RANDOM CODING BOUNDS FOR NONCOHERENT mFSK MULTIPLE-ACCESS CHANNELS

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INTRODUCTION

We investigate a time-varying trellis coded multiple-access scheme using noncoherent mFSK signals. Techniques similar to this were originally proposed by Cohen, Heller, and Viterbi[1] and more recently in a mFSK form by Viterbi.[2] In these multiple-access systems van der Muelen[3] Ahlswede[4] Liao[5], Gaarder and Wolf [6], Kasami and Lin[7], Weldon[8] and Wolf[9] have shown that the decoded symbols of one user can be used to reduce the "multiple-access noise" to other users and thus allow for a larger achievable rate region than one would expect with conventional time division multiple-access techniques. In some cases, specific codes were investigated. Peterson and Costello[10] and Chevillat[11] have extended these earlier works to convolutional and trellis codes. In this case the decoder is designed as a "super" Viterbi decoder that regards all transmitter trellis codes combined to form a single "super" trellis encoder.

In this paper we investigate the noncoherent mFSK scheme discussed by Viterbi[2] and generalize to single level and multi-level energy detectors with a single "super" Viterbi decoder at the receiver. The main results are random coding bounds for the general case where L users each have remotely located time-varying trellis encoders of constraint length K. We assume throughout that the channel is noiseless, and symbol timing synchronization is maintained among the L users. These assumptions are being relaxed in the thesis research of Sorace[12].

CHANNEL MODELS

We assume that during T seconds each of L users will transmit an mFSK tone where the m frequencies are $\{f_1, f_2, \ldots, f_m\}$. By choosing

$$f_{i+1} = f_i + \frac{1}{T} \qquad i = 1,2,\ldots,m-1$$

we have orthogonal tones of total bandwidth B_m given approximately by

$$B_m \cong \frac{m}{T}$$

Conventional frequency division multiple-access (FDMA) divides the m frequencies into L disjoint subsets and assign each user one such subset of frequencies. Time division multiple-access (TDMA) allows all m frequencies to be used by each of the L users on a time-shared basis. In this paper we allow all L users to simultaneously use all m frequencies in a mFSK modulation scheme where each user sends one of m frequency tones every T seconds.

Throughout this paper we assume the channel is noiseless and the receiver's front end consists of m energy detectors that measure the incoming signal energy at each of the frequencies $\{f_1,f_2,...,f_m\}$. All L users are synchronized so that each user transmits a tone at the same fixed T second intervals. Also we assume each of the L tones have the same energy and whenever several tones are transmitted at one frequency the resulting received signal energy during the T second interval is the sum of the individual signal energies (noncoherent combining of signals). The m frequency energy detectors can be implemented by "chirp Z-transform" devices and have been suggested for use on-board satellites.[2]

Figure 1 illustrates the system discussed above. The energy detectors output the vector

$$\underline{e} = (e_1, e_2, \ldots, e_m)$$

at each T second interval where e_i is the measured energy at the frequency f_i during the T second interval. Since the tones are orthogonal we assume there is no energy component in e_i due to tones of frequency f_j where $j \neq i$. Thus, when ℓ tones are transmitted at frequency f_i, we have $e_i = \ell\xi$ where ξ is the energy of each tone. As shown in Figure 1, the energy detectors are followed by quantizers with outputs denoted by $\{n_1,n_2,...,n_m\}$. This completes the model of the mFSK multiple-access channel. We shall consider here two types of quantization.

HARD DECISION CHANNEL

For some threshold $0 < \delta < \xi$ we define the hard quantizer output of the ith detector as

$$n_i = \begin{cases} 1 \; ; & e_i \geq \delta \\ 0 \; ; & e_i < \delta \end{cases} \tag{1}$$

$$i = 1,2,\ldots,m$$

This results in energy detectors that measure the presences of one or more tones at each frequency without regard for the number of tones at each frequency. Every T second interval results in the channel output

$$\underline{n} = (n_1, n_2, \ldots, n_m).$$

If $L \geq m$, the number of possible distinct channel outputs is

$$2^m - 1$$

since (0,0,0,...,0) is not allowed. If $L < m$ then there can be at most L nonzero components in $\underline{n}$. Hence the number of distinct outputs is

$$\sum_{k=1}^{L} \binom{m}{k}.$$

we define the output alphabet size as

$$J_{HD}(L,m) = \begin{cases} \sum_{k=1}^{L} \binom{m}{k} & , \ L < m \\ 2^m - 1 & , \ L \geq m \end{cases} \tag{2}$$

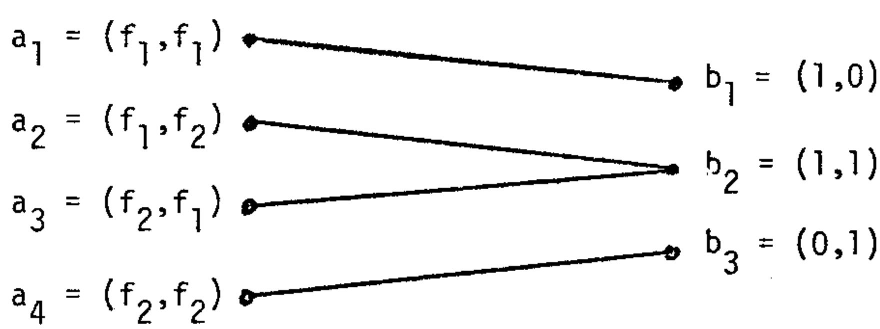
and denote the possible outputs as $b_1, b_2, \ldots, b_j$.

With L users each sending one of m tones, there are

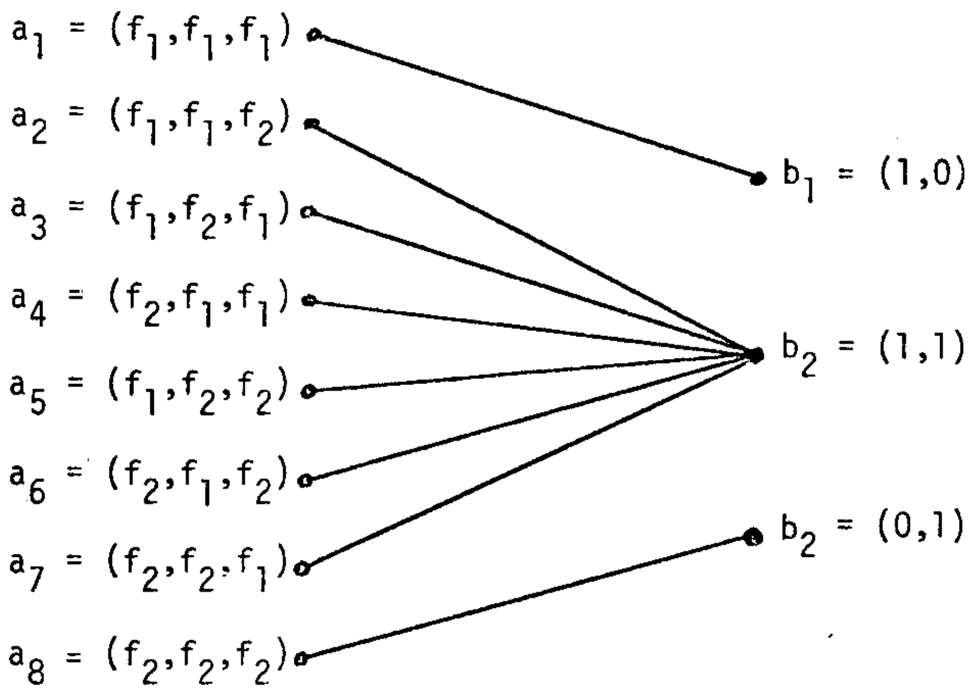
$$K = m^L \tag{3}$$

possible channel inputs which we denote as $a_1, a_2, \ldots, a_K$. Hence we have a K input, $J = J_{HD}(L,m)$ output discrete memoryless channel. We illustrate this with some examples.

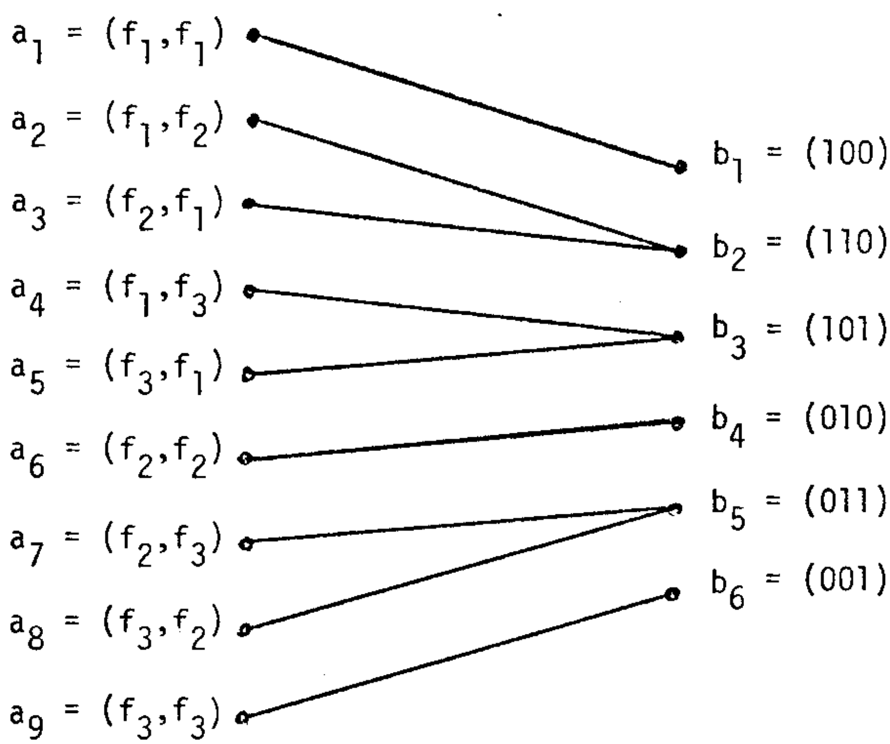
Example: $L = 2$, $m = 2$



Example: $L = 3,\ m = 2$



Example: $L = 2,\ m = 3$



SOFT DECISION CHANNEL

The soft decision channel is one where the ith quantizer output is given by

$$n_i = k \text{ if } k\mathcal{E} \le e_i < (k+1)\mathcal{E};\; k = 0,1,2,\ldots,L \qquad i = 1,2,\ldots,m \tag{4}$$

That is, each quantizer output indicates the number of tones received at that particular frequency. The channel output is thus

$$\underline{n} = (n_1, n_2, \ldots, n_m)$$

where

$$n_i \varepsilon \left\{0,1,2,\ldots,L\right\} \quad i = 1,2,\ldots,m;$$

and

$$n_1 + n_2 + \ldots + n_m = L$$

Let $J_{SD}(L,m)$ be the number of distanct channel outputs. The number of sequences $n_1 n_2 \ldots n_{m-1}$

where

$$n_1 + n_2 + \ldots + n_{m-1} = i$$

is $J_{SD}(i, m-1)$ so that we have the recursion formula,

$$\begin{aligned} J_{SD}(L,m) &= \sum_{i=0}^{L} J_{SD}(k,m-1) \\ &= \sum_{i=0}^{L-1} J_{SD}(i,m-1) + J_{SD}(L,m-1) \\ &= J_{SD}(L-1,m) + J_{SD}(L,m-1). \end{aligned} \tag{5}$$

It is easy to see that end conditions are

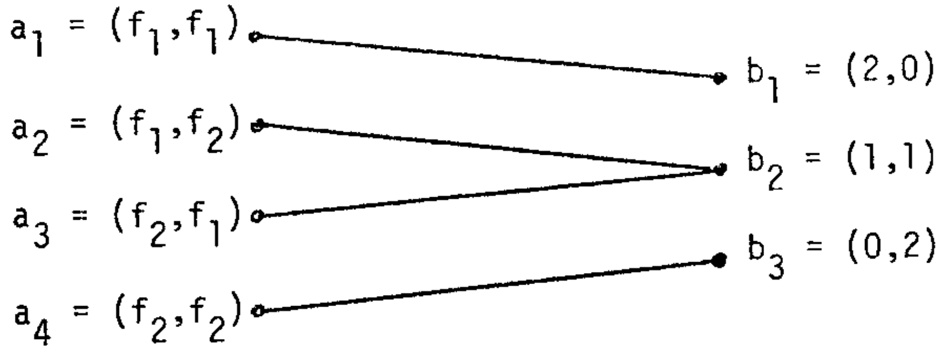
$$J_{SD}(L,2) = \ell + 1$$

and

$$J_{SD}(1,m) = m \ .$$

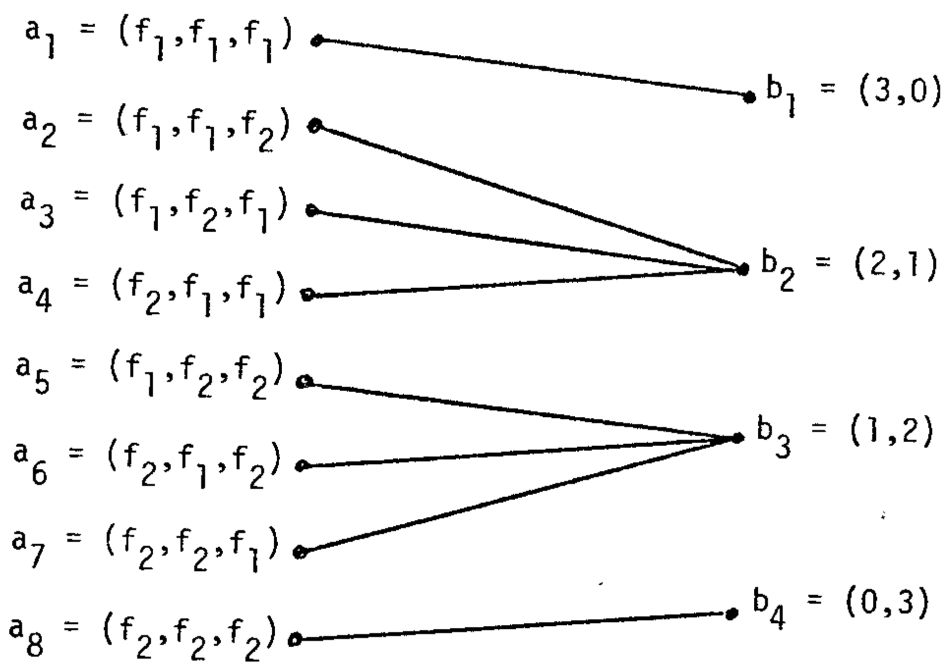
We denote the $J = J_{SD}(L,m)$ outputs as $b_1,b_2,\ldots,b_J$ and the $K = m^L$ inputs as $a_1,a_2,\ldots, a_K$.

Example: L = 2, m = 2



This is identical to the hard decision channel with L = 2, m = 2.

Example: L = 3, m = 2



TIME-VARYING TRELLIS CODES

Let us now return to the L = 2, m = 2 multiple-access channel where now the two users have time-varying trellis codes[14] of constant length υ is the rate r = b/n bits per channel symbol. Here b and n are integres and υ is the encoder memory consisting of υ 2^b-ary sumbols or bυ data bits. Each time b data bits enter the encoder, n FSK signals are transmitted (m = 2 here). Hence data bits enter the trellis encoder at a rate of one bit every nT/b seconds.

If the ℓth trellis code output sequence is

$$\underline{x}^{(\ell)} = \left(\dots, \underline{x}_{-1}^{(\ell)}, x_0^{(\ell)}, x_1^{(\ell)}, \dots\right)$$

$$\ell = 1,2$$

where

$$x_i^{(\ell)} \varepsilon \left\{f_1, f_2\right\}$$

then we can define a "super" trellis code output sequence

$$\underline{z} = (\dots, z_{-1}, z_0, z_1, \dots)$$

where

$$z_i = \left(x_i^{(1)}, x_i^{(2)}\right) \varepsilon \left\{a_1, a_2, a_3, a_4\right\}.$$

belong to the multiple-access channel input alphabet.

The "super" trellis encoder illustrated in Figure 2 consists of the L user trellis encoders and therefore has constraint length υ and rate Lb/n bits per multiple-access channel input symbol. That is, it inputs Lb bits (b bits from each of L sources) and outputs n symbols from, $\mathcal{A} = \left\{a_1, a_2, \dots, a_K\right\}$, the multiple-access channel input alphabet. The maximum likelihood receiver can be realized with the Viterbi algorithm. In practice, since we have a zero-one metric, the sequential decoding algorithms may be more practical. Also sequential decoding can handle the large number of states $2^{Lb\upsilon}$. In the following, however, we shall assume the Viterbi decoding algorithm to examine achievable performance.

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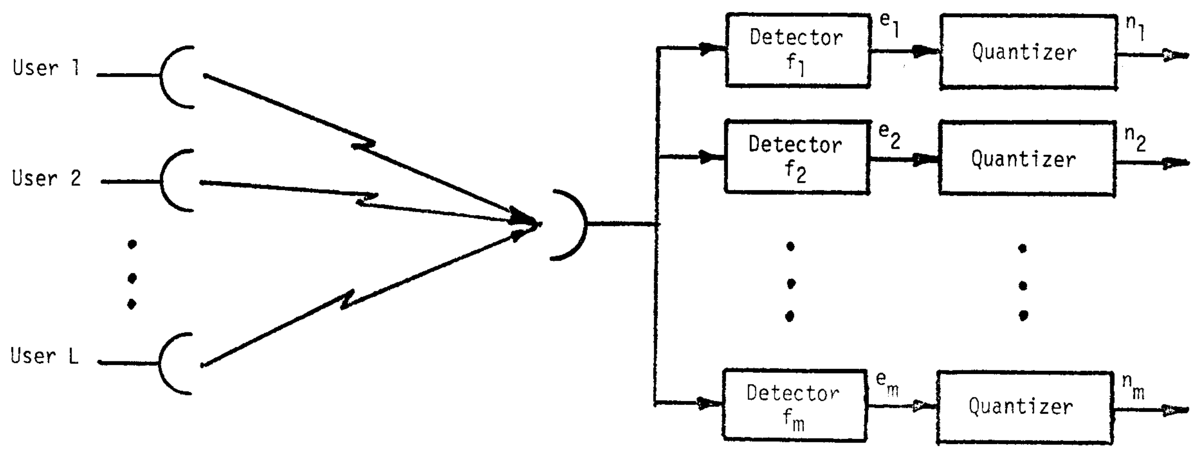

FIGURE 1. THE MULTIPLE-ACCESS CHANNEL

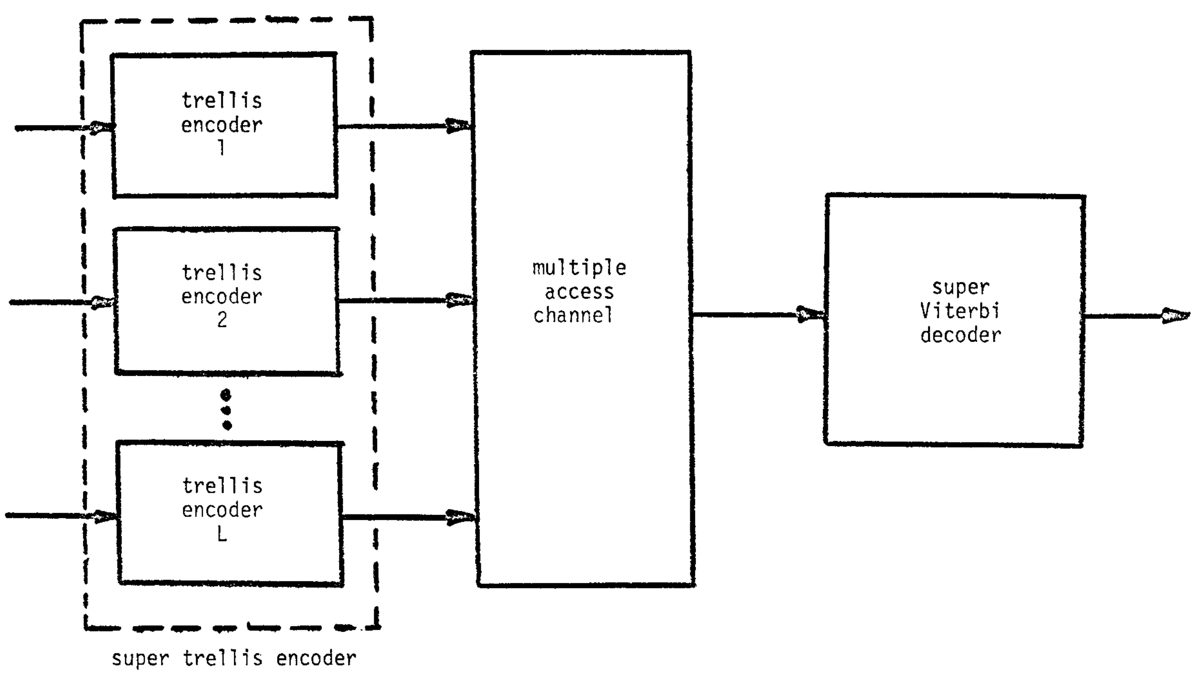

FIGURE 2. TRELLIS CODING SYSTEM