A NONMYSTICAL TREATMENT OF TAPE SPEED COMPENSATION FOR FREQUENCY MODULATED SIGNALS*

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ABSTRACT

In this paper, the problem of non-constant tape speed is examined for frequency modulated signals. Frequency modulation and demodulation are briefly reviewed. Tape speed variation is modeled as a distortion of the independent variable of a frequency modulated signal. This distortion produces an additive amplitude error in the demodulated message which is comprised of two terms. Both depend on the derivative of time base error, which is the flutter of the analog tape machine. The first term depends on the channel's center frequency and frequency deviation constant as well as flutter, while the second depends solely on the message and flutter. The relationship between the additive amplitude error and manufacturer's flutter specification is described. Relative errors and signal-to-noise ratios are discussed for the case of a constant message to gain insight as to when tape speed variation will cause significant errors. An algorithm which theoretically achieves full compensation of tape speed variation is developed. The algorithm is confirmed via spectral computations on laboratory data. Finally, the algorithm is applied to field data. The reference is a temperature signal which is a non-zero constant, and the message is a pressure signal. The spectrum of the uncompensated message is clearly contaminated by the additive amplitude error, whereas the spectrum of the compensated message is not. Incorporation of this algorithm into the data-playback/data-reduction procedures is shown to greatly improve the measurement signal accuracy and quality. The treatment is nonmystical in that all derivations are directly tied to the fundamental equations describing frequency modulation and demodulation.

REVIEW OF FREQUENCY MODULATION AND DEMODULATION

Frequency modulation is a type of angle modulation (1, 2, 3). The form of the carrier is

$$v(t) = A\cos(2\pi f_c t + \mu(t)) \qquad (1)$$

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where A is the amplitude of the carrier, f_c is the center frequency, and μ(t) contains the message. The instantaneous phase is $2\pi f_c t + \mu(t)$, the instantaneous frequency is $d(2\pi f_c t + \mu(t))/dt = 2\pi f_c + d\mu(t)/dt$, and the phase deviation and frequency deviation are μ(t) and dμ(t)/dt, respectively. For frequency modulation, the frequency deviation is varied in proportion to the message; i.e., $d\mu(t)/dt = 2\pi f_d m(t)$, where f_d is the frequency deviationconstant and m(t) is the message. If dπ(t)/dt is measured in radians and m(t) in volts, then f_d must be measured in hertz per volt. By the fundamental theorem of integral calculus,

$$\mu(t) = 2\pi f_d \int_0^t m(\alpha) d\alpha \qquad (2)$$

is an antiderivative for dμ(t)/dt. A device that outputs

$$v(t) = A \cos\left(2\pi f_c t + 2\pi f_d \int_0^t m(\alpha) d\alpha\right) \qquad (3)$$

from input m(t) is called voltage-controlled oscillator. A device that outputs m(t) from input v(t)is a discriminator. The mathematical operations that a discriminator performs are differentiation of the phase deviation and division of the result by $2\pi f_d$. In practice the result is also lowpass-filtered. In fact, most practicing engineers view the lowpass-filtering as part of discrimination. For telemetry work, the range of m(t) is frequently ± 2.5 V. Consider an FM channel with f_c = 176 000 Hz and f_d = 800 Hz/V (Figure 1).
For m(t) = 2.5 V,

$$\mu(t) = 2\pi f_d \int_0^t m(\alpha)\, d\alpha$$

$$= 2\pi\ 800 \int_0^t 2.5 d\alpha$$

$$= 2\pi\ 2000t,$$

and

$$v(t) = A \cos(2\pi\ 176\,000t + 2\pi\ 2000t)$$

$$= A \cos(2\pi\ 178\,000t)\ .$$

Thus a message of 2.5 V corresponds to a carrier with a frequency deviation of 2000 Hz and an instantaneous frequency of 178 000 Hz. Similarly, a message of -2.5 V corresponds to a frequency deviation of -2000 Hz and an instantaneous frequency of 174 000 Hz. For this channel, 178 000 Hz is called the upper band edge (UBE), and 174 000 Hz is called the lower band edge (LBE); 2000 Hz is called the upper deviation limit (UDL), and -

2000 Hz is called the lower deviation limit (LDL). A channel can be specified in terms of UBE and LBE or UDL and LDL, and the range of m(t) can be left unspecified. In these cases, it is understood that the maximum value of m(t), m_u, corresponds to UBE or f_c + UDL, and the minimum value of m(t), M_L, corresponds to LBE or f_c - LDL. Furthermore,

$$f_d = \frac{UBL - LBE}{m_U - m_L} = \frac{UDL - LDL}{m_U - m_L} \qquad (4)$$

is also taken for granted. In fact, the Inter-Range Instrumentation Group (IRIG) calls the above channel 21A (4) and specifies it as 176 000 ± 2000 Hz.

DISCUSSION OF TAPE SPEED VARIATION

When a signal is recorded on analog tape, the independent variable changes from time to distance (Figure 2). If the tape speed is constant, say s inches per second, and both the initial time and distance are 0, then time, t, corresponds to distance, d = st. However, as is well known (5, 6), tape transport systems are not perfect, and as a result the tape speed past the recording head varies. Hence the new independent variable is distorted. When a signal is reproduced from analog tape, a similar phenomenon occurs. The net effect when the signal is reproduced is a distorted independent variable, t + n(t), where n(t) is noise. The noise, n(t), is a result of both the recording and reproduction distortions and is called time base error. The causes of time base error are numerous. The tape transport system has many imperfect rotating mechanical parts (e.g., the capstan, pinch rollers, turnaround idler, reels and bearings) that give n(t) a periodic appearance. The tape vibrates as it slides across fixed guides and heads. The physical condition and characteristics of the tape vary from place to place on the reel. Design of transport systems is complicated by the fact that the mass on each reel is constantly changing. In summary, n(t) is an extremely complex signal composed of both periodic and random processes.

EFFECT OF n(t) ON FREQUENCY MODULATION AND DEMODULATION

Time base error can cause serious amplitude errors in a discriminated message (6, 7, 8). When the FM signal

$$v_m(t) = A\cos\left(2\pi f_{cm} t + 2\pi f_{dm} \int_0^t m(\alpha)\,d\alpha\right) \qquad (5)$$

is recorded and reproduced, the result because of tape speed variation (Figure 3) is

$$r_m(t) = v_m\big(t + n(t)\big)$$

$$= A\cos\left(2\pi f_{cm}\left(t+n(t)\right) + 2\pi f_{dm}\int_0^{t+n(t)} m(\alpha)d\alpha\right)$$

and the discriminator output (cf. Equation (17) in Reference (7) and Equation (34) in Reference (9)) is

$$d_m(t) = \frac{1}{2\pi f_{dm}}\frac{d}{dt}\left(\mu_m(t)\right)$$

$$= \frac{1}{2\pi f_{dm}}\frac{d}{dt}\left(2\pi f_{cm} n(t) + 2\pi f_{dm}\int_0^{t+n(t)} m(\alpha)d\alpha\right)$$

$$= \frac{f_{cm}}{f_{dm}}\frac{d\,n(t)}{dt} + m\left(t + n(t)\right)\left(1 + \frac{d\,n(t)}{dt}\right)$$

using Leibnitz's Rule to differentiate the integral. Upon rearranging, we get

$$d_m(t) = m\left(t + n(t)\right) + \frac{f_{cm}}{f_{dm}}\frac{d\,n(t)}{dt} + m\left(t + n(t)\right)\frac{d\,n(t)}{dt} \qquad (6a)$$

$$= m(t + n(t)) + e_m(t) \qquad (6b)$$

with the amplitude error e_m (t) given by

$$e_m(t) = \frac{f_{cm}}{f_{dm}}\frac{d\,n(t)}{dt} + m\left(t + n(t)\right)\frac{d\,n(t)}{dt}$$

The amplitude error depends on the center frequency, the frequency deviation constant, the message, and the derivative of time base error, dn(t)/dt, which is called flutter. Sometimes e_m (t) is incorrectly called flutter. Actually $e_m(t)$ is a function of flutter. In this paper we will call $e_m(t)$ flutter induced error or simply flutter error. Equation (7) is the key to understanding when tape speed variation will cause a problem. Note that $e_m(t)$ is the sum of two terms. The first term on the right side of Equation (7) is the channel-dependent portion of the flutter error, while the second term is the message-dependent portion. Both terms are dependent on flutter.

The flutter of an analog tape machine is specified by its manufacturer. By using the manufacturer's flutter specification in conjunction with the FM channel specification, it is possible to decide when tape variation is likely to cause significant errors. Flutter specifications published by manufacturers are based on measurements using an unmodulated FM signal $v(t) = A \cos(2\pi f_c t)$. The most common method of specification (10) is that, when this signal is recorded, reproduced and discriminated, 95 percent of the output values correspond to instantaneous frequency deviations within ±b percent of the center frequency, f_c. For a complete specification, it is necessary to know the cutoff frequency of the discriminator output lowpass filter as well as the tape speed. Lower cutoff frequencies remove more flutter. Tape speed variation is less at higher tape speeds. Throughout this paper, we will assume that b = 0.1, which is a good nominal value for a lowpass filter of 10 000 Hz and a tape speed of 60 in./s (152.4 cm/s). Let us reconsider Equation (7). We will assume that the range of m(t) is ± 2.5 V. Under this assumption, f_{cm}/f_{dm} ranges from approximately 2 to more than 200 for IRIG-specified channels. Thus, for some FM channels, the channel-dependent flutter error and message-dependent flutter error are of approximately the same magnitude, while for others the channel-dependent error is much larger than the message-dependent flutter error. Based on our experience, tape speed variation is much more likely to cause a problem in the latter case. In such a case, the message-dependent flutter error can often be assumed to be zero. Equation (7) gives $e_m(t)$ as an absolute error. A more meaningful error measure is a relative one, since the amount of error that can be tolerated is frequently relative to the amplitude of the message. We define the relative error, $re_m(t)$, to be the absolute error, $e_m(t)$, divided by the true value of the message. A relative error measure frequently used by engineers is the signal-to-noise ratio. We define the signal-to-noise ratio, S/N, as the mean square value of the message divided by the mean square value of the absolute error. To illustrate these definitions and gain insight as to when tape speed variation might cause problems, consideration of a constant message is helpful.

CONSTANT MESSAGE EXAMPLE

Let us take m(t) to be the constant, c. Then from Equations (6b) and (7), we get

$$d_m(t) = c + e_m(t), \text{ and}$$

$$e_m(t) = \frac{f_{cm}}{f_{dm}} \frac{d\,n(t)}{dt} + c\,\frac{d\,n(t)}{dt}$$

If flutter were a Gaussian random variable, then b/200 would be its standard deviation; however, flutter is never a perfect Gaussian random variable because it has periodic deterministic components due to the rotating parts of the tape transport System. Nevertheless, the assumption is not a gross error and can lead to some useful insights. Under these assumptions, $e_m(t)$ is a Gaussian random variable with a standard deviation, $\sigma(e_m)$, given by

$$\sigma(e_m) = \left(\frac{f_{cm}}{f_{dm}} + c\right)\frac{b}{200} \tag{8}$$

The discriminator output will lie within ± $2\sigma(e_m)$ V of c for 95 percent of the time. The relative error, $re_m(t)$, satisfies

$$|re_m(t)| = \left|\frac{\frac{f_{cm}}{f_{dm}}\frac{d\,n(t)}{dt} + c\frac{d\,n(t)}{dt}}{c}\right| \tag{9a}$$

$$= \left|\frac{f_{cm}}{cf_{dm}}\frac{d\,n(t)}{dt} + \frac{d\,n(t)}{dt}\right| \tag{9b}$$

$$\leq \left|\frac{f_{cm}}{cf_{dm}}\right|\left|\frac{d\,n(t)}{dt}\right| + \left|\frac{d\,n(t)}{dt}\right| \tag{9c}$$

$$\leq \left(\left|\frac{f_{cm}}{cf_{dm}}\right| + 1\right)\frac{b}{200} \tag{9d}$$

for 95 percent of the time. Equation (9b) clearly shows that the relative error due to channel-dependent flutter error can be quite large, while the relative error due to message-dependent flutter error is always on the order of the flutter, which is nominally 0.1 percent. The signal-to-noise ratio, S/N, is given by

$$S/N = \left(\frac{\frac{1}{T}\int_0^T c^2\,dt}{\sigma^2(e_m)}\right) \tag{10a}$$

$$= \frac{c^2}{\sigma^2(e_m)} \tag{10b}$$

$$= \frac{c^2}{\left(\frac{f_{cm}}{f_{dm}}\frac{b}{200} + c\,\frac{b}{200}\right)^2} \tag{10c}$$

In cases where absolute errors less than $|c\, dn(t)/dt|$ are insignificant, or relative errors less than $|dn(t)/dt|$ are insignificant, we get the following approximations:

$$d_m(t) \approx c + \frac{f_{cm}}{f_{dm}} \frac{d\,n(t)}{dt} \qquad (11a)$$

$$e_m(t) \approx \frac{f_{cm}}{f_{dm}} \frac{d\,n(t)}{dt} \qquad (11b)$$

$$re_m(t) \approx \frac{f_{cm}}{cf_{dm}} \frac{d\,n(t)}{dt} \text{ and} \qquad (11c)$$

$$S/N \approx \frac{c^2}{\left(\frac{f_{cm}}{f_{dm}} \frac{b}{200}\right)^2} \qquad (11d)$$

Equations (11a) through (11d) clearly point out that the most troublesome telemetry channels have large center frequencies and small frequency deviation constants. Many of the IRIG constant bandwidth channels fall in this category. Figure 4 shows re(t) versus f_{cm} with f_{dm} and c fixed at 800 and 1.25, respectively. Figure 5 shows re(t) versus c with f_{dm} and f_{cm} fixed at 800 and 176 000. In both figures, dn(t)/dt is fixed at 0.001.

THEORETICAL BASIS OF TAPE SPEED COMPENSATION

Methods for combatting the amplitude error given by Equation (7) are known as tape speed compensation. A reference FM signal is recorded simultaneously with the message-bearing FM signal. Since these FM signals are recorded at the same time on the same machine, their independent variables undergo almost the same distortion.

We will take r(t) = c as the reference signal. The resulting reference FM signal, $V_r(t) = \cos(2\pi f_{cr} t + 2\pi f_{dc} c)$, has a constant instantaneous frequency. From Equation (6) with m(t) = c, we get

$$d_r(t) = c + \frac{f_{cr}}{f_{dr}} \frac{dn(t)}{dt} + c\,\frac{dn(t)}{dt} \qquad (12a)$$

Equation (12a) can be rearranged to obtain

$$\frac{dn(t)}{dt} = \left(d_r(t) - c\right)\left(\frac{f_{cr}}{f_{dr}} + c\right)^{-1} \qquad (12b)$$

Thus, the flutter of an analog tape machine can be recovered from any FM signal with constant instantaneous frequency provided that f_{cr} f_{dr} and c are known. Observe that by substituting (12b) for the first instance of dn(t)/dt in Equation (6) and then rearranging, we get

$$m\left(t + n(t)\right)\left(1 + \frac{dn(t)}{dt}\right) = d_m(t) - \frac{f_{cm}}{f_{dm}}\left(d_r(t) - c\right)\left(\frac{f_{cr}}{f_{dr}} + c\right)^{-1} \quad (13)$$

Since dn(t)/dt is nominally 0.001 for good recorders, Equation (13) frequently provides adequate tape speed compensation. However, full compensation can be achieved by substituting (12b) for the remaining instance of dn(t)/dt in Equation (13) and then rearranging ,

$$m\left(t + n(t)\right) = \frac{d_m(t) - \frac{f_{cm}}{f_{dm}}\left(d_r(t) - c\right)\left(\frac{f_{cr}}{f_{dr}} + c\right)^{-1}}{1 + \left(d_r(t) - c\right)\left(\frac{f_{cr}}{f_{dr}} + c\right)^{-1}} \quad (14)$$

For the case of c = 0 (Figure 6), a partially compensated message is obtained by subtracting the reference discriminator output, after appropriate scaling, from the message discriminator output. To obtain full compensation, this partially compensated message is then divided by 1 + dn(t)/dt. Note that Equation (13) compensates for only channel-dependent flutter error, whereas Equation (14) compensates for both channel- and message-dependent flutter error.

Consequently, in situations where message-dependent flutter error is neglible, the simpler method can be used. In References (8) and (11), the claim is made that the subtraction compensation technique does not achieve full compensation. Equation (13) supports this claim. However, Equation (14) is a new technique that, at least theoretically, achieves full compensation. In all algorithms that employ the subtraction compensation technique, care must be taken that the signals do not get time-shifted relative to one another. Reference (12) discusses the time shifting problem in detail. Notice that the message in Equation (14) still contains a time base error. If the message is to be digitized, this problem is normally remedied by recording IRIG time along with the other FM signals. At reproduction time, the IRIG time is fed into some circuits that vary the time intervals between the samples to cancel the effects of n(t).

EMPIRICAL CONFIRMATION OF TAPE SPEED COMPENSATION ALGORITHM

The message channel has a center frequency, f_{cm}, of 160 000 Hz with deviation limits of ± 8000 Hz, while the reference channel has a center frequency, f_{cr} of 192 000 Hz with deviation limits of ± 8000 Hz. The input range for both channels is ± 2.5 V. The message, m(t), is the constant -1.25 V, and the reference, r(t), is the constant zero V. Any deviation from zero of the reference discriminator output is noise. The frequency deviation constants for the message channel and reference channel are

$$f_{dm} = f_{dr} = \frac{8000 - (-8000)}{2.5 - (-2.5)} = \frac{16\,000}{5} = 3200 \text{ Hz/V}$$

The discriminator output of the message channel is

$$d_m(t) = -1.25 + \frac{160\,000}{3200}\frac{d\,n(t)}{dt} - 1.25\frac{d\,n(t)}{dt}$$

The discriminator output of the reference channel is

$$d_r(t) = 0 + \frac{192\,000}{3200}\frac{d\,n(t)}{dt} + 0\frac{d\,n(t)}{dt}$$

$$= \frac{192\,000}{3200}\frac{d\,n(t)}{dt}\ .$$

Equation (14) with c = 0 can be rearranged to obtain

$$d_m(t)\left(1 + \frac{f_{dr}}{f_{cr}}\,d_r(t)\right)^{-1} - m\big(t + n(t)\big) = \frac{f_{cm}}{f_{dm}}\frac{f_{dr}}{f_{cr}}\,d_r(t)\left(1 + \frac{f_{dr}}{f_{cr}}\,d_r(t)\right)^{-1} \quad (15)$$

Let x(t) equal the left-hand side of Equation (15), and y(t) the right-hand side of Equation (15) (Figure 7). The coherence between x(t) and y(t) should be 1. (Reference (13) is an excellent source of information on spectral analysis applications.) The amplitude of the frequency response function between x(t) and y(t) should be 1. The phase of the frequency response function between x(t) and y(t) should be 0. The auto spectral densities of x(t)and y(t) should be identical. Figures (8) through (12) show that these claims are closely approximated out to 2000 Hz. The auto spectral densities do not possess the parabolic shape predicted by communication texts (1, 2, 3) for demodulation in the presence of additive noise. The reason for this anomaly is that the flutter-induced noise is much larger than the discriminator output noise induced by additive noise at the demodulator input. Figure (13) shows the auto spectral density of the tape speed compensated message. The parabolic shape indicates that the dominant noise source is additive noise at the demodulator input. We define the noise improvement ratio (NIR(f)) as the auto spectral density of the uncompensated message divided by the auto spectral density of the compensated message; i.e.,

$$\mathrm{NIR}(f) = \frac{\text{auto–spectrum of } d_m(t)}{\text{auto–spectrum of compensated } d_m(t)}$$

Figure (14) shows NIR(f) with decibels as the units of the ordinate. Notice that NIR(f) is very good out to about 1200 Hz, but that past 2000 Hz it falls below 0 dB.

APPLICATION

The message is on IRIG channel 11B, which has a center frequency of 96 000 Hz and deviation limits of ± 4000 Hz. The reference is on IRIG channel 8A, which has a center frequency of 72 000 Hz and deviation limits of ± 2000 Hz. Although the message and reference are a pressure and temperature signal, respectively, we will not convert to the appropriate engineering units because the conversion is of no significance to our discussion. The range of both the pressure and temperature is ± 2.5 V. The form of the temperature at the discriminator output is

$$d_c(t) = c + \frac{72\,000}{800}\,\frac{d\,n(t)}{dt} + c\,\frac{d\,n(t)}{dt}$$

$$\approx c + 90\,\frac{d\,n(t)}{dt}$$

where c is the mean value of $d_c(t)$.

In Equation (13), $d_c(t)$ is used in place of $d_r(t)$ to yield a partially compensated signal. In this application the scaling and subtraction expressed by Equation (13) were done in software on digitized records. Figures (15), (16), and (17) show the uncompensated pressure, compensated pressure, and temperature with its mean adjusted to 0. Note that $d_c(t)$ has both a periodic and random appearance. Figures (18), (19), and (20) show discrete Fourier transforms of the uncompensated pressure, compensated pressure, and temperature, respectively. The peaks at 10 Hz and higher in the uncompensated pressure are flutter induced. One of the peaks between 2 and 4 Hz is also flutter induced.

CONCLUSION

An explicit equation for flutter error has been developed from the fundamental equations describing frequency modulation and demodulation. The equation is used to obtain a tape speed compensation algorithm which theoretically achieves full compensation, even though the instantaneous frequency of the reference FM signal is fixed (but not necessarily at center band). Comparison of this algorithm with the one that varies the pulse width in a pulse-averaging discriminator would probably yield some interesting results. New tape speed compensation methods, for which NIR(f) does not go below 0 dB, could be developed by applying adaptive noise-cancelling techniques (14) to the problem.

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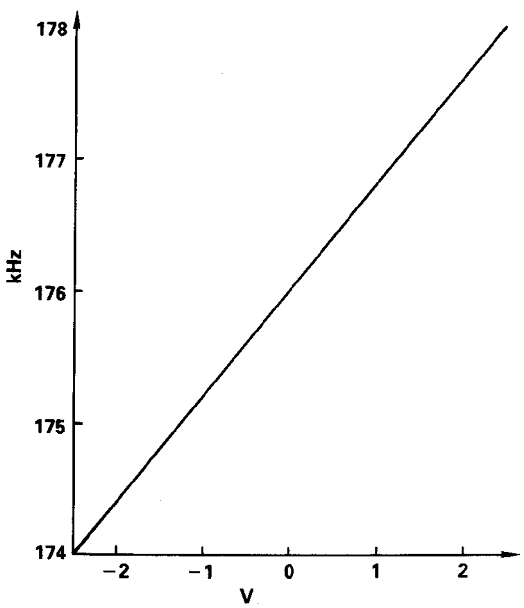


Figure 1. Instantaneous Frequency Versus Message Amplitude

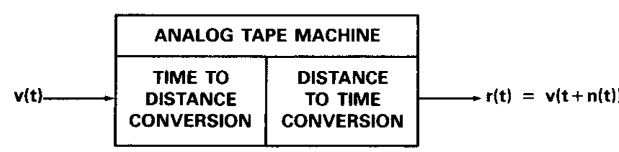


Figure 2. Independent Variable Conversions Introduce the Time Base Error, n(t)

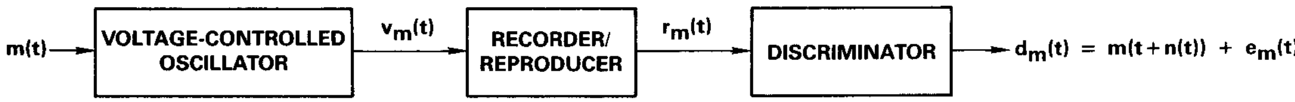


Figure 3. Time Base Error Causes an Additive Amplitude Error, $E_m(t)$

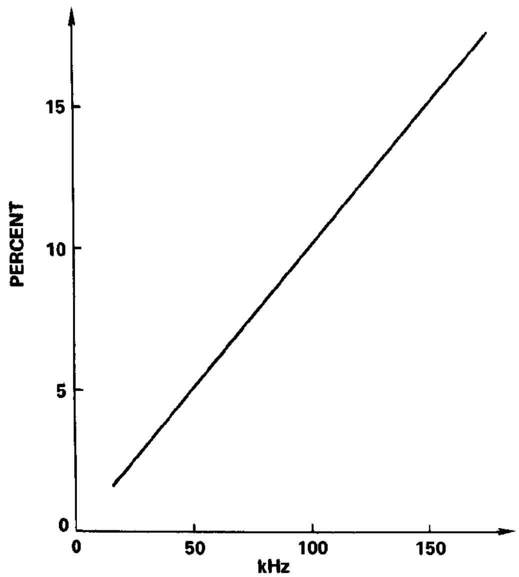


Figure 4. Relative Error Versus Center Frequency for IRIG A Channels

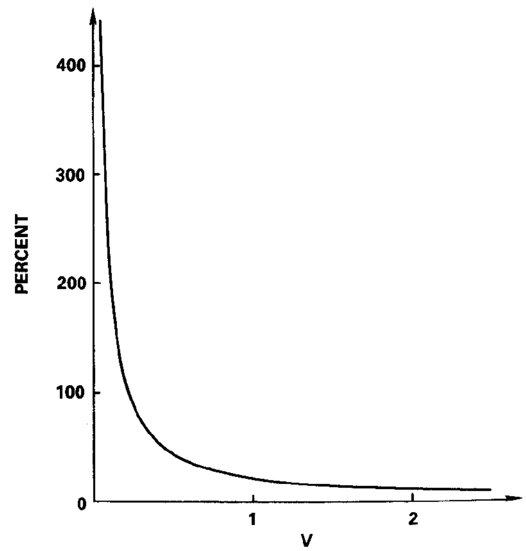


Figure 5. Relative Error Versus Message Amplitude for IRIG Channel 21 A

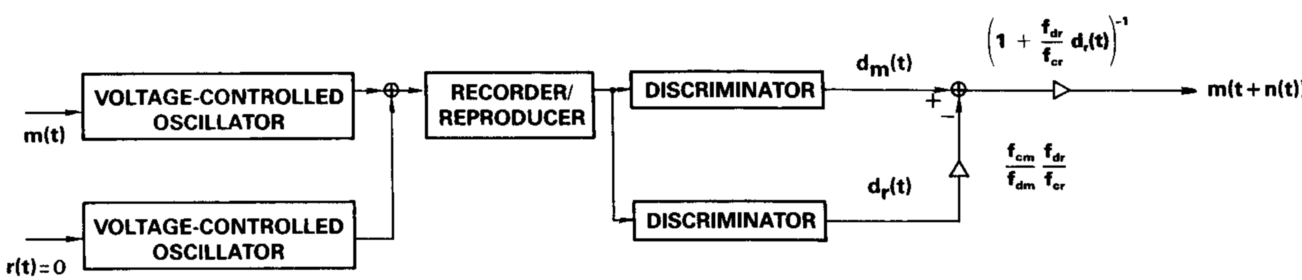


Figure 6. Block Diagram of Tape Speed Compensation Algorithm

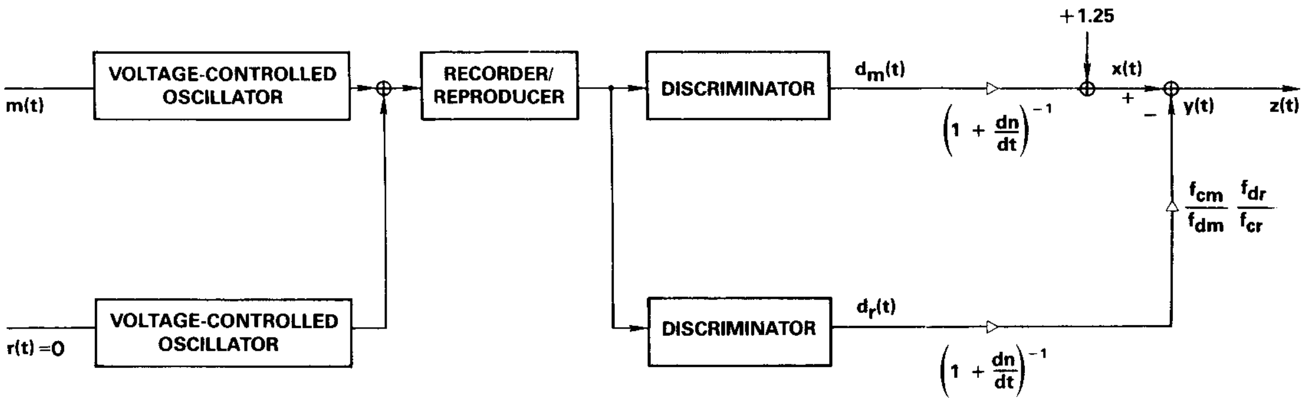


Figure 7. Block Diagram of Empirical Confirmation Experiment

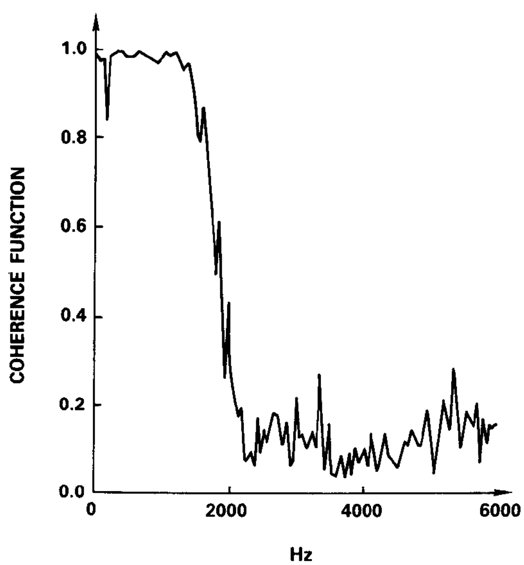


Figure 8. Coherence Function Between x(t) and y(t)

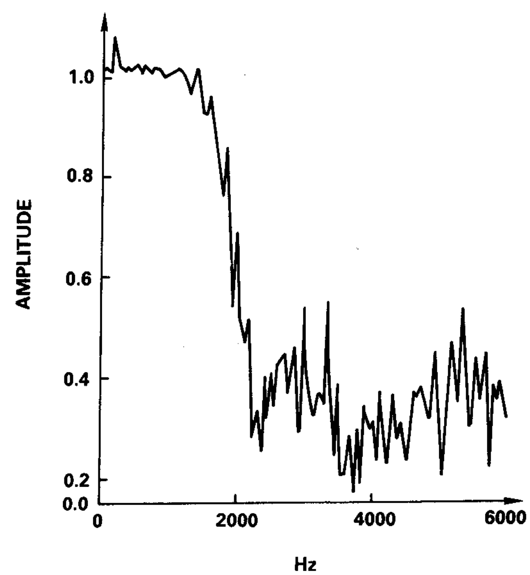


Figure 9. Frequency Response Function Between x(t) and y(t)

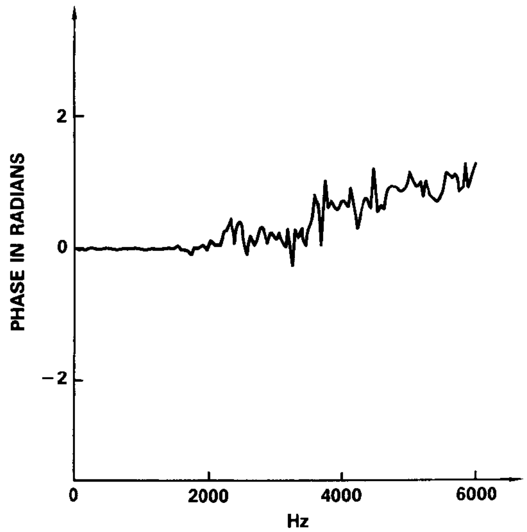


Figure 10. Frequency Response Function Between x(t) and y(t)

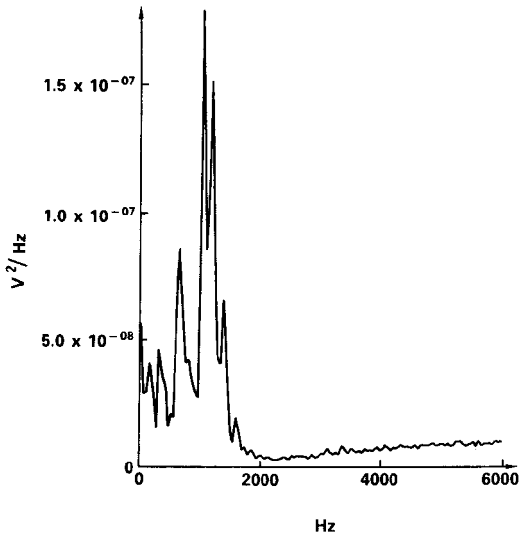


Figure 11. Auto-Spectral Density of x(t)

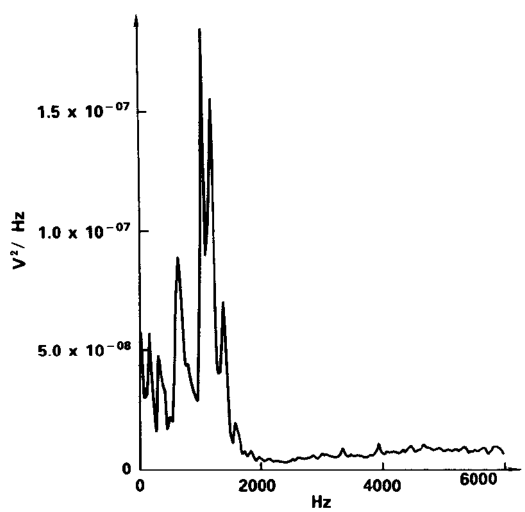


Figure 12. Auto-Spectral Density of y(t)

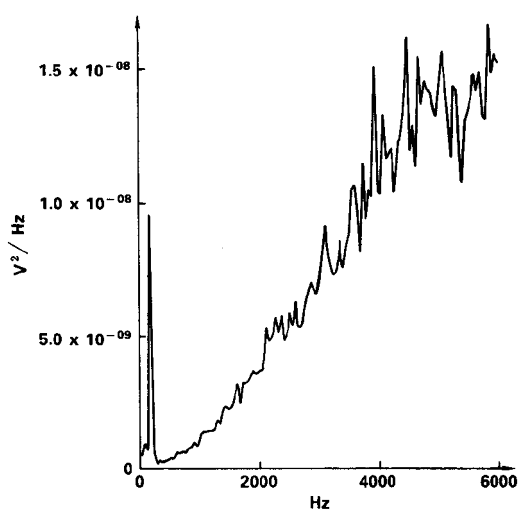
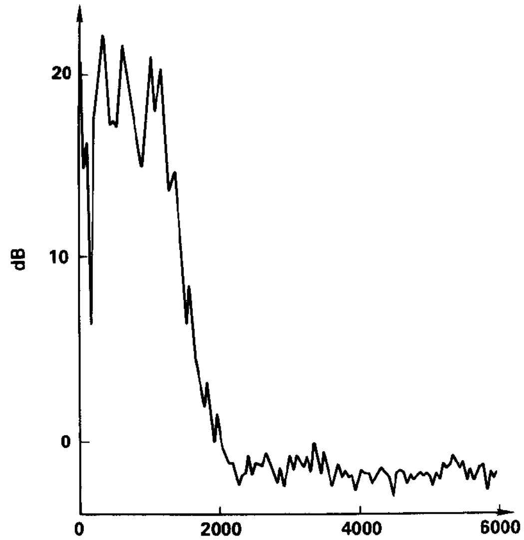
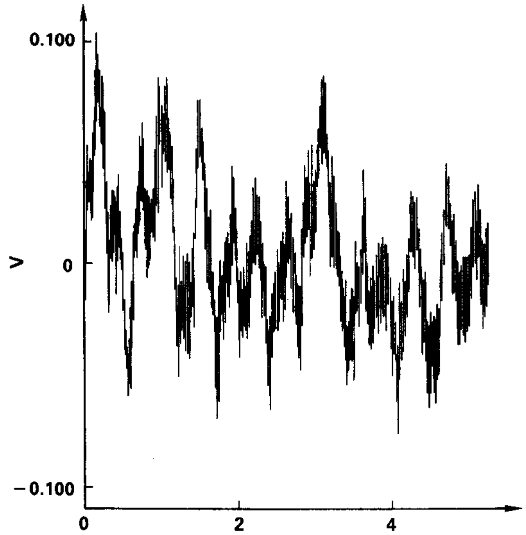


Figure 13. Auto-Spectral Density of Compensated Message



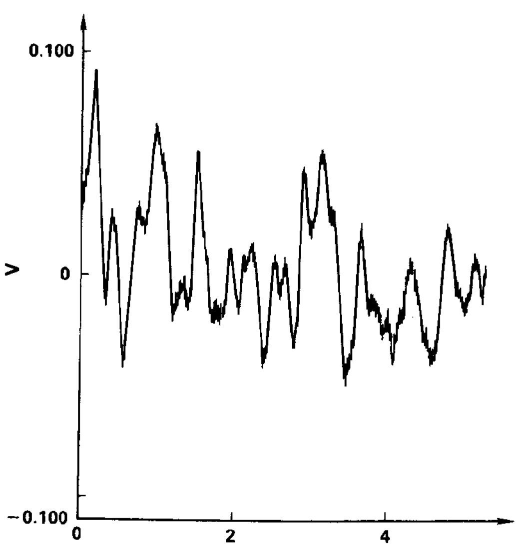
Hz

Figure 14. Noise Improvement Ratio



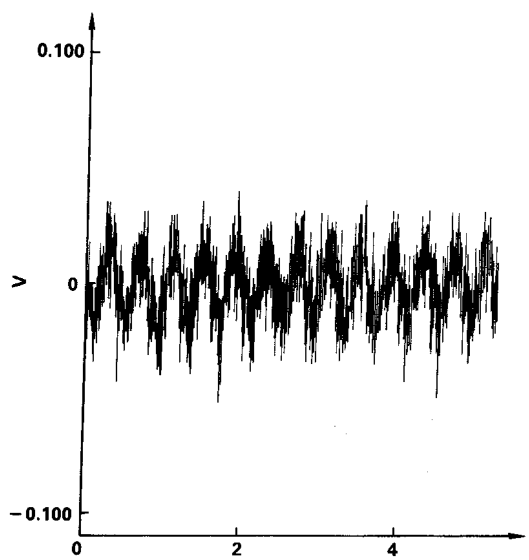
SECONDS

Figure 15. Uncompensated Pressure



SECONDS

Figure 16. Compensated Pressure



SECONDS

Figure 17. Temperature With Mean Value Adjusted to Zero

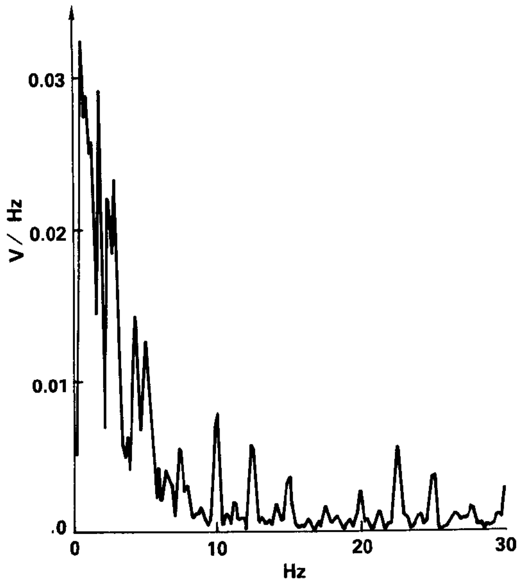


Figure 18. Discrete Fourier Transform of Pressure

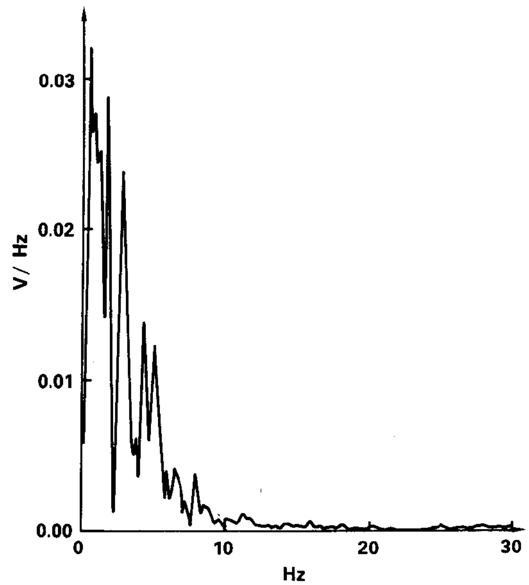


Figure 19. Discrete Fourier Transform of Compensated Pressure

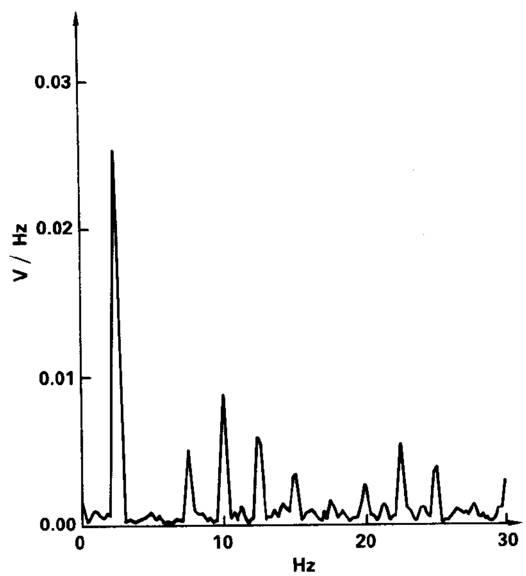


Figure 20. Discrete Fourier Transform of Temperature