PRESAMPLING FILTERING, SAMPLING AND QUANTIZATION EFFECTS ON THE DIGITAL MATCHED FILTER PERFORMANCE

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ABSTRACT

Due to the increased capability and reduced cost of digital devices, there has recently been a growing trend to digitize the matched-filtering data detector in the receiver. Comparing with an idealized integrate-and-dump analog matched filter, the digital matched filter (DMF) requires more E_b / N_o in order to achieve the same bit error rate performance because of the presampling filtering, sampling, and quantization effects. This paper analyzes the performance degradation resulting, separately and jointly, from these three effects.

Quantitative results are provided for commonly chosen sets of design parameters. For a given performance degradation budget and complexity limitation, these results could be applied to choose the optimum DMF design parameters including the presampling filter bandwidth, the sampling rate, the number of quantization bits, and the spacing between adjacent quantization levels.

1.0 INTRODUCTION

The study of sampling and quantization effects on the digital matched filter (DMF) has recently received much attention in evaluating the performance of digital receivers that employ matched-filter detection.[1, 2, 3] A fundamental case of interest is the case when the input to the DMF consists of (1) an NRZ-L PCM baseband signal and (2) an additive white Gaussian noise process. The NRZ-L PCM signal appears in the time domain as a train of rectangular pulses of voltage levels +V or -V (see Figure 1), depending on whether the transmitted data bit is a 0 or a 1. For such a signal plus noise, it is well known that the integrate-and-dump filter is the optimum (or the matched-filtering) detector which results in the minimum error probability as shown in Figure 2. The increased stability, reliability, and flexibility, as well as the decreased size and cost make the digital implementations of many analog matched filters highly desirable. Figure 3 illustrates one possible digital implementation of the integrate-and-dump filter. As evident from the figure itself, the performance of this digital integrate-and-dump (matched) filter depends upon three system parameters:

(1) B (Hz), the bandwidth of the presampling low-pass filter

(2) f_s (samples/bit), the sampling rate of the sampler in samples per data bit

(3) m (bits), the number of bits of the quantizer

Because of presampling filtering, sampling and quantization effects, the DMF requires more E_b/N_o than the analog matched filter. Thus, the degradation factor D of the DMF may be defined as the required increase in E_b/N_o for the DMF in order to yield the same error probability as the analog matched filter. In what follows, the degradation factor is derived in detail with quantatative results presented for commonly chosen sets of design parameters.

2.0 ANALYSIS

This section is devoted to deriving the error probabilities and hence the degradations for the DMF. Refer to the block diagram of the DMF in Figure 3. Let the received signal plus noise at the input to the DMF be expressed as

$$x(t) = s(t) + n(t) \qquad (1)$$

where

n(t) = a stationary white Gaussian noise process of two sided spectral density $N_o/2$

$\sum_j s_j \, u(t-jT)$ = a rectangular pulse train of voltage levels +V or -V

and

$$s_j = \begin{cases} +1 \text{ if the jth data bit is 0} \\ -1 \text{ if the jth data bit is 1} \end{cases}$$

u(t) = a rectangular pulse of amplitude V ana duration T

The energy per bit to one-sided noise density ratio is hence given by

$$E_b/N_o = V^2T/N_o \qquad (2)$$

If the data bits 0 and 1 have equal a priori probabilities, then, by symmetry*, it can be easily proved that the error probability $P(\epsilon)$ is equal to the conditional error probability $P(\epsilon/0)$ [or $P(\epsilon/1)$]:

$$P(\epsilon) = P(0)\,P(\epsilon/0) + P(1)\,P(\epsilon/1) = P(\epsilon/0) = P(\epsilon/1) \tag{3}$$

This simplifies the problem because one can derive $P(\epsilon)$ by computing only one conditional error probability. Assuming that the data bit 0 is transmitted between t=0 and t=T leads to

$$x(t) = u(t) + n(t) \tag{4}$$

If the pre-sampling LPF H(f) is an ideal LPF of bandwidth B

$$H(f) = \begin{cases} 1 & ; \ |f| < B \\ 0 & ; \ |f| > B \end{cases} \tag{5}$$

then at the output of the LPF the signal plus noise becomes (see, for example, [4]) a gaussian random process

$$r(t) = \underbrace{\frac{V}{\pi}[Si(2\pi Bt) + Si(2\pi B(T-t))]}_{s'(t)} + n'(t) \tag{6}$$

Here $$Si(z) \equiv \int_0^z \frac{\sin(y)}{y}\,dy \tag{7}$$

is called the sine integral of z and

> n'(t) = a narrow-band gaussian noise process of zero mean, variance N_oB, and autocorrelation function $R_n(t)$, where

$$R_n(t) = \overline{n(\tau)n(t+\tau)} \equiv N_0 \frac{\sin(2\pi Bt)}{2\pi t} = N_0B\ \mathrm{sinc}(2Bt) \tag{8}$$

Figure 4 illustrates s'(t) for three different values of filter bandwidth B, where s'(t) is the response of an idealized LPF to a rectangular pulse of width T sec and amplitude V. Notice that the larger the value of B compared with the bit rate 1/T, the greater s'(t)

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* A commonly used odd-symmetical quantizer in the DMF is implied.

resembles the input rectangular pulse u(t). Note also that the effect of intersymbol interference would tend to enhance the magnitude of S'(t) at one time (provided that neighboring bits are also 0) but to reduce it at another (provided that the neighboring bits are 1) with equal probabilities.

The filtered signal plus noise r'(t) is then fed through an A/D converter consisting of a sampler and a quantizer. Synchronous* and uniform sampling is assumed such that if the sampling rate is f_s samples/bit, the sampling instants t_i are

$$t_i = \frac{(2i-1)}{2 f_s} \qquad ; \; i = 1, 2, ----, f_s \tag{9}$$

and

$$r_i = r(t_i) = s'(t) + n'(t) \tag{10}$$

For an m-bit uniform-step quantizer as shown in Figure 5, the jth threshold q_j and the jth output level v_j are given by

$$q_j = -L + \frac{(j-1)L}{2^{m-1}-1} \qquad ; \; j = 1, 2, ---, 2^m-1 \tag{11}$$

$$v_j = -1 + \frac{2(j-1)}{2^m-1} \qquad ; \; j = 1, 2, ---, 2^m \tag{12}$$

where L= the magnitude of the maximum threshold = $-q_1 = q_{2^m-1}$ (See Figure 5) (13)

The output $\hat{r}_i$ of tne quantizer can thus be represented as

$$\hat{r}_i = Q(r_i) = v_j \text{ if } q_{j-1} < r_i \leq q_j \tag{14}$$

where j = 1, 2, ---, 2^m ; q_o and q_2m

Refer back to Figure 3, the quantized samples r_i over one-data bit period (i.e., i = 1, 2, ---, fs) are summed and dumped by an accumulator to approximate the integrate-and-dump operation in the analog domain:

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* That is, the sampler is properly synchronized with respect to the bit rate and the bit transition time of the input signal.

$$y = \sum_{i=1}^{f_s} \hat{r}_i \tag{15}$$

Synchronous timing is again supposed to be available at the accumulator. The output switch closes momentarily at $t = kT^-$ right before the dumping action to sample the value of y, and a hard decision is then made based upon the sign of y. Reviewing Eq.(3), one readily writes the probability of error for the DMF as

$$P_d(\epsilon) = P_d(\epsilon/o) = P[y<o/s(t) = +u(t)] \tag{16}$$

Because the error probability $P_a(\epsilon)$ for the optimum (analog integrate-and- dump) filter is given by

$$P_a(\epsilon) = 1/2 \text{ erfc } \left(\sqrt{E_b/N_o} \right) \tag{17}$$

or, conversely,

$$E_b/N_o = [\text{erfc}^{-1} [2P_a(\epsilon)]]^2 \tag{18}$$

the effective E_b/N_o for the DMF may be defined similarly as

$$(E_b/N_o)_d = [\text{erfc}^{-1}[2P_d(\epsilon)]]^2 \tag{19}$$

where

$$\text{erfc } (z) \equiv \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-u^2} du = 1 - \text{erf } (z) \tag{20}$$

is the complementary error function and $\text{erfc}^{-1}(z)$ is the inverse of erfc(z) such that

$$\text{erfc}^{-1} [\text{erfc}(z)] = z \tag{21}$$

The physical meaning of $(E_b/N_o)_d$ for the DMF can be understood as follows. Suppose for a given input E_b/N_o the error probability at the output of the DMF is $Pd(\epsilon)$. If the DMF is replaced with the analog intergrate-and-dump (or the optimum) filter, then it requires only $(E_b/N_o)_d$ ($<E_b/N_o$) at the input to maintain the same error probability performance at the output. In other words, the DMF needs a factor of

$$D = (E_b/N_o)/(E_b/N_o)_d \tag{22}$$

more energy per bit to one-sided noise density ratio than the optimum filter in order to attain the same error probability performance, Where D -- according to Section 1 -- is defined as the degradation factor of the DMF.

2.1 INFINITE-BIT QUANTIZATION

Consider the situation when the quantizer in Figure,3 is an infinite-bit quantizer. Then $\hat{r}_i$ is passed through without distortion

$$\hat{r}_i = r_i = s'(t_i) + n'(t_i)$$
$$= \text{a gaussian random variable with mean } s'(t_i) \text{ and variance } N_oB \tag{23}$$

This is equivalent to the case when the only nonlinear device -- the quantizer -- in Figure 3 is removed, thereby simplifying greatly the subsequent analysis. Incorporating (23) into (15) leads to

$$y = \sum_{i=1}^{f_s} r_i = \sum_{i=1}^{f_s} [s'(t_i) + n'(t_i)] \tag{24}$$

Because y is a summation of gaussian random variables r_i, it is also gaussian-distributed With mean

$$\overline{y} = \sum_{i=1}^{f_s} s'(t_i) \tag{25}$$

and variance

$$\sigma_y^2 = \overline{\left[\sum_{i=1}^{f_s} n'(t)\right]^2} = f_s N_o B + 2 \sum_{i=1}^{f_s-1} (f_s - i) R_n[(iT)/f_s] \tag{26}$$

Inserting (8) into (26) brings about

$$\sigma_y^2 = N_o B f_s + 2 N_o B \sum_{i=1}^{f_s-1} (f_s - i) \ \text{sinc}\ [(i2BT)/f_s] \tag{27}$$

The second term in the above equation is contributed by the fact the noise samples $n'(t_i)$ are dependent; it is zero only if f_s = BT or 2 BT.

For such a gaussian-distributed y, the error probability takes the form

$$P_d(\epsilon) = P[\, y > o / s(t) = +u(t)] = 1/2 \text{ erfc } [\sqrt{(\bar{y}^2/2\sigma_y^{\,2})}] \tag{28}$$

According to (19) and (22), $(E_b/N_o)_d$ and D becomes

$$(E_b/N_0)_d = (\bar{y}^2/2\sigma_y^{\,2}) \tag{29}$$

and

$$D = [(V^2T/N_0)/(\bar{y}^2/2\sigma_y^{\,2})] \tag{30}$$

Substituting (25) and (27) into (30) and making use of (6) and (9) gives rise to

$$D = D(BT, f_s) = \frac{2\pi^2 BT\,[f_s + 2 \sum_{i=1}^{f_s-1} (f_s - i)\,\text{sinc}\,[(i2BT)/f_s]}{\left(\sum_{i=1}^{f_s} Si[[(2i-1)2\pi BT]/2f_s] + Si[2\pi BT[1-(2i-1)/2f_s]]\right)^2} \tag{31}$$

which is a function of Doth BT and f_s.

2.2 m-BIT QUANTIZATION

Consider now the case when the quantizer in Figure 3 is an m-bit uniform-step quantizer. The output of the quantizer r_i is then a discrete random variable with probabilities

$$\begin{aligned} P[\hat{r}_i = v_j] &= P\,[q_{j-1} < r_i \le q_j] \\ &= 1/2\,[\text{erf}\,[(q_j - s'(t_i))/(\sqrt{2N_0B})] - \text{erf}\,[(q_j - s'(t_i))/(\sqrt{2N_0B})]\,; \\ & j = 1, 2, ---, 2^m \end{aligned} \tag{32}$$

where q_j and $s'(t_i)$ are given by (11) and (6), respectively. For simplicity of analysis, assume f_s = 2BT so that the noise samples $n'(t_i)$ and hence $\hat{r}_i$ are independent.* The output of accumulator, y, in (15) is thus a sum of independent (discrete) random variables r_i. Invoking the theorem that the probability density function (p.d.f.) of the sum of independent random variables is equal to the convolution of p.d.f.'s of the individual random variables, one can write the p.d.f. of y as

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* As will be seen shortly in Section 3, the performance of the DMF is degraded negligibly by setting f_s= 2BT instead of . In other words, increasing the sampling rate from 2BT to any other higher values results in very little improvement in the performance of DMF. Of course this assumes there is no synchronous time error.

$$P(y) = P(\hat{r}_1) * P(\hat{r}_2) * - - - * P(\hat{r}_{f_s}) \quad (33)$$

Here the random variable y is also of discrete type With ($f_s 2^m$ - f_s+1) possible values, ranging from -f_s to +f_s (including 0). Substituting (32) into (33), one can calculate P(y) with the aid of a computer. The so-called FFT (fast Fourier transform) technique may be used to reduce the computational time required for convolution (see, e.g. [5]). Once P(y) is known, the error probability $P_d(\epsilon)$ can be readily obtained via

$$P_d(\epsilon) = P[y < o/s(t) = +u(t)] + 0.5P[y = o/s(t)=+u(t)] \quad (34)$$

The second term of the above equation, which could be non-zero for a discrete y, is included to account for the ambiguous case in which y is equal to the threshold (=0) of the decision device. In such an ambiguous case, the decision is made by tossing a fair coin -- as reflected by the factor 0.5 in (34). Once $P_d(\epsilon)$ is known, $(E_b/N_o)_d$ and the degradation D can then be computed via (19) and (22).

3.0 RESULTS

Here, results obtained by numerically computing the degradation D derived in the previous section for various values of BT, f_s, and m are present. Recall, again, that D is defined as the additional E_b/N_o required for the DMF to attain same error probability performance as the optimum (analog integrate-and-dump) filter. In order to compare the magnitudes of degradations arising from three different effects, the overall degradation D in dB will be written as the sum of D_B, D_S, and D_Q:

$$D = D_B + D_S + D_Q \text{ (dB)} \quad (35)$$

where D_B, D_S, and D_Q are degradations in dB arising from tne bandwidth limiting effect of the presampling filter, the finite sampling effect, and the quantization effect, respectively.

3.1 DEGRADATION FOR INFINITE-BIT QUANTIZATION

First of all, the degradation D in the case of infinite-bit quantization (see Eq. (31)) is computed and shown in Table 1 and Figure 6 for BT = 1, 5, 10, 20. Also included in Table 1 is the case BT = 2. For such an infinite bit quantizer (or, equivalently the quantizer in Figure 3 is removed), all the degradation is contributed by (1) the bandwidth limiting effect (namely, the effect that signal energy outside the bandwidth B is rejected) and (2) the finite sampling-rate effect. As BT $\rightarrow\infty$ (i.e., the pre-sampling filter in Figure 3 is removed) and $f_s \rightarrow\infty$ (i.e., the discrete summation becomes a continuous integration), the

DMF is effectively an analog integrate-and-dump filter; the degradation for this limiting case is, of course, 0 dB by definition.

Several remarks may be drawn from Table 1 and Figure 6:

(1) If only f_s (rather that both f_s and BT) $\rightarrow\infty$, then all the degradations are due to the bandwidth limiting effects (i.e., BT being finite):

$D = D_B(BT)$ for $m \rightarrow\infty$ and $f_s \rightarrow\infty$

According to the last row of Table 1, which is reproduced in Table 2, such degradations at BT = 1, 2, 5, 10 and 20 are 0.444, 0.223, 0.0887, 0.0442, and 0.0221 dB, respectively.

(2) A useful result from (1) above is that in order to avoid more than 0.2 dB degradation caused by the bandwidth limiting effect, the bandwidth B of the pre-sampling LPF has to be at least twice the bit rate of the input binary signal (i.e. $BT \geq 2$).

(3) For a fixed value fo $BT(\geq 1)$, the degradation D drops rapidly as the sampling rate f_s increases from 1 to 2 BT samples/bit. However, as f_s increases further from 2BT to infinite samples/bit, D in dB is reduced by no more than 8%. As shown by Table 3, the degradations resulting from $f_s = 2BT$ instead of infinite samples/bit are only 0.017 dB for BT = 1 and about (0.036/BT) dB for $BT \geq 2$.

(4) A corollary of (3) above is that for a fixed sampling rate f_s, the optimum choice of the bandwidth B (that leads to the minimum degradation for the DMF) is $B = f_s/2T$.

3.2 DEGRADATION OF DMF WITH m-BIT UNIFORM QUANTIZATION AND f_s = 2BT

For a finite-bit instead of infinite-bit quantizer, one expects some,additional degradation to be introduced by the quantization effect. Figure 7 and 8 illustrate the overall degradation D in dB vs L/N_o, the normalized maximum threshold of the quantizer for $E_b/N_o \leq -10$ dB, $f_s = 2BT$, m = 1, 2, 3, 4, 5[*], and BT = 1, 5, 10. Figures 7(a)-(c) (corresponding to $E_b/N_o \leq -10$ dB) are replotted in Figures 9(a)-(c) with the addition of two other cases: $E_bN_o = 0$ and 10 dB. For $f_s = 2BT$ and various values of m, BT, and E_b/N_o, Table 4 lists the

[*] The case m = 1 is equilvalent to ghe case when L = 0 for any m-bit quantizer.

minimum degradations[*] D in dB along with the optiumum values of $L/\sqrt{N_o}$ (given in parentheses). Several points worth noting are:

(1) For all cases investigated, the degradation D near the optimum value of $L/\sqrt{N_o}$ is essentially constant over a wide range of values of $L/\sqrt{N_o}$. This allows for a gain variation in AGC (automatic gain control circuitry with controls the threshold settings) of ±20% without significant performance degradation.

(2) According to Table 4 (and also Figure 9), both the minimum degradation D and the optimum value of the maximum threshold $L/\sqrt{N_o}$ (for a given set of values of BT and m) stays primarily unchanged when E_b/N_o increases from minus-infinity to 0 dB. As E_b/N_o increases further from 0 to 10 dB, both the minimum D and the optimum $L/\sqrt{N_o}$ increase slightly. Such increases, however, are almost entirely negligible for BT $\geq$ 2.

(3) The minimum degradation D_Q due to m-bit instead of infinite-bit quantization can be easily derived from Table 4 and are listed in Table 5. It is evident from Table 5 that D_Q depends strongly on the value of m, weakly on the value of BT, and very weakly on the value of E_b/N_o. Given BT $\geq$ 1 and $E_b/N_o \leq$ 10 dB, D $\leq$ 3.0, 0.8, 0.2, 0.06, and 0.02 dB for m=1, 2, 3, 4, and 5, respectively.

4.0 CONCLUSIONS

Using Table 2, 3, and 5, one can (a) examine the relative contributions to the overall degradations by the three different effects, (b) perform presampling-bandwidth vs sampling-rate vs quantization-bit trades, and thereby (c) choose the optimum values of BT, f_s, and m for a given requirement of the overall degradation and a given limitation on the hardware complexity.

Several major conclusions that can be drawn from all presented results are summarized here:

- A presampling filter bandwidth B on the order of once (or twice) the data bit rate is adequate because it contributes only about 0.4 (or 0.2) dB to the overall degradation of the DMF.

- A sampling rate f_s (samples/bit) = 2 BT is almost as good as f_s equal infinity, provided that there is no timing (including bit synchronization) error at the sampler.

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[*] The minimum degradation is defined as the degradation value corresponding to the optimum quantizer threshold settings.

- 3-(or 4-)bit uniform-step quantization with optimum threshold settings recovers most of the digital implementation degradation. Using infinite-bit instead of 3-(or 4-)bit quantization improves the DMF degradation by no more than ~0.2 (or 0.06) dB.

- The degradation effect is quite sensitive to the quantizer threshold settings with respect to the received noise level when low ($\leq$ 3) quantization bits are used.

- The preceding conclusions are fairly insensitive to variations of E_b/N_o from minus-infinity to 10 dB.

- Finally, listed in Table 6 are several efficient combinations of BT, f_s and m, which for a given D require the minimum number of bits (=m + $\log_2 f_s$) for the accumulator in Figure 3.

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TABLE 1 THE DEGRADATION D IN dB FOR INFINITE-BIT QUANTIZATION

fs \ BT	1	2	5	10	20
1	1.580	6.909	9.658	13.188	16.109
BT	1.580	2.006	2.444	2.664	2.806
2BT	0.461	0.238	0.0956	0.0478	0.0239
3BT	0.447	0.225	0.0898	0.0448	0.0224
4BT	0.445	0.224	0.0891	0.0444	0.0222
5BT	0.444	0.223	0.0889	0.0443	0.0221
INFINITE	0.444	0.223	0.0887	0.0442	0.0221

TABLE 2
THE DEGRADATION D_B IN dB RESULTING FROM THE BANDWIDTH LIMITING EFFECT OF PRESAMPLING FILTER FOR VARIOUS VALUES OF BT

BT	1	2	5	10	20
D_B	0.444	0.233	0.0887	0.0442	0.0221

TABLE 3 DEGRADATION Ds IN dB RESULTING FROM FINITE SAMPLING RATE

fs \ BT	1	2	5	10	20
1	1.136	6.686	9.596	13.144	16.087
BT	1.136	1.783	2.355	2.620	2.784
2BT	0.017	0.015	0.0069	0.0036	0.0018
3BT	0.003	0.002	0.0011	0.0006	0.0003
4BT	0.001	0.001	0.0004	0.0002	0.0001
5BT	$<10^{-3}$	$<10^{-3}$	0.0002	0.0001	$<10^{-4}$
INFINITE	0	0	0	0	0

TABLE 4
THE MINIMUM DEGRADATION D IN dB ALONG WITH THE OPTIMUM VALUES OF L//No (GIVEN IN PARETHESES) FOR fs=2BT AND VARIOUS VALUES OF m, BT, AND Eb/No

BT	Eb/No (dB) \ m (BIT)	1	2	3	4	5	∞
1	≤−10	3.471	1.178 (0.8)	0.652 (1.3)	0.515 (1.7)	0.477 (2.0)	0.461
	0	3.471	1.197 (0.8)	0.657 (1.4)	0.516 (1.7)	0.477 (2.0)	
	10	3.471	1.308 (1.1)	0.696 (1.6)	0.525 (2.0)	0.478 (2.3)	
5	≤−10	2.275	0.695 (0.9)	0.276 (1.7)	0.150 (2.2)	0.112 (2.7)	0.0956
	0	2.284	0.699 (0.9)	0.276 (1.7)	0.150 (2.2)	0.112 (2.7)	
	10	2.372	0.722 (1.0)	0.282 (1.7)	0.152 (2.3)	0.112 (2.7)	
10	≤−10	2.118	0.624 (1.0)	0.221 (1.7)	0.100 (2.3)	0.0638 (2.7)	0.0478
	0	2.123	0.626 (1.0)	0.221 (1.7)	0.101 (2.3)	0.0639 (2.7)	
	10	2.175	0.637 (1.0)	0.225 (1.7)	0.101 (2.3)	0.0639 (2.8)	

TABLE 5
THE MINIMUM DEGRADATIONS DQ IN dB DUE TO QUANTIZATION EFFECT FOR fs = 28T AND VARIOUS VALUES OF m, BT AND Eb/No

BT	Eb/No (dB) \ m (BIT)	1	2	3	4	5	∞
1	≤−10	3.010	0.717	0.191	0.0540	0.0160	0
	0	3.010	0.736	0.196	0.0550	0.0160	
	10	3.010	0.847	0.235	0.0640	0.0170	
5	≤−10	2.179	0.599	0.180	0.0544	0.0164	0
	0	2.188	0.603	0.180	0.0544	0.0164	
	10	2.275	0.626	0.186	0.0564	0.0164	
10	≤−10	2.070	0.576	0.173	0.0522	0.0160	0
	0	2.075	0.578	0.173	0.0532	0.0160	
	10	2.127	0.589	0.177	0.0532	0.0161	

TABLE 6
SEVERAL EFFICIENT COMBINATIONS OF THE VALUES OF BT, fs AND m ALONG WITH THE RESULTANT VALUES OF D

BT	fs (samples/data bit)	m (bits)	m+log2 fs (bits)	D (dB)
1	2	1	2	3.5
1	2	2	3	1.3
1	2	3	4	0.7
2	4	3	5	0.45
2	4	4	6	0.3
4	8	4	7	0.2
8	16	3	7	0.2
8	16	4	8	0.1

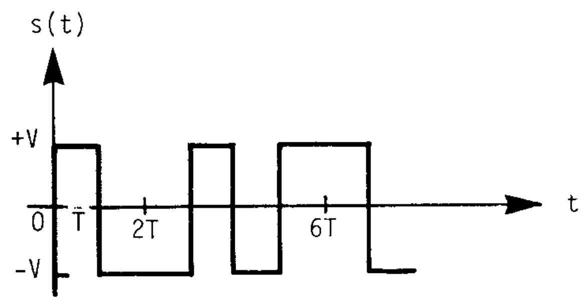

FIGURE 1 A BINARY-ENCODED NRZ-L PCM BASEBAND SIGNAL

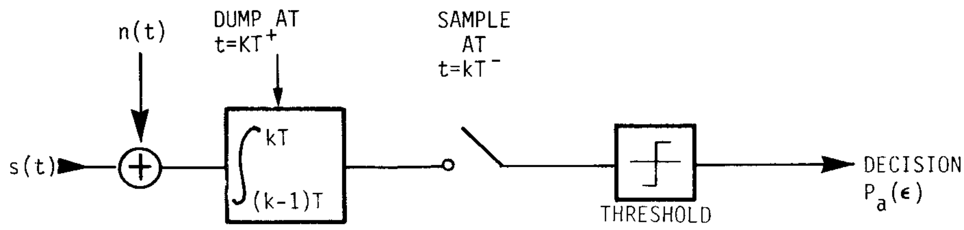

FIGURE 2 THE OPTIMUM (INTEGRATE-AND-DUMP) DETECTOR FOR A BINARY-ENCODED PCM SIGNAL

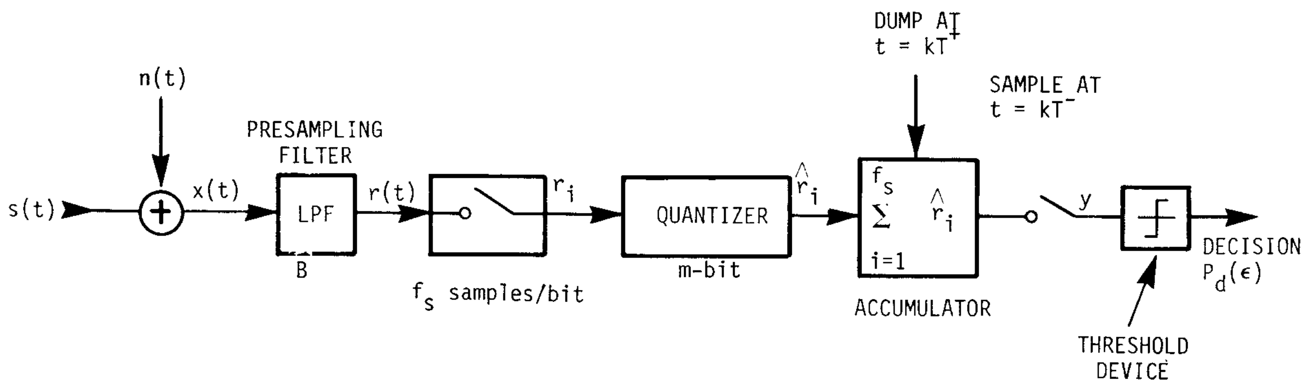

FIGURE 3 BLOCK DIAGRAM OF THE DIGITAL INTEGRATE-AND-DUMP (MATCHED) FILTER

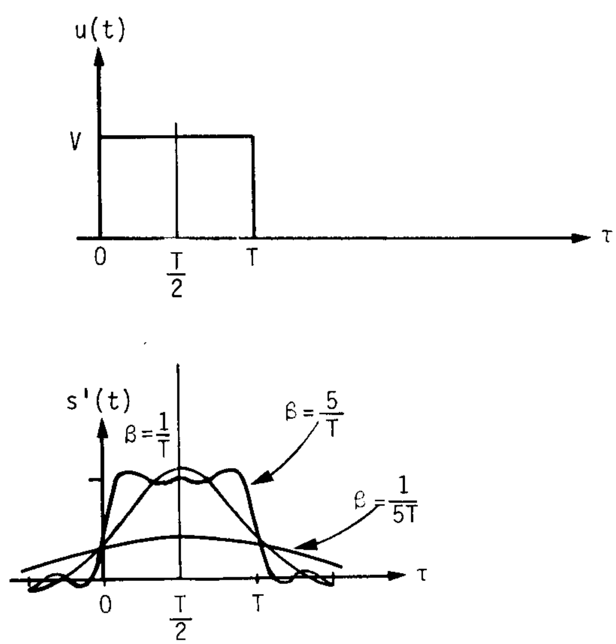

FIGURE 4 RESPONSE s'(t) OF AN IDEAL LOW-PASS FILTER TO A RECTANGULAR INPUT PULSE u(t)

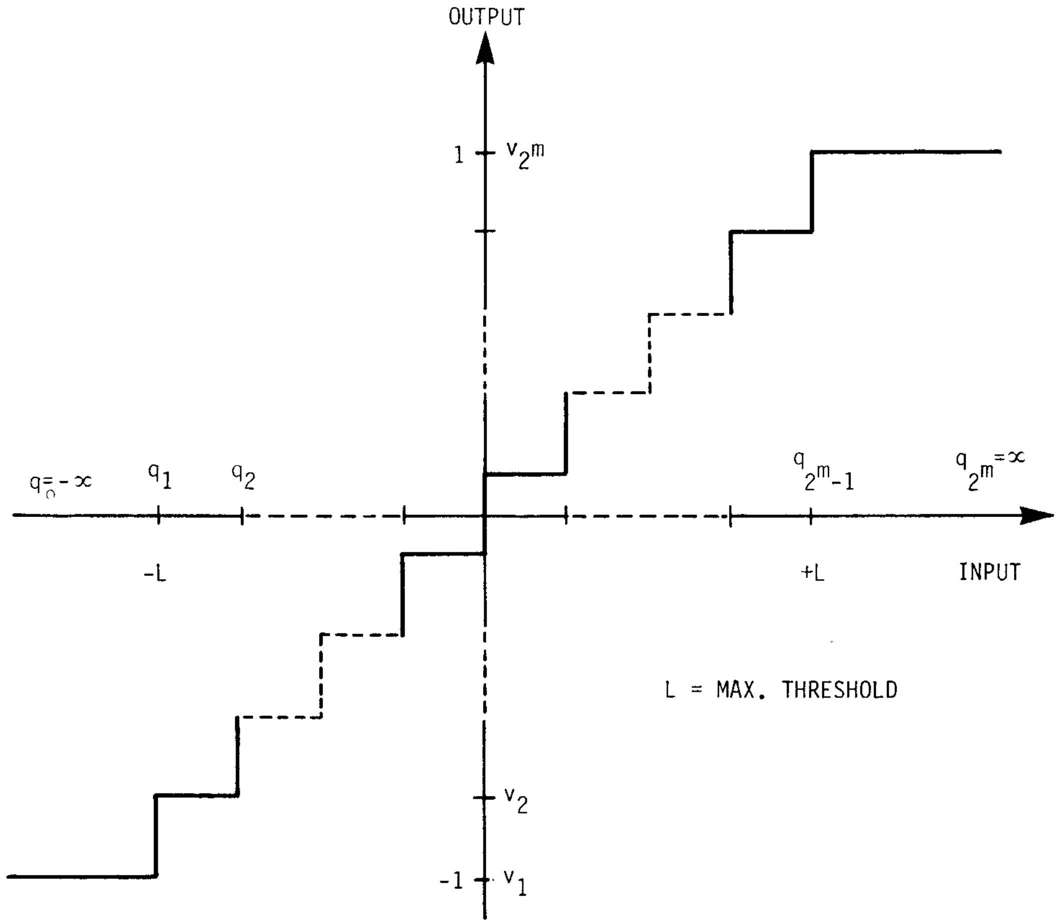

FIGURE 5 INPUT-OUTPUT CHARACTERISTICS OF AN m-BIT UNIFORM-STEP QUANTIZER

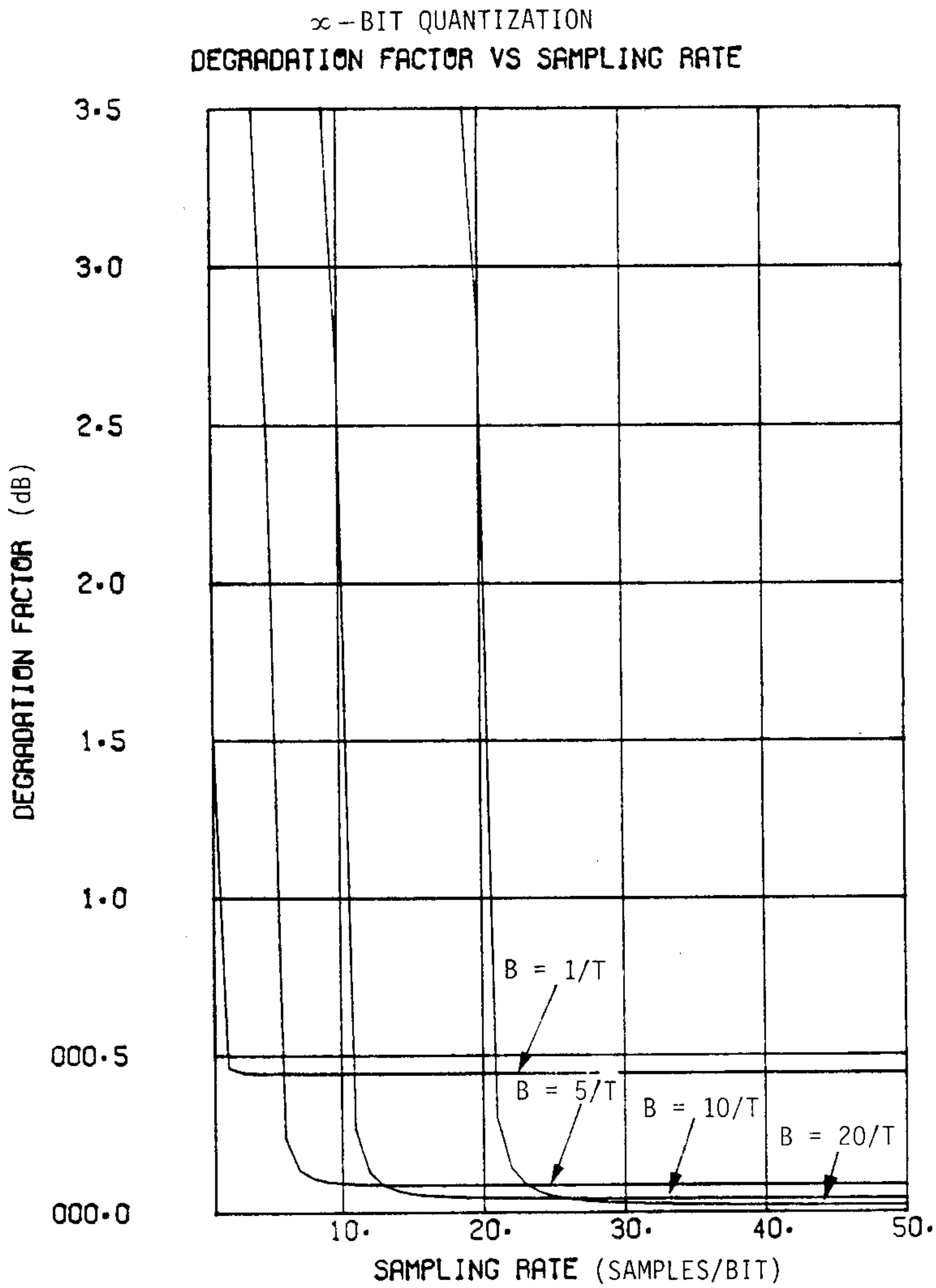

FIGURE 6 DEGRADATION D IN dB VS SAMPLING RATE FOR x-BIT QUANTIZATION

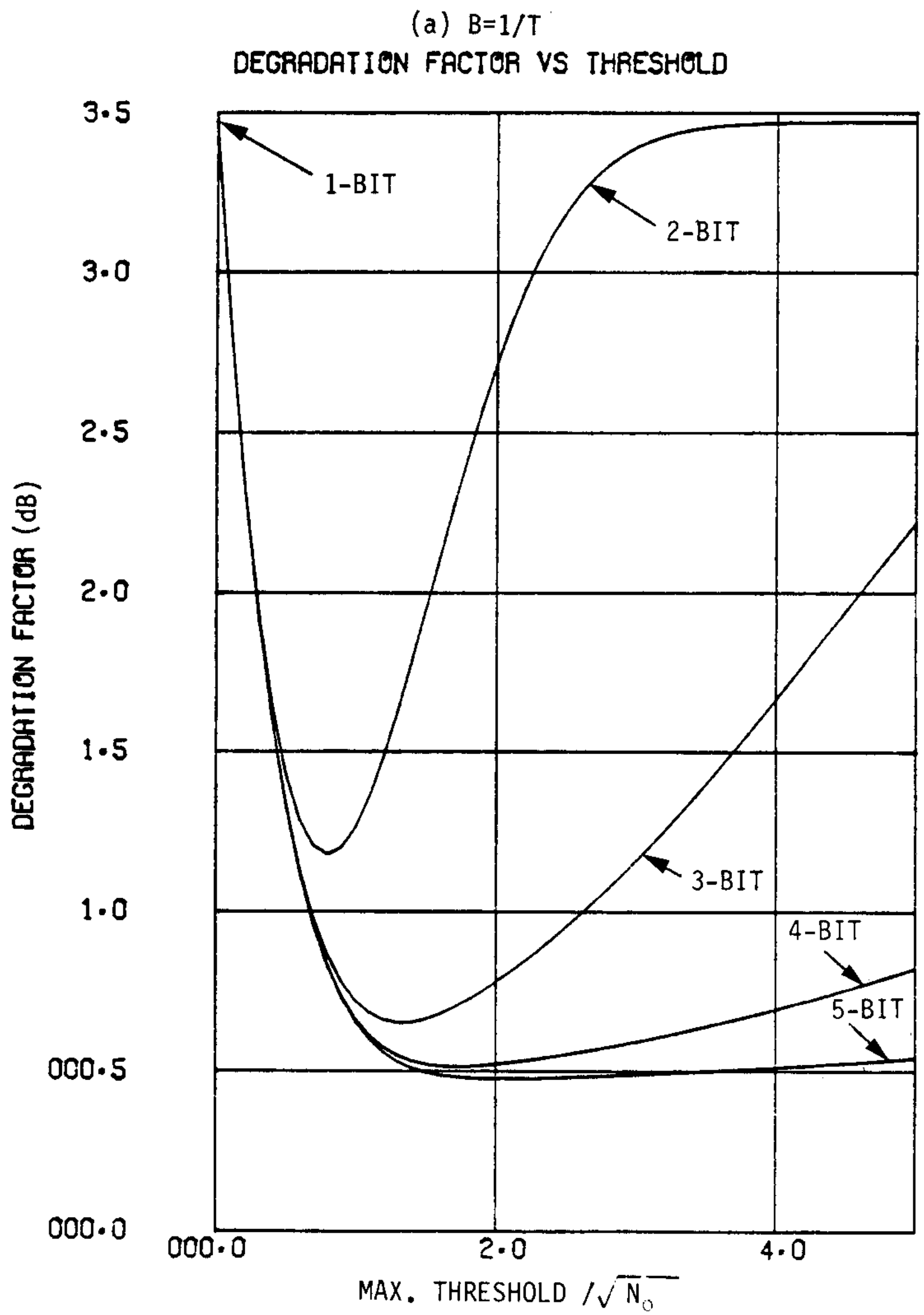

FIGURE 7 DEGRADATION D IN dB VS L /$\sqrt{N_o}$ FOR f_s = 2BT AND $E_b/N_o \leq$-10dB WITH 1,2,3,4, AND 5-BIT QUANTIZATION

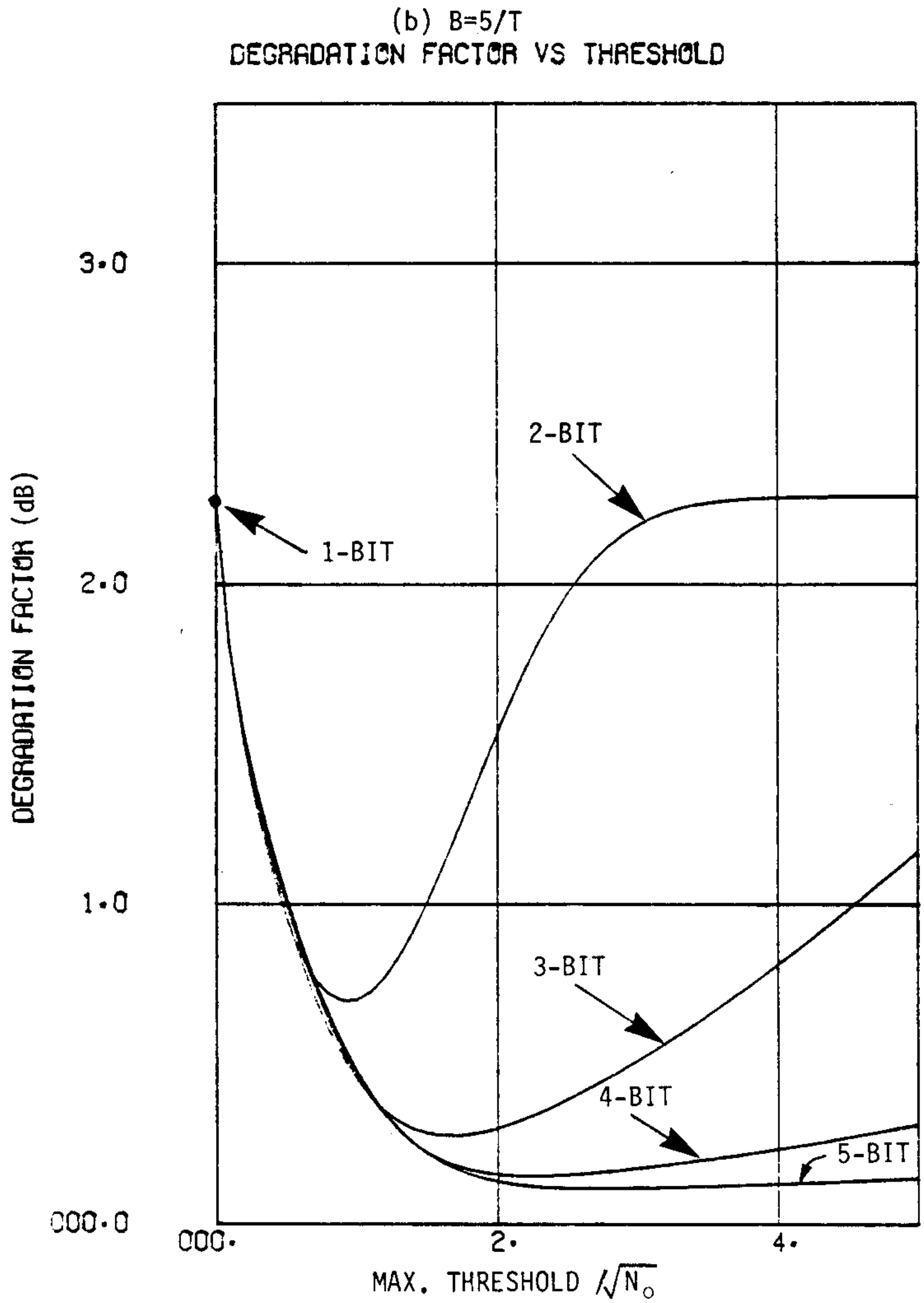

FIGURE 7 CONTINUED

(c) B=10/T

DEGRADATION FACTOR VS THRESHOLD

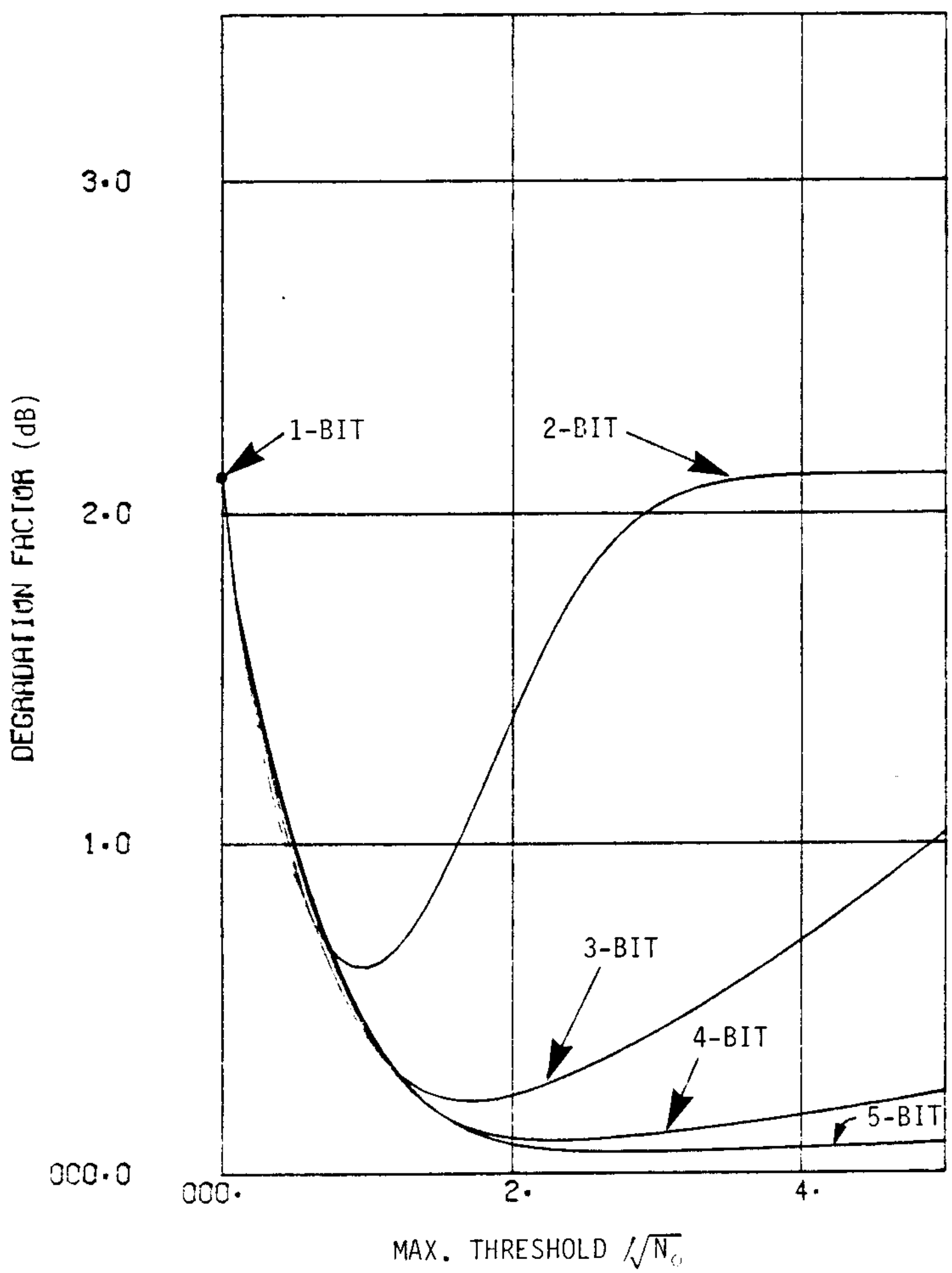

FIGURE 7 CONTINUED

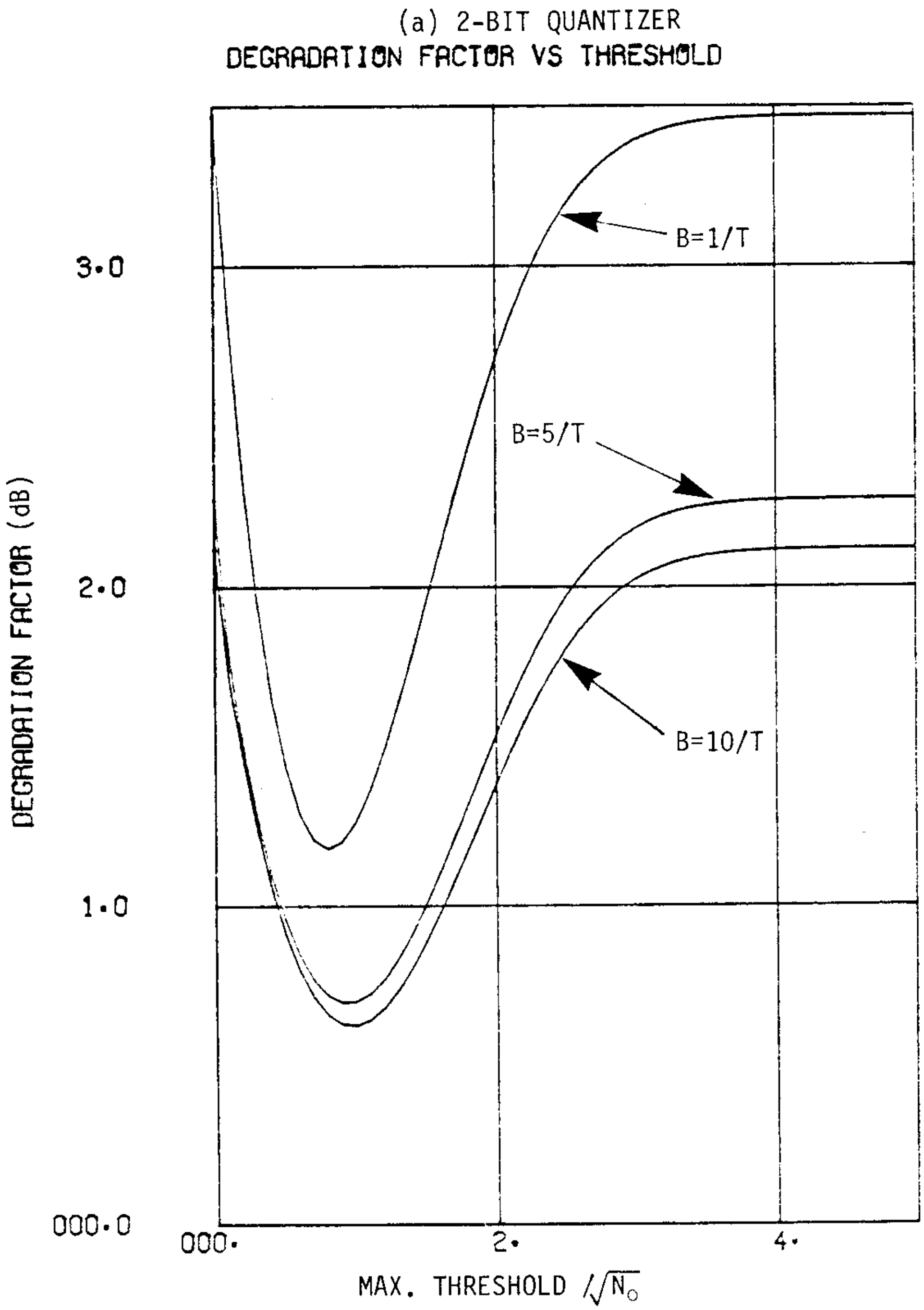

FIGURE 8 THE DEGRADATION D IN dB VS L $/\sqrt{N_o}$ FOR f_s =2B AND $E_b/N_o \leq$-10dB

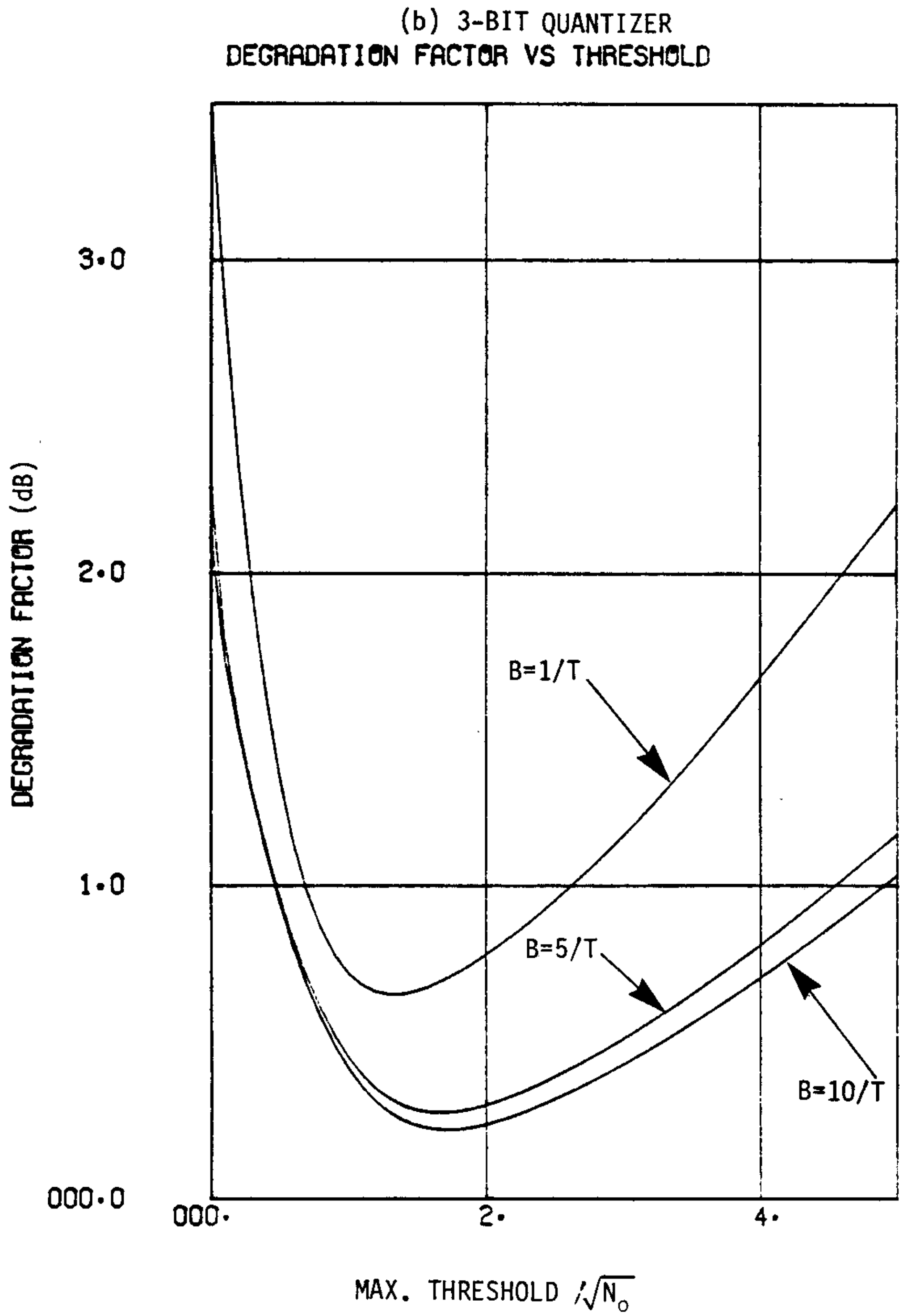

FIGURE 8 CONTINUED

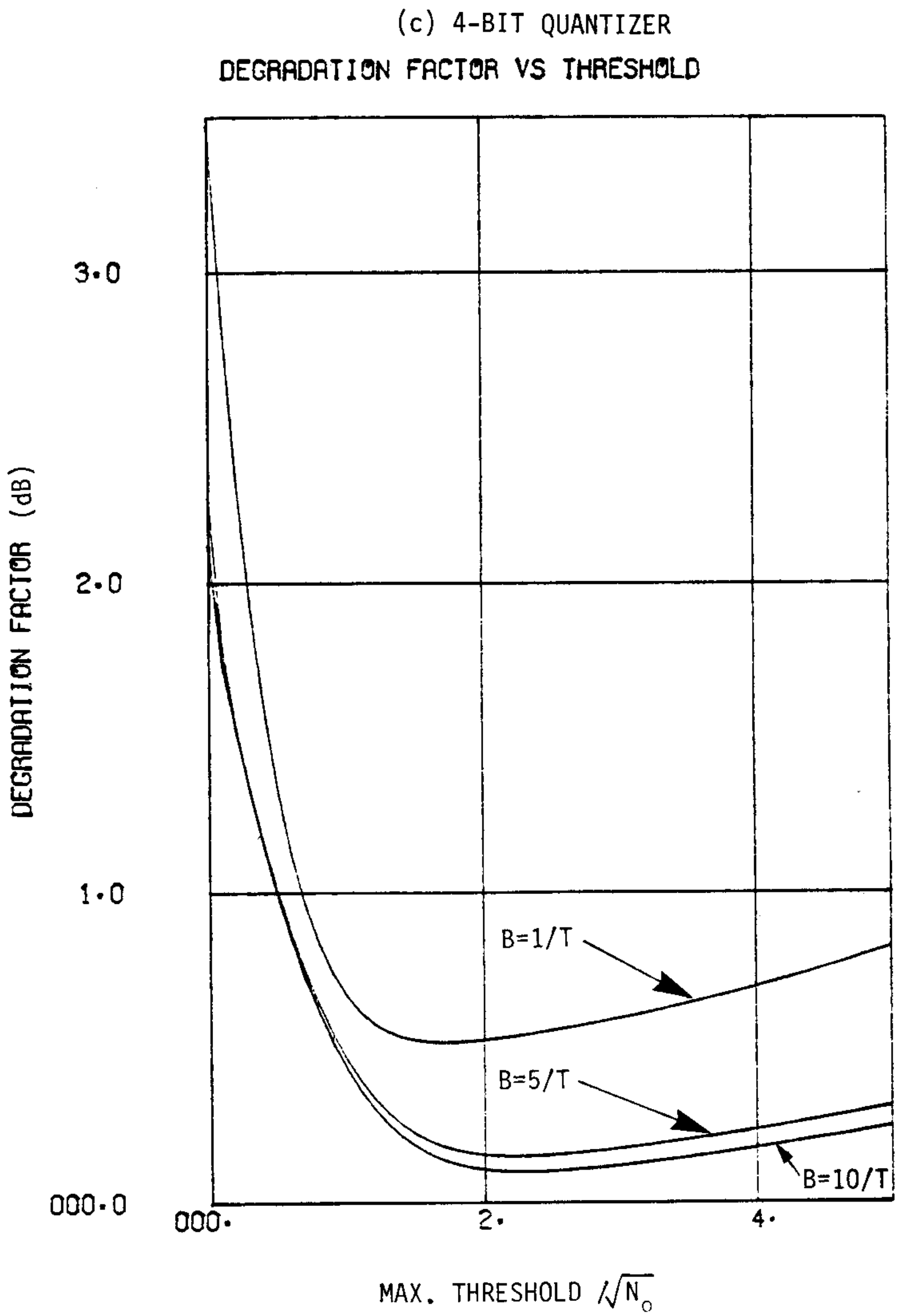

FIGURE 8 CONTINUED

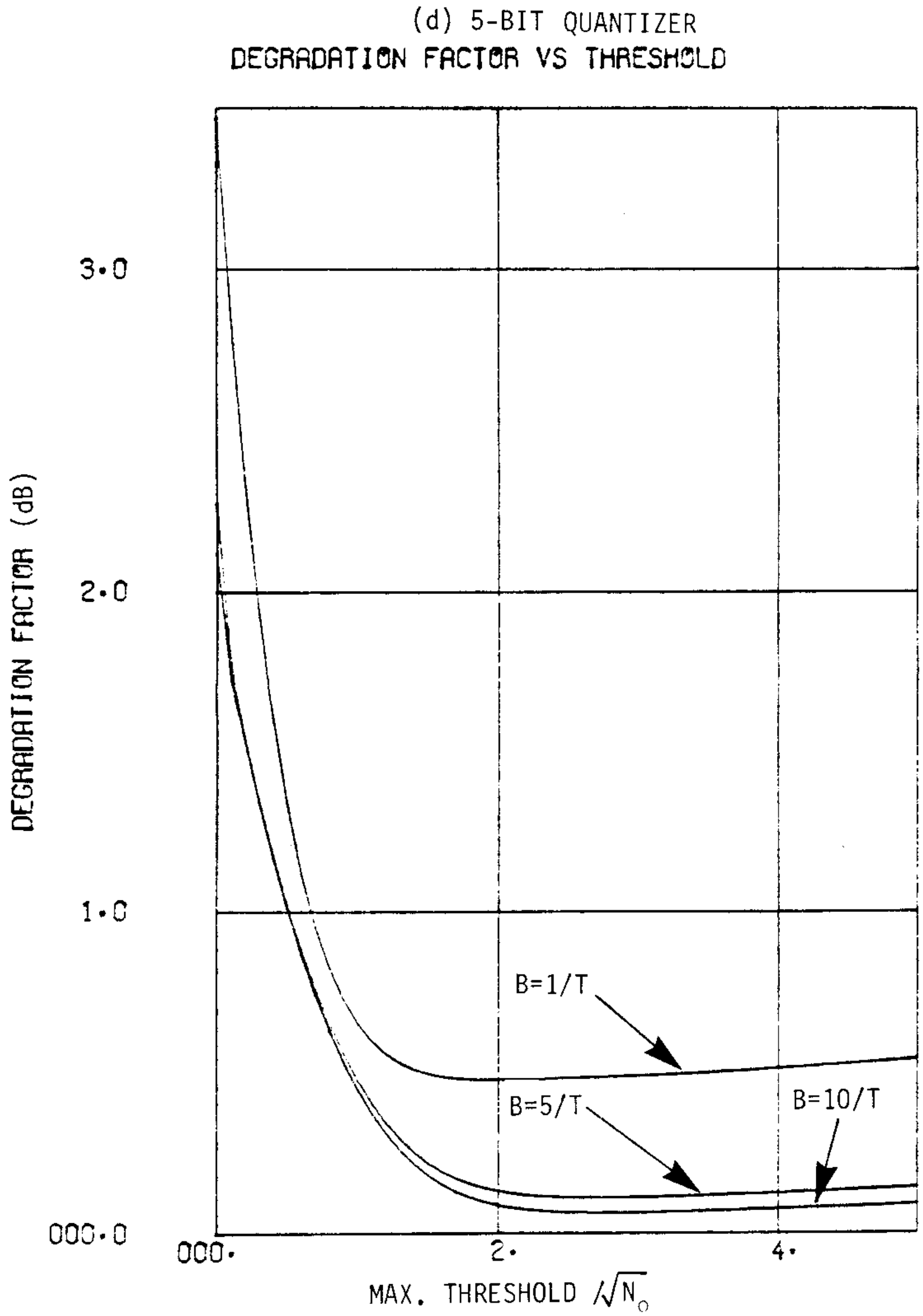

FIGURE 8 CONTINUED

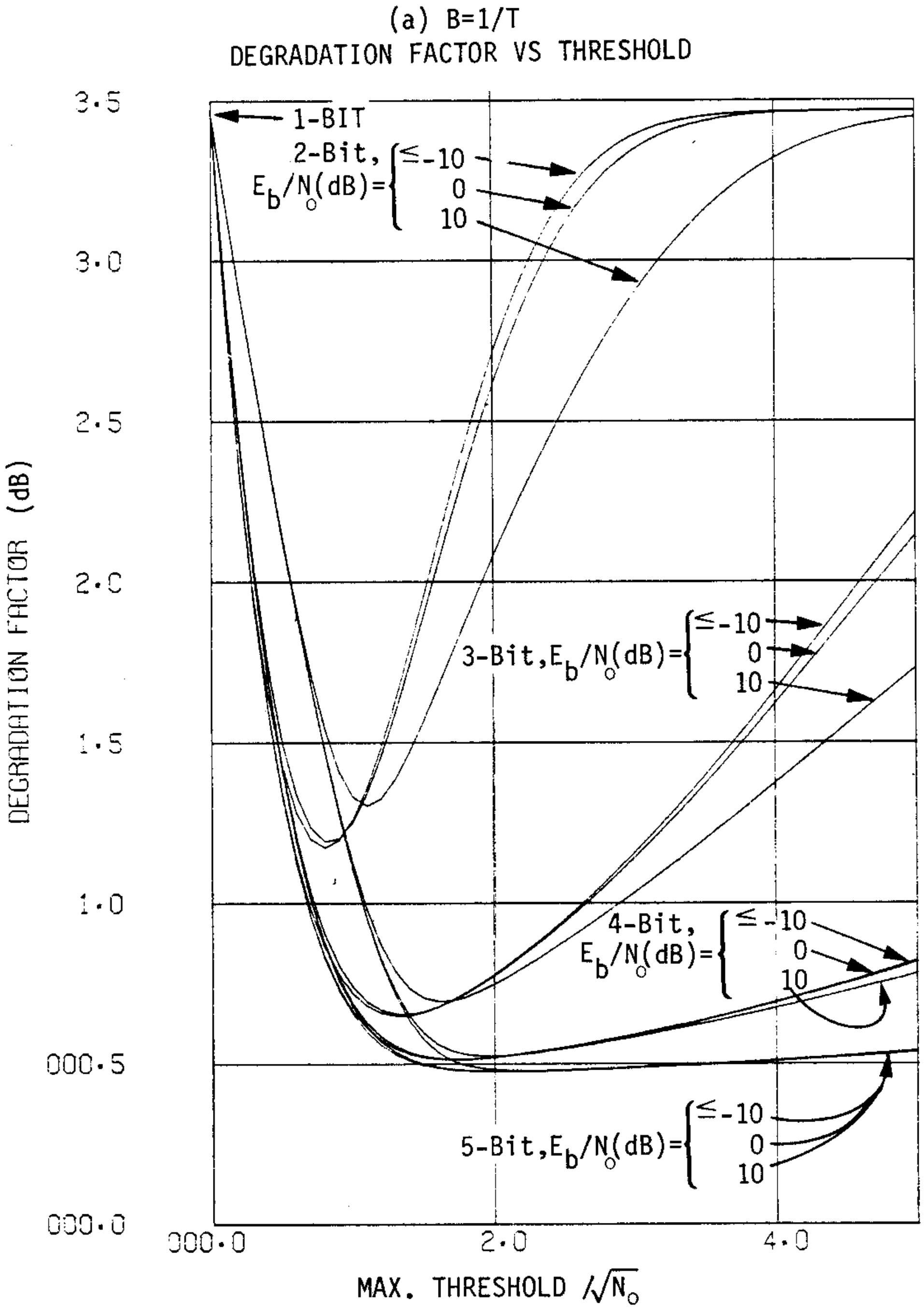

FIGURE 9 THE DEGRADATION D IN dB VS L $/\sqrt{N_o}$ FOR f_s =2BT WITH 1,2,3,4,AND 5-BIT QUANTIZATION

(b)B=5/T
DEGRADATION FACTOR VS THRESHOLD

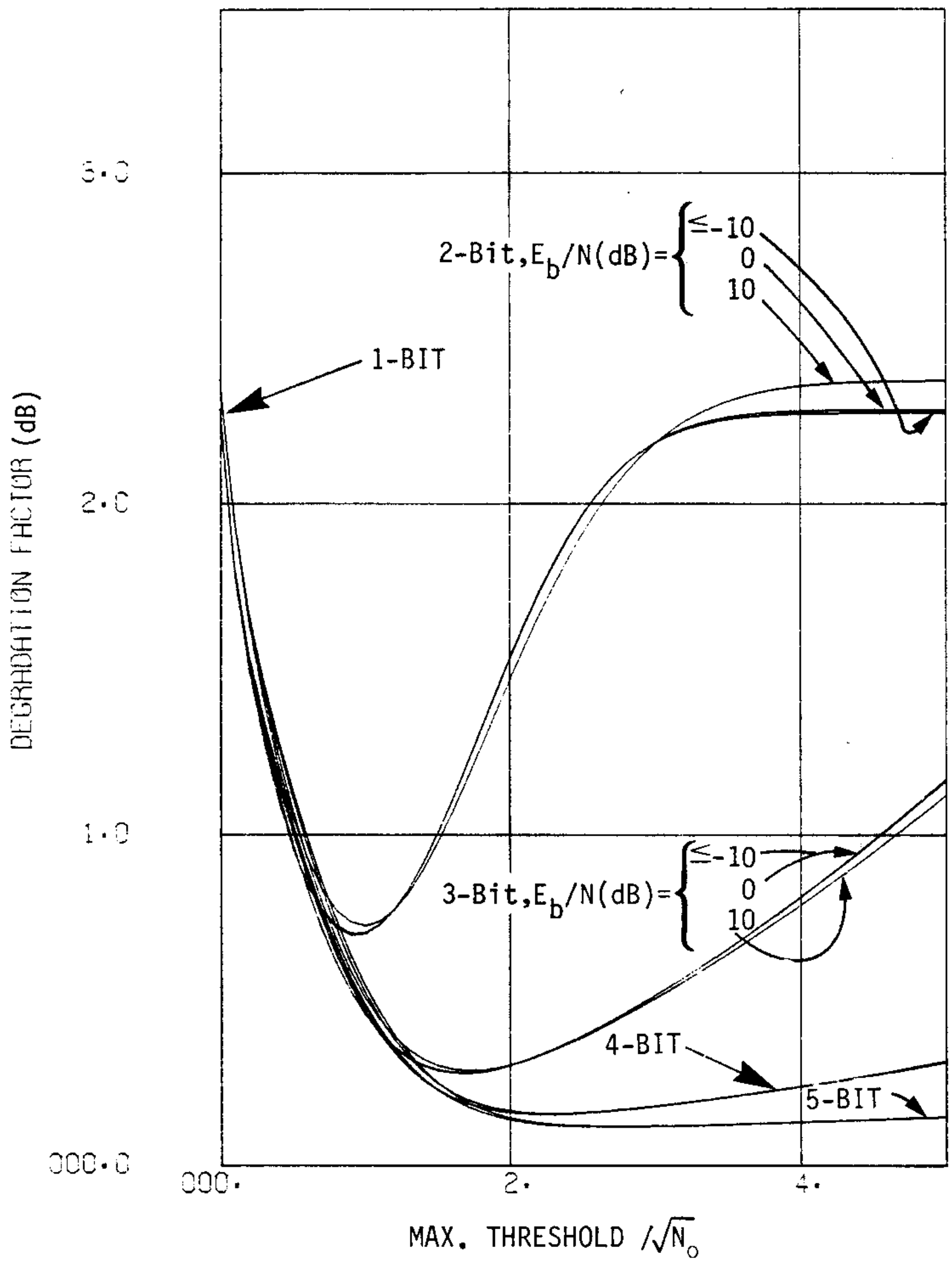

FIGURE 9 CONTINUED

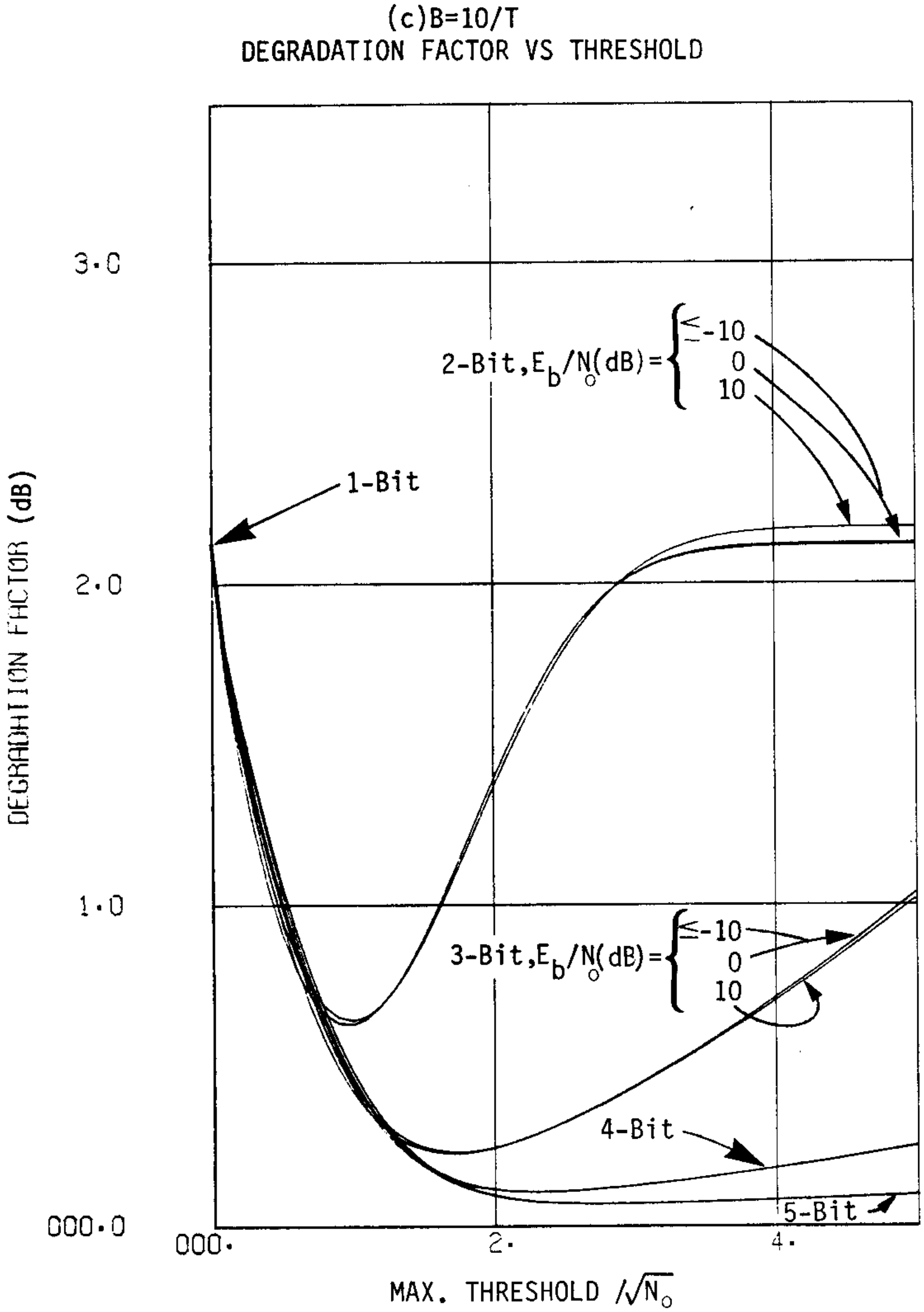

FIGURE 9 CONTINUED