THE PARTICULATE NOISE POWER SPECTRUM OF A MAGNETIC TAPE RECORDER/REPRODUCER

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ABSTRACT

This paper presents a theoretical derivation of the noise power spectrum of a magnetic tape recorder/reproducer. The theoretical results are shown to be in good agreement with experimental data.

INTRODUCTION

Particulate noise is that fraction of recorder output due to deviation from the mean magnetization. Mallinson (reference 1) and Thurlings (reference 2) have reported on this component. Based on his predictions, and measurements of the root mean square signal to noise ratio in the passband of the recorder, Mallinson has concluded that particulate noise is dominant in a well designed recorder, and, with others, (references 3, 4, 5) has projected optimum track widths of 1 mil, or less if mechanical implementation is feasible, for high density digital recording.

Experimental data discloses that when using a 50 mil track width, a tape velocity of 30 ips, bias recording and Ampex 795 tape, peak particulate noise power density exceeds the reproduce amplifier first circuit noise by 3 dB. Particulate noise power density must be a linear function of track width and tape velocity. For a tape speed by track width product of two square inches per second or less, the rms particulate noise power should be less than rms background noise in the passband. The result might be different for other modes of recording and other tape bases.

The following discussion introduces the concept that the separation effect described by Wallace (reference 6) is different from signal separation for noise. This concept is suggested by data presented by Lemke (reference 7) which show that the effective signal separation for a tape base, with bias recording, is a function of the record gap length, decreasing for the narrower record gaps. With this concept, and some revision of Mallinson’s theory, essential agreement between theory and experiment can be obtained.

THEORY

The reproduce model is that described by Wallace, and is shown here in figure 1 with a slightly modified coordinate system. It is a semi-infinite half space of constant permeability around which there is one turn of wire of negligible cross section. Below this is the magnetically recorded medium of thickness "d" separated from the half space by the distance "s".

It is assumed that magnetic particle size is small compared to the shortest wavelength on tape; that the tape is smooth and free of foreign matter on its surface; that magnetic particle population density is uniform throughout the volume; that, for signal, magnetization is constant as a function of depth in the recorded medium, and that, for noise, magnetization is normally distributed throughout the medium. Wallace calculated the magnetomotive force and flux density in the half space, then finding the total flux linking the turn of wire. Induced voltage is the time rate of change of flux, modified by a constant. These operations can be described, in their result, as:

$$E(k) = E\cos(kvt)e^{-ks}\int_0^{\infty}\int_0^{kd} e^{-p-q}\,dp\,dq \qquad (1)$$

where:

E = a constant voltage,
k = wave number (2π/wavelength),
s = separation between the recorded medium and the half space,
d = depth of recording,
v = tape velocity,
t = time, and

$$p = k(z + d + s) \qquad (2)$$
$$q = kz_o$$

For signal this results in the well-known relationship which was first stated by Wallace:

$$E(k) = E\cos(kvt)e^{-ks}(1 - e^{-kd} \qquad (3)$$

The exponential term involving "s" Wallace termed the "separation effect," and the term in parentheses involving "d" the "depth effect."

Wallace assumed the phase of the magnetization to be constant, as well as amplitude, in a plane normal to the velocity vector (X-axis) within the recording medium. This cannot be true for particulate noise. If the particulate noise output of the reproduce head is a Gaussian process, Thomas (reference 10, pg. 154-160) shows that at a fixed value of wave number the amplitude distribution of noise voltage components must be Rayleigh, and the phase distribution uniform.

We inquire as to the expected result when the depth of the emulsion is changed and the wave number is very small. From eq. (3):

$$E(k) = E \cdot kd \cdot \cos(kvt) \qquad (4)$$

$$kd,\ ks \ll 1. \qquad (5)$$

The signal components from each lamina add coherently - signal voltage is a linear function of "kd." Noise components will add randomly - noise power is a linear function of "kd'," where d' is the depth of the emulsion, which can be greater than the depth of recording. Then:

$$|E'(k)|^2 = E'^2\ kd'/a \qquad (6)$$

where the constant "a" is required for a generalized result.

For large values of wave number, changing the depth of the emulsion has very little effect on either signal or noise, since the separation effect controls, and response is derived from those particles close to the reproduce head, i.e., within less than one wavelength. We conclude that:

$$|E'(k)|^2 = E'^2\ e^{-2ks'}\ (1 - e^{-kd'/a}) \qquad (7)$$

where prime values refer to noise related parameters. The constant "a" is deduced from experimental data, and the manufacturer's quoted nominal depth of the recording medium, to be 2.

Particulate noise power density is a linear function of tape speed, v, the magnetic particle volume density, n, and the track width, w. Then:

$$|E'(k)|^2 = E'^2\ (vnw)e^{-2ks'}\ (1 - e^{-kd'/2}) \qquad (8)$$

From eq. (3) signal attenuation at small wave numbers increases at 6 dB per octave of inverse wave number, while from eq. (8) noise attenuation at small wave numbers will increase 3 dB per octave of inverse wave number. At large wave numbers both signal and noise attenuation in dB are proportional to wave number. It is neither necessary nor required that effective separation and/or depth be the same for both signal and noise.

The practical reproduce head is finite in width and depth. Tape length in contact with the head is finite, which makes separation variable. Width, depth and contact length influence signal and noise response at small wave numbers. It is expected that the functions described in eqs. (3) and (8) will be modified should any of these parameters become effective.

EXPERIMENT

The experimental method involves observation of the power in a bandpass filter whose bandwidth is much smaller than its center frequency over a period of time sufficiently long to obtain a statistically reliable result. The observation is made at the output terminals of a perfectly equalized recorder with the input terminals to the record amplifier shorted and with tape moving, P(l), then with the tape stopped, P(2). If:

NPRP = particulate noise power density for a perfect medium, $d' = \infty$ $s' = o$,

NPRS = noise power density in the first circuit of the reproduce amplifier,

G(k) = reproduce gain schedule required to correct for signal response,

$$= \text{Constant}\cdot e^{2ks} (1 - e^{-kd})^{-2} \quad (9)$$

R(k) = reproduce power gain schedule required to correct for the reproduce gap length,

$$= [(kr/2)/ \sin(kr/2)]^2 \quad (10)$$

r = reproduce gap length,

N(k) = power transfer function for particulate noise,

$$= \text{Constant}\cdot e^{-2ks'} (1 - e^{-kd/2}) \quad (11)$$

Then:

$$P(1) = [NPRP\cdot N(k)\cdot R(k)^{-1} + NPRS]\, G(k)\cdot R(k) \quad (12)$$

$$P(2) = NPRS\cdot G(k)\cdot R(k) \quad (13)$$

From eqs. (12) and (13):

$$N(k)\cdot NPRP/NPRS = [P(1)/P(2) - 1]\ R(k) \qquad (14)$$

$$G(k)\cdot NPRS = P(2)\cdot R(k)^{-1} \qquad (15)$$

If it can be shown that NPRP and NPRS are white, the right side of eq. (14) is the particulate noise power transfer function, independent of the reproduce gain schedule. The right side of eq. (15) is the signal power equalization function. The most convenient method of testing the "whiteness" of NPRP and NPRS is to perform the experiment at a number of substantially different tape speeds. If these functions are "white" the same functions of wave number G(k) and N(k) will be found for the several tape speeds used.

It is possible that high frequency bias might have some effect on particulate noise. This was explored by evaluating P(l), eq. (12), using degaussed tape with and without standard bias (reference 8). A Honeywell, Model 7600, recorder/reproducer was used for all tests. The nominal record and reproduce gaps were 80 and 30 microinches, respectively. The reproduce gap was measured using the first null extinction method and found to be approximately 26 µin. At each tape speed cw signals were used at slightly less than standard level to calibrate variations from perfect equalization.

DATA PROCESSING

Data corrected for imperfect equalization and the reproduce gap length, as indicated in eqs. (14) and (15), is shown and plotted in figures 2 through 4. A log (wave number) abscissa is used for small wave numbers, while linear (wave number) is used for large wave numbers. Two straight lines then describe the data approximately. The slope of the line at large wave numbers is proportional to separation, and the location of the line at small wave numbers is a function of depth. The signal data was processed to find the best fit to eq. (9), and the noise data to find the best fit to eq. (11).

It is more important for the small wave number data to obtain a fit in the region between 1200 and 2400 µin wavelength than elsewhere. It is in this region that the results are least influenced by the finite dimensions of the record and reproduce systems. It is also in this region that one obtains a sufficiently large ratio P(1)/P(2) to provide reliable results.

DISCUSSION

The data shown in detail in figures 2 through 4 is summarized in table 1. The same functions of wavelength are obtained at all tape speeds, within experimental error, so we infer that NPRP and NPRS are both "white."

For the oriented particle tapes used in these tests the separation for noise is much less than the effective separation for signal. The separation parameter with bias is, near enough, the same as separation without bias. Particulate noise power density with bias is approximately 3 dB greater than noise for degaussed tapes. The 786 tape base produces higher noise levels than the 795 tape base at all wave numbers used in the tests.

The effective depth for signal for both tape bases is less than the effective depth for noise. The signal uses less than the full depth of the emulsion. Noise uses the full depth, not being influenced by the record process. The data at 7.5 ips is necessarily less accurate than the data at higher tape speeds because of the smaller ratio of particulate noise to background noise.

The data demonstrates that the gain schedule required to equalize signal response at small wave numbers is 6 dB per octave of inverse wave number; while particulate noise attenuation is 3 dB per octave of inverse wave number. Particulate noise power density at all wave numbers is a linear function of tape velocity.

Lemke (reference 7) performed an experiment in which he used the head and tape combinations shown in table 2.Two substantially different tape bases were used. The first was a currently available gamma iron oxide base; the second an unoriented particle base with exceptionally small particles, processed for smoothness. He reports the use of IRIG standard bias recording.

The data given by Lemke have been corrected for the (quoted) reproduce gap length effect before arriving at the best straight line fit for attenuation at large wave numbers. The effective separation for the oriented particle tape in line 1 of table 2 is not too different from the value shown in table 1 obtained under the same conditions.

However, using the same tape base and a very narrow record gap Lemke obtains a much smaller attenuation as a function of wave number, indicating a much smaller separation, about 10 microinches. This is comparable to the separation for noise measured for this tape base in the experiments reported here. For the unoriented particle tape table 2 shows a much smaller separation than for the oriented particle tape for the wide (85 μin) record gap. There is a further decrease in effective separation for the unoriented particle tape for the very narrow record gap.

The point is that if effective signal separation decreases as the record gap becomes smaller then magnetic particles are certainly present sufficiently close to the reproduce head to be useful. Particulate noise data confirms this inference. Separation for noise with clean degaussed tape is a measure of the combined effects of tape roughness and uniformity of magnetic particle distribution in the near surface layers.

CONCLUSIONS

Theory and experiment are in agreement. Particulate noise power density decreases at 3 dB per octave of inverse wave number at small wave numbers. Separation for noise is different from effective separation for signal with bias recording, and is much smaller. Particulate noise with bias recording is greater than particulate noise with degaussed tape. The particulate noise power level is such that it is less than current reproduce amplifier noise when the product of track width by tape speed is less than two square inches per second for the 795 tape base. Particulate noise is a factor in selecting the tape speed at which a recorder/reproducer operates. For a track width of 1 mil and a tape speed of 1000 ips it should be a minor concern. For 17 mil tracks, standard for 42 tracks on 1" tape, the effect will be present at tape speeds of 60 ips.

Of greater importance in digital recording, to realize the data storage potential of magnetic tape, is the management of available signal to noise margins (reference 9), the properties of codes, particularly the match between the power spectral density of a code and the transfer function of the recorder; and the selection of the tape base, the mode of recording and the length of the record gap to minimize signal attenuation due to the separation effect.

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TABLE 1

Summary of Signal and Noise Separation and Depth Data

Tape Base	Tape Speed	Function	Separation, Microinches	Depth, Microinches
795	7.5	Signal	18.5	100
795	30	Signal	19.8	110
795	60	Signal	20.4	120
795	30	Signal	16.5	110
795	7.5	Noise	6.5	200
795	30	with	7.0	200
795	60	bias	8.8	200
795	30		6.6	200
795	7.5	Degaussed	6.0	200
795	30	tape	5.0	200
795	60		7.0	200
795	30		4.0	200

TABLE 2

Record Gap Data

Tape Base	Reproduce Gap, microinches	Record Gap, microinches	Effective Separation, microinches
Fe_2O_3	28	80	20
Fe_2O_3	10	10	10
SP	10	80	10
SP	10	10	4

The separation values are deduced from Lemke's data (reference 7) after correction of the data for the quoted reproduce gap length.

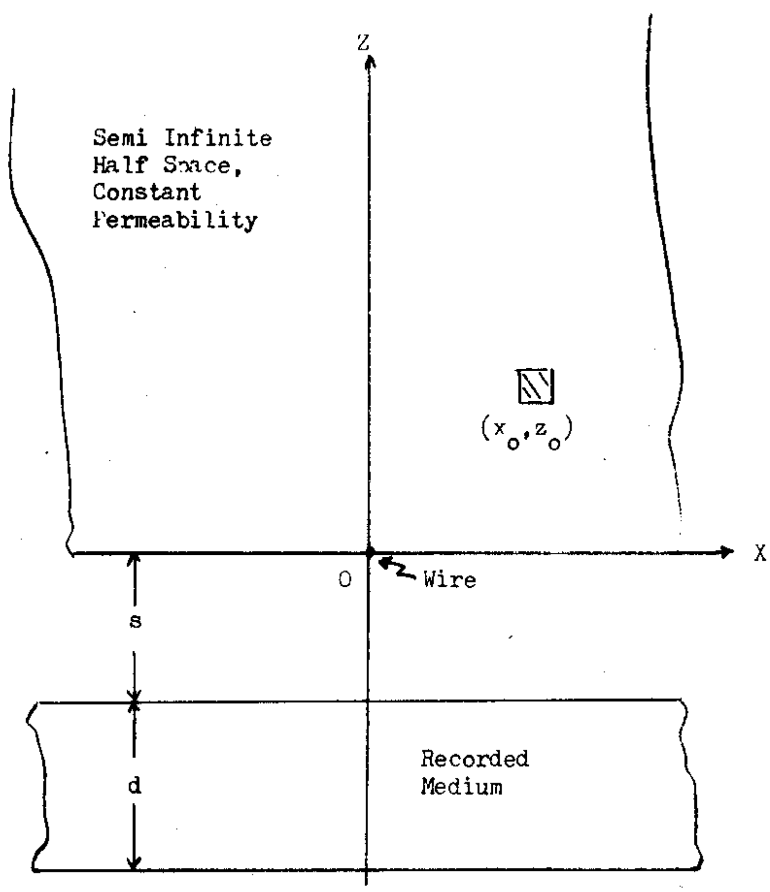

Fig. 1. Reproduce Geometry.

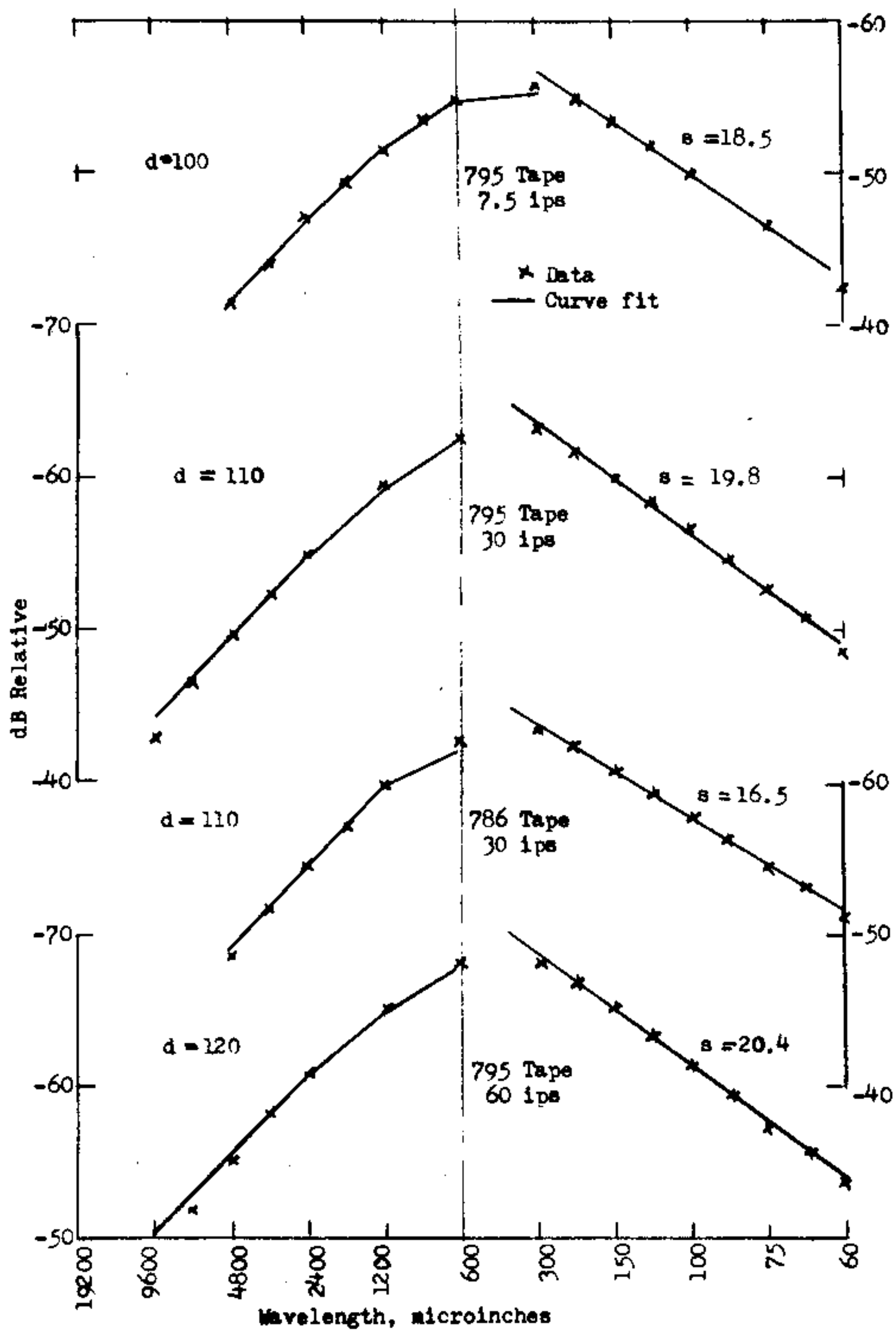

Fig. 2. Signal Schedule

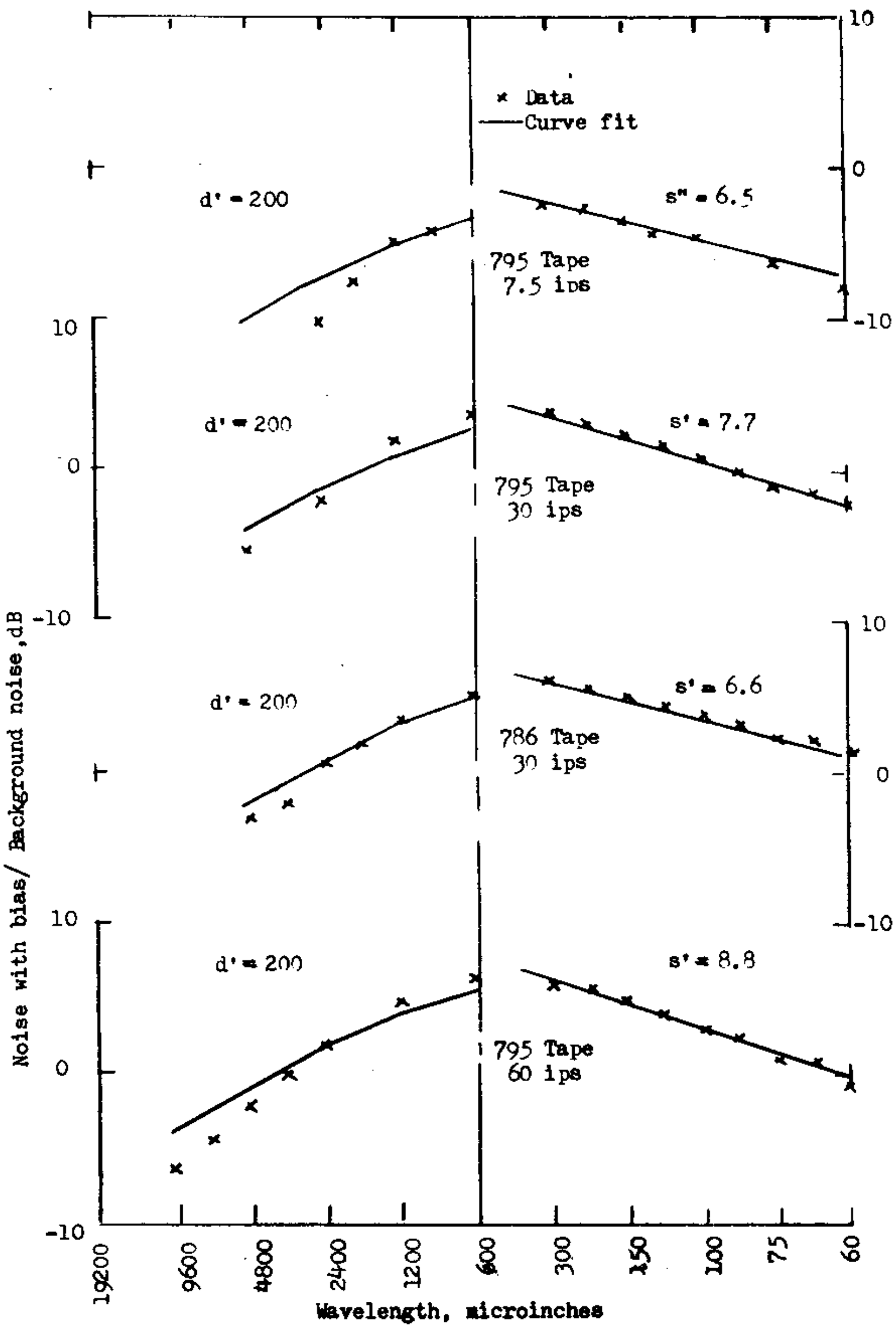

Fig. 3. Noise with Bias

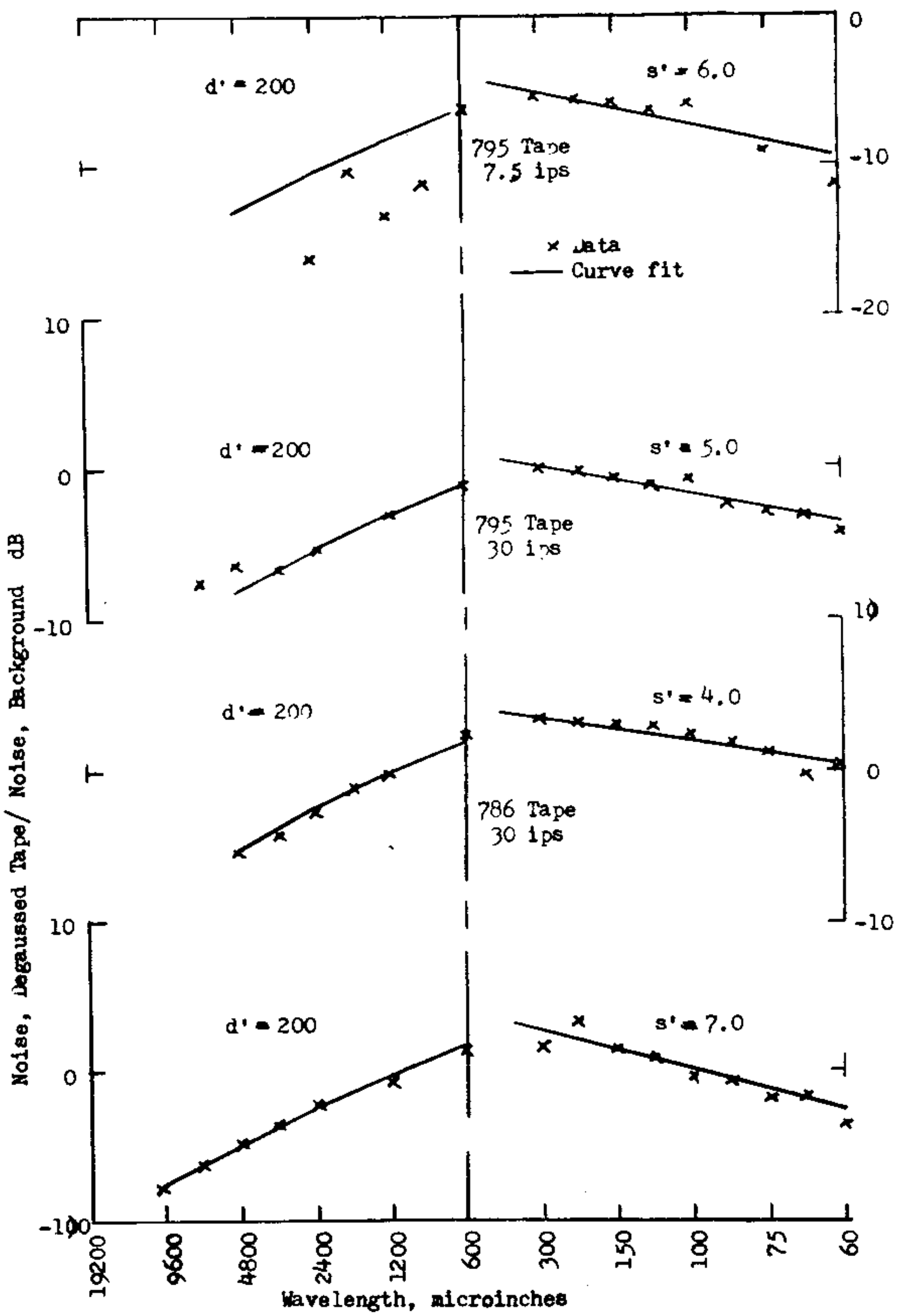

Fig. 4. Noise with Degaussed Tape