Subcarrier Placement in a PCM-FM-FM/FM Modulation Scheme

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Abstract

In a PCM-FM-FM/FM modulation scheme, one pulse code modulated (PCM) signal is added to a series of FM subcarrier modulated signals, and the sum is modulated on one FM carrier frequency. After the signal is carrier demodulated at the receiver and the signals modulating the subcarriers are individually filtered and demodulated, the information carried by the subcarrier frequencies may be distorted or lost due to interference power of the PCM signal that is passed by the subcarrier signals' bandpass filters. The effect of the interference power may be reduced when the subcarrier frequencies are chosen to coincide with the zero crossing frequencies of the PCM signal. It will be shown that this choice results in a lower interference power than when the subcarrier frequencies come between the zero crossings. The PCM signal used in this study is of polar nonreturn to zero format.

Introduction and Background Information

Before discussing PCM-FM-FM/FM, the schemes of PCM-FM and FM/FM will be introduced. PCM-FM modulation modulates a pulse-code modulated (PCM) signal $m_D(t)$, of a polar nonreturn-to-zero format, amplitude A, bit rate R_b, and

period T, on the FM carrier frequency f_c. A block diagram of the circuit that produces this modulation scheme is given in Figure 1.

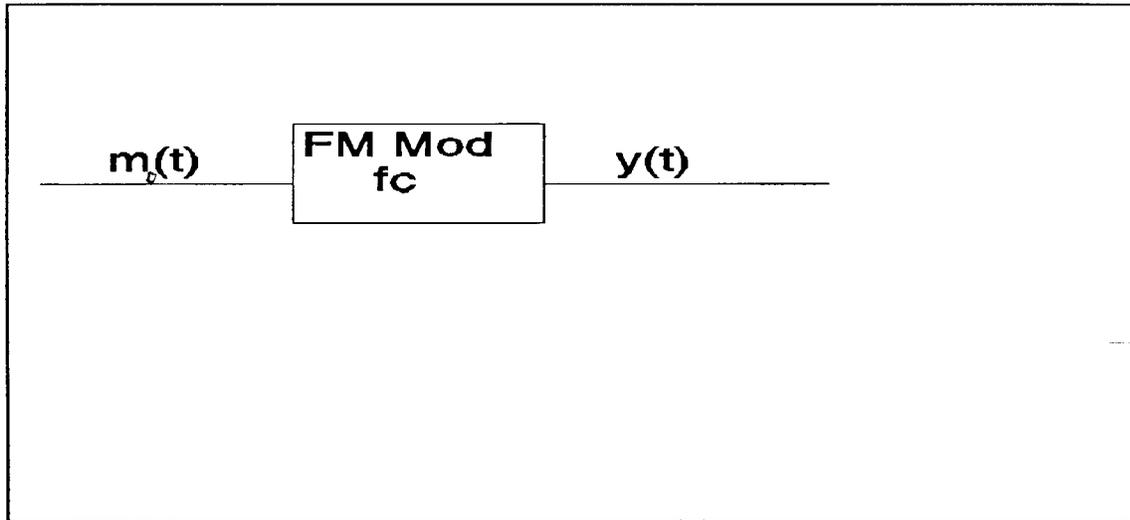Figure 1: PCM Modulation Scheme

The equation describing the modulated signal y(t) is given as

$$y(t) = \cos\left(2\pi\left(f_c t + k\int_0^t m(t')\,dt'\right)\right)$$

(1)

where k is the gain of the FM modulation box. k has units of Hertz per volt. The derivative of the argument of the cosine gives the instantaneous frequency of the modulated signal.

This instantaneous frequency shows that the peak deviation in frequency is given by

$$f_d = Ak$$

(2)

The FM/FM modulation scheme is shown in the black box design in Figure 2. FM/FM has n signals $m_n(t)$ modulating n FM subcarrier frequencies. The signals are then summed, and the sum modulates a single FM carrier frequency. The FM/FM signal is described as Equation 3.
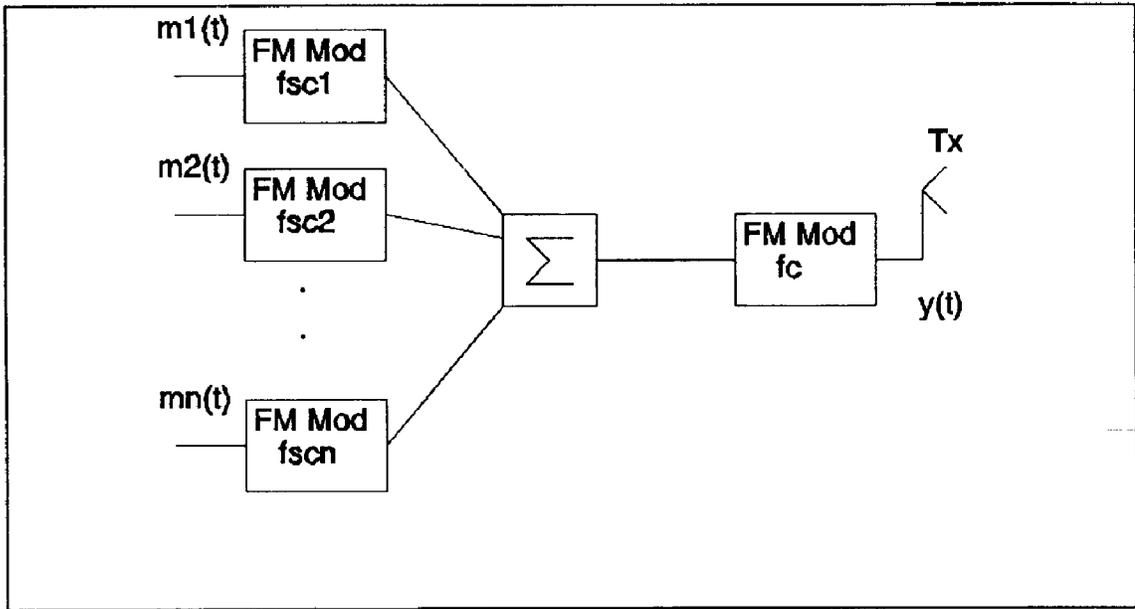
Figure 2: FM/FM Modulation Scheme

$$y(t) = kcos\left[2\pi\left(f_c t + f_{d1}\int^t cos\,\omega_{sc1}t'dt' + f_{d2}\int^t cos\,\omega_{sc2}t'dt' + \cdots\right.\right.$$

$$\left.\left.\cdots + f_{dn}\int^t cos\,\omega_{scn}t'dt'\right)\right] \tag{3}$$

In Equation 3, $f_{dn} = A_n$, where A_n is the amplitude of $m_n(t)$ and k is the gain, in Hz/volt, of the FM carrier modulation box.

PCM-FM-FM/FM is like the FM/FM modulation scheme, but the PCM signal $m_D(t)$ is summed with the other n signals after they modulate the n subcarriers. Figure 3 is the black box design of the new modulation scheme, and Equation 4 is the PCM-FM-FM/FM signal.

$$y(t) = Acos\left[2\pi\left(f_c \pm f_d\right)t + \frac{f_{dc1}}{f_{s1}}\sin\left(2\pi f_{s1}t\right) + \frac{f_{dc2}}{f_{s2}}\sin\left(2\pi f_{s2}t\right) + \cdots\right.$$

$$\left.\cdots + \frac{f_{dcn}}{f_{sn}}\sin\left(2\pi f_{sn}t\right)\right] \tag{4}$$
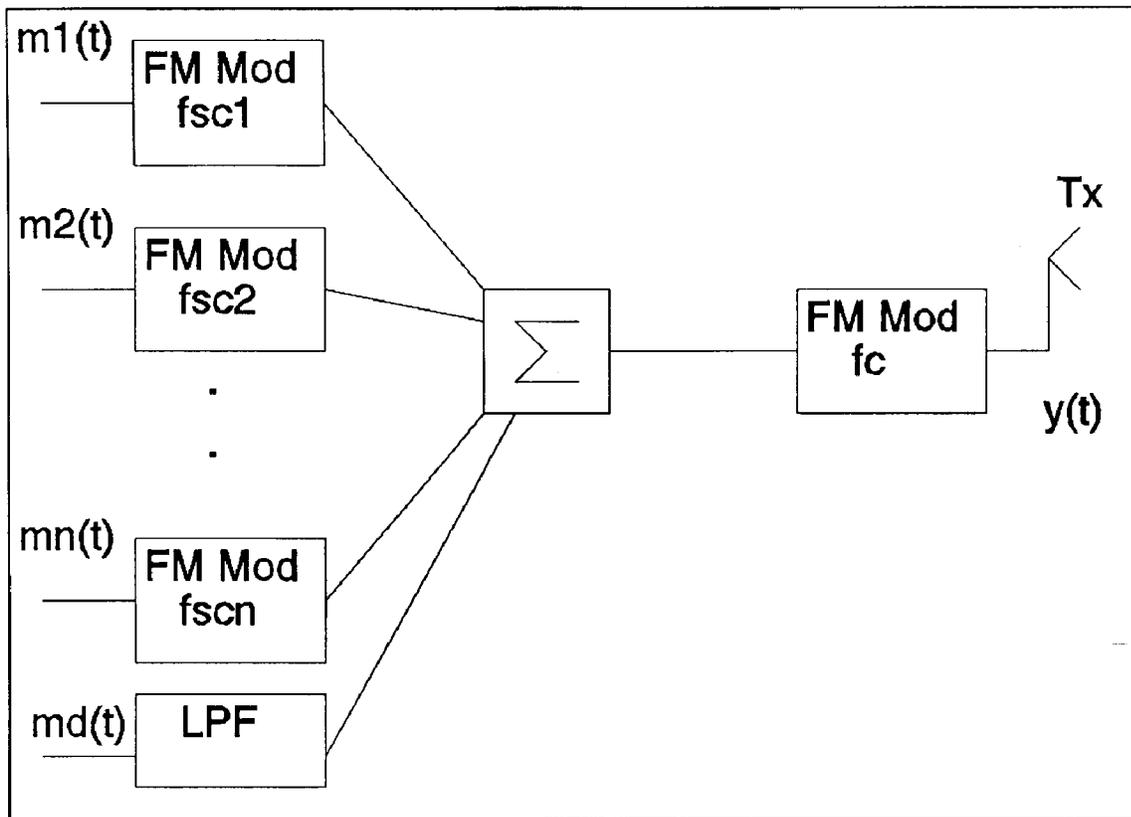
Figure 3: PCM-FM-FM/FM Scheme

Equation 4 introduces several new parameters. f_c is the carrier frequency, and f_d is the peak frequency deviation of $m_D(t)$, as introduced in Equation 2. f_{dcn} is the frequency deviation of the carrier by the nth subcarrier frequency. For $m_D(t)$, f_d is usually chosen to be $.35R_b$, where R_b is the bit rate of the signal. f_{dcn} is determined according to the design's objectives, and the subcarrier frequencies f_{scn} are specifications of the modulation scheme design.

For the remainder of this paper, the lowest frequency subcarrier signal will be considered. The rms power associated with the lowest frequency subcarrier of the PCM-FM-FM/FM signal, $m_1(t)$, after carrier demodulation is given in Equation 5. This shows that the weight of the subcarrier in the frequency domain is f_{ds1} (See Figure 5).

$$p_{sc1} = \frac{(f_{ds1})^2}{\sqrt{2}}$$

(5)

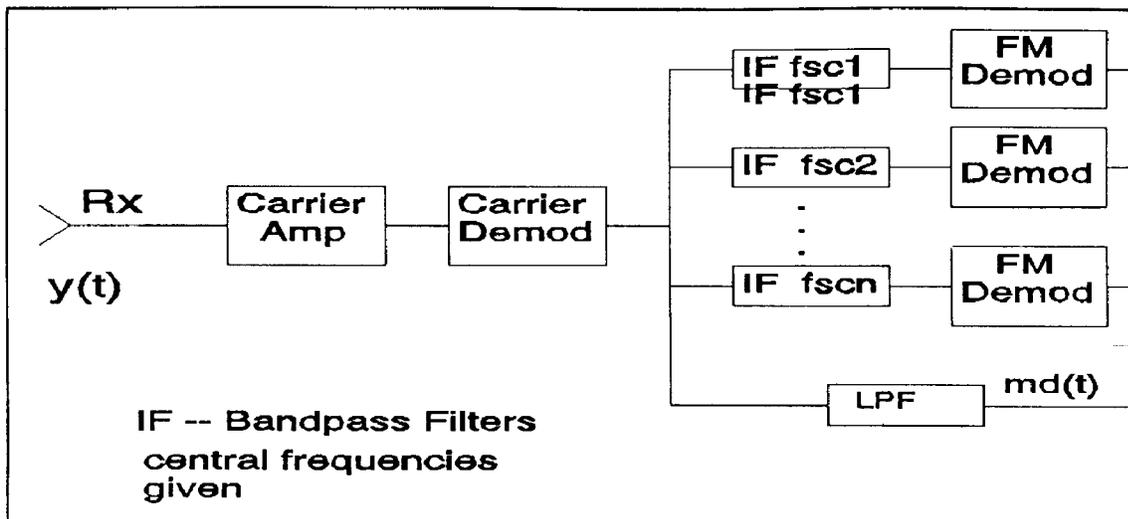The PCM-FM-FM/FM demodulation scheme is shown in Figure 4.

Figure 4: PCM-FM-FM/FM Demodulation

After carrier demodulation, the subcarrier modulated by $m_1(t)$ is filtered by an IF filter, which is a bandpass filter with a central frequency equal to f_{sc1}, and with a bandwidth of 15% of the central frequency, 7.5% on each side. This filter may be considered ideal. These percentages are the IRIG standard bandwidths of bandpass filters. This filter also passes any components of $m_D(t)$ that fall within the bandwidth, which could interfere with the power of the subcarrier signal and either distort or destroy the information associated with $m_1(t)$.

The interference power associated with the components of $m_D(t)$ that fall within the subcarrier's IF filter is the integration over the IF filter bandwidth of the energy spectral density (ESD) of $m_D(t)$ divided by T, the period of $m_D(t)$. This expression is shown as Equation 6,

$$P_{PCM} = \frac{2}{T} \int_{f_1}^{f_2} S(f) |H(f)|^2 df,$$ (6)

where $S(f)$ is the ESD of one period of $m_D(t)$. Note that the integration is multiplied by 2 to achieve the double sided ESD of $m(t)$. $S(f)$ is expressed as

$$S(f) = (f_d T)^2 Sinc^2(fT)$$ (7)

If any premodulation filtering was performed on the PCM signal before it was added to the other signals and

modulated (see Figure 3), then H(f) in Equation 6 is the transfer function of the filter. If no premodulation filtering was done, then H(f) is unity, or the filter is an all pass filter.

A comparison of the power associated with the components of $m_D(t)$ within the bandpass filter's bandwidth to the power of the subcarrier modulated by $m_1(t)$ is best expressed as a ratio of the powers. This ratio is shown here in dB.

$$10 \log \left(\frac{p_{PCM}}{f_{ds1}^2} \right) \qquad (8)$$

Employing logarithmic properties, remembering the IF filter is ideal, and using the expression for power in Equation 7, the ratio becomes

$$10 \log \left(\frac{f_d^2}{f_{ds1}^2} \int 2 T sinc^2 (fT) |H(f)|^2 df \right)$$

where the integration is performed over the bandwidth of the IF

$$= 10 \log \left(\frac{f_d^2}{f_{ds1}^2} \right) + 10 \log \left(\int 2 T sinc^2 (fT) |H(f)|^2 df \right) \qquad (10)$$

filter. Notice the first term of Equation 10 does not depend on f_{sc1} but the second term does because it contains the integration over the bandwidth of the IF filter. If f_{sc1}, and consequently the IF filter, were changed, any change in the ratio of PCM signal power to subcarrier signal power would show in this second term only. It is the second term that is used to monitor the effect of changing the location of the subcarrier in the PCM-FM-FM/FM modulation scheme.

Approach

A sample PCM signal used in this analysis is of polar nonreturn to zero format, with amplitude A = 1, bit rate R_b = 11 kbits/second, and T = $1/R_b$. The gain of the FM carrier modulation box is set to k = 1 Hz/volt. Three different

premodulation filters are employed in the modulation scheme of Figure 3. The first has a transfer function H(f) = 1, to pass all frequencies. The second filter is a single-pole low pass filter with a transfer function of

$$H(f) = \frac{1}{1 + j\dfrac{f}{R_b}} \qquad (11)$$

where R_b, the -3dB cutoff frequency, is the bit rate of $m_D(t)$. The final filter is a six-pole Bessel low pass filter, designed for the same -3dB cutoff frequency and normalized. Its transfer function is

$$H(f) = \frac{\left(10395\dfrac{\omega_c}{2.7}\right)^6}{b_{n1}b_{n2}b_{n3}}, \qquad (12)$$

where w_c is R_b expressed in radians/sec, and

$$b_{n1} = [(j\omega)^2 + 8.49672j\omega + 18.80113]\left(\frac{\omega_c}{2.7}\right)^2$$

$$b_{n2} = [(j\omega)^2 + 7.47142j\omega + 20.85282]\left(\frac{\omega_c}{2.7}\right)^2$$

$$b_{n3} = [(j\omega)^2 + 5.03186j\omega + 26.51402]\left(\frac{\omega_c}{2.7}\right)^2$$

The subcarrier frequencies were first chosen to fall between the zero crossings of the frequency representation of $m_D(t)$. This puts the first subcarrier frequency at f_{sc1}=16.5 kHz. Figure 5 is a sketch of the frequency domain representation of the PCM-FM-FM/FM signal after carrier demodulation.

For each H(f) , the second term of Equation 10 was evaluated for f_{sc1}. The integration was performed over the limits of the bandwidth of the IF filter centered at f_{sc1}.

Then the subcarrier frequencies were chosen to coincide with the zero crossings, so f_{sc1} = R_b = 11 kHz. This is shown in Figure 6.
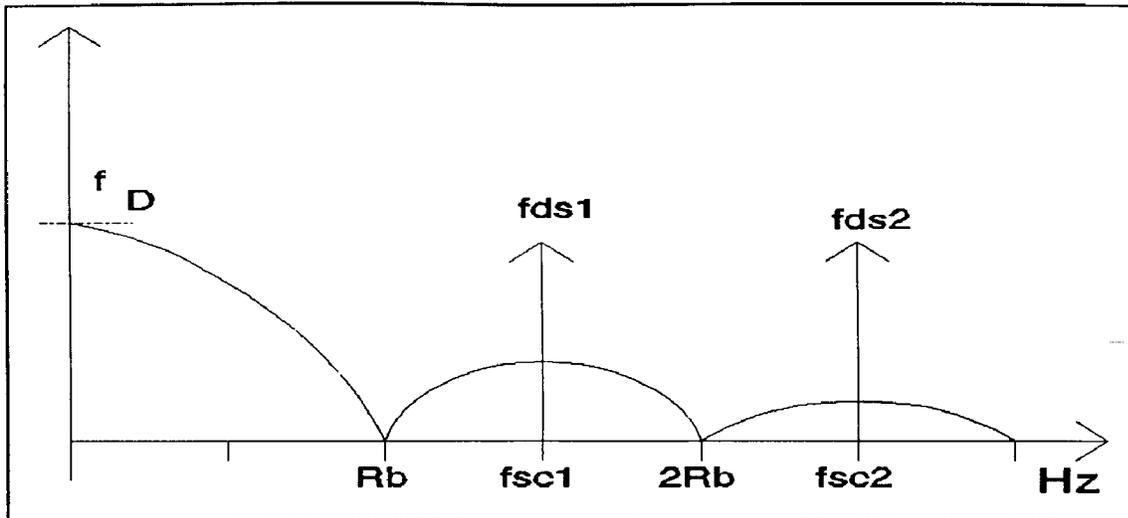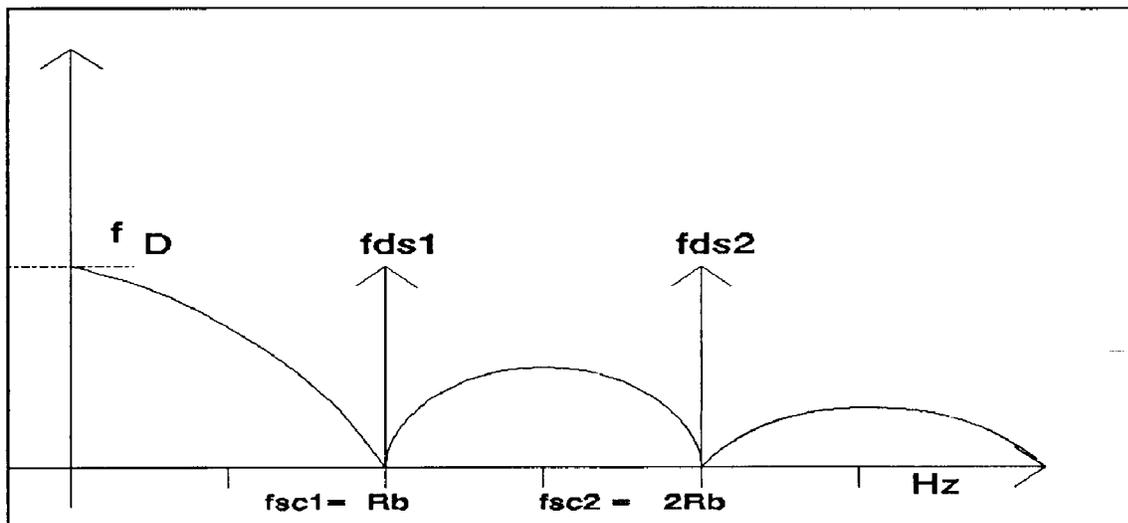
Figure 5

Figure 6

Again, for each H(f), the PCM signal was integrated over the bandwidth of the IF filter centered at f_{sc1}. The following table gives the calculations just described. All powers are expressed in dB, according to Equation 10. The calculations were performed using the Matlab software package (see Appendix).

Results

Type of Premodulation Filter	Second Term of (10) (dB)	
	f_{sc1}=16.5 kHz	f_{sc1}=11kHz
All Pass (H(f)=1)	-17.0997	-32.5030
Single Pole	-22.1467	-35.4769
Six Pole	-24.3180	-35.4610

The data in the table shows that placing f_{sc1} at the zero crossing Of $m_D(t)$ causes the PCM signal power-to-subcarrier power ratio to be lower in dB. The ratios calculated for the single pole low pass filter show a drop of over 10 dB when f_{sc1} coincides with $m_D(t)$'s zero crossing. The lower the PCM signal power being passed through the IF filter, the lower is the distortion of the subcarrier signal. The table data also shows that a high order premodulation filter helps reduce the power ratio. For example, in the second column of the table, the ratio associated with the all pass filter is higher than that of the single pole low pass filter by over 5 dB.

There is an anomaly in the results. The calculation of the ratio given f_{fs1} = R_b and the single pole low pass filter is lower than that for the same subcarrier frequency and the six pole filter. The difference is small, however, and may have come from using two different versions of Matlab software for the calculations. Another run of all the Matlab files on the same computer should correct this error.

Conclusions

The results of the calculations performed do show that subcarrier frequencies placed at the zero crossings of the PCM signal in a PCM-FM-FM/FM format result in lower interference power. Choosing a higher order premodulation filter for the PCM signal also lowered the interference power. Making these choices when designing a PCM-FM-FM/FM system will help to insure that the signals $m_n(t)$ are received with minimal interference by $m_D(t)$. Choosing a higher order premodulation filter for $m_D(t)$ should not

compromise the signal, since the components of $m_D(t)$ associated with frequencies higher than the filter's cutoff frequency are small.

This paper examined an example of a PCM-FM-FM/FM modulation scheme of R_b = 11 kbits/sec. For an actual case of PCM-FM-FM/FM using IRIG standards, the bit rate would have to be chosen as either 10.5 or 14.5 kbits/sec. The subcarriers' IF filters could not be ideal in an actual case, but could be chosen as high order filters and considered ideal for analysis purposes.

References

Carlson, A. Bruce. <u>Communication Systems: An Introduction to Signals and Noise in Electrical Communication</u>, New York: McGraw-Hill, Inc., 1986.

Haykin, Simon. <u>Digital Communications</u>, New York: John Wiley and Sons, 1988.

Stremler, Ferrel G. <u>Introduction to Communication Systems</u>, Reading, Mass: Addison-Wesley Publishing Co, 1990.

Van Valkenburg, M. E. <u>Analog Filter Design</u>, New York: CBS College Publishing, 1982.

Appendix

Included in the Appendix are all the Matlab command files used in computing the data presented in the Results section. PC-Matlab and Matlab for Macintosh were used.

```
%   beslpowr.m                              julie
%                                           itc paper
%   function files used:
%           PISINCSQ.m--ESD of the bit stream
%           BESLOUT.m--ESD of the output of bessel filter
%   result of this program is the output power
%
fc=11000;
a=1;
tt=1/fc;
wc=2*pi*fc;
%
pwrzro=(2*(a^2)*tt)*quad('beslout', 10175, 11825)
pwrmid=(2*(a^2)*tt)*quad('beslout',  15262.5, 17737.5)
end




    function y = beslout(f)                  %julie moser
%                                           itc paper
% 6-pole bessel lpf designed for -3db at fc
% function returns the output of lpf
%
fc=11000;
tt=1/fc;
w=2.7*f/fc;
wc=2*pi*fc;
bn61=((i*w) .^2 + 8.49672*i*w + 18.80113)*(wc/2.7).^2;
bn62=((i*w) .^2 + 7.47142*i*w + 20.85282)*(wc/2.7).^2;
bn63=((i*w) .^2 + 5.03186*i*w + 26.51402)*(wc/2.7).^2;
bn6=bn61 .* bn62 .* bn63;
h2=abs(10395*(wc/2.7).^6 ./ bn6) .^2;
y=h2 .* pisincsq(tt,f);
```

```
%              lpfout.m                    julie
%                                          itc paper
%        power of output of the single pole lpf
%        function file:  SO2.m---ESD of output of single
%                            pole lpf
a=1;
fc=11000;
rc=1/(2*pi*fc);
tt=1/fc;
powzro=(2*(a^2)*tt)*quad('so2', 10175, 11825)
powmid=(2*(a^2)*tt)*quad('so2', 15262.5, 17737.5)
end




function y = so2(f,rc,tt)              % Julie moser
%                                        ITC paper
% returns ESD of  output of 1-pole lpf
%            PISINCSQ.m---ESD of PCM signal
%
tt=1/11000;
rc=1/(2*pi*11000);
x=pisincsq(tt,f);
z=abs(1 ./ (1+i*2*pi*f*rc)) .^ 2;
y= x .^ z;




function y = pisincsq(tt,f)            % julie moser
%                                        ITC paper
%  PISINCSQ is PISINC squared
tt=1/11000;
y=pisinc(tt,f) .^ 2;
```

```matlab
function y=pisinc(tt,x)
%SINC(X)   sinc(X) is the function   sin(x)/x
% If x is a vector, it performs an element by element evaluation
%(same as sa(x)

%By Scott Dunn  2-13-91
%modification to include PI in argument by JULIE MOSER
%
a = find(x==0);
if isempty(a)
   y=sin(pi*x*tt)./(pi*x*tt);
else
          if a(1)==1
   y(1)=1;
 else
   y(1:a(1)-1)=sin(pi*tt*x(1:a(1)-1))./(pi*tt*x(1:a(1)-1));
   y(a(1))=1;
 end
 L=length(a);
 if L>=2
  for i=2:L
           fr=a(i-1)+1;
   to=a(i)-1;
   y(fr:to)=sin(pi*tt*x(fr:to))./(pi*tt*x(fr:to));
   y(a(i))=1;
  end
 end
 Lx=length(x);
 if a(L)~=Lx
  y(a(L)+1:Lx)=sin(pi*tt*x(a(L)+1:Lx))./(pi*tt*x(a(L)+1:Lx));
 end
end
```

```matlab
%      subcarrier.m
%      power of bit associated with the ideal BPF
%      of the width f1-f2;
%      Notice the original bit rate changes the
%        sinc function
f1=11000 - 11000*.075;
f2=11000*.075 +11000;
fc=11000;
tt=1/11e3;
lim1=f1*tt*pi;
limc=fc*tt*pi;
lim2=f2*tt*pi;
a=i;
pow1=(2*a^2/pi)*quad('sincsq', lim1, limc);
pow2=(2*a^2/pi)*quad('sincsq', limc, lim2);
power=pow1 +pow2
```

```
%       subcarrier.m
%       Notice the bit rate is RB=11kbits/sec
f1=16500 - 16500*.075;
f2=16500 + 16500*.075;
fc=16500;
tt=1/11e3;
a=1;
pow1=(2*a^2/pi)*quad('sincsq',f1*tt*pi, fc*tt*pi);
pow2=(2*a^2/pi)*quad('sincsq',fc*tt*pi, f2*tt*pi);
power=pow1+pow2
end
```