DECENTRALIZED CONTROL FOR LARGE COMMUNICATION SATELLITES BY MODEL ERROR SENSITIVITY SUPPRESSION

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ABSTRACT

The rapid growth in world demand for satellite telecommunications and the limited number of positions in the geostationary arc are leading inexorably to larger, higher capacity communications satellites. This trend, coupled with the projected weight and volume capability of the Space Transportation System (STS), will lead to satellites in the 80s weighing 5000 kg and measuring 50 to 100 feet across. By the end of the century these figures could increase again by an order of magnitude. Such large, low-density structures tend to have closely spaced, low-frequency dynamic modes. At the same time, multibeam-frequency reuse antennas (MBFRA) projecting narrow-spotbearns require pointing stability within a hundredth of a degree or so. The combination of low structural natural frequency and more stringent pointing requirements imposes the need for an entirely fresh approach to dynamic control of communications satellites. This paper outlines such an approach.

A modern optimal control methodology is advanced that provides decentralized modular control for large communication satellites. The fundamental property of the control algorithm is its ability to stabilize certain subsets of vibration modes without disturbing others.

This decoupling action allows the control task to be implemented in a modular or building block fashion so that different modal subsets are stabilized by separate controllers. Decentralization according to functional task is also possible such that noninteracting rigid body and elastic body control is achieved. Thus, the technique provides a solution for the problem of rigid body control in the presence of low frequency elastic modes that are in the rigid body controller bandwidth. The design methodology, termed Model Error

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Sensitivity Suppression (MESS), is a derivative of modern optimal control and estimation theory. Several examples illustrate the capability of the design algorithm.

1. INTRODUCTION

The finite number of positions in the geostationary arc, coupled with the ever-increasing demand for telecommunication systems, will inevitably lead to larger, more complex communication satellites, with ever more stringent pointing requirements. Multibeam-frequency reuse antennas (MBFRA) will require pointing accuracies on the order of 10^{-2} degrees. The advent of the Space Transportation System (Shuttle) will allow the construction of ever larger communication satellites in the attempt to meet this demand.

These large systems will present a new class of problems for the control engineer, for these satellites will be of low density (mass to volume ratio) which tends to produce vibration modes of extremely low frequency. These low frequency modes will be closely spaced and will be located within the rigid body controller bandwidth, which compounds the control problem. Therefore, high performance design for this new regime of space systems requires control methods which differ from the usual frequency domain techniques of classical control theory.

In addition to low frequency vibration modes, the large size and possible modular construction of these systems will create another class of problems to be addressed. As shown in Figure 1, the basic space system could be modular in nature, with new modules being added during system life. Thus, another aspect of the control problem becomes evident: the control system must have inherent add-on capability to cope with this modular nature. As new modules are added or structure geometry is changed, the controller must be capable of reflecting these changes.

Now consider the large size of the advanced structures. This large size makes centralized control somewhat troublesome due to the long signal paths for actuation and sensing signals. Control of a large number of modes will involve a large number of signal paths.

To adequately cope with these problems, a decentralized control methodology is advanced which allows noninteracting control with distributed micro processing. Such an approach uses the large size of these systems to positive advantage. Noninteracting decentralization also mirrors the possible modular nature of these structures in that, for many cases, the same control hardware would be employed, and computer control gains would be

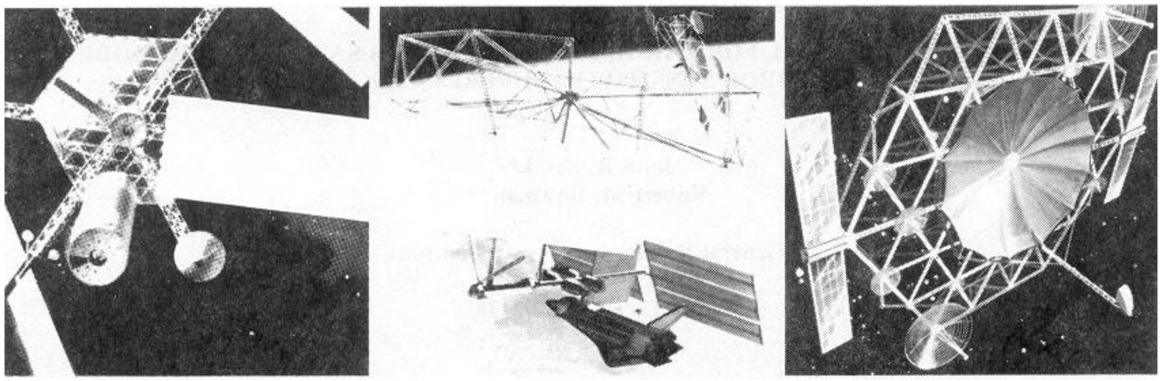


Figure 1. Large communication satellite concepts.

recomputed to account for changing structure geometry. Consequently, additional control hardware would be needed in only the new or add-on modules.

The Model Error Sensitivity Suppression algorithm, Ref. 1 through 9, is amenable to decentralized control, providing a solution for the control problem just discussed. The basic controller which is derived from optimal regulator theory is shown in Figure 2.

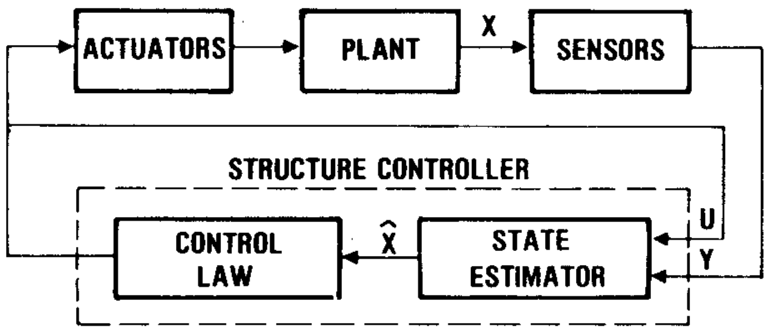


Figure 2. Active controller structure.

2. THE FLEXIBLE SATELLITE CONTROL PROBLEM

Simply stated, the flexible spacecraft control problem involves multivariable control in the presence of modeling error. If the linear, constant-coefficient differential and algebraic equations (1) and (2)

$$\dot{x} = Ax + Bu + v \tag{1}$$

$$y = Cx + w \tag{2}$$

provided an accurate model of the dynamic processes governing flexible spacecraft behavior, we would implement the theories in our textbooks and the flexible spacecraft control problem would be solved. (Here, x is the state vector; u is the control vector; y is the observation vector; A, B, and C are constant matrices; and v and w represent white, Gaussian noise.) Our analysis task is to find ways to correct for the deficiencies of this model. In other words, our approach is to reduce the model error sensitivity of the system.

It is helpful to classify the model errors as follows:

1. Model reduction (truncation) errors
   a. Truncation of known modes for implementation of the estimator
   b. Omission of unmodeled modes; truncation of unreliably computed modes

2. Parameter errors

3. Nonlinearities

4. Unmodeled disturbances

On the basis of studies in the control literature, Ref. 10 through 19, it appears that the significance of these errors is in the order indicated: truncation, parameter errors, nonlinearities, and disturbances. For particular systems these priorities may vary.

3. MODEL ERROR SENSITIVITY SUPPRESSION

MESS is a method adapted from Ref. 1 that very effectively resolves the two most critical model error problems: truncation of known modes, and parameter errors. We speak of model truncation error sensitivity suppression and model parameter error sensitivity suppression. In effect, this method permits the suppression or elimination of truncation and parameter error sensitivity in the estimation and control system. This is accomplished without the augmentation of the state vector or the introduction of special signal compensation, such as comb filters. This paper addresses model truncation error. References 1, 4, and 7 address parameter error.

We first partition the state equations for the modeled system into separate "controlled states" x_c and "suppressed states" x_s as shown by equations (3) and (4).

$$\begin{bmatrix} \dot{x}_C \\ \dot{x}_S \end{bmatrix} = \begin{bmatrix} A_C & 0 \\ 0 & A_S \end{bmatrix} \begin{bmatrix} x_C \\ x_S \end{bmatrix} + \begin{bmatrix} B_C \\ B_S \end{bmatrix} u \qquad (3)$$

$$\begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} C_c & C_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} \tag{4}$$

Our first task is to suppress the "control spillover," Ref. 18 and 19, $B_s u$ so that we can control the states x_c via $B_c u$ and minimally excite the suppressed states x_s.

We consider only the low-order problem

$$\dot{x} = A_c x_c + B_c u \tag{5}$$

$$y = C_c x_c \tag{6}$$

but we modify the usual performance index

$$J_1 = \int_0^\infty [x_c{}^T Q_c x_c + u^T R_c u] \, dt \tag{7}$$

and substitute

$$J_2 = \int_0^\infty [x_c{}^T Q_c x_c + u^T R_c u + (B_s u)^T R_s (B_s u)] dt \tag{8}$$

By penalizing the control spillover $B_s u$, we can suppress this term. In many eases, we can make $B_s u$ approach zero without seriously constraining $B_c u$.

In practice, one obtains desired stability characteristics by incorporating an α-shift in the problem, Ref. 20, introducing

$$\dot{x}_c = \begin{bmatrix} A_c + \alpha I \end{bmatrix} x_c + B_c u \tag{9}$$

with

$$J = \int_0^\infty e^{2\alpha t} [x_c{}^T Q_c x_c + u^T R_c u \qquad 00$$
$$+ (B_s u)^T R_s (B_s u)] \, dt \tag{10}$$

This refinement is important for practical application, although not necessary to an understanding of the concept. Analogous procedures are used in estimator design which is designed by duality theory, Ref. 20 through 22.

The final control law is realized via estimated state feedback

$$u = -K\hat{x}_c \tag{11}$$

where $\hat{x}_c$ is an estimate of the state vector x_c and K is the optimal controller gain matrix. Defining estimation error e_c as

$$e_c = x_c - \hat{x}_c \tag{12}$$

the control vector, equation (11), becomes

$$u = -K(x_c - e_c)$$

and it is possible to write the state dynamics, equation (3), as

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c(-K\hat{x}_c) \\ &= A_c x_c - B_c K(x_c - e_c) \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{x}_s &= A_s x_s + B_s(-K\hat{x}_c) \\ &= A_s x_s - B_s K(x_c - e_c) \end{aligned} \tag{14}$$

Similarly the estimator dynamics (error form) satisfy

$$\begin{aligned} \dot{e}_c &= \dot{x}_c - \dot{\hat{x}}_c \\ &= A_c x_c + B_c u - A_c\hat{x}_c - B_c u \\ &\quad - G(C_c x_c + C_s x_s - C_c\hat{x}_c) \\ &= A_c e_c - GC_c e_c - GC_s x_s \end{aligned} \tag{15}$$

where G is the optimal observer gain matrix.

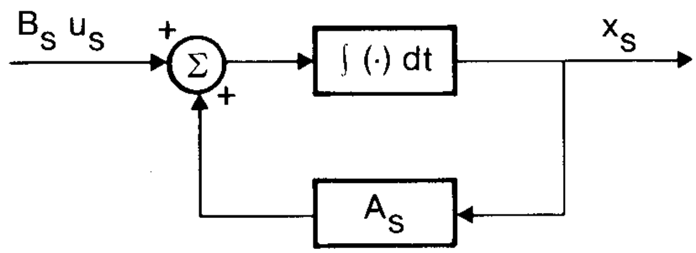
Assembling equations (13 through 15) in matrix form yields

$$\begin{bmatrix} \dot{x}_c \\ \dot{e}_c \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} A_c - B_cK & B_cK & 0 \\ 0 & A_c - GC_c & -GC_s \\ -B_sK & B_sK & A_s \end{bmatrix} \begin{bmatrix} x_c \\ e_c \\ x_s \end{bmatrix} \qquad (16)$$

The excitation of suppressed states occurs through the matrix B_sK, the sensing of the suppressed states occurs through the matrix GC_s; but these matrix entries are precisely those which are constrained by the algorithm. For certain classes of systems, these terms can be made zero, Ref. 23. Figure 3 illustrates the decoupling action of the controller.

- **INPUT TO SUPPRESSED SYSTEM CONSTRAINED**

$$\dot{x}_s = A_s x_s + B_s u_s \qquad (1)$$



OPTIMALITY CONSTRAINS INTERNAL SYSTEM DRIVER

$$J_s = \int_0^{\infty} (B_s u)' R_s (B_s u)\, dt \qquad (2)$$

Figure 3. Controller decoupling action.

4. DECENTRALIZED CONTROL

Although an elastic body has essentially an infinite number of normal mode coordinates, it is possible to model with acceptable accuracy only a comparatively modest number (surely less than 100). In most applications, the unmodeled states can be safely truncated and ignored in control system design. However, there remains the problem of designing a controller for a physical system which is characterized by a number of states and is generally too large to be accommodated in a state estimator operated by a flight computer. Moreover, it may not be desirable to develop a centralized estimation and control system even if it was possible. A decentralized concept which permits many independent estimation and control systems to assume individual control responsibilities for a subset of the system states has four advantages: (1) feasibility with small flight computers, or even individual micro-processors; (2) reliability, since subsystem failure need not imply system failure, and redundancy of model coverage is easily implemented; (3) adaptability, since design changes or model revisions can be accommodated by changes in individual subsystem controllers; (4) economy, since the greater design flexibility permits consideration of more economical alternatives.

The model error sensitivity suppression method can be incorporated into a decentralized control system quite easily, as illustrated by the following example. Suppose that an elastic body (fully characterized by an infinite number of coordinates) is adequately described in terms of a model having 12 degrees of freedom, as defined by 12 normal mode coordinates for the uncontrolled body. If each of these coordinates is judged to be dynamically significant, but a twenty-fourth order state estimator is judged to be infeasible or undesirable, one can construct instead three eighth-order estimators (or four sixth-order estimators, or any other such combination). Each of the three subsystem controllers can be charged with eight states, or four normal modes, grouped in any convenient manner (perhaps sequentially, or geographically, or by similarity of mode shape). The critical requirement is, of course, that any one of the subsystem controllers exert only minimal influence on those states with which it is not charged. This requirement can be met by assigning each subsystem controller not only eight controlled states (x_c) but also sixteen suppressed states (x_s); applying the method of the MESS algorithm.

Figure 4 illustrates the concept for two decentralized controllers. Application of the MESS algorithm tends to decouple the original system into component subsystems as shown.

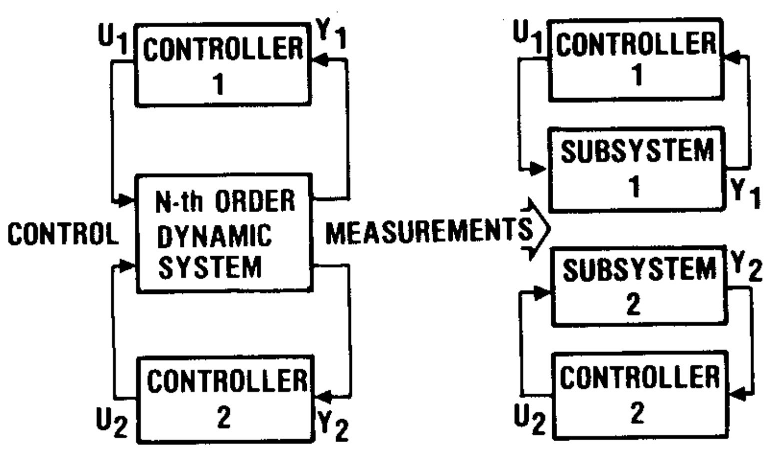


Figure 4. Decentralized control - two subsystems.

Each subsystem is stable when isolated, and the magnitude of subsystem interaction is constrained; i.e., each controller is "blind" to the other controllers and "sees" only its own subsystem. The remaining portions of the global system dynamics remain "invisible" to it. This decoupling effect of the MESS algorithm eliminates the need for coordination and information exchange among the controllers. The decoupling between the subsystems is not absolute but for many systems can be minimized to the extent that each controller views the actions of the other controller as a slight disturbance.

The closed-loop equations for the two controller systems are given below.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} A_{11} - B_1K_{11} & -\delta_1 & -B_{11}K_{11} & -\delta_1 \\ -\delta_2 & A_{22} - B_{22}K_{22} & -\delta_2 & -B_{22}K_{22} \\ 0 & \delta_1 + \delta_3 & A_{11} - G_{11}C_{11} & -\delta_1 \\ \delta_2 + \delta_4 & 0 & \delta_2 & A_{22} - G_{22}C_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ e_1 \\ e_2 \end{bmatrix} \quad (17)$$

$$\delta_1 = B_{12}\, K_{22} \qquad \delta_3 = G_{11}\, C_{12}$$

$$\delta_2 = B_{21}\, K_{11} \qquad \delta_4 = G_{22}\, C_{21}$$

In these equations, x_1 is the state vector associated with controller I, and x_2 is the state vector associated with controller II. The Greek "dels" indicate matrices constrained by the control and observation algorithms. Smallness in these matrices entries implies decoupling. Zero entries imply total decoupling.

5 DESIGN EXAMPLES — A SPACE PLATFORM

Figure 5 illustrates the large space structure used as a baseline vehicle for test applications of the model error sensitivity suppression method. The control task is the damping of the vibratory modes by two colocated sensor/actuator pairs at corners labeled A and B in Figure 5. Each pair is capable of torquing in roll and pitch and sensing local angular rates about these axes. For the studies presented, the platform is modeled by its four lowest frequency elastic modes with zero damping. Except for the decentralized altitude and elastic body controller, the three rigid body modes are omitted in order to facilitate computations. A state estimater, designed by dual MESS, is employed in all examples.

EXAMPLE 1 — NO MESS. This example illustrates that the platform is not an overly benign system. Modes 3 and 4 are neglected during the design process (truncated from the model) and an optimal controller/estimator is designed for modes 1 and 2. Figure 6 illustrates the unstable result — all modes, controlled and suppressed, are unstable.

EXAMPLE 2 — MESS APPLIED. The design algorithm is now applied to the platform. Figure 7 illustrates the results. The suppressed modes receive very little excitation from the controller and oscillate at low amplitudes. These modes ring because they are modeled with zero damping; the controller is *not* to damp these modes.

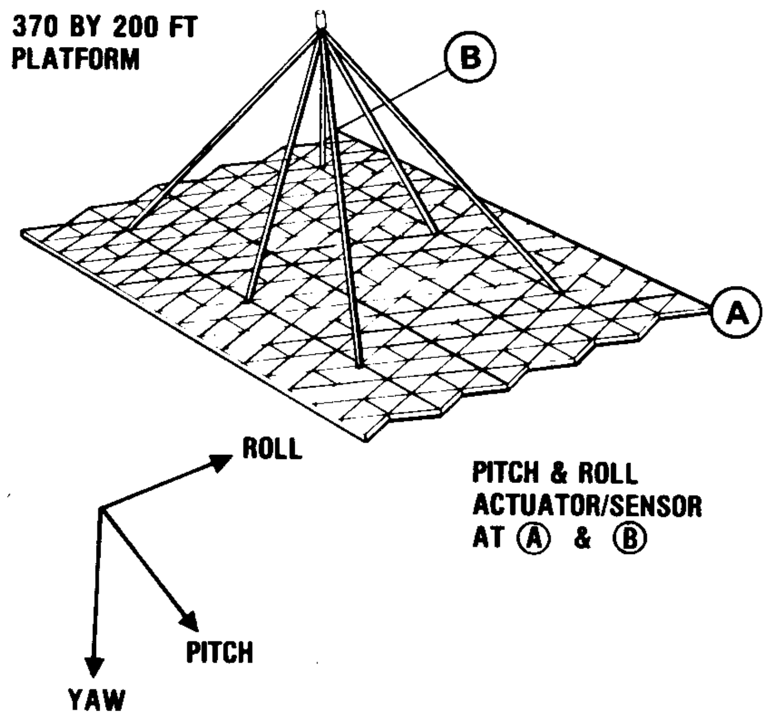


Figure 5. Large space platform for design examples.

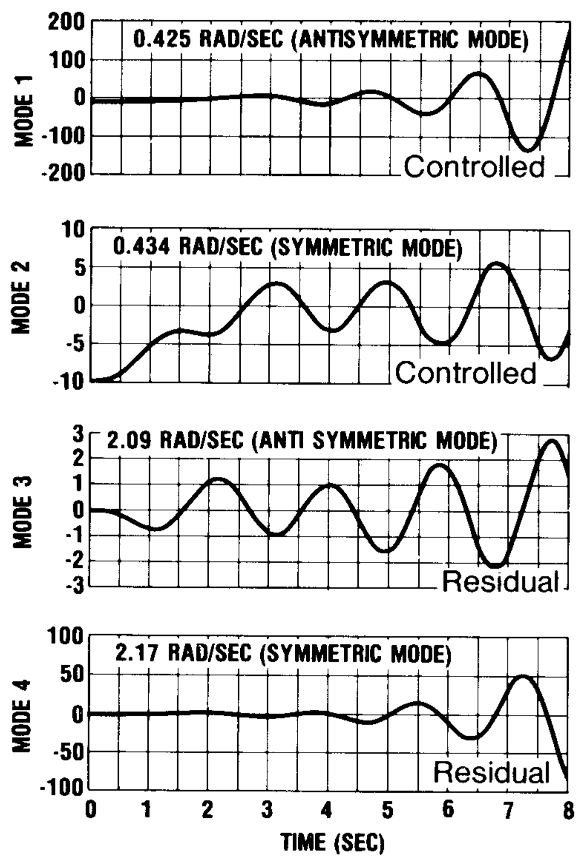


Figure 6. Attempted design by modal truncation unstable.

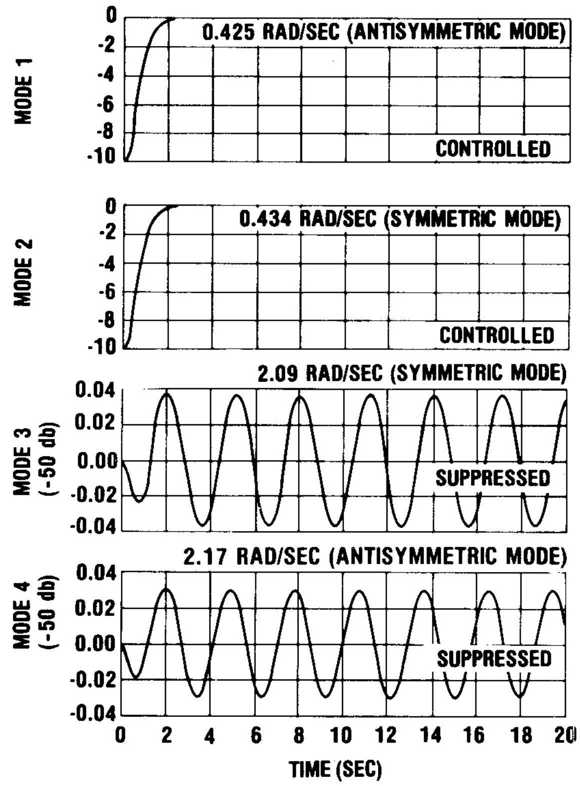


Figure 7. Suppression of two modes by the MESS algorithm.

EXAMPLE 3 — SEPARATE RIGID BODY AND ELASTIC MODE CONTROL. This example illustrates the capacity for decentralization that is inherent in the MESS algorithm. As shown in Figure 8, we wish to address the problem of elastic body modes whose frequencies are within the rigid body controller bandwidth. First we design a rigid body controller using MESS. (For this example, a position sensor was placed in the center of the platform to sense rigid body motion.) Figure 9 illustrates the rigid body controller action. The elastic modes are suppressed and ring with low amplitude.

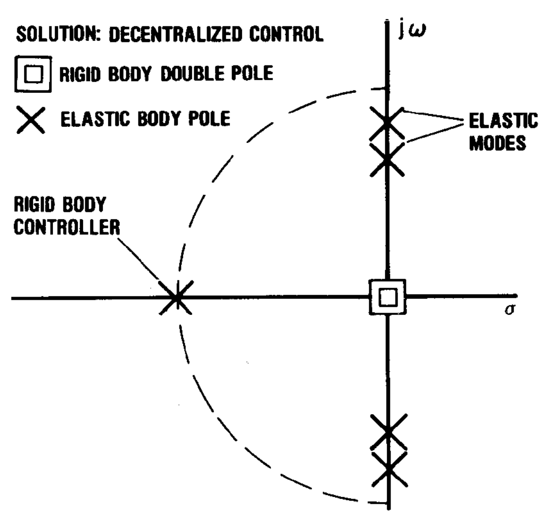


Figure 8. Elastic modes in the rigid body controller bandwidth.

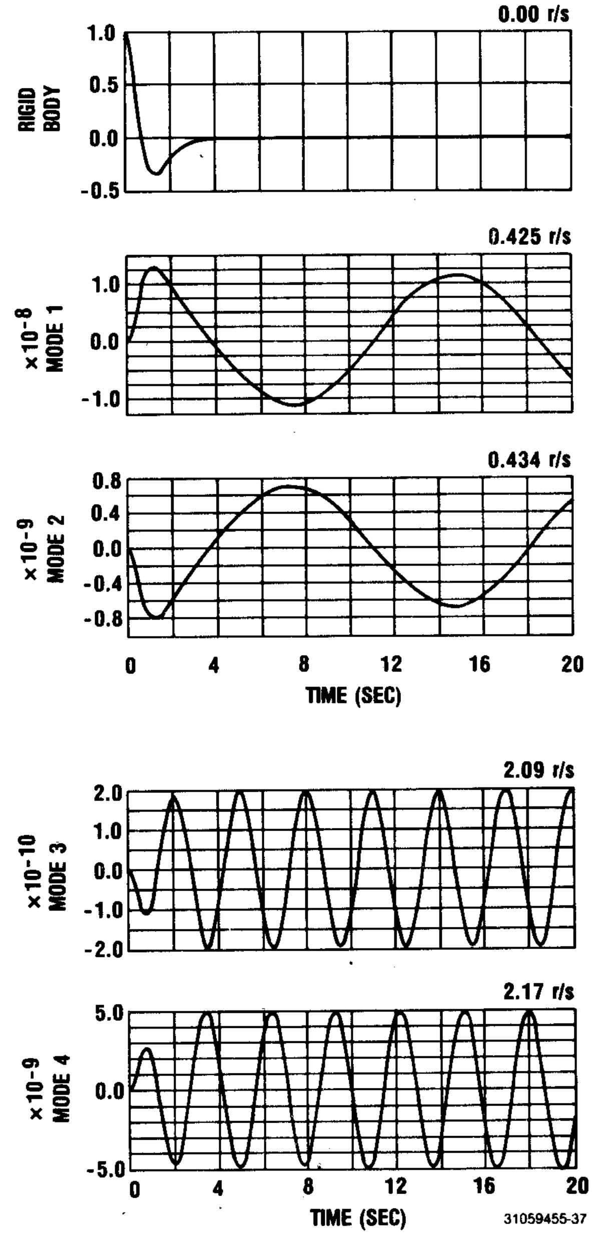


Figure 9. Decentralized rigid body controller response.

The next step is the application of the MESS algorithm for elastic body control. Figure 10 illustrates the elastic body controller action. The rigid body mode rings at very low amplitude and frequency it is stable).

The final step of the procedure is the simultaneous placing of both the rigid body and elastic body controllers on the platform. Figure 11 illustrates the results and demonstrates that the MESS algorithm can provide noninteracting rigid body and elastic body control, although the modal frequencies are in the rigid body controller bandwidth. Figure 12 shows the open and closed-loop pole-zero plots.

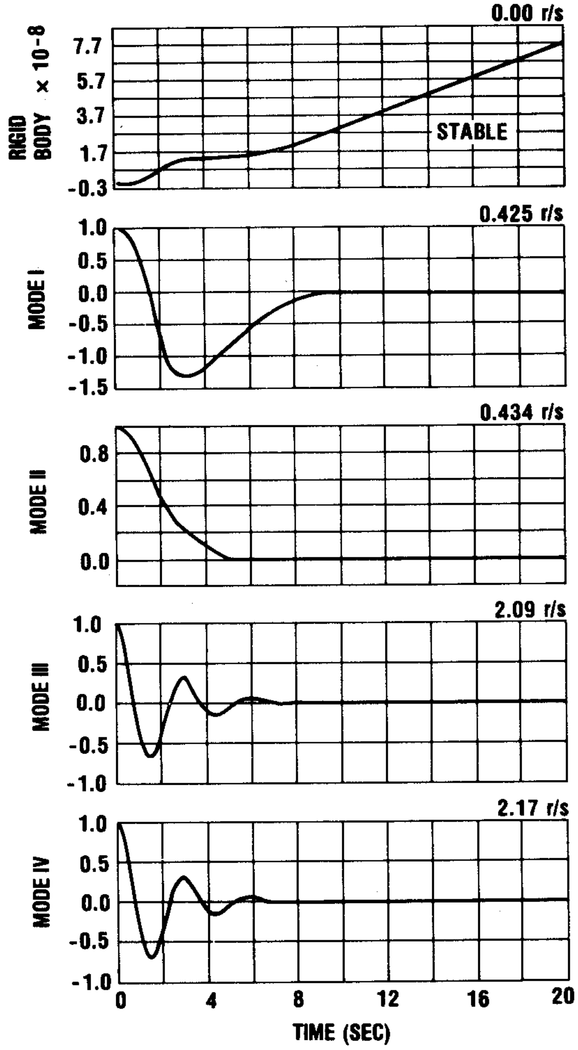


Figure 10. Decentralized elastic body controller response.

EXAMPLE 4 — FREQUENCY DISCRIMINATION. For large structures, the modal frequencies of controlled and suppressed modes will be closely spaced. As a result, the controller should not be overly sensitive to small frequency differences. To test this sensitivity for MESS designs, identical frequencies of 1.0 radian were placed in a modified model of the platform and a MESS design was performed. Figure 13 illustrates these results. The algorithm is capable of providing control with identical modal frequencies.

EXAMPLE 5 — PARAMETER SENSITIVITY STUDY. A practical system must not be overly sensitive to parameter variations. To examine the closed-loop parameter sensitivity, a MESS controller for three controlled modes and one suppressed mode was designed. Figure 14 illustrates system performance for parameter variations of standard deviation for 30% from the baseline vehicle. (All matrix entries were varied randomly according to a normal distribution with 30% standard deviation.) This study and others, Ref. 1 and 4, have indicated that MESS controllers are not overly sensitive to parameter error. A more exhaustive study including the effect of actuator number was undertaken in Ref. 24.

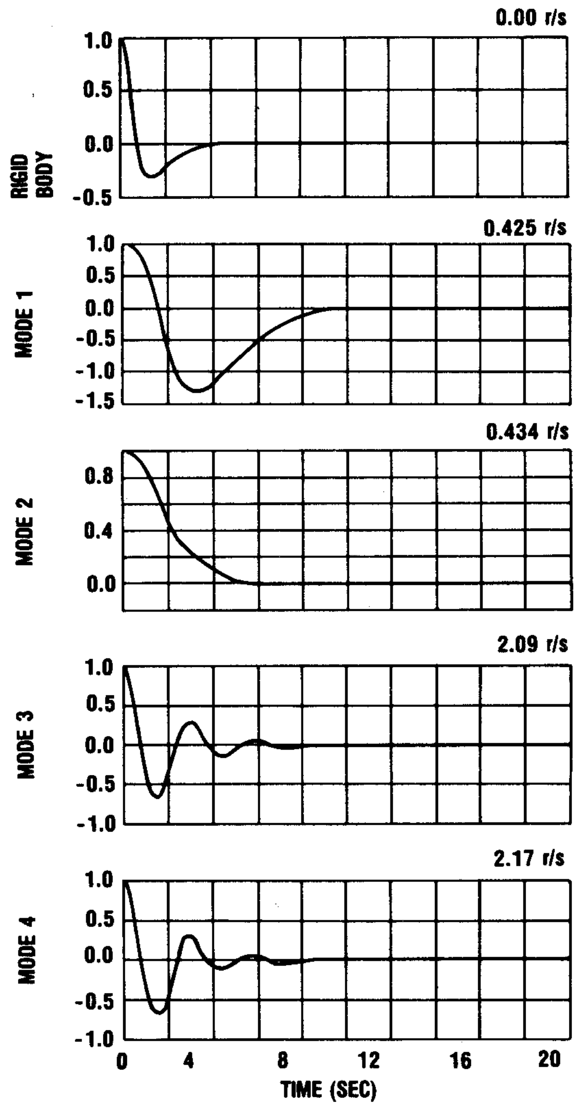


Figure 11. Decentralized rigid body and elastic body controller - two controllers.

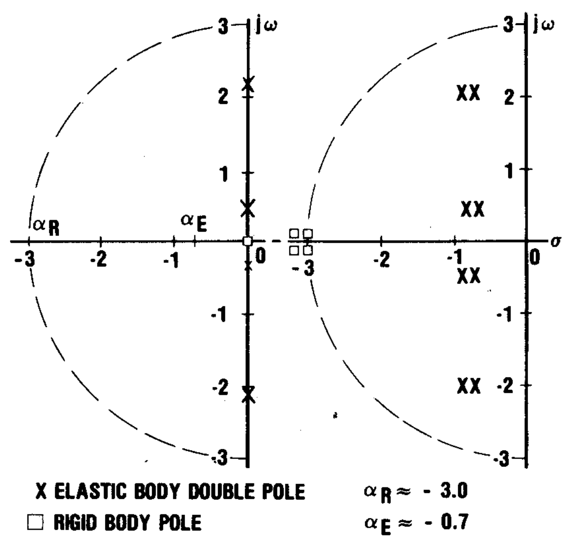


Figure 12. Pole-zero plots for the decentralized controller.

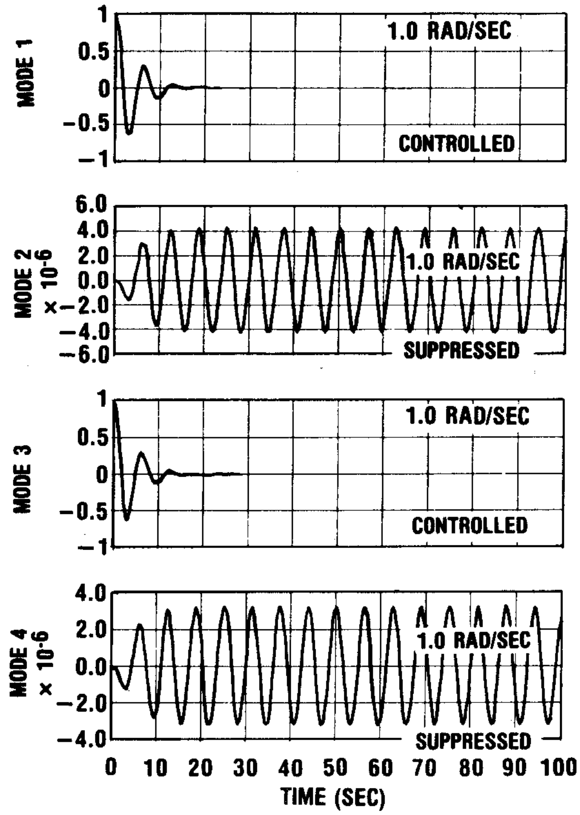


Figure 13. Suppression of modes with identical frequencies.

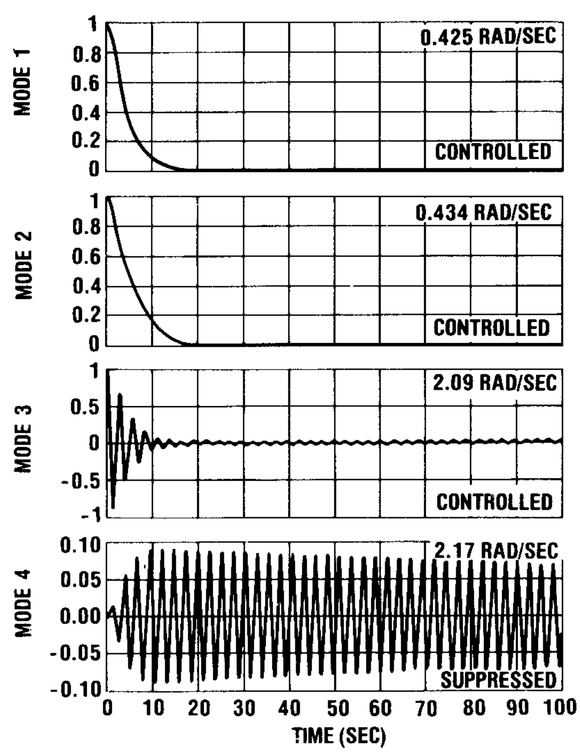


Figure 14. Parameter error study. All system parameters varied according to 30% standard deviation.

EXAMPLE 6 — UNMODELED MODES. Ultimately, every controller will have to function in the presence of unmodeled modes. To evaluate the effect of unmodeled modes, a two controlled and two suppressed mode controller was designed, and three unmodeled modes were added to the system. Figure 15 illustrates the results. Stability, suppression, and controlled mode stabilization were maintained in the presence of unmodeled modes.

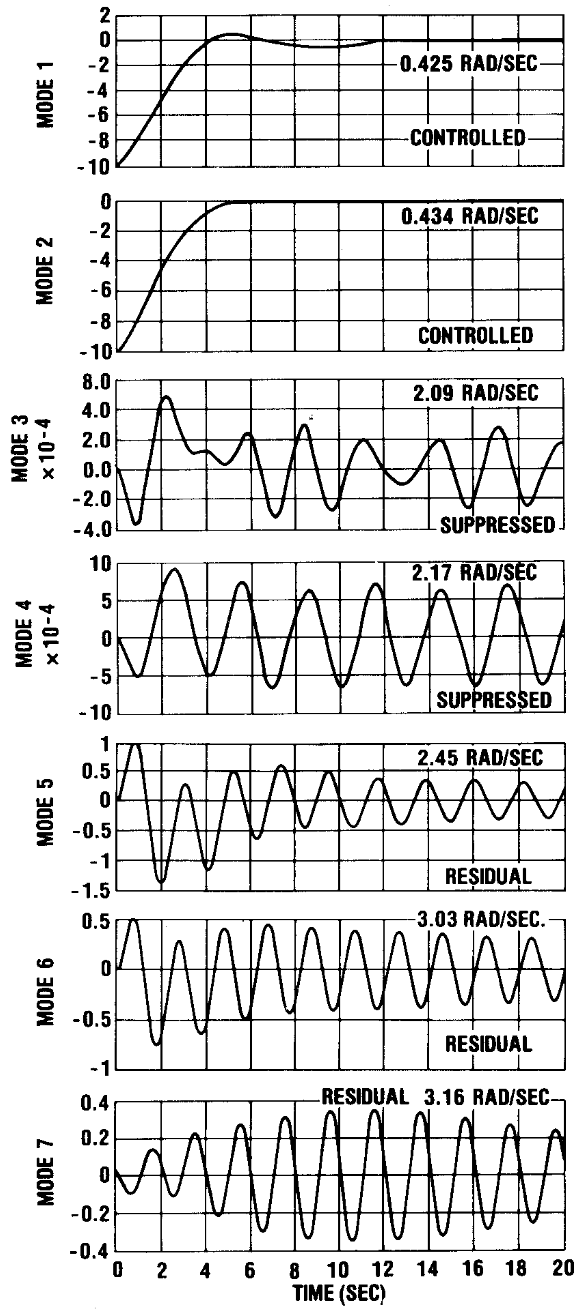


Figure 15. Truncation error study — three unmodeled modes.

6. CONCLUSIONS

Although more experience is required before all the capabilities and limitations of MESS are fully appreciated, it seems evident that this approach permits a practical solution to the error sensitivity problem. For a class of physical systems, at virtually no cost in control system complexity, this method provides substantial reduction in control and observation spillover for truncated modes having known characteristics. On the basis of preliminary empirical evidence, the resulting system performance seems to have relatively little sensitivity to uncertainties in these modal characteristics.

The MESS approach is particularly advantageous in a decentralized control mode of application in which each subsystem controls certain modes of deformation with minimal spillover into other subsystem modes.

These advantages of the MESS technique make it an ideal candidate to satisfy the stringent requirements for control of the large communications satellites of the next two decades. It seems unlikely that conventional techniques will be able to satisfy the pointing requirements of large, multibeam antennas attached to large, lightweight structures having several closely spaced low-frequency dynamic modes. We therefore expect such systems to implement decentralized control schemes incorporating some version of the Model Error Sensitivity Suppression technique described above.

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APPENDIX

This appendix contains the mathematical model for the structure, matrices A, B, and C. The control gain matrix K, and the observer gain matrix G, are included for a three-controlled-mode, one-suppressed-mode controller (Figure A-1).

MATRIX A (8, 8) COLUMNS 1 TO 8 (NOMINAL)

ROW 1 =	0.	1.000	0.	0.	0.	0.	0.	0.
ROW 2 =	-.1808	0.	0.	0.	0.	0.	0.	0.
ROW 3 =	0.	0.	0.	1.000	0.	0.	0.	0.
ROW 4 =	0.	0.	-.1884	0.	0.	0.	0.	0.
ROW 5 =	0.	0.	0.	0.	0.	1.000	0.	0.
ROW 6 =	0.	0.	0.	0.	-4.393	0.	0.	0.
ROW 7 =	0.	0.	0.	0.	0.	0.	0.	1.000
ROW 8 =	0.	0.	0.	0.	0.	0.	-4.694	0.

MATRIX B (8, 4) COLUMNS 1 TO 4 (NOMINAL)

ROW 1 =	0.	0.	0.	0.
ROW 2 =	-9.1686E-04	-1.3571E-03	9.1686E-04	-1.3571E-03
ROW 3 =	0.	0.	0.	0.
ROW 4 =	6.4858E-04	1.5854E-03	6.4858E-04	-1.5854E-03
ROW 5 =	0.	0.	0.	0.
ROW 6 =	1.4347E-03	-1.0473E-03	1.4347E-03	1.0473E-03
ROW 7 =	0.	0.	0.	0.
ROW 8 =	2.0359E-03	-8.0632E-04	-2.0359E-03	-8.0632E-04

MATRIX C (4, 8) COLUMNS 1 TO 8 (NOMINAL)

ROW 1 =	0.	-1.8337E-03	0.	1.2972E-03	0.	2.8693E-03	0.	4.0719E-03
ROW 2 =	0.	-2.7141E-03	0.	3.1708E-03	0.	-2.0945E-03	0.	-1.6126E-03
ROW 3 =	0.	1.8337E-03	0.	1.2972E-03	0.	2.8693E-03	0.	-4.0719E-03
ROW 4 =	0.	-2.7141E-03	0.	-3.1708E-03	0.	2.0945E-03	0.	-1.6126E-03

MATRIX K4 (4, 6) COLUMNS 1 TO 6 (REG. — 4 ACTUATORS)

ROW 1 =	-56.41	-161.2	67.58	175.0	67.09	327.2
ROW 2 =	-142.4	-406.9	125.7	373.7	-178.1	-214.4
ROW 3 =	56.41	161.2	67.58	175.0	67.09	327.2
ROW 4 =	-142.4	-406.9	-125.7	-373.7	178.1	214.4

MATRIX G4 (6, 4) COLUMNS 1 TO 4 (EST. — 4 SENSORS)

ROW 1 =	203.8	514.6	-203.8	514.6
ROW 2 =	-92.10	-232.5	92.10	-232.5
ROW 3 =	-241.1	-433.1	-241.1	433.1
ROW 4 =	102.8	212.7	102.8	-212.7
ROW 5 =	-10.34	25.35	-10.34	-25.35
ROW 6 =	188.4	-120.1	188.4	120.1

MATRIX EIGV2 (14, 2) COLUMNS 1 TO 2

CLOSED-LOOP EIGENVALUES
NOMINAL SYSTEM
4 ACTUATORS

ROW 1 =	-1.9895E-13	2.166
ROW 2 =	-1.9895E-13	-2.166
ROW 3 =	-.8000	2.096
ROW 4 =	-.8000	-2.096
ROW 5 =	-.7000	2.096
ROW 6 =	-.7000	-2.096
ROW 7 =	-.7000	.4252
ROW 8 =	-.7000	-.4252
ROW 9 =	-.8000	.4251
ROW 10 =	-.8000	-.4251
ROW 11 =	-.7000	.4340
ROW 12 =	-.7000	-.4340
ROW 13 =	-.8000	.4340
ROW 14 =	-.8000	-.4340

Figure A-1. System matrices.

The procedure used to calculate the optimal gains is outlined in Table A-1. The internal driver minimization form of the algorithm is used to alleviate the need for calculating any inverse matrices of As, Ref. 1 and 2.

The state weighting matrix Q_c is taken to be an identity matrix of proper dimension. The same is true of the control weighting matrix R_c. The suppressed mode weighting matrix R_s equals $\beta 1$, where I is an identity matrix and β is a scalar equal to 10^{10}. As noted in the text, the performance index of equation (3) Table A-1 is further modified by the α-shift (equation 3 - text). Alpha for the regulator is 0.35 and α for the estimator is 0.40. Therefore, the design procedure is: choose the suppressed modal system, form the performance index of equation (3), and further modify equation (3) by the α-shift technique. Mode (4), the highest frequency mode, is suppressed for the example.

Typical system responses of the four actuator systems at nominal values are shown in Figure A-2.

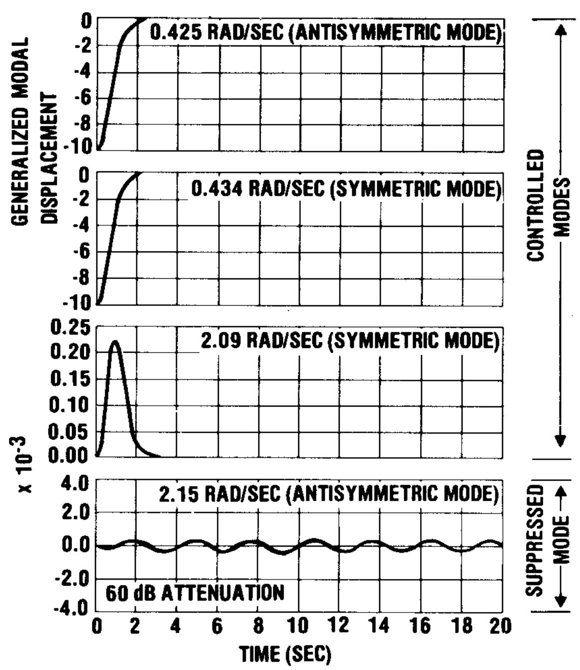

Figure A -2. System responses of the four actuator nominal system.

INITIAL FORMULATION (STANDARD TRUNCATED DESIGN)

$$\min_{u} J = \int_0^{\infty} (x_c' Q_c x_c + u' R_c u) dt \quad (1)$$

SUBJECT TO

$$\dot{x}_c = A_c x_c + B_c u$$

ADD INTERNAL DRIVER TO PERFORMANCE INDEX

$$u' R_c u \rightarrow u' R_c u + u' B_s' R_s B_s u \quad (2)$$

MINIMIZE THE NEW PERFORMANCE INDEX

$$J_n = \int_0^{\infty} \{ x_c' Q_c x_c + u' [R_c + B_s' R_s B_s] u \} dt \quad (3)$$

Table A -1. Performance under modification for internal driver minimization.