified Hermite polynomials was introduced. [4] The first six modified Hermite polynomials are:

$$H_0(t) = \frac{1}{\pi^{1/4}\ a^{1/2}}\ e^{-t2/2\ a^2}$$

$$H_1(t) = \frac{2^{1/2}\ t}{\pi^{1/4}\ a^{3/2}}\ e^{-t2/2\ a^2}$$

$$H_2(t) = \frac{1}{\pi^{1/4}\, 2^{1/2}\, a^{5/2}} (2t^2 - a^2) e^{-t^2/2a^2}$$

$$H_3(t) = \frac{2^{1/2}}{\pi^{1/4}\, 6^{1/2}\, a^{7/2}} (2t^3 - 3a^2 t) e^{-t^2/2a^2}$$

$$H_4(t) = \frac{1}{2\pi^{1/4}\, 6^{1/2}\, a^{9/2}} (4t^4 - 12a^2 t^2 + 3a^4) e^{-t^2/2a^2}$$

$$H_5(t) = \frac{1}{2\pi^{1/4}\, 15^{1/2}\, a^{11/2}} (4t^5 - 20a^2 t^3 + 15a^4 t) e^{-t^2/2a^2}$$

(5)

It is still too complex for engineering practice.

AN INTRODUCTION TO BRIDGE FUNCTION

When symmetric copying of Walsh function and the shift mode of block pulse is combined, the construction of the bridge function is obtained. The method is as follows:

(1) If i represents ith number of bridge function, it can be expressed in binary code $i_{p-1}\ i_{p-2} \ldots i_{j-1} \ldots i_1\ i_0$

(2) The P binary code is divided into two parts:
a) the j binary code at the right side $i_{j-1}\ i_{j-1} \ldots i_1\ i_0$ is used as shift information.
b) the (P-j) binary code at the left side is used as copy information.

(3) Sequence shift is done at first, then sequence copy is done.
A) The value of the original sequence is always +1 in the 0 interval, (+ in short) in other (L-1) intervals the values are zero, where $L=2^j$.
b) According to the shift information $i_{j-1} \ldots i_0$, the original sequence "+" is shifted to the right side.
c) The symmetric copying mode is used, according to the information $i_{p-1} \ldots i_j$, the copy is done one by one. At the first time, i_{p-1} is taken as the information, then, i_{p-2} is taken as the information, and so on, until it is taken (P-j) times.

The process of forming bridge function is shown in table, where i=0... 15, for shift number j=1.

MATHEMATICAL EXPRESSION OF BREDGE FUNCTION

We observe that in the case of trignometric, exponential, and logarithmic functions, many of notations accepted as “standard” consist of three letters. Examples of these include sin, cos, exp, erf, and log. Thus it is reasonable to introduce a similar scheme to denote nonsinusoidal functions. In addition, it is necessary to distinguish continuous functions from the corresponding discrete functions.

Discrete bride function is represented as Bri, continuons bridge function is given as bri. To form a bridge function, not only is the copy information required, but also the shift information is needed. So there are four independent variables for bridge function. For discrete bridge function, it may be represented as

$$Bri_w\ (i, j, P, t) \tag{6}$$

where t is the independent variable, time, in the interval (0, 1).
$N=2^P$ it represents the total umbers of the intervals in (0, 1).
$L=2^j$ it represents the numbers of the intervals occupied by the shift, $L \leq N$
i is the i^{th} number of bridge function.
subscript w denotes Walsh ordering.

If the value in a interval is constant, the discrete bridge function becomes continuous bridge function. Continuous bridge function is represented as

$$bri_w\ (i, j, P, t) \tag{7}$$

The meaning lof the four independence variables and subscript are the same as the discrete bridge function.

WAVEFORMS OF BRIDGE FUNCTION

The waveforms of the bri_w (i, 1, 3, t), i=0...7 are shown in figure3, and the waveforms of the bri_w (i, 2, 3, t), i=0...7 are shown in figure 4. For the poupose of comparison, the waveforms of wal_w (i,t), i=0...3 are shown in f igure 5.

Comparing the waveforms of the figure with the waveforms of the figures, the following features can be found:

(1) When non-zero values of bri_w (i, 1, 3, t), i=0-7, are merged in the figure 3, four different waveforms appear. Their shape is identical with the waveform. of wal_w (i,t), i=0-3. When non-zero values of bri_w (i, 2, 3, t), i=0-7, are merged in the figure 4, two different waveforms appear. Thair shape is identical with the waveforms of wal_w(i,t), i=0-1.

(2) In general: when non-zero values of bri_w(i, j, P, t) are merged, $2^{(P-j)}$ different waveforms appear. For example, the waveforms, of bri_w(i, 1, 3,t) are identical with those of wal_w(i,t), $^{i=0-3}$. The waveforms of bri_w (i, 2, 3, t) are identical with those of wal_w(i,t), i=0-1.

(3) In general:, when non-zero values of bri_w (i,j,P,t) are merged, the waveforms are identical with those of wal_w ($[\frac{i}{L}]$, t). where $[\frac{i}{L}]$ represents that i is devided by L ($L=2^j$) then the idteger is taken. For example, the waveform of bri_w (7, 1, 3, t) is identical with that of wal_w (3, t), the waveform, of br_w (5, 2, 3, t) is identical with that of wal_w (1,t)..

ORTHOGONALITY OF BRIDGE FUNCTION

For simplicity, we take discrete bridge function as example to show the orthogonality of bridge function. Let Bri_w (i_1, j, P, t) and Bri_w (i_2,j, P, t) be discrete bridge functions. It is shown as follows:

(1) $i_1 \neq i_2$ (mod L), where $L=2^j$

In such a case, the part of non-zero value of a bridge function happen to correspond with the part of zero value of another bridge function. So the sum of the product of these two bridge function is zero. i.e.

$$\sum_{tk=0}^{N-1} Bri_w(i_1, j, P, t_k)\, Bri_w(i_2, j, t_k)=0 \qquad (8)$$

(2) $i_1 \equiv i_2$ (modL), but $i_1 \neq i_2$

In such a case, the parts of non-zero value of two bridge functions are correspondent to each other, so the parts of zero value.

In such a case, the sum of the product of two bridge function may be divided into two parts, i.e. the part of non-zero value and the part of zero value. The product of two parts of zero value is zero. The part of non-zero value may be translated into the sum of the walsh function's product.

Let $\{t_1\}$ denote a set of interval with non-zero value.

$$\sum_{t_k=0}^{N-1} Bri_w(i, j, P, t_k)\, Bri_w(i_2, j, P, t_k)$$

$$= \sum_{t_k \in \{t_1\}} Bri_w(i_1, j, P, t_k)\, Bri_w(i_2, j, P, t_k) + 0$$

$$= \sum_{t_k=0}^{\frac{N}{L}-1} Wal_w\left(\left[\frac{i_1}{L}\right], t_k\right) Wal_w\left(\left[\frac{i_2}{L}\right], t_k\right),$$

where symbol [ ] means that the integer is only taken.

$$\therefore \sum_{t_k=0}^{\frac{N}{L}-1} Wal_w\left(\left[\frac{i_1}{L}\right], t_k\right) Wal_w\left(\left[\frac{i_2}{L}\right], t_k\right) = 0 \qquad (9)$$

(3) $i_1 = i_2 = i$

$\because i_1 = i_2$, $\therefore i_1 = i_2 \pmod{L}$, $\left[\frac{i_1}{L}\right] = \left[\frac{i_2}{L}\right]$

In such a case, two bridge functions are the same. The parts with non-zero value of two bridge functions are not only correspondent to each other, but also the parts with zero value of two bridge functions are correspondent each other. It is similar to the case (2), The sum of the product of two bridge functions may be divided into two parts, i,e, the part of non-zero value and the part of zero value. The product of the part with zero value is zero. The part with non-zero value may be translated into the sum of walsh function's product.

$$\sum_{t_k=0}^{N-1} Bri_w(i, j, P, t_k)\, Bri_w(i, j, P, t_k)$$

$$= \sum_{t_k \in \{t_1\}} Bri_w(i, j, P, t_k)\, Bri_w(i, J, P, t_k) + 0$$

$$= \sum_{t_k=0}^{\frac{N}{L}-1} Wal_w\left(\left\{\frac{i}{L}\right\}, t_k\right) Wal_w\left(\left\{\frac{i}{L}\right\}, t_k\right) = \sum_{t_k=0}^{\frac{N}{L}-1} Wal_w(0, t_k) = \frac{N}{L} \quad (10)$$

Consider simultaneously the above equations (8), (9) and (10), the orthogonality of bridge function is expressed as follows.

$$\sum_{t_k=0}^{N-1} Bri_w(i_1, j, P, t_k)\, Bri_w(i_2, j, P, t_k)$$

$$= \begin{cases} 2^{(P-j)} & i_1 = i_2 \\ 0 & \left\{ \begin{array}{l} i_1 \neq i_2 \\ i_1 \equiv i_2 \pmod{L} \\ i_1 \not\equiv i_2 \pmod{L} \end{array} \right. \end{cases} \quad (11)$$

CONCLUSION

Since the bridge functions are orthogonal, it may be used as subcarriers. According to the principles of orthogonal multiplexing, it is possible to form a new telemetry system based on bridge functions. The construction of the system is similar to that of SDM.

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Table 1 The process of forming bridge function i=0-15, f or shift number j=1 .

order number	binary code	copy Infor-mation	shift Infor-mation	original sequence	sequence after shift	first copy sequence	second cop sequence	third cop sequence
i	$i_3i_2i_1i_0$	$i_3i_2i_1$	i_0			i_3	i_2	i_1
0	0 0 0 0	0 0 0	0	+ 0	+ 0	0 +	+oo+	+oo++oo+
1	0 0 0 1	0 0 0	1	+ 0	0 +	+ 0	0++0	0++00++0
2	0 0 1 0	0 0 1	0	+ 0	+ 0	0 +	+00+	−00−−00−
3	0 0 1 1	0 0 1	1	+ 0	0 +	+ 0	0++0	0−−00−−0
4	0 1 0 0	0 1 0	0	+ 0	+ 0	0 +	−00−	−00−+00+
5	0 1 0 1	0 1 0	1	+ 0	0 +	+ 0	0−−0	0−−00++0
6	0 1 1 0	0 1 1	0	+ 0	+ 0	0 +	−00−	+00+−00−
7	0 1 1 1	0 1 1	1	+ 0	0 +	+ 0	0−−0	0++00−−0
8	1 0 0 0	1 0 0	0	+ 0	+ 0	0 −	−00+	+00−−00+
9	1 0 0 1	1 0 0	1	+ 0	0 +	− 0	0−+0	0+−00−+0
10	1 0 1 0	1 0 1	0	+ 0	+ 0	0 −	−00+	−00++00−
11	1 0 1 1	1 0 1	1	+ 0	0 +	− 0	0−+0	0−+00+−0
12	1 1 0 0	1 1 0	0	+ 0	+ 0	0 −	+00−	−00+−00+
13	1 1 0 1	1 1 0	1	+ 0	0 +	− 0	0+−0	0−+00−+0
14	1 1 1 0	1 1 1	0	+ 0	+ 0	0 −	+00−	+00−+00−
15	1 1 1 1	1 1 1	1	+ 0	0 +	− 0	0+−0	0+−00+−0

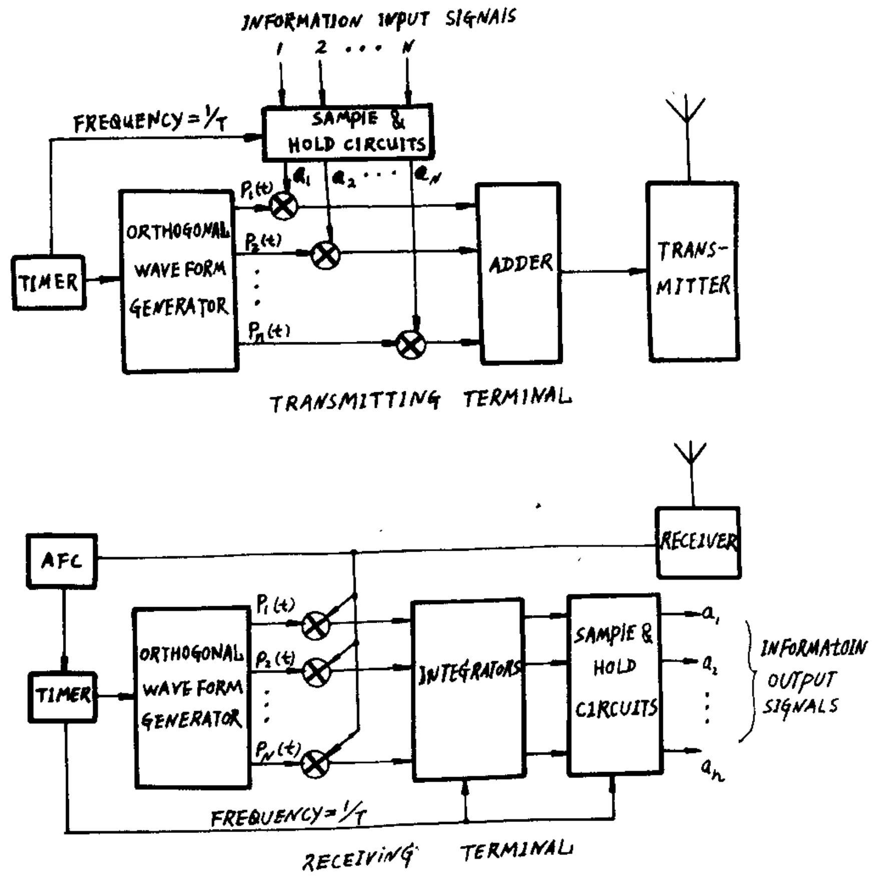

Figure 1, optimum multiplex system

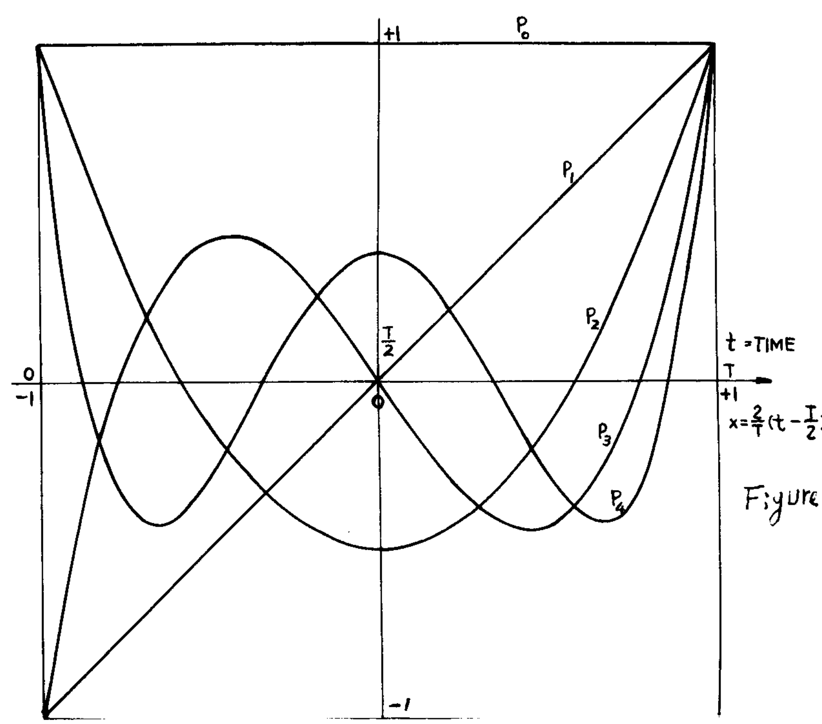

Figure 2

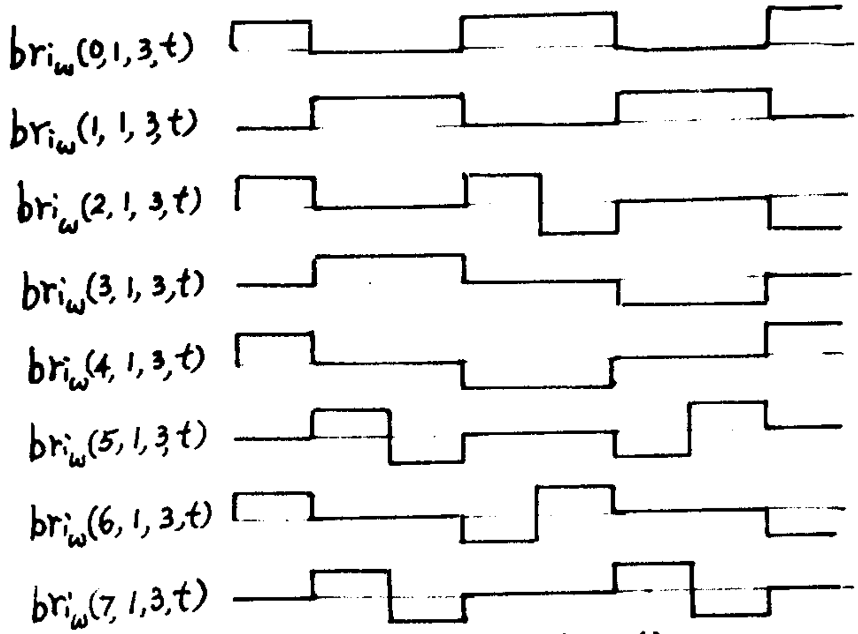

Figure 3 $bri_{\omega}(i,1,3,t)$, $i=0-7$.

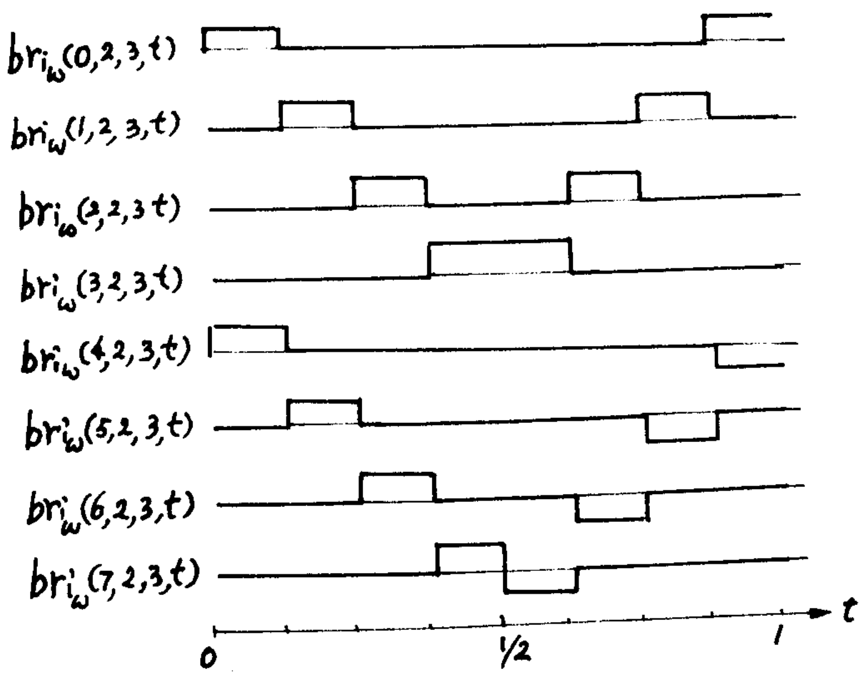

Figure 4 $bri_{\omega}(i,2,3,t)$, $i=0-7$.

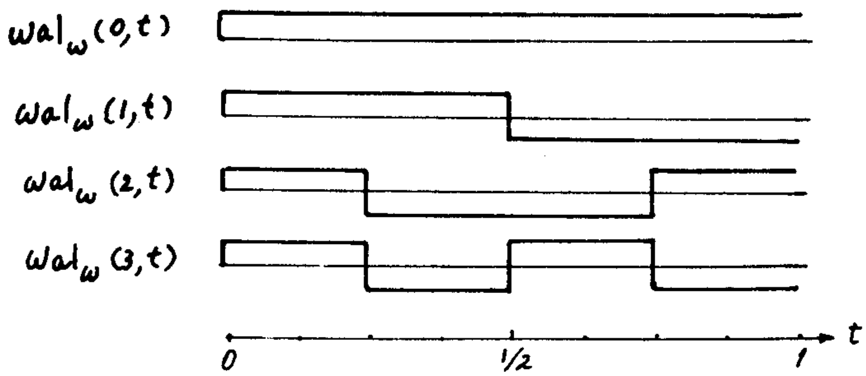

Figure 5 $wal_\omega(i,t)$, $i = 0 \sim 3$.