SPECTRAL SHAPE PREDICTOR FOR PCM/FM TRANSMISSIONS

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ABSTRACT

This paper will demonstrate a computer application program to predict the spectral shape and IRIG-RF bandwidth required for a PCM/FM telemetry system.

A filtered PCM bit stream is constructed from a Fourier series and used to FM-modulate the RF carrier. The Fourier transform of the FM-wave is calculated numerically, and voltage amplitude levels are plotted on a Tektronix graphic display. Several PCM bit patterns are used and a composite display is formed to approximate an actual PCM/FM spectrum. A comparison of predicted and actual spectra will be presented.

INTRODUCTION

All types of RF transmissions at Government-operated test ranges must adhere to IRIG (Inter-Range Instrumentation Guideline) requirements. Bandwidth requests made to the test range should be as accurate as possible. Overstating the requirements may cause longer delays in order to clear the larger-than-needed bandwidth. On the other hand, if the requested bandwidth is exceeded there is the possibility that the test may be constrained or cancelled by the range.

The program used in this paper follows a digital modulating waveform (PCM) through the RF modulation (FM) to predict the RF signal spectra. Since a PCM source will generate varying bit patterns, several bit patterns are used and the results averaged to produce a composite PCM/FM RF spectrum.

METHOD

The following steps are performed in predicting a PCM/FM signal spectrum:

1. The maximum PCM bit rate will be used to define a 1010 PCM bit pattern. A Fourier series is used to define a periodic modulating waveform:

$$f(t) = -1 + \frac{2\tau}{T} + \frac{4\tau}{T} \sum_{n=1}^{\infty} \left(\frac{\sin n\pi\ \tau/T}{n\pi\ \tau/T}\right) \cos \left(\frac{2\pi\ nt}{T}\right)$$

With pulse width = τ and pulse period = T.

Each harmonic is filtered as the waveform is calculated, thereby producing a filtered time-domain waveform.

A Gaussian premodulation filer is used with filter gain

$$H(f) = e^{-0.35 \left(\frac{nf_1}{f_g}\right)^2}$$

where f_1 = fundamental frequency

f_g = cutoff frequency (3 dB down)

n = harmonic number of f ($f = nf_1$)

2. The filtered time waveform is used to FM modulate the RF carrier:

$$e_0(t) = \sin \left[2\pi f_c t + 2\pi\ \Delta f \int_0^t g(t)dt\right]$$

where $e_0(t)$ = FM modulated signal

f_c = Carrier frequency

Δ_f = Carrier deviation

g(t) = Filtered PCM signal

1. The Fourier integral is calculated at integer multiples of the PCM modulating frequency, giving sideband amplitudes around the RF carrier.

The Fourier integral is:

$$e(j\omega) = \int_{-T/2}^{T/2} e_0(t)\ [\cos\ \omega t - j\sin\ \omega t]dt$$

Sideband amplitudes are

$$e_{n,f_1} = \frac{2}{T} \, |e(jnf_1)|.$$

2. The line spectrum is plotted for the resultant FM modulated carrier.

3. Alternate PCM bit patterns are selected, and steps 1 through 4 are performed. Frequencies within a 3-KHz band are averaged, and a composite spectrum is plotted. The IRIG bandwidth requirement is calculated for the composite.

   IRIG states the bandwidth at $(55.0 + 10*\log_{10} P_t)$ dB below the unmodulated carrier must be within amount requested (where P_t = transmitted power).

   Eight different PCM patterns are used:

   τ = PCM bit duration (sec)

   T = Pattern period (sec)

   a. 10101010 - square wave - $\tau/T = 1/2$, $f_1 = 1/2\tau$

   b. 11001100 - $\tau/T = 1/2$, $f_1 = 1/4\tau$

   c. 111000111000 - $\tau/T = 1/2$, $f_1 = 1/6\tau$

   d. 111100001111 - $\tau/T = 1/2$, $f_1 = 1/8\tau$

   e. 1111100000 - $\tau/T = 1/2$, $f_1 = 1/10\tau$

   f. 111111000000 - $\tau/T = 1/2$, $f_1 = 1/12\tau$

   g. 111111001111 - $\tau/T = 6/8$, $f_1 = 1/8\tau$

   h. 100000001000 - $\tau/T = 1/8$, $f_1 = 1/8\tau$

   With case a of highest frequency, and case f of lowest frequency.

TEST CASE

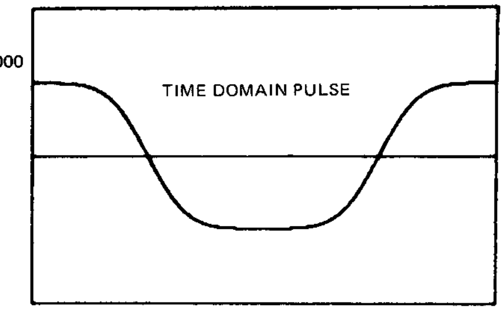
A recent vehicle telemetry system used a bipolar PCM unit at 670.255 kbps FM modulating the RF carrier of 2.2315 GHz with 234-KHz deviation/volt. A Gaussian premodulation filter with cutoff (3 dB down) at 670.255 KHz was used. The average transmitted power was 2 watts.

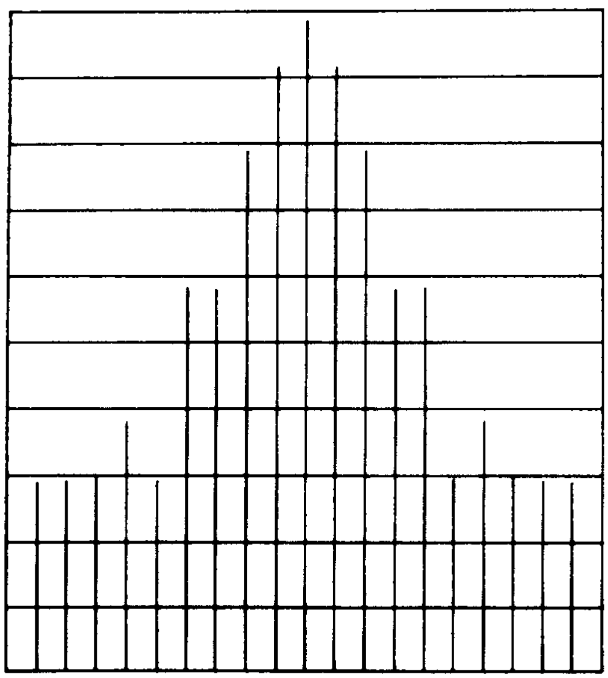
Figure 1 shows the first PCM bit pattern used (1010) and its result. Figure 2 shows the composite spectrum after all the bit patterns described earlier are used. Figure 3 was taken from a spectrum analyzer display during a field measurement of the RF bandwidth.

BIPOLAR PCM/FM/NRZ ANALYSIS FOR PATTERN = 10101010

PULSE WIDTH = 0.1491969E-05
NUMBER OF HARMONICS USED = 20
CARRIER FREQUENCY = 0.2231500E+10
FILTER CUTOFF FREQUENCY = 0.6702550E+06
MODULATION FREQUENCY = 0.3351275E+06
TRANSMITTED POWER (WATTS) = 2.00

PERIOD = 0.2983939E-05
NUMBER OF FOURIER INTERVALS = 4000
CARRIER DEVIATION = 0.2340000E+06





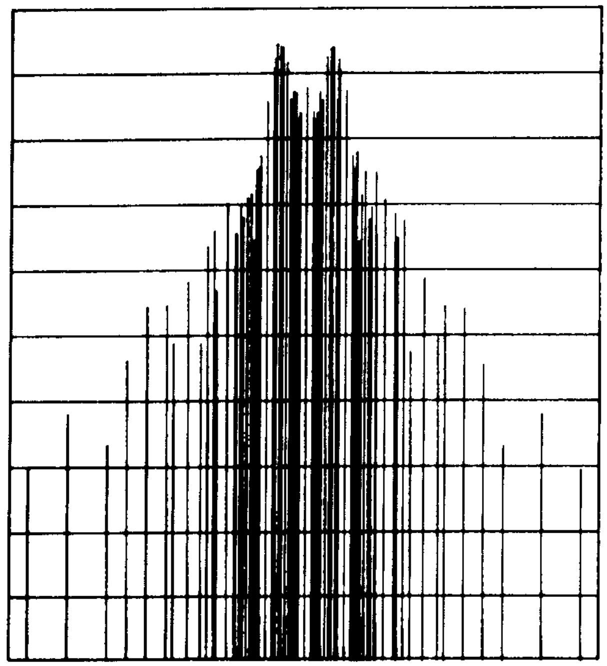
FREQUENCY	AMPLITUDE	DB FROM UNMOD. CARRIER	
0.2228149E+10	0.2815052E-03	-71.01	SIDEBAND 10
0.2228484E+10	0.2811060E-03	-71.02	SIDEBAND 9
0.2228819E+10	0.2867136E-03	-70.85	SIDEBAND 8
0.2229154E+10	0.3198585E-03	-69.90	SIDEBAND 7
0.2229489E+10	0.7984024E-03	-61.96	SIDEBAND 6
0.2229824E+10	0.2911174E-03	-70.72	SIDEBAND 5
0.2230159E+10	0.8043454E-02	-41.89	SIDEBAND 4
0.2230495E+10	0.7912190E-02	-42.03	SIDEBAND 3
0.2230830E+10	0.8681380E-01	-21.23	SIDEBAND 2
0.2231165E+10	0.3725537E+00	-8.58	SIDEBAND 1
0.2231500E+10	0.8409341E+00	-1.50	CARRIER
0.2231835E+10	0.3724768E+00	-8.58	SIDEBAND 1
0.2232170E+10	0.8685783E-01	-21.22	SIDEBAND 2
0.2232505E+10	0.7905544E-02	-42.04	SIDEBAND 3
0.2232841E+10	0.8042888E-02	-41.89	SIDEBAND 4
0.2233176E+10	0.2997035E-03	-70.47	SIDEBAND 5
0.2233511E+10	0.8137899E-03	-61.79	SIDEBAND 6
0.2233846E+10	0.3071661E-03	-70.25	SIDEBAND 7
0.2234181E+10	0.2864834E-03	-70.86	SIDEBAND 8
0.2234516E+10	0.2823996E-03	-70.98	SIDEBAND 9
0.2234851E+10	0.2809505E-03	-71.03	SIDEBAND 10

START = 0.2228149E+10 HZ   STOP = 0.2234851E+10 HZ
SPAN = 0.6702560E+07 HZ
10 DB/VERTICAL DIVISION
BANDWIDTH = 0.2681376E+07 AT -58.01 DB
FROM UNMODULATED CARRIER

Figure 1. Spectral Prediction for Maximum Bit Rate

The telemetry system was configured such that the modulation could not be turned off during bandwidth testing, and therefore the relation between the modulated and unmodulated carrier could not be experimentally performed. By approximating the filtered PCM patterns with a sine wave and averaging the corresponding J_0 Bessel function terms, the modulated carrier is found to be $\approx$10 dB down from the unmodulated carrier. The predicted value of -12.19 dB is found in Table I corresponding to the carrier frequency of 2.2315 GHz.

NEW FREQUENCY SPAN ?
YES
NEW PLOTTING SPAN (HZ) ?
5.0E+6
COMPOSITE OF PCM PATTERNS
CARRIER FREQUENCY = 0.2231500E+10 CARRIER DEVIATION = 0.2340000E+06
FILTER CUTOFF FREQUENCY = 0.6702550E+06
TRANSMITTED POWER (WATTS) = 2.00
PCM BIT RATE = 670255 BITS/SECOND
DISPLAY COMPOSED OF AVERAGE OF AMPLITUDES



START = 0.2229000E+10 HZ STOP = 0.2234000E+10 HZ
SPAN = 0.5000000E+07 HZ
10 DB/VERTICAL DIVISION
BANDWIDTH = 0.3012736E+07 AT -58.01 DB
FROM UNMODULATED CARRIER

Figure 2. Spectral Prediction for PCM Composite

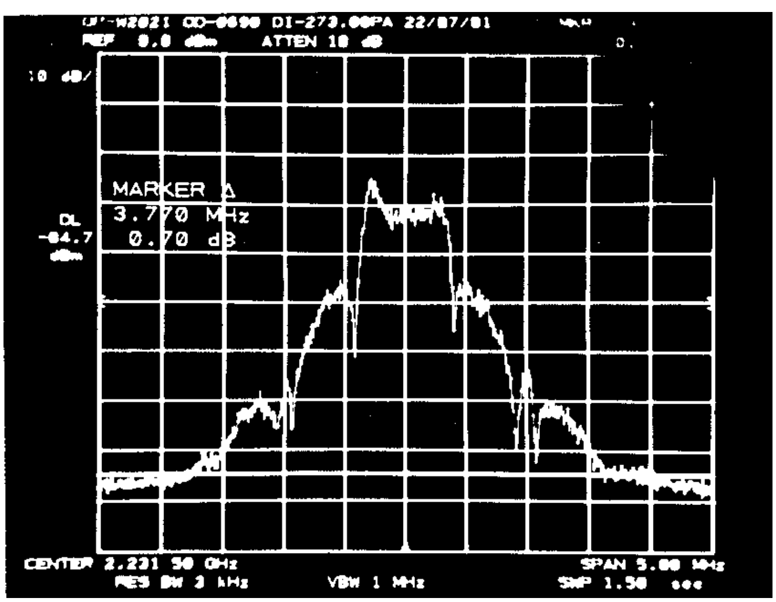


Figure 3. Spectrum Analyzer Display

Table I. PCM Composite Values (Page 1 of 2)

Composite of PCM patterns
Carrier frequency = 0.2231500E+10 Carrier deviation = 0.2340000E+06
Filter cutoff frequency = 0.6702550E+06
Transmitted power (watts) = 2.00
PCM bit rate = 670255 bits/second
Display composed of average of amplitudes

Frequency	Amplitude	DB from unmod. carrier
0.2228149E+10	0.2815052E-03	-71.01
0.2228484E+10	0.2811060E-03	-71.02
0.2228819E+10	0.2867136E-03	-71.85
0.2229154E+10	0.3198585E-03	-69.90
0.2229489E+10	0.7984024E-03	-61.96
0.2229824E+10	0.4700736E-03	-66.56
0.2229992E+10	0.2023081E-02	-53.88
0.2230159E+10	0.5228543E-02	-45.63
0.2230327E+10	0.5415784E-02	-45.33
0.2230383E+10	0.2765352E-02	-51.16
0.2230495E+10	0.8205358E-02	-41.72
0.2230606E+10	0.2758632E-02	-51.19
0.2230662E+10	0.1530495E-01	-36.30
0.2230718E+10	0.2004726E-01	-33.96
0.2230746E+10	0.6998275E-02	-43.10
0.2230830E+10	0.3249869E-01	-29.76
0.2230897E+10	0.1881898E-01	-34.51
0.2230914E+10	0.1906468E-01	-34.40
0.2230941E+10	0.2990776E-01	-30.48
0.2230964E+10	0.2496915E-01	-32.05
0.2230997E+10	0.3537595E-01	-29.03
0.2231031E+10	0.3833299E-01	-28.33
0.2231053E+10	0.1694567E-01	-35.42
0.2231081E+10	0.5696936E-01	-24.89
0.2231098E+10	0.6070417E-01	-24.34
0.2231109E+10	0.7429257E-01	-22.58
0.2231165E+10	0.1886713E+00	-14.49
0.2231221E+10	0.3367525E+00	-9.45
0.2231232E+10	0.4116872E+00	-7.71
0.2231249E+10	0.5219916E+00	-5.65
0.2231277E+10	0.5058855E+00	-5.92
0.2231299E+10	0.4899905E+00	-6.20
0.2231332E+10	0.3793966E+00	-8.42
0.2231366E+10	0.2011260E+00	-13.93
0.2231388E+10	0.2309812E+00	-12.73
0.2231416E+10	0.2251551E+00	-12.95
0.2231433E+10	0.1426258E+00	-16.92
0.2231444E+10	0.1597057E+00	-15.93
0.2231500E+10	0.2456722E+00	-12.19
0.2231556E+10	0.1612491E+00	-15.85
0.2231567E+10	0.1426374E+00	-16.92
0.2231584E+10	0.1592613E+00	-15.96
0.2231612E+10	0.2299149E+00	-12.77
0.2231634E+10	0.2007715E+00	-13.95
0.2231668E+10	0.4238525E+00	-7.46
0.2231701E+10	0.4900588E+00	-6.20
0.2231723E+10	0.5058204E+00	-5.92
0.2231751E+10	0.2013417E+00	-13.92
0.2231768E+10	0.4118540E+00	-7.71
0.2231779E+10	0.3351176E+00	-9.50
0.2231835E+10	0.2331055E+00	-12.65
0.2231891E+10	0.7600643E-01	-22.38
0.2231902E+10	0.6051686E-01	-24.36
0.2231919E+10	0.8099684E-01	-21.83
0.2231947E+10	0.1703327E-01	-35.37
0.2231969E+10	0.3818651E-01	-28.36
0.2232003E+10	0.5706299E-01	-24.87
0.2232036E+10	0.2504191E-01	-32.03

Table I. PCM Composite Values (Page 2 of 2)

Frequency	Amplitude	DB from unmod. carrier
0.2232059E+10	0.3065473E-01	-30.27
0.2232086E+10	0.5710557E-01	-24.87
0.2232103E+10	0.1891144E-01	-34.47
0.2232170E+10	0.3531460E-01	-29.04
0.2232254E+10	0.2777859E-01	-31.13
0.2232282E+10	0.1820171E-01	-34.80
0.2232338E+10	0.2452384E-01	-32.21
0.2232394E+10	0.2429753E-02	-52.29
0.2232505E+10	0.8812162E-02	-41.10
0.2232617E+10	0.3175510E-02	-49.96
0.2232673E+10	0.5458460E-02	-45.26
0.2232841E+10	0.5227816E-02	-45.63
0.2233008E+10	0.1966799E-02	-54.12
0.2233176E+10	0.4654382E-03	-66.64
0.2233511E+10	0.8137899E-03	-61.79
0.2233846E+10	0.3071661E-03	-70.25
0.2234181E+10	0.2864834E-03	-70.86
0.2234516E+10	0.2823996E-03	-70.98
0.2234851E+10	0.2809505E-03	-71.03

Bandwidth = 0.3012736E+07 at -58.01 db
from unmodulated carrier

Using the value of the modulated carrier to be -10 dB from the unmodulated carrier, the bandwidth from Figure 3 is found to be approximately 3 MHz. This compares favorably to the predicted value of 3.012 MHz.

In comparing the predicted versus actual spectral shapes, a couple of points should be mentioned:

1. The sample time of the spectrum analyzer was 1.5 sec, allowing a large number of modulated PCM pulses to build the display; the predicted spectrum used the eight cases mentioned earlier and applied equal weight to each case to determine the average.

2. The effects of the spectrum analyzer, including averaging, sampling methods, and display characteristics were not considered.

CONCLUSION

A tool for the prediction of PCM/FM IRIG RF-bandwidth and RF spectral shape has been demonstrated to be closely matched to the actual RF spectrum. Although this paper has limited itself to PCM/FM, the program method and coding are applicable to many forms of communications types. A copy of the FORTRAN source code can be obtained through the author.