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Model Choice in Multiobjective Decision Making in Water and Mineral Resource Systems

by

Mark Elliott Gershon

University of Arizona

Technical Reports on
Natural Resource Systems

Collaborative Effort Between:

Hydrology and Water Resources
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This report is published to provide detailed data and methodological developments and is not meant to replace publication in refereed journals. It presents the results of doctoral research of the author.

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TABLE OF CONTENTS

TABLE OF CONTENTS--Continued

LIST OF TABLES

LIST OF TABLES--Continued

LIST OF ILLUSTRATIONS

ABSTRACT

The problem of model choice in multiobjective decision making, that is, the selection of the appropriate multiobjective solution technique to solve an arbitrary multiobjective decision problem, is considered. Classifications of the available techniques are discussed, leading to the development of a set of 27 model choice criteria and an algorithm for model choice. This algorithm divides the criteria into four groups, only one of which must be reevaluated for each decision problem encountered. Through the evaluation of the available multiobjective techniques with respect to each of the model choice criteria, the model choice problem is modeled as a multiobjective decision problem. Compromise programming is then used to select the appropriate technique for implementation.

Two case studies are presented to demonstrate the use of this algorithm. The first is a river basin planning problem where a predefined set of alternatives is to be ranked with respect to a set of criteria, some of which cannot be quantified. The second is a coal blending problem modeled as a mathematical programming problem with two linear objective functions and a set of linear constraints. An appropriate multiobjective solution technique is selected for each of these case studies.

In addition, an approach for the solution of dynamic multiobjective problems, one area where solution techniques are not available, is

presented. This approach, known as dynamic compromise programming, essentially transforms a multiobjective dynamic programming problem into a classical dynamic programming problem of higher dimension. A dynamic programming problem, modeled in terms of three objectives, is used to demonstrate an application of this technique.

CHAPTER 1

INTRODUCTION

Recent emphasis in the field of multiobjective decision making has focused on classification of the multiobjective solution techniques. These classifications, however, have fallen short of prescribing how to choose one technique to solve some given problem. Attempts at evaluation and comparison of techniques have not succeeded. Despontin and Spronk (1979) point out that evaluation of multiple objective decision methods is useful, but not always necessary because different problems may require different solution techniques.

The purpose of the research documented here is to develop an algorithm by which the most suitable multiobjective solution technique can be selected for application to a multiobjective problem. Of explicit concern is the matching of a given problem with the appropriate technique. In addition, a practical area of multiobjective decision making, where little work has been done, is investigated; namely, the application of multiobjective decision making techniques to dynamic problems.

Three case studies are used to illustrate these concerns. The first presents a dynamic multiobjective problem while the next two provide examples of model choice problems. A multiobjective problem involving bauxite mining, water supply, and environmental impacts is chosen as the dynamic problem. The model choice algorithm is applied to two problems, one represented by a finite number of predefined alternatives which are to be ranked, and another represented by a mathematical modeling formulation (continuous set of alternatives).

The research has potential benefits to the user groups involved in the areas of the three case studies, as well as to any systems analyst using the multiobjective decision making techniques discussed in the literature review. With respect to the latter, the applied systems analyst is often at a loss to determine which model to choose when confronted with a real problem. This ambiguity may cause inappropriate selection of a multiobjective solution technique with a resulting misspecification of the design problem in terms of its criteria. For example, Cohon and Marks (1975), in their classification of multiobjective decision making techniques as applied to water resource problems, state that the ELECTRE method is not applicable to these problems. In a subsequent discussion, Krzysztofowicz, Castano, and Fike (1977) point out that ELECTRE has been applied quite successfully to water resource problems and its use in this area should be continued. Given this apparent contradiction, it is not surprising to see a mismatch between problem and methodology in practice.

There are three possible consequences of this situation. First, results stemming from poorly matched problems and solution techniques will be misleading or suboptimal. Second, useful models may be judged harshly due to the poor results obtained when they are applied incorrectly and might then fade into obscurity. Finally, a general trend away from the use of multiobjective decision making techniques may occur.

The second potential beneficiaries of this research are the user groups working in the areas of the case studies or related fields. Solution techniques in these areas will be improved and the studies will serve as a basis for applying the approach to other problems. Incorporation of several criteria, where possibly only one has been considered in the past, may change the approach to problem solving in these areas. Improved solutions may lead, for example, to substantial savings in the conservation of energy, water, or raw materials.

Cohon (1978) suggests that multiobjective planning can improve problem solving in at least three ways. First, multiobjective planning can help in the decision process by encouraging appropriate roles for the decision makers. Second, multiobjective planning typically identifies a larger range of alternatives than does single objective planning. Finally, multiobjective planning better portrays reality.

In summary, the following question is addressed in this research:

> How does a system analyst, when confronted with a decision problem defined in terms of multiple objectives, select the best multiobjective decision making technique to solve that problem?

1. Organization of the Dissertation

The chapters that follow present a survey of the field of multiobjective decision making. This survey serves as the foundation for the development and application of the model choice algorithm.

The terminology used in this field has not yet stabilized to the point where all researchers are using a standard set of terms. Chapter 2 attempts to answer the following questions:

1. What is multiobjective decision making?
2. What concepts must be understood prior to studying multiobjective decision making?
3. What are the important definitions in multiobjective decision making?

Once identified, these concepts and definitions are explained, providing reference for the remainder of the text.

Chapter 3 consists of a review of the multiobjective literature. Empahsis is placed on what has been accomplished and identifying areas where more work needs to be done (e.g., dynamic or stochastic problems). Chapter 2 described what multiobjective decision making is and Chapter 3 describes how it is accomplished.

Chapter 4 attempts to fill one of the gaps identified in Chapter 3 by describing and demonstrating the use of a dynamic approach to multiobjective decision making (Szidarovszky, Gershon, and Bardossy 1981). This approach transforms the multiobjective dynamic problem into a classical dynamic programming problem of higher dimension.

Chapter 5 develops the set of criteria upon which the selection of the best solution technique for a given problem is to be based. These criteria are based on the criteria used in various classifications of techniques as well as the criteria which have been used to compare and evaluate these techniques in the past.

Chapter 6 uses these criteria to develop a procedure by which the best multiobjective solution technique can be selected based on knowledge of the problem which must be solved and the decision maker responsible for the solution. The criteria are grouped in such a way as to minimize the effort required to select the solution technique. The model choice is achieved through the application of compromise programming to a techniques versus criteria array.

In Chapter 7 the model choice procedure is demonstrated. Two case studies are provided for this purpose. A river basin planning problem serves as the example defined in terms of a finite set of alternative solutions. The example defined in terms of a continuous set of alternative solutions is a coal blending problem. The available techniques are ranked as to their applicability for solving these problems. Finally, conclusions are reached concerning the research.

CHAPTER 2

CONCEPTS IN MULTIOBJECTIVE DECISION MAKING

The basic ideas and concepts used in multiobjective decision making (MODM) are introduced in this chapter. Use is made of these concepts in many of the various approaches to MODM to be discussed in the chapters that follow. Those presented here appear throughout the field of MODM and any definitions that are useful only to the understanding of a particular approach will be presented with the discussion of that approach.

Various authors have used different terms in reference to the same concept. "Goal point" (Ignizio 1976), "ideal point" (Zeleny 1973), "utopia point" (Yu 1973), and "aspiration level" (Monarchi 1972, Johnsen 1968) are all used to convey the same idea. "Aspiration levels" as used by Wierzbicki (1979) carries another meaning. A unified set of concepts for multiobjective decision makers is sought here. These concepts are listed, followed by an explanation of each.

1. Decision Problem
2. Decision Maker and Analyst
3. Multiple Objectives
    a. Noncommensurable
    b. Conflict
    c. Trade-offs
4. Preference Structure

5. Goals, Aspiration Levels
6. Ideal Solution
7. Efficient Solution
    a. Weakly Efficient
    b. Strongly Efficient
8. Marginal Solution
9. Decision Space vs. Objective Space
10. Compromise Solution
11. Generating Method
12. Preference Determination Method
13. Solution Technique

1. Decision Problem

The allocation of scarce resources to conflicting interests in a manner that achieves the greatest benefit for all is the desired result of the decision problem dealt with in multiobjective decision making. This implies that the system under study has been properly defined using, for example, the framework for system definition given in Wymore (1976). No emphasis will be placed on problem definition since this research is concerned with methods of solving previously defined problems. Specifically, systems design and systems analysis are the two classes of problems encountered. In the former, a mathematical model of the problem is formulated and solved to achieve the best possible allocation. The solution techniques are called mathematical programming techniques. For the latter, a finite set of solutions (systems) are developed and the goal is to select that solution from which the greatest benefits are obtained. An example of each kind of problem is modeled and solved in later chapters, but the emphasis of this chapter is placed on the mathematical programming approach.

Example

Hashimoto (1980) deals with a problem concerning the allocation of scarce water resources to municipal, agricultural, and recreational uses. A satisfactory allocation is achieved by using nonlinear programming.

2. Decision Maker and Analyst

The decision maker (DM) is that person or group whose preferences must be satisfied by the outcome of the decision process. It is the responsibility of the decision maker to specify the objectives of the decision problem. The analyst is responsible for conducting that decision process and presenting the results. This requires that a wide range of operations research tools (Hillier and Lieberman 1967, Wagner 1969, Taha 1976) be at his disposal.

The level of interaction between the analyst and the decision maker is a characteristic of the decision process which is influenced by the mathematical sophistication of the decision maker as well as his preference for taking an active or passive role in that process. The minimum interaction requirements are that the decision maker be able to identify the problem, the objectives, and the preference structure and then decide if the solutions presented to him by the analyst are acceptable.

Example

Zwirnmann, Kindler, and Golsbev (1980) describe a problem concerning control of the nonpoint nitrate pollution of municipal water supply sources. In this case, the authors serve as the analysts and the decision maker is the group responsible for implementing the proposed alternatives.

3. Multiple Objectives

The presence of multiple objectives in decision problems has been widely recognized (Keeney 1973, Major 1977). However, not all decision problems should be modelled in terms of multiple objectives. If one objective can be found which, when satisfied, incorporates all of the desires of the decision maker, it is then more convenient to use single objective optimization methods such as linear (Gass 1969), nonlinear (Himmelblau 1972), dynamic (Bellman 1957), or integer (Taha 1975) programming. In the language of mathematical programming, the single objective problem is known as "scalar" optimization and the multiobjective problem is known as "vector" optimization.

Example

Bertier and de Montgolfier (1973) use multiple objectives to model a problem in ecological planning (forest management). Objectives include the protection of green areas, improvement in public transportation, and the creation of more jobs.

A problem should be modelled using multiple objectives if two more concepts are present; namely, the objectives are noncommensurable and they are in conflict with one another. A solution resulting from any multiobjective analysis of conflicting objectives must "trade-off" among those objectives.

A. Noncommensurable Objectives

Objectives are noncommensurable if their level of attainment cannot be measured in common units. When a common unit can be found, it may be possible to combine the objectives into a single objective function. If not, a multiobjective approach must be taken.

Example. The objectives of an inventory problem (Johnson and Montgomery 1974) might be the minimization of ordering costs, carrying costs, and shortage costs. However, a single objective approach is recommended because a common unit, say dollars, can be found. The single objective, then, is the minimization of costs.

Example. Szidarovszky, Bogardi, and Duckstein (1978) model a water resource-mining system in which two of the objectives are economic costs and environmental risks. Clearly these two objectives cannot be measured in like units and treated as a single objective.

B. Conflict

When optimization of one objective can be achieved only by decreasing the level of attainment of another objective, it is said that these objectives are in conflict. Usually, conflict arises when the attainment of each of the objectives requires the use of a given shared resource and there is a limited amount of that resource available.

Example. Kallio, Lewandowski, and Orchard-Hays (1980) model a problem with two objectives: forestry income and industrial income. Decisions regarding the pricing of raw wood can satisfy either objective at the expense of the other objective. Both cannot be improved at the same time.

C. Trade-off

In order to reach a solution which achieves, at least to some degree, all of the objectives, it is necessary to accept a less than optimal attainment of some or even all of these objectives. Decreasing the level of attainment of one objective may lead to an improvement in some of the other objectives. When such a solution is reached, it is said that a "trade-off" among the objectives has been accomplished.

Example. Figure 2.1 shows the following constraints:

$$X_1 + 3X_2 \leq 6$$
$$3X_1 + X_2 \leq 6.$$

The objectives are:

(1) max X_1

(2) max X_2.

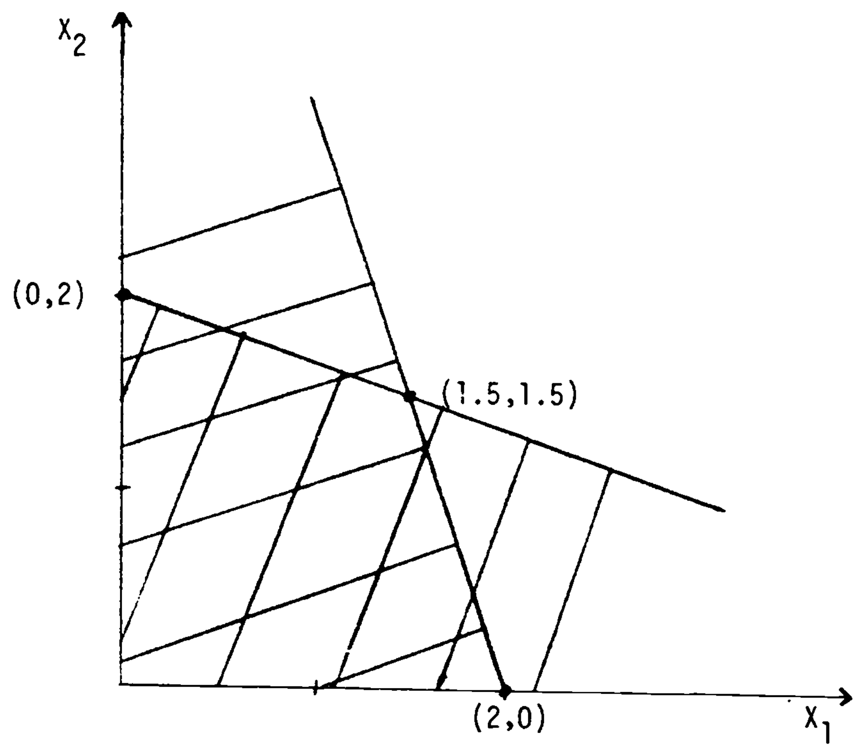

Figure 2.1. Trade-off Solutions

Clearly, the optimal solution for the first objective is the point (2,0) and for the second objective it is the point (0,2). By selecting the point (1.5,1.5) as the solution, a trade-off is put into effect. While neither objective is optimized, this solution may be fair to both.

4. Preference Structure

In seeking a trade-off solution, how does one determine which objectives are more important than the other objectives? This question is answered by determining the preference structure of the decision maker for the problem. The preference structure can take the form of a weighting factor assigned to each objective or a utility function (Fishburn 1970, LaValle 1970) can be derived. The former case is merely a special case of the latter in that the assignment of weights implies a linear utility function for each objective.

Example

The Tisza River Basin development plans were evaluated by David and Duckstein (1976) using a weighted preference structure and by Keeney and Wood (1977) using a utility approach. The utility approach is more difficult and time consuming but could yield more useful results. In this example the ordering of the systems was very similar for both approaches.

Note: It could occur that the decision maker may not like the solution which is based on his stated set of weights. In this case, the weights might be changed in order to reach a more acceptable solution. The process of changing the weights until an acceptable solution is reached can be of great value to a decision maker in that he learns his "true" preference structure. This information will be of assistance to him in future decision problems.

5. Goals, Aspiration Levels

As mentioned earlier, the terms "goal," "ideal," "utopia," and "aspiration level" have all been used to describe a measure of attainment that the decision maker would like to achieve. "Goal" is the term to be used here. It carries the same meaning with one restriction. The levels specified for goals must be such that they cannot be achieved simultaneously for all the objectives. That is, the goal point is not a feasible solution.

When the goal point is a feasible solution, it is referred to here as an "aspiration level." This is consistent with the terminology of Johnsen (1968) used later by Wierzbicki (1980). The aspiration level is that level for each objective which must be achieved for any solution to be acceptable to the decision maker.

The goal point and the aspiration level, then, can be viewed as the solution that the decision maker would like to achieve, if possible, and the minimal acceptable solution.

Example

A technique known as goal programming (Charnes and Cooper 1961, Lee 1972) attempts to find a solution which minimizes the distance from (is closest to) the goal point. Any technique making use of the aspiration level tries, in some sense, to find the solution which is the farthest from that minimum level. In Figure 2.1, if the goal point is (3,3), then the point (1.5,1.5) is (3-1.5)+(3-1.5) or three units away. The point (2,0) is (3-2)+(3-0) or four units away.

6. Ideal Point

The definition of a goal point is too general for the decision maker to use effectively. A more prescriptive definition is desired. If each objective is optimized without regard to the other objectives, the point having these optimal values as its elements is referred to as an "ideal point." This is a very special goal point that is often used in practice (Zeleny 1975). Its use eliminates the need for the decision maker to specify his desired value for each objective.

Example

In Figure 2.1, the ideal point would be (2,2). This results from observing that the optimal solution to the first objective is $X_1 = 2$ and to the second objective it is $X_2 = 2$.

7. Efficient Solution

The term "efficient solution" is the one seen in the mathematical programming literature. However, the same concept is well known in economics where it is referred to as the "Pareto optimal" solution and in decision theory where it is known as the "nondominated" or "non-inferior" solution.

An efficient solution is one from which the level of attainment of all of the objectives cannot be improved simultaneously. This general definition is sometimes referred to as "weakly efficient." A solution is "strongly efficient" if the improvement of any one objective requires a decrease in the level of attainment of at least one other objective. A strongly efficient solution is also weakly efficient.

The solution accepted by a decision maker should come from the set of efficient solutions. It would not make sense to choose any other solution since that solution could be improved upon for every objective. More specifically, the solution should come from the set of strongly efficient solutions because a weakly efficient solution could be improved in at least one objective without hurting the others.

Example

In Figure 2.1, the set of efficient solutions is the line [(0,2), (1.5,1.5),(2,0)]. All points on this line are strongly efficient.

Example

Figure 2.2 shows the existence of weakly efficient solutions which are not strongly efficient. The constraint

$$X_1 \leq 1.75$$

is added to the system of Figure 2.1. The points on line [(0,2),(1.5,1.5),(1.75,0.75)] are strongly efficient. Those on line ((1.75,0.75),(1.75,0)] are weakly efficient because no improvement can be made in both objectives; they are not strongly efficient, however, because from any of these points the second objective can be improved without adversely affecting the first objective.

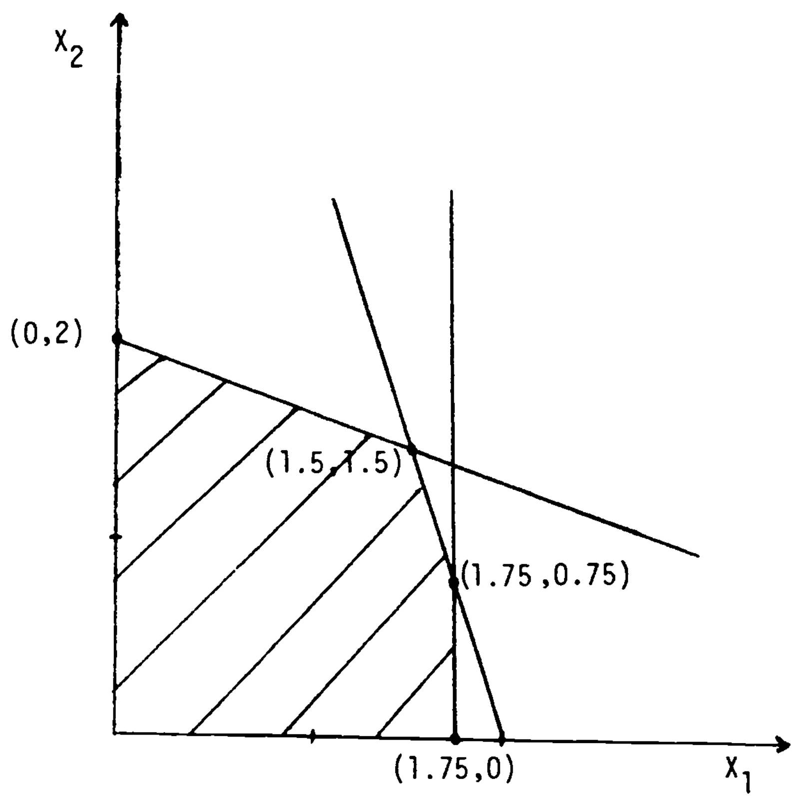

Figure 2.2. Weakly Efficient Solutions

8. Marginal Solution

Given an efficient solution, if one or more of the objectives are at their optimal solution, then this solution is known as a "marginal" solution or a "marginally efficient" solution. All marginal solutions are efficient but most efficient solutions are not marginal. This concept is useful because it is relatively easy to find these solutions. They are just the result of optimizing one objective at a time.

Example

In the problem of Figure 2.1, optimizing the first objective yields point (2,0) as the solution. This point and point (0,2), the result of optimizing the second objective, are the two marginal solutions of this problem.

In Figure 2.2, the marginal solutions are the point (0,2) and the line [(1.75,1.42),(1.75,0)].

At this point in the discussion, it is helpful to define some notation and restate some definitions in terms of this notation.

Define:

X_i, $i = 1,\ldots,n$	the set of decision variables
$\underline{X} = (X_1,\ldots,X_n)$	the decision vector
$f_K(\underline{X})$, $K = 1,\ldots,p$	the objective functions
$g_j(\underline{X}) \leq 0$, $j = 1,\ldots,m$	the problem constraints
L	feasible region defined by $g_i(\underline{X})$

Definition 2.1. Ideal Point

Let $f_K^*(\underline{X})$ be the optimal solution of the Kth objective subject to all the constraints $g_j(\underline{X})$. The point $\underline{X}^* = (f_1^*(\underline{X}), f_2^*(\underline{X}), \ldots, f_p^*(\underline{X}))$ is the ideal point.

Definition 2.2. Weakly Efficient Point

Let $\underline{X}^* \varepsilon L$. $\underline{X}^*$ is weakly efficient if there exists no $\underline{X} \varepsilon L$ such that $f_K(\underline{X}) > f_K(\underline{X}^*)$ for every K.

Definition 2.3. Strongly Efficient Point

Let $\underline{X}^* \varepsilon L$. $\underline{X}^*$ is strongly efficient if there exists no $\underline{X} \varepsilon L$, $\underline{X} \neq \underline{X}^*$, such that $f_K(\underline{X}) \geq f_K(\underline{X}^*)$ for every K.

Definition 2.4. Marginal Solution

Define P subproblems:

$$\text{optimize } f_K(\underline{X}) \qquad K = 1,\ldots,P$$
$$\text{subject to } \underline{X} \varepsilon L$$

$\underline{X}_k^*$ is a marginal solution if it solves this problem.

Definition 2.5. Multiobjective Decision Problem

The general multiobjective decision problem is:

$$\text{optimize } f_K(\underline{X}) \qquad K = 1,\ldots,P$$
$$\text{subject to } \underline{X} \varepsilon L$$

9. Decision Space vs. Objective Space

When the level of attainment, $f_K(\underline{X})$, of an objective is known, that level can be plotted on a graph in multidimensional space. The graph of these values, with the $f_K(\underline{X})$ as its axes, is plotted in objective space. This is to be differentiated from decision space, where all of the decision variables, X_i, comprising the objective functions are plotted. Single objective decision making deals in decision space without the need for a plot in objective space (objective space would just be a straight line). However, the presence of more than one objective necessitates a plot of the values of one objective versus the others.

This plot is very helpful in presenting to the decision maker the options that are open to him. When referring to the feasible region in multiobjective decision making, that region will always be discussed in terms of the objective space (defined as region H corresponding to region L in decision space). Of course, for every point in decision space there is a corresponding point in objective space. The mapping F from one space to the other is defined by the equation $F(\underline{X}) = (f_1(\underline{X}),\ldots,f_p(\underline{X}))$.

Example

In Figures 2.1 and 2.2, decision space and objective space are equivalent. This is because the objective functions $f_1(\underline{X})$ and $f_2(\underline{X})$ are just the decision variables X_1 and X_2.

Example

Figure 2.3 shows both decision space and objective space for the following problem:

$$\max f_1(\underline{X}) = X_1 + 4X_2$$

$$\max f_2(\underline{X}) = 3X_1 - X_2$$

$$\text{subject to } g_1(\underline{X}) = X_1 - 3 \leq 0$$

$$g_2(\underline{X}) = X_2 - 2 \leq 0$$

$$g_3(\underline{X}) = X_1 + X_2 - 4 \leq 0$$

10. Compromise Solution

A marginal solution probably will not be an acceptable solution to the decision maker. He is looking to satisfy many objectives, but the marginal solution has taken only one of them into account. The trading off among objectives discussed earlier has been ignored. A solution is sought which can be viewed as a compromise between the competing objectives. This solution, which is the result of a trade-off, is known as a compromise solution.

This notion of seeking something other than an optimal solution was first discussed by Simon (1953) who called this kind of solution a "satisficing" solution. The solution to a multiobjective decision problem is never called an "optimal solution." This term is only used for solutions to single objective problems.

Example

In Figure 2.1, any efficient solution other than (0,2) and (2,0), the marginal solutions, can be called a compromise solution.

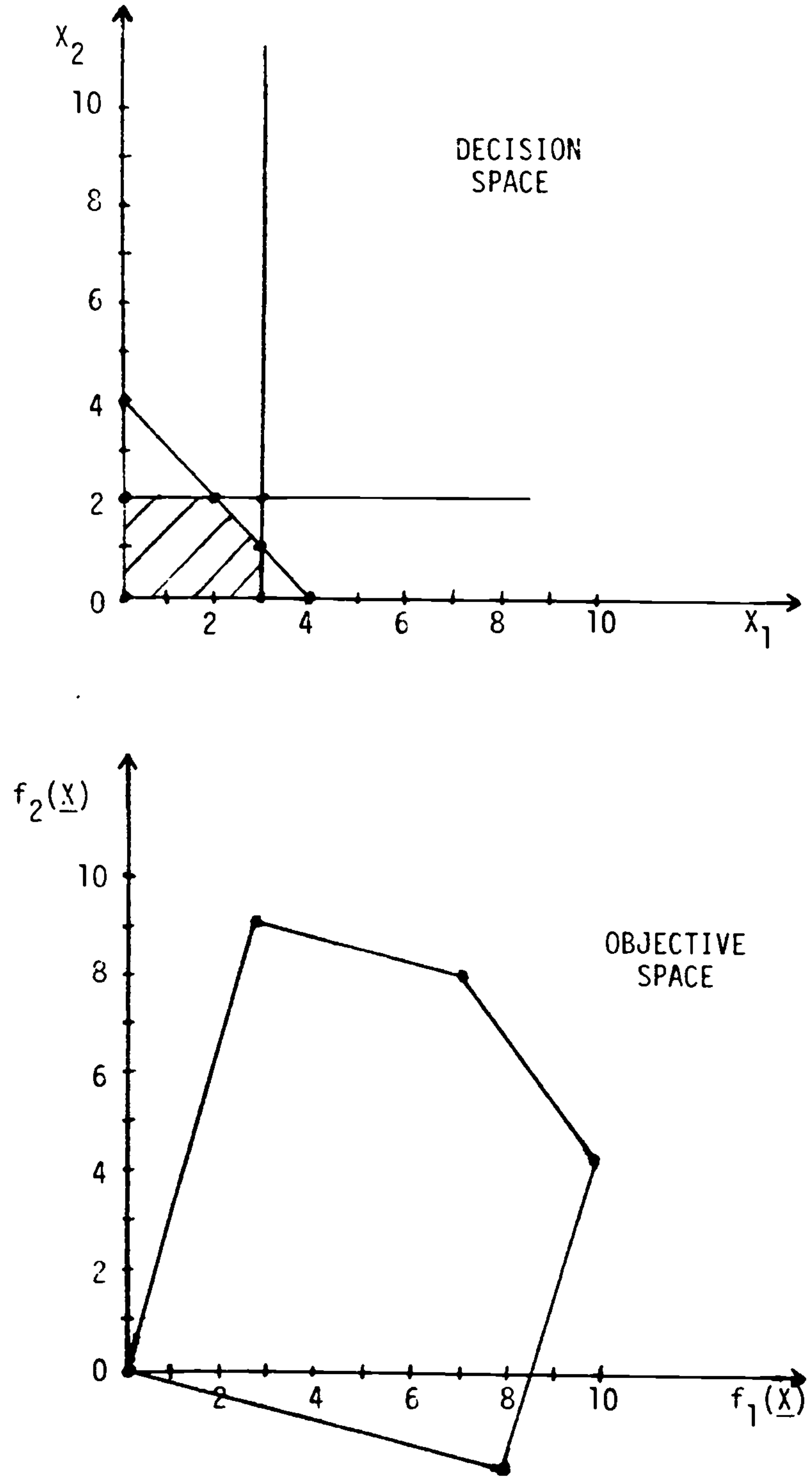

Figure 2.3. Decision and Objective Spaces

Two important steps in the multiobjective decision process have now been alluded to: the determination of the set or a subset of efficient solutions and the determination of the preference structure of the problem. The third step is to make use of this preference structure for the purpose of choosing a solution from among the set of efficient solutions.

11. Generating Method

The task of determining the set of efficient solutions is accomplished by means of a generating method. Specific methods to locate this set will be discussed in the next chapter. While it is not always necessary to identify the entire set of efficient solutions, Yu and Zeleny (1975) point out that it is desirable to do so. It is current wisdom that a good consultant suggests a number of good solutions to the decision maker along with his recommendation as to which is the best. He does not make the decision. The alternatives must come from the efficient set.

There are two problems encountered by generating methods. First, it is computationally difficult to locate the entire set of efficient solutions. For this reason, only a carefully selected subset of these is often explored. Second, if there are more than two objectives, the display of this set becomes very difficult. Although display mechanisms for graphical data have been developed for up to five variables (Loucks 1979, Schmidt and Soloman 1980), these may be too complicated and confusing to be worthwhile to a decision maker.

Example

The efficient set has been previously identified in Figure 2.1. For such a simple example, graphical observation is sufficient.

12. Preference Determination Method

The preference determination methods are those methods that help the analyst to develop a preference structure with the decision maker. The most accurate of these methods is to assess the complete utility function for each objective (Keeney 1972). However, this is both time consuming and difficult. The simplest approach, referred to as prior assessment of weights (Cohon 1978), implies that the emphasis on any objective does not change regardless of the level of attainment which has been achieved for that objective.

Example

In many fields where decisions are required, the weights placed on various objectives are a function of the need to improve upon those objectives. Politics is one such field. In politics, very little emphasis is placed on improving a given objective (e.g., full employment) until the lack of achievement creates a crisis situation. Then, that objective receives top priority (highest weight).

13. Solution Technique

The solution technique is the method or algorithm used to select a compromise (satisficing) solution from the set of efficient solutions. Techniques are available which solve multiobjective problems with or without enumerating the entire set of efficient solutions. Techniques are also available for dealing with quantitative or qualitative objectives and many other situations which arise in decision problems. Although many widely differing techniques have been developed, all of them seek to find the best compromise solution. These techniques will be described in the next chapter.

CHAPTER 3

LITERATURE REVIEW

1. Historical Development

The first ideas leading to multiobjective analysis of decision problems were developed by Pareto (1896) and the Pareto optimal solution is the basis for the concept of the efficient solution. Koopmans (1951) introduced this concept into the field of operations research (activity analysis) and Kuhn and Tucker (1951) formalized the concept in a rigorous mathematical fashion, labeling their result "proper efficiency." Simon (1953) introduced the notion that decision makers do not really seek an optimal solution when confronted with a complex problem. Rather, they seek a "satisficing" solution. The basis for this idea is the psychological theory that people only strive towards a goal until their personal needs are satisfied (Johnsen 1968). The work of these men laid the groundwork for the introduction of multiobjective techniques in decision problems.

While it was not thought of as a multiobjective technique at the time, it is now recognized that goal programming (Charnes and Cooper 1961, Ijiri 1965, Ignizio 1976) is a special case of more general techniques (Zeleny 1973, Szidarovszky 1979). Thus, goal programming can be considered to be the first multiobjective technique. It also remains the most popular. As interest in the subject began to accelerate in the late sixties, new techniques were developed by Bod (1963), Benayoun, Roy, and Sussman (1966), Marglin (1967), and Geoffrion (1967). The first book on the subject (Johnsen 1968) was also published during this time.

The early seventies witnessed a proliferation of new techniques in this field (Roy and Bertier 1972, Monarchi 1972, Philip 1972, Zeleny 1973, Haimes 1973). Recognition came as a new and distinct branch of operations research with the first conference devoted entirely to multiobjective decision making (Cochrane and Zeleny 1973). Impetus for further work in the area came when the United States Water Resources Council adopted a set of rules prescribing the use of multiple objectives in the evaluation of future federally funded water projects (Federal Register 1973).

In recent years, emphasis has centered on attempting to classify these techniques (Starr and Zeleny 1977, Cohon and Marks 1975, Wallenius 1975) as well as to demonstrate their applicability (Major 1974, 1977; Werczberger 1976, Loucks 1977). Areas such as dynamic multiobjective programming (Tauxe, Inman, and Mades 1979a, Opricovic 1979, Szidarovszky 1979) and stochastic multiobjective programming (Wilhelm 1975, Goicoechea 1977, Haimes 1978), which have been ignored in the earlier developments, are just now under investigation and must be further developed (Duckstein 1978).

The first textbooks on the subject (Haimes, Hall, and Freedman 1975, Cohon 1978, Rietveld 1980) have been published and more are in preparation at this time (Goicoechea, Hansen, and Duckstein 1981, Szidarovszky 1981). The books by Cohon and Goicoechea et al. provide an overview of the field while the remaining three give heavy emphasis to one particular approach.

2. Theoretical Foundations

In order for multiobjective decision making to gain widespread acceptance, a strong theoretical foundation had to be built. This task was begun by Kuhn and Tucker (1951), who showed the existence of a set of optimality conditions for the "vector maximization" problem (i.e., maximization of multiple objectives):

Given a vector maximization problem written as

$$\text{maximize } \underline{f}(\underline{X}) = [f_1(\underline{X}), f_2(\underline{X}), \ldots, f_p(\underline{X})]$$

$$\text{subject to } g_i(\underline{X}) \leq 0 \qquad i = 1, \ldots, m$$

$$\underline{X} \geq 0,$$

$$\text{or} \qquad \underline{X} \in L$$

if a solution $\underline{X}^*$ is efficient, then there exist two sets of multipliers

$$u_i \geq 0 \qquad i = 1,2,\ldots,m$$

$$W_K \geq 0 \qquad K = 1,2,\ldots,p$$

and

$$X^* \in L$$

$$u_i g_i(\underline{X}^*) = 0 \qquad i = 1,2,\ldots,m$$

$$\sum_{K=1}^{p} W_K \nabla f_K(\underline{X}^*) - \sum_{i=1}^{m'} u_i \nabla g_i(\underline{X}^*) = 0$$

Geoffrion (1968) further refined the mathematical definition of efficient solution, essentially adding a smoothing condition. His assumption that all weights be strictly positive indicates that the refinement is similar to the distinction between strong and weak efficiency made previously.

Parallel to the development of mathematical optimization for the scalar problem, a duality theory has been developed for the vector case. Isermann (1977) and Kornbluth (1975) have demonstrated the existence of a multiobjective duality theory for linear problems and Brumelle (1979) has presented the more general nonlinear duality theory. The presentation by Brumelle is closely patterned after the work of Rockafellar (1970) which only deals with single objective optimization.

Example

Linear case--two objectives

$$\text{maximize } \underline{\underline{c}}\underline{x}$$

$$\text{subject to } \underline{\underline{A}}x \leq \underline{b}$$

$$x \geq 0$$

$$\text{where } \underline{\underline{c}} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \underline{\underline{A}} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 6 \\ 15 \end{bmatrix}$$

The primal problem is shown in Figure 3.1. Through use of a simplex algorithm or by graphical inspection, three efficient extreme points are located ((5,0),(3,3),(0,6)). A dual problem, corresponding to each of these points, is of the form

$$\text{minimize } \underline{\underline{U}}\underline{b}$$

$$\text{subject to } \underline{\underline{U}}\underline{\underline{A}}\underline{w} \geq \underline{\underline{c}}\underline{w}$$

where the u_{ij} are the dual variables and w_i are the weights associated with the given extreme point. The dual solutions are given in Table 3.1.

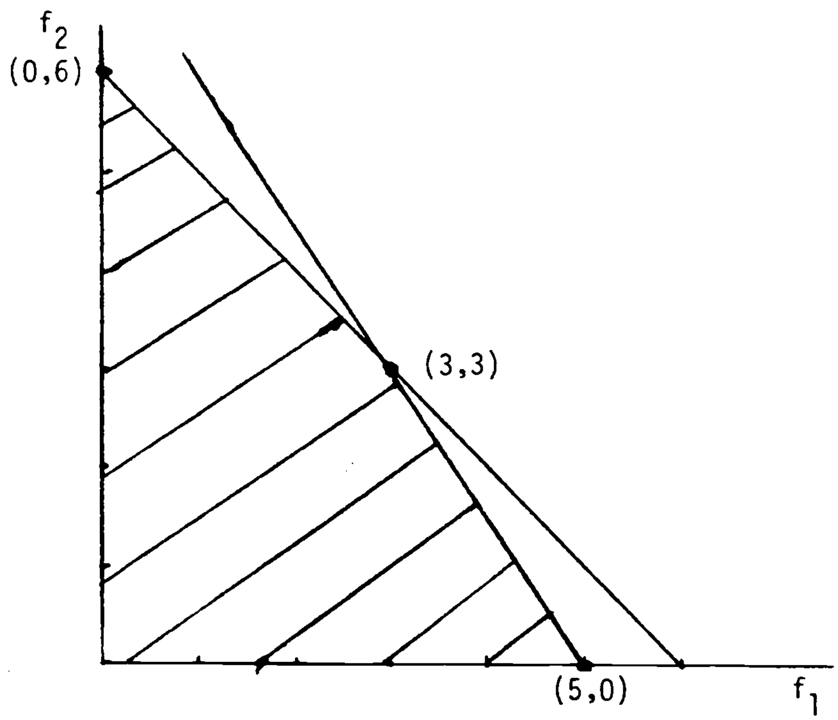

Figure 3.1. Primal Feasible Region

TABLE 3.1. DUALS TO PRIMAL SOLUTIONS

Primal	Dual
(5,0)	$\begin{bmatrix} 0 & 2/3 \\ 0 & 1/3 \end{bmatrix}$
(3,3)	$\begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix}$
(0,6)	$\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$

The economic interpretation of the dual variables for the multiobjective case is that u_{ij} is the value of the jth resource for the ith objective.

Mathematical programming is not the only field whose theory has been used in the advancement of multiobjective decision making. Other fields include game theory (Von Neumann and Morgenstern 1947, Nash 1953), utility theory (Fishburn 1974, Keeney 1974), and decision analysis (Raiffa 1968).

3. Methods for Generating Efficient Solutions

A. Weighting Method

The weighting method is the oldest and simplest of the generating techniques, first proposed by Zadeh (1963). It consists of optimizing a weighted sum (linear combination) of the objective functions to obtain the first efficient solution, varying the weights to get a second solution and continuing until the entire efficient set is represented.

Mathematically, this can be written as:

$$\text{maximize } \underline{w}\underline{Z}(\underline{x}) = w_1 Z_1(\underline{x}) + w_2 Z_2(\underline{x}) + \ldots + w_p Z_p(\underline{x})$$

$$\text{subject to } x \in L,$$

$$\text{where} \quad \underline{w} \geq 0, \text{ and } \underline{w} \neq 0.$$

The solution to this problem is guaranteed to be a weakly efficient solution and if $w_i > 0$ for all i, then the solution is strongly efficient. One should be aware, however, that it may not be possible to generate all efficient points using this method. In general, if the feasible region (L) is nonconvex, all efficient solutions cannot be found through the use of a linear objective function. In a special case, if L is a polyhedron and the objective functions are all linear, then all nondominated solutions can be obtained by using this method with positive weights.

The ability to observe directly how the solution is affected by changing the weights is the prime motivation for a decision maker to use this method. A more theoretical motivation is provided by the fact that any point derived in this manner satisfies the Kuhn-Tucker conditions for the vector maximization problem. In addition, this approach is more computationally efficient than the other generating techniques.

B. Constraint Method

Marglin (1967) developed the constraint method, which consists of optimizing one objective function while the others are all constrained to a level acceptable to the decision maker. The process is repeated until all objectives have been optimized in this manner. The appeal of this method is that it allows the decision maker to vary these constraint levels and see how the resulting solution is affected. The ε-constraint method (Haimes 1973) is another example of this approach.

The problem is written:

$$\begin{aligned} &\text{maximize } Z_{k_0}(\underline{x}) \\ &\text{subject to } \underline{x} \in L' \\ &\qquad Z_k(\underline{x}) \geq Z_k^* \quad (k \neq k_0) \end{aligned}$$

One algorithm (Cohon 1978) for using this method is presented here:

1. Construct a payoff table.

   a. Maximize each objective individually. The solution for objective k is (marginal solution) $\underline{X}^k = (X_1^k, X_2^k, \ldots, X_n^k)$.

   b. Compute the value of each objective at each of the p solutions: $Z_1(x^k), Z_2(x^k), \ldots, Z_p(x^k)$.

   c. Construct a p×p matrix of these values with the rows corresponding to the solutions and columns corresponding to the objectives.

   d. For k = 1,2,...,p, find the largest and smallest entry in the kth column and denote them M_k and n_k respectively.

2. Convert the multiobjective programming problem into its corresponding constrained problem of the above form.

3. The n_k and M_k from step 1 define a range for objective k in the efficient set:

$$n_k \leq Z_k \leq M_k$$

   This range applies to Z_k^* as well. Choose the number (r) of different Z_k^*'s that will be used in the generation of efficient solutions.

4. Solve the constrained problem for every combination of values of Z_k^*, $k = 1,2,\ldots,k_0-1,k_0+1,\ldots,p$, where

$$Z_k^* = n_k + [t/(r-1)](M_k - n_k) \quad t = 0,1,2,\ldots,(r-1).$$

   Each constrained problem for which a feasible solution exists will yield an efficient solution.

C. Multiobjective Simplex

Multiobjective simplex is the name applied to many approaches to the linear multiobjective problem (Benayoun et al. 1972, Philip 1972, Holl 1973, Evans and Steur 1973, Zeleny 1974), but that of Zeleny seems to be the most widely known. Based on the simplex technique (Dantzig 1963) for the solution of linear programming problems, this technique is only applicable when all of the objective functions and all of the constraints are linear. An algorithm, flowchart, and computer program listing can be found in Zeleny (1974).

While the methods which have been discussed previously derived efficient points by making use of only one objective function at a time, this method uses all objectives simultaneously. The standard simplex tableau is used except that the row corresponding to the objective function is replaced by a matrix where each row of the matrix represents one objective function.

This algorithm is capable of finding all efficient points (due to the linearity of the problem). However, for large problems, only enough solutions are generated to yield a close approximation to the efficient set. In addition, none of the solutions generated is dependent upon the decision maker setting bounds or weights on the objectives. The elimination of these tasks and the availability of a computer code make this a highly desirable approach.

D. Noninferior Set Estimation

The method of noninferior set estimation (Cohon, Church, and Sheer 1979) places its emphasis on approximating the efficient set.

This is in recognition of the fact that the other methods available, while capable of generating the entire set, usually find only an approximation to it. This method converges quickly and the accuracy of the approximation can be controlled through the use of a predetermined error criterion, which is compared to the maximum possible error at every iteration of the method. All objectives must be linear and the feasible region must be a convex set for this method to be applied. Cohon et al.'s algorithm for a bicriterion problem requires the following definitions:

P_t = the t^{th} efficient extreme point in objective space

S_i = the efficient point with the ith best value of the second objective

B_i = the solution to the ith weighted objective function (bounds the error)

T = maximum tolerable error

$\psi_{i,i+1}$ = maximum possible error in the line segment connecting S_i and S_{i+1}.

The algorithm follows:

1. Maximize the two objectives individually. In objective space, the optimum for objective Z_1 is P_1 and for Z_2 is P_2. Let $S_1 = P_2$, $S_2 = P_1$, and set $n = 2$. Compute ψ_{12}.
2. If $\psi_{i,i+1} \leq T$ for $i = 1,2,\ldots,n-1$, then STOP. Otherwise go to step 3.
3. Search the $\psi_{i,i+1}$, $i = 1,2,\ldots,n-1$, for the largest value. For the i,i+1 that yields the largest maximum possible error, solve the weighted objective problem:

$$\text{maximize } Z(x_1,\ldots,x_n;i,i+1) = [Z_2(S_i)-Z_2(S_{i+1})]Z_1(x_1,\ldots,x_n) + [Z_1(S_{i+1})-Z_1(S_i)]Z_2(x_1,\ldots,x_n).$$

Compute $B_{i,i+1}$ at the weighted objective function evaluated at S_i or S_{i+1}. If the resulting $B_{i,i+1} = Z(x_1,\ldots,x_n;i+1)$, set $\psi_{i,i+1} = 0$ and return to step 2. If $B_{i,i+1} < Z(x_1,\ldots,x_n;i+1)$, designate the new noninferior solution as P_{n+1} and proceed to step 4.

4. Reorder the points P_t, $t = 1,2,\ldots,n+1$ by the following reordering scheme:

$$S'_t = S_t \qquad t = 1,2,\ldots,i$$

$$S'_{i+1} = P_{n+1}$$

$$S'_{t+1} = S_t \qquad t = i+1,\ldots,n.$$

The ψ terms must also be relabeled:

$$\psi'_{t,t+1} = \psi_{t,t+1} \qquad t = 1,2,\ldots,i-1 \quad (\text{if } i \geq 1)$$

$$\psi'_{t+1,t+2} = \psi_{t,t+1} \qquad t = i+1,\ldots,n-1 \quad (\text{if } i \leq n-2).$$

Compute $\psi'_{i,i+1}$ and $\psi'_{i+1,i+2}$. Increment n by one and return to step 2.

Application of this method is limited to linear programming problems. In addition, it is most suited to bicriterion problems. The approximations become difficult to visualize in more than two dimensions.

E. Use of the Multiobjective Dual

Hannan (1978) has developed an algorithm using a dual form of the multiobjective problem to generate the efficient set. The method is based on the multiobjective simplex approach of Evans and Steur (1973).

The primal problem (with weights λ)

$$\text{maximize } \lambda cx$$

$$\text{subject to } Ax \leq b$$

$$x \geq 0, \quad \lambda \geq 0, \quad \sum_i \lambda_i = 1$$

is converted to the dual

$$\text{minimize } \pi b$$

$$\text{subject to } \pi A \geq \lambda c$$

$$\pi \geq 0, \lambda \geq 0, \quad \sum_i \lambda_i = 1.$$

This is still a linear programming problem which can then be solved by a multiobjective simplex algorithm to generate efficient solutions. Use of the dual in this manner has the advantage of serving as an alternate solution technique to the primal problem if the dual can be solved more easily.

4. Preference Structure Determination Methods

A. Multiattribute Utility Theory

The nineteenth century philosopher Jeremy Bentham (1879) was the first to view utility as a measurement of individual preferences. Von Neuman and Morgenstern (1944) developed the axiomatic utility theory upon which many decision analysis techniques are now based (Raiffa 1968).

Utility theory in the presence of multiple objectives is known as multiattribute utility theory. The subject was first addressed by Aumann (1964) and Briskin (1966) but it was Keeney (1969) and Raiffa (1969) who developed the theory for assessing multiattribute utility functions.

The theory for assessing single attribute utility functions is well known (Raiffa 1968, Fishburn 1970, LaValle 1970) and consists of lotteries presented to the decision maker by an analyst. Keeney and Raiffa (1976) provide a detailed review of the multiattribute utility theory and Keeney (1979) demonstrates the complete preference structure assessment for a real problem with this method.

The two axioms sufficient for the use of multiattribute utility theory and the two most popular forms of the multiattribute utility function are presented here:

Axiom 1: Preferential Independence

Let G_i, $i = 1,2,\ldots,p$, be the attributes. The pair $\{G_1,G_2\}$ is preferentially independent of the remaining attributes $\{G_3,G_4,\ldots,G_p\}$ if preferences between $\{G_1,G_2\}$ given that $\{G_3,G_4,\ldots,G_p\}$ are held fixed, do not depend on the level where $\{G_3,G_4,\ldots,G_p\}$ are fixed.

Axiom 2: Utility Independence

The attribute G_1 is utility independent of the remaining attributes $\{G_2,G_3,\ldots,G_p\}$ if preferences among lotteries over G_1 given $\{G_2,G_3,\ldots,G_p\}$ are fixed, do not depend on the level where those attributes are fixed.

Additive Utility Function:

$$u[f_1(\underline{X}),\ldots,f_p(\underline{X})] = k_1u_1[f_1(\underline{X})] + k_2u_2[f_2(\underline{X})] + \ldots + k_pu_p[f_p(\underline{X})]$$

Multiplicative Utility Function:

$$1 + ku[f_1(\underline{X}),\ldots,f_p(\underline{X})] = \prod_{i=1}^{p}(1+kk_iu_i[f_i(X_i)])$$

These functions are used to combine the single attribute utility functions into a multiattribute utility function which then represents the preference structure of the decision maker. The slope of the utility function is the marginal rate of substitution between the objectives. The first axiom implies the use of the additive utility function while the second axiom allows the use of the multiplicative form.

B. Prior Assessment of Weights

It has been mentioned previously in the description of the weighting method that if the decision maker is able to assign a weight to each of the objectives, then the multiple objectives can be reduced to a single objective. Known as prior assessment of weights (Cohon 1978), this method is very easy to apply, a fact that has led to its widespread use (Marglin 1967, Major 1969). These researchers have advocated that this method be used for public decision making problems (UNIDO 1972) and have suggested that a set of weights be devised for each country upon which all design and policy formulation decisions would be based.

The use of constant weights means that the marginal utility for each objective does not change with the level of the objective.

This assumption does not usually hold and the decision maker should be aware of that prior to selecting this method. Zeleny (1974) presents arguments against the use of this method, basing his arguments on the limitations of a human being to give an overall evaluation of the interfaces between many (not more than seven plus or minus two) objectives (Shepard 1964).

C. Fuzzy Sets

Zeleny (1973) proposes the use of fuzzy set theory (Zadeh 1965) to obtain weights. Fuzzy set theory can make use of "fuzzy" statements such as "the weight for objective one should be low" or "much higher than .3." This approach is useful when the decision maker cannot state precisely the value of a given weight. The procedure is interactive and iterative so that the decision maker eventually converges upon his "optimal" weights. The algorithm (Zeleny 1973) follows:

1. Divide the scale of each weight λ_i, $i = 1,\ldots,p$, into convenient units (e.g., .01, .05, .1) and set reference values k_i for each λ_i (e.g., $k_i = 1/p$).
2. The decision maker gives a fuzzy statement about the relationship between λ_i and k_i. L_i is the fuzzy set and $M_{L_i}(\lambda)$ is the degree of membership in that set.
3. Construct the set L and evaluate all $\lambda \varepsilon L$ with respect to $M_L(\lambda) = \min_{i=1,\ldots,p} \{M_{L_i}(\lambda)\}$. The set L corresponds to a subset of the efficient set.

4. Find max $M_L(\lambda)$ for all λ. These values are used as the reference values for the next iteration. The procedure terminates when no additional information improves or reduces the set.

This approach has been used by Roy (1977) and Orlovsky (1978).

D. Simplex-Based Methods

Zeleny (1973) provides a method whereby an optimal set of weights is found which is associated with a given efficient point. The efficient points are found by multiobjective simplex and then the weights are found for each point. This is shown to be a more efficient generating method than the weighting method (Yu and Zeleny 1975).

The optimal set of weights is defined as the optimal solution to the linear programming problem:

$$\text{maximize } \sum_k w_k c\underline{x}^i$$

$$\text{subject to } x \varepsilon L$$

$$\text{and } \sum_k w_k = 1$$

where

w_k = weight of the kth objective

c = matrix of objective coefficients

$\underline{x}^i$ = the ith efficient extreme point

While multiobjective simplex can only be used for linear problems, the objectives need not be linear here since this objective is linear in terms of the weights. This approach is also related to a dual method where the dual variables can be viewed as optimal weights. This can be seen either by comparing the above formulation with that of Brumelle (1979) discussed earlier or by viewing this as a Lagrangian formulation.

E. Scaling

When the range of values that any given objective can take on is scaled, the decision maker is implicitly specifying his preference structure. Since this is done in many of the multiobjective methods presented in the next section, a brief discussion is now given.

The simplest of the procedures available for scaling, the ranking method, is very similar to the method of prior assessment of weights. The major difference lies in the fact that scales (either cardinal or ordinal) are assessed instead of weights. The most complex procedures involve the use of complicated lotteries. These are known as standard gamble or decision analysis methods. Dyer and Sarin (1977) have developed a procedure for scaling which is easier to apply than decision analysis yet is closely related to and as rigorous as utility theory. Fishburn (1967) presents a survey of 24 scaling methods with a short description of each.

5. Solution Techniques

A. Comparative Techniques

I. ELECTRE. This method, which can only be applied to discrete sets of alternatives, consists of two phases: ELECTRE I and ELECTRE II. Its primary advantage over other techniques is its ability to incorporate qualitative data into the analysis.

The idea in ELECTRE I (Benayoun et al. 1966) is to choose those systems which (1) are preferred for most of the criteria and (2) do not cause an unacceptable level of discontent for any one criterion. The concepts of concordance and discordance are introduced to determine if these conditions are satisfied.

The concordance between any two actions i and j is a weighted measure of the number of criteria for which action i is preferred to action j and is given as:

$$c(i,j) = \frac{\text{sum of weights for criteria where } i > j}{\text{total sum of weights}}$$

where i > j means i is preferred to j and the weights are elicited from the decision maker. Concordance can be thought of as the weighted percentage of criteria for which one action is preferred to another.

To compute the discordance, an interval scale common to each criterion is first defined. The scale is used to compare the discomfort caused between the "worst" and the "best" of each criterion. Each criterion can be assigned a different range. Given this information, the discordance index is defined to be:

$$D(i,j) = \frac{\text{maximum interval where } j > i}{\text{total range of scale}}$$

To synthesize both the concordance and discordance matrices, threshold values (p,q) are defined. These values must be between zero and one because no action is dominated by any other action for every criterion (any such action is excluded from the analysis). In choosing a value of p, the decision maker specifies the amount of concordance that is desired. In specifying q, he specifies the amount of discordance he is willing to tolerate. The result of ELECTRE I is a preference graph representing a partial ordering of the alternative systems. ELECTRE II (Roy and Bertier 1972) may then be used to obtain a complete ordering.

ELECTRE II requires two preference graphs representing the strong and weak preferences of the decision maker. The strong preference graph results from the use of stringent threshold values; that is, the decision maker is asked to select a high level of concordance and a low level of discordance. For the weak preference graph, the decision maker is asked to relax his threshold values (lower p, higher q). These relaxed threshold values can be viewed as lower bounds on the system performance that the decision maker is willing to accept. The algorithm for ordering the systems from these preference graphs is presented by Abi-Ghanem, Duckstein, and Hekman (1978).

II. Q-Analysis. Q-analysis is a technique which can be applied to a class of problems very similar to that of ELECTRE (Kempf, Duckstein, and Casti 1978). That is, it can be applied to rank a discrete set of alternatives with respect to a set of criteria and can incorporate qualitative data into the analysis. Where ELECTRE made use of grapn theory in its analysis, Q-analysis uses a multidimensional graph theory known as polyhedral dynamics.

This technique is applied by examining the geometric structure of a multidimensional object called a simplicial complex. After developing the simplicial complex, the property of q-connection is defined on the simplices of the complex. Q-connection defines an equivalence relation on the simplices and leads to an ordering imposed over the set of simplices (Atkin 1974).

Those alternatives which are shown to have high q-levels are preferred to those having lower q-levels. Any alternatives which are q-connected are indistinguishable with regard to choice. If the objective of the analysis is to select a single, most preferred alternative, the highest satisfaction level, α, at which a non-q-connected simplex occupies the high q position and at which the dimensional level of the high q simplex is such that the maximum number of criteria are being considered must be found. The project represented by this simplex is the preferred choice.

III. Surrogate Worth Trade-off. The surrogate worth trade-off method (Haimes et al. 1975, Haimes 1979) can be applied to multiobjective mathematical programming problems when all functions are differentiable. A single objective problem is constructed through the development of surrogate worth functions with the cooperation of the decision maker.

The first step in the procedure is to construct an array like that discussed in the constraint method for generating the set of efficient solutions. All trade-offs developed will then be among the efficient set. This search along the efficient frontier distinguishes this method from cooperative game theory which approaches the efficient set from within the feasible region and compromise programming which approaches it from the infeasible region.

The trade-off function for any two objectives and a given efficient solution $\underline{x}$ is defined as follows:

$$T_{ij}(\underline{x}) = \frac{\partial f_i(\underline{x})}{\partial f_j(\underline{x})}$$

Computing these trade-offs allows the decision maker to make a systematic pairwise comparison of objectives.

The surrogate worth function is determined in a similar manner (Haimes and Hall 1974). This function, w_{ij}, $i \neq j$, $j = 1,2,\ldots,n$, can be defined as a function of λ_{ij} for estimating the desirability of trade-off λ_{ij}. For example, the scale could range from -10 to +10. A -10 would indicate that λ_{ij} marginal units of objective i are worth very much less than one marginal unit of objective j, a +10 means the opposite, and a zero would indicate an even trade. The optimum is found when all surrogate worth functions are equal to zero.

<u>IV. Step Method</u>. The step method (Benayoun et al. 1971) is an iterative technique which converges to the "best" efficient solution in no more than p steps, p being the number of objectives. It is useful for application to linear problems, makes use of an ideal point, and solves a minimax problem as its definition of "best." The algorithm (Cohon 1978) follows:

1. Construct a table of marginal solutions, or payoff table, by optimizing the objectives individually.
2. Compute α_k for each objective:

$$\alpha_k = \frac{M_k - n_k}{M_k}\left[\sum_{j=1}^{n} (c_j^k)^2\right]^{-1/2}$$

where M_k and n_k are the largest and smallest values of objective k and the c_j^k are the objective function coefficients. Set i = 0.

3. Compute $\pi_k = \frac{\alpha_k}{\sum_k \alpha_k}$ and solve the minimax problem. Call the solution x(i).

4. Show the solution to the decision maker:

   a. If satisfied, STOP.

   b. If not satisfied and i < (p-1), go to step 5.

   c. If not satisfied and i = (p-1), STOP. A different procedure is required.

5. The decision maker selects an objective which the solution satisfies and determines the amount by which it can be decreased in order to improve the objectives which have not been satisfied. If this cannot be done, again, some other procedure is required.

6. Define a new constraint relaxing the objective selected in step 5. Set $\alpha_k = 0$ for that objective, increment i by one, and go to step 3.

B. Distance-Based Techniques

I. Compromise Programming. Compromise programming (Zeleny 1974) is an approach which defines the "best" solution as that point which minimizes the distance from a goal point (often the ideal point is used) to the set of efficient solutions. By restricting the goal point so that it is greater than or equal to the ideal point (see Figure 3.2), the affect of choosing this point is minimized. The motivation behind this method is the desire to achieve a solution that is as "close" as possible to some "ideal."

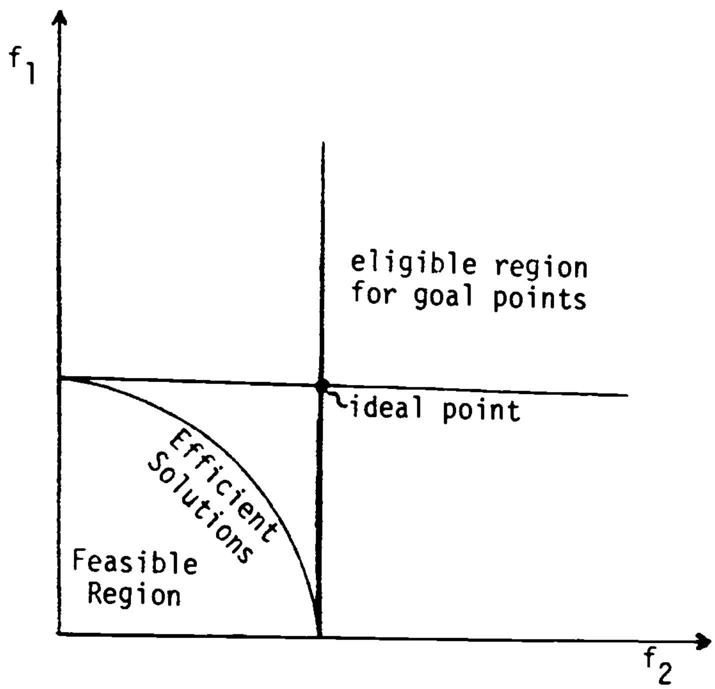

Figure 3.2. Selection of Goal Points

The distance measure used in compromise programming is the family of L_p-metrics. This metric is given as:

$$L_p(\underline{x}) = \left[\sum_{i=1}^{n} \alpha_i^p \left| \frac{f_i^* - f_i(\underline{x})}{f_i^* - f_{i\text{-min}}} \right|^p \right]^{1/p}$$

where the α_i are the weights, f_i^* is the optimal value of the ith criterion, $f_{i\text{-min}}$ is the worst value obtainable for criterion i, and $f_i(\underline{x})$ is the result of implementing decision $\underline{x}$ with respect to the ith criteria. For $p = 1$, all deviations from f_i^* are taken into account in direct proportion to their magnitudes. For $2 \leq p < \infty$, the largest deviation has the greatest influence. For $p = \infty$, the largest deviation is the only one taken into account (minimax criterion). Goal programming (Lee 1972) is analogous to compromise programming when p = 1 and an arbitrary goal point is chosen.

<u>II. Cooperative Game Theory</u>. Cooperative game theory is another distance-based approach. However, in this method, rather than minimizing the distance to a goal point, the "best" solution is that which maximizes the distance from some "status quo" point, aspiration, or minimum level. The distance measure used is the geometric distance:

$$g(x) = \prod_{i=1}^{n} (f_i(\underline{x})-f_i^*)^{\alpha_i}$$

where the α_i are again the criteria weights, f_i^* is the ith element of the status quo point, and $f_i(\underline{x})$ is defined as before.

Szidarovszky et al. (1978) generalizes the cooperative game concept of Nash (1953) for two-person games to the case of n-person games. Seven axioms are developed defining a cooperative solution and it is proven that a unique solution exists. This solution is found by the methods of nonlinear programming. An application is presented in Appendix A.

The appeal of this method stems from the existence of a unique solution and the unique way in which the multiobjective problem is viewed. Rather than speaking of competing objectives, the objectives are treated as if they are working in cooperation to achieve a mutually satisfactory solution. Another advantage is that any intermediate solution obtained is feasible. This is a desirable feature due to the interactive nature of nonlinear programming algorithms.

III. Multiattribute Utility Theory. In the previous section, multiattribute utility theory was used to obtain a preference structure in the form of a utility function. Here, that function is used to determine the solution which provides the highest degree of utility with respect to all of the objectives. Since the utility function is usually nonlinear, nonlinear programming techniques are again employed to find the solution.

Oppenheimer (1978) reviews the two prevalent approaches to the utility maximization problem: the global approach and the local approach. The global approach (Keelin 1976) consists, as described above, of the determination of one multiattribute utility function and maximizing that function. However, such a procedure "may force the decision maker to fit a function not truly representing" the preference structure over all regions of the curve.

The local approach (Geoffrion, Dyer, and Feinberg 1972) avoids this problem. In place of specifying one utility function, a sequence of local linear approximations to it are generated, each yielding a trial solution. Each trial solution is an improvement over its predecessor, so eventually the sequence reaches its optimum.

C. Other Methods

An approach based on metagame analysis (Hipel, Ragede, and Unny 1976) has been used to choose among a predefined set of systems in the presence of either quantitative or qualitative criteria. Lexicographic ordering (MacCrimmon 1973), concordance analysis (Nijkamp and Vos 1977), and the method of Zionts (1977) are used for this same purpose.

Linear programming approaches, in addition to those already elaborated upon, include LINMAP, ORDREG, and conjoint measurement (Srinivasan and Shocker 1973, Shocker and Srinivasan 1974) and the interactive algorithm of Zionts (1976). Other interactive techniques include interactive goal programming (Dyer 1972), the method of Geoffrion et al. (1972), sequential multiobjective programming system (SEMOPS) (Monarchi, Kisiel, and Duckstein 1973), the game theory based method of Belenson and Kapur (1973), the Lagrangian method of Neuman and Krzysztofowicz (1977) and the method of the displaced ideal (Zeleny 1975).

Approaches relying on nonlinear optimization include TRADE (Goicoechea, Duckstein, and Fogel 1976) and the reference objective method (Wierzbicki 1975, 1979). TRADE is a nonlinear version of the step method; it makes use of a cutting plane algorithm (Kelley 1960) to linearize the objectives. The reference objective method uses the concept of aspiration levels rather than an ideal point and solves the problem using penalty methods (Zangwill 1969). Convexity of objectives and constraints is required for the use of these methods.

6. Dynamic Multiobjective Programming

The development of techniques to solve dynamic multiobjective problems is still in its early stages. Some have recently been presented, but these have not been applied as yet to real problems. These methods are discussed in this section.

The early work in this area involves utility functions over time (Bell 1972, Anderson and Rohrbaugh 1979) and a similar approach called preference order dynamic programming (Mitten 1974). Bell refers to a two-attribute utility function with time as one of the attributes so this is really a single objective dynamic programming problem. Mitten avoids real-valued utility functions by using an interactive scheme where only preferences among outcomes are measured.

Later approaches focus on developing the efficient set, where the efficient solutions over time are referred to as efficient trajectories or efficient policies. Yu and Leitman (1974) consider properties of domination structures and efficient trajectories. Villarreal and Karwin (1978) and Tauxe et al. (1979b) present techniques which locate these types of solutions. The former is an integer programming technique in an interactive mode while the latter is essentially the constraint method in a dynamic programming format. None of these techniques finds the "best" compromise solution, instead, letting the decision maker do this after reducing the choice set. Harboe, Schultz, and Duckstein (1980) also go only so far as a reduction of the efficient set.

One approach which has been developed into a dynamic multiobjective method is the nested Lagrangian multiplier method (Seo and Sakawa 1979). Elements of mathematical programming and decision analysis are included in a two-layer analysis. Only selected solutions from the efficient set are found by mathematical programming. These are determined through the use of a subjective utility assessment at each stage of the process. The method has been applied to a dynamic problem with two objectives (Seo and Sakawa 1980), but may be limited to relatively small problems.

An extension of compromise programming to dynamic problems seems to hold the most promise as a dynamic multiobjective programming solution technique. Two algorithms (Szidarovszky 1979, Opricovic 1979) have been developed for this purpose. The formulations of these two methods are very similar, but the method of Szidarovszky allows the multiobjective problem to be completely transformed into a dynamic programming problem. The method of Opricovic requires that an optimization problem be solved as a subproblem at each step of the recursive dynamic procedure. The method of Szidarovszky will be discussed in more detail and applied to a real problem in the next chapter.

7. Stochastic Multiobjective Programming Methods

While the subject of multiobjective decision making for nondeterministic problems has been addressed many times (Wilhelm 1975, Haimes et al. 1975, Duckstein and Bogardi 1978), only two methods have been published, so far, to solve such problems: the PROTRADE method of Goicoechea, Duckstein, and Fogel (1979) and an extension of the surrogate worth trade-off method known as the multiobjective statistical method (Haimes et al. 1980). Duckstein (1978) points to the need for more research on this problem.

PROTRADE is a probabilistic trade-off development algorithm, providing a framework in which the decision maker can articulate his preferences, generate alternative solutions, develop trade-offs among these, and eventually arrive at a satisfactory solution if one exists. The twelve step algorithm can be found in Goicoechea (1977). Basically, an initial solution is found using a surrogate objective function, a multiattribute utility function is formed leading to a new surrogate objective function and a new solution; the solution is checked to see if it is satisfactory to the decision maker, and if so, the procedure terminates. If not, a new surrogate objective function is obtained and the process is repeated until a satisfactory solution is found.

The problem described in PROTRADE has a probabilistic objective function. However, objectives and constraints can be interchanged (Lasdon 1970). Therefore, a problem having probabilistic constraints can be solved by incorporating these constraints into the objective function and applying PROTRADE.

Another approach to this problem is to maximize expected utility and Vajda (1972) has done this for the single objective case. The multiobjective statistical method is this kind of approach. In this method, all random quantities are discretized with a probability assigned to each set of occurrences. A new objective is defined in terms of the expected values. This is then a deterministic multiobjective problem and can be solved by the surrogate worth trade-off method.

8. Applications

Unfortunately, a large portion of the applications of the multiobjective methods have been performed by the authors of those methods themselves. All of the applications of the surrogate worth trade-off method (Haimes 1977, Das and Haimes 1979) and Q-analysis (Atkin 1974, Kempf et al. 1978, Casti et al. 1979) are of this type, and this is also true of the methods listed under "other methods" and cooperative game theory for which no applications have yet appeared. This is hardly an effective way to evaluate these methods, and it will be necessary for others to apply them to real problems before their use can be recommended.

Multiattribute utility theory is a method for which a number of applications have been published (Keeney 1973, DeNeufville and Keeney 1973, Major 1974, Keeney and Wood 1977, Keeney 1979), but here again, the originator of the method is predominant among the authors. ELECTRE, too, has only been applied by a small group of researchers (Roy and Bertier 1972, David and Duckstein 1976, Abi-Ghanem et al. 1978), and one of the rare applications of the step method is that of Loucks (1977).

While compromise programming has been applied by researchers other than Zeleny (Duckstein, Bogardi, and Szidarovszky 1979, Duckstein and Opricovic 1980), it is under the name of "goal programming" that it has achieved the most recognition. Goal programming is the single most widely used multiobjective method (Charnes, Cooper, and DeVoe 1968, Lee and Clayton 1972, Werczberger 1976, Lee and Moore 1977).

Based on the number of applications, it is evident that compromise programming and multiattribute utility theory are those multiobjective methods which have been accepted by the largest percentage of researchers in this field. Those applications referred to above do not constitute a comprehensive listing, but they do provide a representative sampling. For more extensive bibliographies, see Johnsen (1968), Zeleny (1974), OWRT/WRSIC (1975), and Hwang and Masud (1979).

CHAPTER 4

DYNAMIC COMPROMISE PROGRAMMING

In this chapter, the dynamic compromise programming methodology (Szidarovszky 1979) is applied to determine a mineral extraction policy for a bauxite mine under water hazard. This methodology has been developed for dynamic multiobjective problems that are fully quantifiable. It makes use of the LPNORM concept of multiobjective programming which includes goal programming, compromise programming, and the cooperative game theory approach, essentially transforming the dynamic multiobjective problem into a classical dynamic programming problem of increased dimensionality.

The regional groundwater system under consideration is located in the Transdanubian Mountain region of Hungary. The system is modeled in terms of three objectives: bauxite production (economic), water supply, and thermal water recharge (environmental). Bauxite deposits are located below the underground water level, and this underground water is the principal source of water supply in the region. Attainment of these first two objectives (themselves in conflict) has the effect of lowering the groundwater level. However, this would have a detrimental effect on the thermal baths and springs of Budapest, which receive their natural recharge from this same source. As a result, trade-offs must be made in order to reach a solution which achieves a reasonable level of attainment of all three objectives.

1. Methodology

Consider a discrete multiobjective dynamic programming problem having the general form

$$s_m = f_m(s_{m-1}, x_m) \quad (s_0 \text{ is given})$$

$$\sum_{m=1}^{M} g_{mi}(s_m, x_m) \to \max \quad (i = 1,2,\ldots,n)$$

where s_m are the state variables, and x_m are the decision variables. Remember that no restrictions are made about the decision and state spaces. By transforming our dynamic programming problem into the generalized goal programming problem (Szidarovszky 1979), we have

$$s_m = f_m(s_{m-1}, x_m) \quad (s_0 \text{ is given})$$

$$\sum_{m=1}^{M} g_{mi}(s_m, x_m) \geqq \phi_{i*} \quad (i = 1,2,\ldots,n)$$

$$u = \left\{ \sum_{i=1}^{n} v_i(\phi_i^* - \sum_{m=1}^{M} g_{mi}(s_m, x_m)) \right\} \to \min$$

where ϕ_i^* and ϕ_{i*} are the maximal and minimal values of the ith objective, the v_i are arbitrary functions, and u is a distance metric. Since function u is strictly monotone increasing or decreasing, this problem is equivalent to finding an optimal (maximal or minimal) point of the function

$$z = \sum_{i=1}^{n} v_i(\phi_i^* - \sum_{m=1}^{M} g_{mi}(s_m, x_m))$$

subject to the same constraints. By introducing the new variables

$$\phi_i^* = \sum_{m=1}^{M} \phi_{mi}^* \qquad (\forall_i)$$

$$d_{mi} = \phi_{mi}^* - g_{mi}(s_m,x_m) \qquad (\forall_i,\forall_n)$$

$$D_{mi} = \sum_{\ell=1}^{m} d_{\ell i}, \quad D_{0i} = 0, \quad (\forall_i,\forall_m)$$

function z can be rewritten as

$$z = \sum_{i=1}^{n} v_i(D_{Mi}) = \sum_{i=1}^{n} \sum_{m=1}^{M} [v_i(D_{mi})-v_i(D_{m-1,i}) + \sum_{i=1}^{n} v_i(0)$$

$$= \sum_{m=1}^{M} \sum_{i=1}^{n} [v_i(D_{mi})-v_i(D_{mi}-\phi_{mi}^*+g_{mi}(s_m,x_m))] + \sum_{i=1}^{n} v_i(0)$$

Thus the optimization problem of function z is equivalent to optimizing the function

$$\sum_{m=1}^{M} \left\{ \sum_{i=1}^{n} [v_i(D_{mi})-v_i(D_{mi}-\phi_{mi}^*+g_{mi}(s_m,x_m))] \right\}.$$

Let us define functions G_m by the following equation

$$G_m(s_m,D_{m1},\ldots,D_{mn},x_m) = \sum_{i=1}^{n} [v_i(D_{mi})-v_i(D_{mi}-\phi_{mi}^*+g_m(s_m,x_m))]$$

then our problem has the form

$$s_m = f_m(s_{m-1}, x_m) \quad (s_0 \text{ is given})$$

$$D_{mi} = D_{m-1,i} + \phi^*_{mi} - g_{mi}(f_m(f_m(s_{m-1}, x_m), x_m), \quad D_{0i} = 0 \quad (\forall_i)$$

$$D_{Mi} \leqq \phi^*_i - \phi_{i*} \quad (\forall_i)$$

$$\sum_{m=1}^{M} G_m(s_m, D_{m1}, \ldots, D_{mn}, x_m) \rightarrow \text{opt},$$

which is a regular dynamic programming problem with decision variable x_m and state variables $s_m, D_{m1}, \ldots, D_{mn}$ at stage m. There will be one additional state variable (D) added for every objective.

2. Problem Formulation

The problem discussed here deals with N bauxite mines, S_0 water supply points, and R natural recharge sites. Among the N mines, N_0 of them are in need of water protection ($N_0 \leq N$). The annual requirements for bauxite production (both quality and quantity) are given for the planning horizon of M years. Water requirements at each supply point are also provided for the next M years as well as the ideal recharges necessary at each recharge site for this time frame.

The goal is to find efficient policies of production and water protection for the planning horizon of M years under the following three conflicting objectives:

(1) Economic objective to meet the bauxite requirements (quality and quantity) with minimal costs.

(2) Water supply objective to efficiently use water pumped from the mines to meet the water requirements of the region.

(3) Environmental objective to minimize environmental damage caused to the thermal baths and springs by the dewatering of the mines.

A multiobjective model is constructed in terms of the following variables:

State Variables

d(i,k) mining level in mine i during year k. An initial value, d(i,0), must be given, and this is updated as a function of the amount mined.

Z(i,k) water level in mine i during year k. This variable is updated as a function of the amount of water pumped from the mine.

CP(i,k) capacity of mine i during year k. This variable is a function of the amount mined previously from the given mine and its overall maximal capacity.

H(r,k) amount of natural recharge at the rth thermal bath during year k. It is a function of the amount of water pumped from the mines and the artificial recharge supplied.

Decision Variables

X(i,k) amount of bauxite produced from mine i during year k.

Q(i,k) amount of water pumped from mine i during year k.

v(k) amount of artificial recharge during year k.

t(i,s,k) amount of water conveyed from mine i to requirement point s during year k.

Inputs

W(k) total bauxite requirement (quantity) during year k.

A(k) bauxite aluminum content requirement during year k.

S(k) bauxite silicon content requirement during year k.

P(s,k) water requirements at point s during year k.

HI(r,k) ideal recharge requirements for site r during year k.

f(i,k) unit mining operational cost of mine i during year k.

fv(k) unit cost of artificial recharge during year k.

fq(i,k) unit cost of pumping from mine i during year k.

b(i,k) investment costs for mine i during year k. This and all other costs used include discount factors.

mcp(i) maximal bauxite mining capacity of mine i.

qcp(i) capacity of pumping water at mine i.

vcp artificial recharge capacity.

K(i) initial year of construction of mine i.

a(i) average bauxite aluminum content of mine i.

s(i) average bauxite silicon content of mine i.

MX(i) initial bauxite resources of mine i.

Next, the state variables, decision variables, and state transition function (law of motion) are defined.

The state variable can be given in vector form as

$$\underline{X}(k) = [X(1,k),X(2,k),X(3,k),X(4,k)]$$

$$= \left[\sum_{m=1}^{k} X(i,m),Z(j,k),CP(i,k),H(r,k)\right]$$

$$i = 1,\dots,N;\ j = 1,\dots,N_0;\ r = 1,\dots,R$$

The decision variable (policy) is the following:

$$\underline{P}(k) = [P(1,k),P(2,k),P(3,k),P(4,k)]$$

$$= [x(i,k+1),Q(j,k+1),v(k+1)t(i,s,k+1]$$

$$i = 1,\ldots,N;\ j = 1,\ldots,N_0;\ s = 1,\ldots,S_0$$

In words, the state of the system is completely defined by the amount which has so far been mined from each mine, the water level in each mine, the capacity of each mine, and the recharge at each spring. The decision consists of how much to mine from each mine, how much water to pump from each mine, how much artificial recharge should be done, and how much to pump from each mine to each water supply point.

The state transition function (law of motion) is:

$$\underline{X}(k+1) = R(\underline{X}(k),\underline{P}(k),k) = \begin{bmatrix} X(1,k) + P(1,k) \\ X(2,k) + F[\underline{X}(k),\underline{P}(k)] \\ G[X(3,k)] \\ H[X(4,k),\underline{P}(k)] \end{bmatrix} = \begin{bmatrix} \sum_{m=1}^{k+1} X(i,m) \\ Z(j,k+1) \\ CP(i,k+1) \\ H(r,k+1) \end{bmatrix} \quad (1)$$

Here, function F gives the new water levels:

$$Z(j,k+1) = F[j,Z(j,k),Q(j,k),v(k)].$$

Function G gives the new mining capacities:

$$CP(i,k+1) = G[mcp(i),K(i),k].$$

Function H gives the recharges at the thermal baths and springs:

$$H(r,k+1) = H[r,Z(j,k),Q(j,k),v(k)].$$

F, G. and H are assumed to be known functions.

The economic objective is to minimize the costs of meeting all of the system requirements in each time period. This task is accomplished through the solution of a constrained nonlinear dynamic programming problem.

The loss function consists of all costs incurred as a result of operating the system. This function is to be minimized.

$$L[\underline{X}(k), \underline{P}(k)] = \sum_{i=1}^{N} f(i,k)x(i,k) + \sum_{i=1}^{N_0} fq(i,k)Q(i,k) + fv(k)v(k) + \sum_{i=1}^{N} b(i,k) \quad (2)$$

That is, the loss incurred at time k is the sum of the following costs: mining, pumping, artificial recharge, and investment costs. Here,

$$b(i,k+1) = B[mcp(i),K(i),k]$$

where function B is assumed to be known.

The goal is to minimize these costs summed over the planning horizon of M time periods.

$$\min: \sum_{k=1}^{M} L[\underline{X}(k),\underline{P}(k)]$$

There are $2N + 2N_0 + 4$ constraints; namely,

(1) Bauxite quantity requirements must be met in each period.

$$\sum_{i=1}^{N} x(i,k) \geq W(k) \tag{3}$$

(2) Bauxite quality requirements must be met in each period.

$$\sum_{m=1}^{k} a(i)x(i,m) \geq A(k)W(k) \tag{4}$$

$$\sum_{m=1}^{k} s(i)x(i,m) \leq S(k)W(k) \tag{5}$$

(3) The amount mined from any mine is constrained by the total bauxite resources available at that mine.

$$\sum_{m=1}^{k} x(i,m) \leq MX(i) \quad i = 1,\ldots,N \tag{6}$$

(4) The mining level must be above the water level.

$$Z(i,k) \leq d(i,k) \quad i = 1,\ldots,N_0 \tag{7}$$

where

$$d(i,k) = g\left[\sum_{m=1}^{k} x(i,m)\right]$$

(5) There are physical limitations on the amount mined from any mine during any time period.

$$x(i,k) \leq CP(i,k) \quad i = 1,\ldots,N \tag{8}$$

(6) The pumping capacity at each mine is limited.

$$Q(i,k) \leq qcp(i) \quad i = 1,\ldots,N_0 \tag{9}$$

(7) The amount of artificial recharge is constrained.

$$v(k) \leq vcp \tag{10}$$

The water supply objective is to maximize the amount of pumped water available for regional water supply. The water requirement is given for the next M years at S_0 points in the region of the mines. This requirement is to be met with pumped mine water.

The loss function is expressed in terms of maximizing the amount of water pumped from the mines in the region. The pumped water for the year k is:

$$\max \quad Y(k) = \sum_{i=1}^{N_0} Q(i,k) \tag{11}$$

There are $N_0 + S_0$ constraints. They can be written as:

(1) Water requirements must be met.

$$\sum_{i=1}^{N_0} t(i,s,k) = P(s,k) \quad s = 1,\ldots,S_0 \tag{12}$$

(2) The total received at all supply points S_0 from any given mine may not exceed the amount pumped from that mine.

$$\sum_{s=1}^{N_0} t(i,s,k) \leq Q(i,k) \quad i = 1,\ldots,N_0 \tag{13}$$

From the previous objective, Q(i,k) is constrained by the pumping capacity.

The environmental objective is to minimize the environmental damage to the thermal baths and springs caused by the mine water withdrawals. Ideal natural recharges at the thermal baths and springs are given. The goal is to minimize the maximum difference between the ideal recharge and the realized recharge for any year over the planning horizon of M years. For R thermal baths and springs, therefore, there are R separate minimax problems of the following form:

$$\text{minimize } D(r) \tag{14}$$

The constraints are:

$$H(r,k) = H[r,Q(i,k),v(k),Z(i,k-1)] \qquad r = 1,\ldots,R \tag{15}$$

$$HI(r,k) - H(r,k) - D(r) \leq 0 \tag{16}$$

and

$$H(r,k) \leq HI(r,k) \quad r = 1,\ldots,R \tag{17}$$

This problem can be solved as a linear programming problem if the functions $H(r,\cdot)$ are linear.

The multiobjective model consists of the three objectives developed above [(2), (11), and (14)], subject to all the constraints [(3)-(10), (12), (13), and (15)-(17)] restricting any one of these. Also included are the state transition functions (1).

This formulation is converted into the LPNORM framework by the addition of three constraints in terms of three new variables, L(1,k), L(2,k), and L(3,k). These variables are equal to $L[\underline{X}(k),\underline{P}(k)]$, Y(k), and D(k), respectively. The three constraints may be written as:

$$
\begin{aligned}
&L(1,k) - L[\underline{X}(k),\underline{P}(k)] = 0 \\
&L(2,k) - Y(k) = 0 \qquad (18) \\
&L(3,k) - D(r) = 0
\end{aligned}
$$

The new objective for the kth time period is the L_p-metric.

$$\sum_{i=1}^{3} \left[\frac{\alpha_i^P [L(i,k) - L^*(i,k)]^P}{[RANGE(i,k)]^P} \right]^{1/P}$$

where:

α_i = the weight assigned to the ith objective.

$L^*(i,k)$ = the optimal solution to the ith problem in the kth period.

RANGE(i,k) = the scale of values over which objective i is defined in period k.

Following Szidarovszky (1979), the introduction of the new variables:

$$L^*(i) = \sum_{k=1}^{M} L^*(i,k),$$

$$d(i,k) = L(i,k) - L^*(i,k),$$

and

$$D(i,k) = \sum_{m=1}^{k} d(i,k) \quad D(i,0) = 0,$$

transforms the multiobjective problem into a dynamic programming problem having the additional state variables D(1,k), D(2,k), and D(3,k) at stage k.

This problem has the state transition function given in Equation (1) plus the following additional transition function for the additional state variables:

$$D(i,k) = D(i,k-1) + L(i,k) - L^*(i,k); \quad D(i,0) = 0.$$

The new objective is to minimize the summation of the distance measure over the M time periods.

3. Numerical Results

The problem described in the previous section is solved using the data provided in Table 4.1. This data consists of all the necessary model inputs as well as the transition functions.

All of the constraints for this problem are linear and linear transition functions are also present. The L_1 metric is used, yielding a linear programming formulation.

Prior to the solution of the multiobjective problem, it is necessary to obtain the "ideal points," L*(i,k), for use in the multiobjective objective function. This is done by solving each of the three individual problems in isolation of the others. The optimal solutions to these problems are then used as the "ideal points." The linear programming technique is used to solve these single objective problems, yielding the results of Table 4.2.

TABLE 4.1. MODEL INPUTS

N = 7	M = 3	R = 1	S_0 = 7
N_0 = 5	f(1,k) = 850,000	p(1,1) = 8	cp(1,1) = 1030
W(1) = 1680	f(2,k) = 1,328,000	p(1,2) = 8	cp(1,2) = 1030
W(2) = 4373	f(3,k) = 910,000	p(1,3) = 8	cp(1,3) = 1030
W(3) = 5691	f(4,k) = 1,020,000	p(2,1) = 30	cp(2,1) = 1640
A(k) = 52.5	f(5,k) = 1,384,000	p(2,2) = 46	cp(2,2) = 2050
S(1) = 6.87	f(6,k) = 1,050,000	p(2,3) = 46	cp(2,3) = 2050
S(2) = 6.95	f(7,k) = 1,173,000	p(3,1) = 10	cp(3,1) = 764
S(3) = 7.35	fq(2,k) = 1.02	p(3,2) = 15	cp(3,2) = 1910
a(1) = 54.7	fq(3,k) = 1.50	p(3,3) = 15	cp(3,3) = 1910
a(2) = 52.9	fq(5,k) = 1.09	p(4,1) = 19	cp(4,1) = 0
a(3) = 51.1	fq(6,k) = 1.71	p(4,2) = 19	cp(4,2) = 412
a(4) = 52.5	fq(7,k) = 1.80	p(4,3) = 19	cp(4,3) = 1030
a(5) = 52.4	fv(k) = 0.84	p(5,1) = 16	cp(5,1) = 0
a(6) = 48.4	b(1) = 106	p(5,2) = 16	cp(5,2) = 0
a(7) = 49.1	b(2) = 4000	p(5,3) = 16	cp(5,3) = 1640
s(1) = 5.46	b(3) = 1838	p(6,1) = 0	cp(6,1) = 0
s(2) = 5.88	b(4) = 850	p(6,2) = 10	cp(6,2) = 0
s(3) = 8.82	b(5) = 4023	p(6,3) = 15	cp(6,3) = 1059
s(4) = 6.41	b(6) = 1373	p(7,1) = 0	cp(7,1) = 0
s(5) = 5.99	b(7) = 2171	p(7,2) = 5	cp(7,2) = 0
s(6) = 10.08	mx(1) = 1472	p(7,3) = 10	cp(7,3) = 353
s(7) = 9.14	mx(2) = 3936	Z(2,1) = 80	qcp(2) = 450
d(2,1) = 150	mx(3) = 3646	Z(3,1) = 155	qcp(3) = 50
d(3,1) = 280	mx(4) = 2686	Z(5,1) = 80	qcp(5) = 450
d(5,1) = 180	mx(5) = 4631	Z(6,1) = 155	qcp(6) = 50
d(6,1) = 180	mx(6) = 3771	Z(7,1) = 160	qcp(7) = 50
d(7,1) = 180	mx(7) = 3986	vcp = 50	HI = 27

$$Z(2,k+1) = Z(2,k) + 6.9557 - .03455Q(2,k) - .00007Q(3,k) + .01958v(k)$$

$$Z(3,k+1) = Z(3,k) - .00008Q(2,k) - .014553Q(3,k) + .00006v(k)$$

$$Z(5,k+1) = Z(5,k) - .03455Q(2,k) - .00007Q(3,k) + .01958v(k)$$

$$Z(6,k+1) = Z(6,k) - .00008Q(2,k) - .014553Q(3,k) + .00006v(k)$$

$$Z(7,k+1) = Z(7,k) - .00002Q(2,k) - .02437Q(3,k) + .00002v(k)$$

$$d(2,k+1) = d(2,k) - .035x(2,k)$$

$$d(3,k+1) = d(3,k) - .05x(3,k)$$

TABLE 4.2. IDEAL POINTS

		Objective		
		1	2	3
Time Period	1	2,823,141,783	500	0
	2	5,903,471,184	500	0
	3	6,712,664,057	1050	0

TABLE 4.3. SEQUENCE OF DECISIONS

x(i,k)

i \ k	1	2	3
1	211	685	193
2	884	1661	1390
3	584	1615	1437
4	-	412	1030
5	-	-	1640
6	-	-	0
7	-	-	0

Q(i,k)

i \ k	1	2	3
2	349	450	119
3	0	5	10
5	-	-	230
6	-	-	0
7	-	-	0

$v(k) = 0 \quad k = 1,2,3$

The multiobjective dynamic objective is then formulated as described previously and the problem is solved via linear programming. The sequence of decisions which should be taken is shown in Table 4.3.

4. Discussion and Recommendations

The results provided are those found by linear programming. A problem which can be solved by this method is the easiest constrained example found. The presence of a nonlinear objective would make the dynamic compromise programming formulation more difficult to apply, and if many of the constraints are nonlinear, it would be virtually impossible to solve large problems in this manner. Using classical dynamic programming reduces the number of decision variables, but its use is limited due to the increased dimension of the vector of state variables in this formulation. The applicability of dynamic programming is dependent on the number of state variables to a much larger extent than on the number of decision variables.

A dynamic programming method which overcomes, to some degree, this "curse of dimensionality" is differential dynamic programming (Jacobson and Mayne 1970). Ohno (1978) and Murray and Yakowitz (1979) have applied this method to constrained dynamic programming problems. The problem considered in this chapter is a constrained dynamic programming problem, and it would seem that differential dynamic programming holds great promise for solving dynamic multiobjective problems.

CHAPTER 5

CRITERIA FOR MODEL CHOICE

When choosing among any set of alternatives, the choice will be dependent upon the set of criteria chosen to evaluate them. This is true for the choice of a solution technique for solving a multiobjective problem. In this chapter, criteria for choosing among multiobjective solution techniques are discussed. Khairullah and Zionts (1979) and Despontin and Spronk (1979) have performed experiments in evaluation of these techniques and many of the criteria discussed here were used in their evaluations.

The techniques are classified with respect to five different groups of criteria. This is followed by a discussion of the characteristics of the problem to be solved, the techniques available for solution, the decision maker, and his role in the decision process. These characteristics will serve as model choice criteria to achieve a match between the problem and the appropriate technique. The general discussion of this chapter is formalized in Chapter 6.

1. Classification of Solution Techniques

Recently, emphasis in the multiobjective decision making literature has been given to classification of the solution techniques (MacCrimmon 1973, Cohon and Marks 1975, Starr and Zeleny 1977). These efforts, while attempting to make some sense of the myriad techniques which have been published, stop short of prescribing which method should be implemented under a given set of conditions. However, examining the criteria used for classifying the methods is an important first step. Some of the classification criteria are now discussed, namely,

A. Mathematical Programming versus Decision Analysis

B. Quantitative versus Qualitative Criteria

C. Timing of Preference Determination

D. Interactive versus Noninteractive

E. Method of Comparing Alternatives

A. Mathematical Programming Versus Decision Analysis Techniques

Either a continuous set of alternatives (mathematical programming) or a discrete set (decision analysis) must be evaluated; such a criterion for classifying the techniques is advocated by MacCrimmon (1973). It is particularly desirable because it arises from two characteristics of the decision process. First, the analyst responsible for implementing the solution technique will probably be trained in one of these areas and will slant his selection toward that group. Second, the nature of the problem will probably be such that it will lead to a solution by a technique from one of these groups, but not both of them.

Characteristics of the mathematical programming techniques include:

(1) an infinite or very large set of alternatives which are described by a set of constraints.

(2) a set of technological (or sometimes preference) constraints.

(3) an objective function vector either local or global.

(4) an algorithm to generate more preferred points in order to converge to an optimum.

Characteristics of the decision analysis techniques include:

(1) a set of available alternatives with specified attributes and attribute values.

(2) scalings, perhaps only ordinal, of attribute values and, in some cases, an ordering across attributes.

(3) a set of constraints across attributes.

(4) a process for sequentially comparing alternatives on the basis of attribute values so that alternatives can be either eliminated or retained.

Table 5.1 presents a partial listing of the techniques classified with respect to this criterion. Note that many of the mathematical programming techniques can be applied to the decision analysis problem.

B. Quantitative Versus Qualitative Criteria

In many decision problems, the presence of criteria which either cannot or should not be quantified makes this classification (Duckstein 1979) a logical one. Cost, tensile strength, and annual rainfall are examples of criteria which are easily quantified. Examples of criteria which cannot be so easily quantified include environmental impacts, political preferences, and aesthetics.

The techniques are classified according to this criterion in Table 5.2. Those which can handle qualitative criteria are categorized by some sort of ordinal scalings over these criteria. This has been

TABLE 5.1. MATHEMATICAL PROGRAMMING--
DECISION ANALYSIS CLASSIFICATION

Mathematical Programming	Decision Analysis
Compromise Programming	ELECTRE
Goal Programming	Q-Analysis
Cooperative Game Theory	Metagame
Multiattribute Utility Theory	Concordance Analysis
Surrogate Worth Trade-off	Ziont's Method
Dynamic Compromise Programming	Compromise Programming
STEP Method	Cooperative Game Theory
TRADE	Multiattribute Utility Theory
PROTRADE	
SEMOPS	
Reference Objective	
Displaced Ideal	
Lagrangian Methods	
Local MAUT	

TABLE 5.2. QUANTITATIVE-QUALITATIVE CLASSIFICATION

Only Quantitative	Quantitative or Qualitative
Compromise Programming	ELECTRE
Goal Programming	Q-Analysis
Cooperative Game Theory	Metagame
Multiattribute Utility Theory	Concordance Analysis
Surrogate Worth Trade-off	Ziont's Method
Dynamic Compromise Programming	
STEP Method	
TRADE	
PROTRADE	
SEMOPS	
Reference Objective	
Displaced Ideal	
Lagrangian Methods	
Local MAUT	

mentioned previously as a characteristic of the decision analysis techniques. Therefore, it is not surprising to find that this classification is very similar to the previous one.

C. Timing of Preference Determination

Cohon and Marks (1975) divide multiobjective techniques into two groups: those which rely on prior articulation of preferences and those which rely on progressive articulation of preferences. They include goal programming, multiattribute utility theory, ELECTRE, and the surrogate worth trade-off method in the first group. The second group includes the STEP method, iterative weighting methods, and sequential multiobjective problem solving (SEMOPS).

The ability of a decision maker to specify his preference structure gives rise to this classification. Many decision makers would prefer to refine their preferences as more information becomes available to them while others, relatively sure of their preferences, would prefer to receive solutions based on their original estimates. Well-known techniques such as compromise programming and multiattribute utility theory have been developed for both situations. A summary of the techniques, classified in this manner, appears in Table 5.3. The reference objective method is unique in that no preference structure is utilized.

TABLE 5.3. PRIOR-PROGRESSIVE ARTICULATION OF WEIGHTS CLASSIFICATION

Prior Articulation	Progressive Articulation
Compromise Programming	STEP Method
Goal Programming	Iterative Weighting
Cooperative Game Theory	SEMOPS
Multiattribute Utility (global)	Compromise Programming (Displaced Ideal)
ELECTRE	Interactive Goal Programming
Surrogate Worth Trade-off	Multiattribute Utility (local)
Q-Analysis	TRADE
Metagame	PROTRADE
Dynamic Compromise Programming	Nested Lagrangian
	Metagame

D. Interactive Versus Noninteractive

Due to the trade-offs inherent in multiobjective decision problems, many of the available techniques have specifically been designed as interactive. These techniques allow the decision maker to play an active role in the decision process and also to be more aware of those trade-offs involved in accepting an eventual solution. The classification scheme of Starr and Zeleny (1977), distinguishing between decision outcome-oriented approaches and decision process-oriented approaches, is very close to this one with the interactive techniques considered to be the decision process-oriented approaches. The techniques, classified according to this criterion, are shown in Table 5.4. The interactive decision process is very closely related to the progressive articulation of the preference structure, so these two classifications are nearly identical. This is due to the fact that the purpose of the interactive process is often to obtain a progressive articulation of the weights.

E. Classification According to How Solutions are Compared

The previous classifications were motivated primarily by differences in the methods as perceived by those who make use of them in their work. Here, a new classification is proposed which is based on the kind of solution that is desired by the decision maker. Confronted with a multiobjective problem, a decision maker wants to do one of three things:

TABLE 5.4. INTERACTIVE-NONINTERACTIVE CLASSIFICATION

Interactive	Noninteractive
Compromise Programming (Displaced Ideal)	Compromise Programming
Interactive Goal Programming	Goal Programming
Multiattribute Utility (local)	Cooperative Game Theory
STEP Method	Multiattribute Utility (global)
SEMOPS	ELECTRE
TRADE	Q-Analysis
PROTRADE	Metagame
Surrogate Worth Trade-off	Dynamic Compromise Programming
Nested Lagrangian	

(1) He wants to implement that solution which shows the greatest improvement over his present solution or with respect to an aspiration level.

(2) He wants to implement the best solution resulting from a comparison of the alternatives with one another.

(3) He wants to implement that solution which is the closest to a goal point.

Techniques are available to accomplish these three tasks for multiobjective problems. The terms "aspiration level" and "goal point" have been defined in Chapter 2. A summary of this classification is presented in Table 5.5.

2. Characteristics Describing the Problem

The selection of an appropriate multiobjective solution technique is dependent upon the nature of the problem under consideration. Two characteristics describing the problem have been used as classification criteria. First, the problem can be expressed either in mathematical programming or decision analysis form. Second, the criteria can all be quantified or some can remain in qualitative form. Assessing a problem with respect to both of these problem characteristics eliminates a number of available techniques from consideration.

If there is uncertainty in the problem data, the techniques used for solution must somehow take this into account. This limits the analyst to selecting from those few techniques mentioned earlier for stochastic problems (Goicoechea et al. 1979, Haimes et al. 1980). Dynamic problems are another special case. Again, only a few techniques

TABLE 5.5. DIRECT COMPARISON CLASSIFICATION

Comparison with Aspiration Level	Direct Comparison	Comparison with Goal Point
Cooperative Game Theory	ELECTRE	Compromise Programming
Multiattribute Utility Theory (local, global)	Q-Analysis	Goal Programming
Reference Objective	Surrogate Worth Trade-off	Dynamic Compromise Programming
TRADE	Metagame	Displaced Ideal
PROTRADE	Concordance Analysis	
SEMOPS	Ziont's Method	
Lagrangian Methods	STEP Method	

are available for problems of this type (Mitten 1974, Szidarovszky et al. 1981).

Another important problem characteristic is the size of the problem in terms of the number of objectives, alternative systems, and in programming formulations, decision and state variables and constraints. Problem size places severe limitations on dynamic techniques and techniques using nonlinear programming. Use of dynamic techniques is limited by the number of state variables, and nonlinear programming techniques are limited by the number of constraints and decision variables.

The nature of the problem variables, whether they are integer or continuous, is another characteristic. Integer formulations are much more difficult to solve and their presence would limit the number of possible solution techniques (Armijo 1981).

3. Characteristics Describing the Techniques

A complete knowledge of the capabilities and limitations of the techniques is necessary in order to decide which one to apply to a given problem. Again, some of these capabilities have been used as classification criteria. The ability of a technique to make use of qualitative or ordinal criteria data in its analysis is very important as is its ability to analyze discrete or continuous sets of systems. The latter ability essentially labels the technique as either one for mathematical programming or decision analysis.

The characteristics of the solution obtained by the techniques is another classification criterion. The solution is either the closest one to a goal point, the farthest from an aspiration level, or the result of a direct comparison between alternatives. Certain techniques will always yield a strongly efficient solution while others can only be shown, in general, to yield a weakly efficient solution.

Additional characteristics of the solution include consistency of results, robustness, and the degree of ranking provided. If the results from application of a given technique are consistent with those of other techniques, this is at least an indication that the technique is useful for application to similar problems. The robustness of the solution with respect to parameter changes measures the power of the technique to discern among alternative solutions. A technique which yields a complete ranking of alternatives is usually preferable to one yielding only a partial ranking and a cardinal ranking is preferable to an ordinal ranking since it gives information as to the magnitude of the differences between alternatives.

Ease of use is the final technique characteristic. This includes the solution time, time for implementation, and the amount of interaction time that is required with the decision maker. While the solution time refers in most cases to computer time, the time for implementation refers to the time and effort required to determine all the necessary problem parameters. If the decision maker has very little time available for interaction in the decision process, those techniques requiring a large amount of such time cannot be used.

4. Characteristics Describing the Decision Maker

The determination of the preference structure of the decision maker is more difficult with certain techniques than with others. The presence of a group decision maker as opposed to an individual significantly increases this difficulty. Techniques which require a large amount of effort to accomplish this task may not be suitable when there is a group decision maker due to the excessive time requirements (McAniff 1980).

The amount of time that the decision maker has available for interaction and his desire either to involve himself or to avoid an interactive process are characteristics which can be matched with the appropriate technique characteristics. A similar match can be made concerning his ability to state his weights at the outset of the decision process. If this cannot be done, a technique relying on progressive articulation of weights would seem inappropriate.

The level at which the decision maker understands the functioning of the multiobjective techniques may also limit the usage of some of them. A technique requiring a great deal of educational preparation may be inappropriate if a more intuitive technique yields similar results. This is because a thorough understanding of the technique helps the decision maker to understand fully the meaning of the solution. Some of the techniques, for example, the surrogate worth trade-off method, require a large degree of sophistication on the part of the decision maker. In contrast, ELECTRE is one technique which requires very little background with multiobjective decision problems.

5. Summary

The characteristics discussed in this chapter are necessary for a complete description and evaluation of the techniques. The elements which must fit together in order to match a multiobjective solution technique with a particular problem and decision maker have been discussed. The criteria are summarized in Table 5.6. The missing element, the algorithm to bring about this match (see Figure 5.1) is developed and applied in the remaining chapters.

TABLE 5.6. MODEL CHOICE CRITERIA

1. Ability to handle qualitative criteria.
2. Ability to choose among discrete sets of alternatives.
3. Ability to choose among continuous sets of alternatives.
4. Ability to solve dynamic problems.
5. Ability to solve stochastic problems.
6. Comparison to goal point.
7. Comparison to aspiration level.
8. Direct comparison.
9. Strongly efficient solution.
10. Complete ranking.
11. Cardinal ranking.
12. Ability to handle integer variables.
13. Computer time required.
14. Implementation time required.
15. Interaction time required.
16. Decision maker's level of knowledge required.
17. Consistency of results.
18. Robustness of results.
19. Applicability to case of group decision maker.
20. Number of objectives.
21. Number of systems.
22. Number of constraints.
23. Number of variables.
24. Decision maker's level of knowledge.
25. Time available for interaction.
26. Desire for interaction.
27. Confidence in original preference structure.

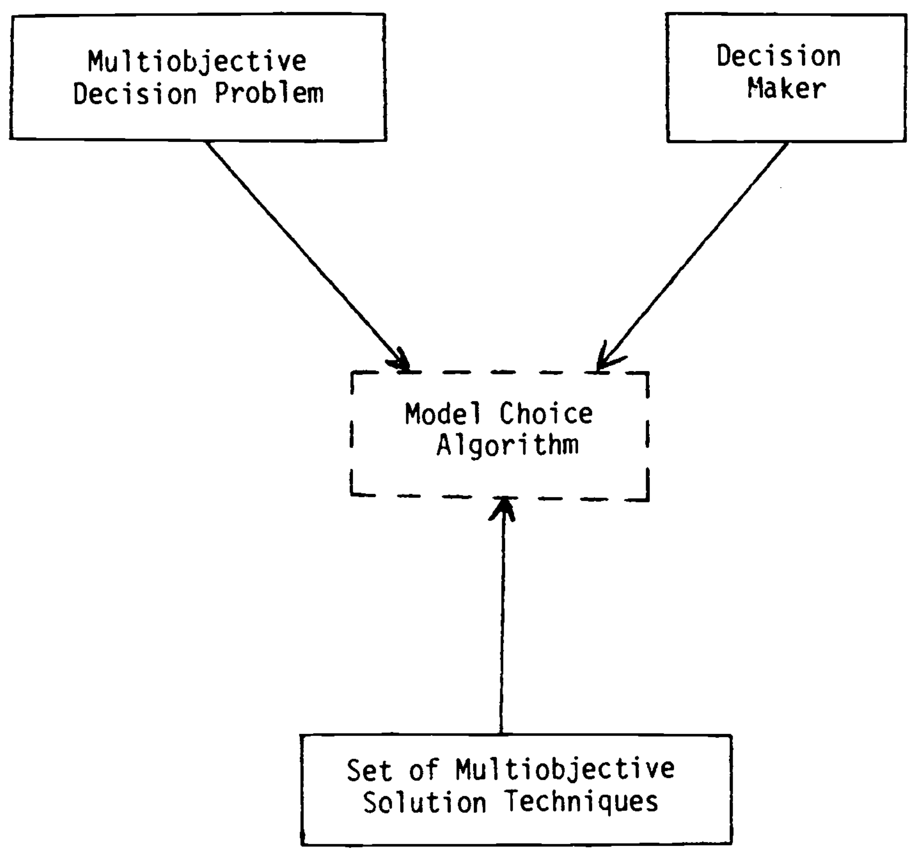

Figure 5.1. Inputs to Model Choice

CHAPTER 6

MODEL CHOICE ALGORITHM

The evaluation of alternative multiobjective solution techniques is itself a multiobjective decision problem. That is, it is a problem where it is desired to optimize several objectives simultaneously and these objectives are in conflict with one another. As an example, a multiobjective technique which performs best with respect to one criterion (possibly a minimum amount of required computer time) may not do well with regard to another criterion (perhaps the interaction requirements between the decision maker and the analyst).

Kisiel and Duckstein (1972) recommend the use of the cost-effectiveness approach (Kazanowski 1968) for the purpose of attacking model choice problems and apply that approach to the problem of choosing among appropriate hydrologic models. The steps of the cost-effectiveness approach, tailored to the multiobjective model choice problem, are outlined below.

(1) Define the desired goals, objectives, or purposes that the multiobjective techniques are to fulfill.

(2) Establish technique evaluation criteria that relate technique capabilities to objectives, and hence, to goals.

(3) List alternative multiobjective techniques available for attaining the goals.

(4) Determine capabilities or performance of the alternative techniques in terms of the evaluation criteria.

(5) Generate a techniques versus criteria array.

(6) Analyze the merits of alternative multiobjective solution techniques with respect to the given problem.

The cost-effectiveness approach only provides a useful framework to the model choice algorithm that is sought here. Essentially, Step 6 is the implementation of that algorithm.

1. Model Choice Criteria

The criteria upon which the choice of an appropriate solution technique is to be made have been discussed in the preceding chapter. These criteria are listed in this section in four groups, categorized according to their role in the model choice algorithm as follows:

(1) Mandatory 0-1 criteria.

(2) Nonmandatory 0-1 criteria.

(3) Technique dependent criteria.

(4) Application dependent criteria.

The mandatory 0-1 criteria are those criteria for which a technique can be evaluated by the answer to a yes-no type of question. In addition, the answer to the question for any technique must be positive in order for that technique to remain in consideration. No technique, for example, can be used to solve a problem with qualitative criteria if it provides no means of dealing with this type of criteria. The criteria in this group are:

(1) Ability to handle qualitative criteria.

(2) Ability to choose among discrete sets of alternatives.

(3) Ability to choose among continuous sets of alternatives.

(4) Ability to solve dynamic problems.

(5) Ability to solve stochastic problems.

Nonmandatory 0-1 criteria again are those which can be evaluated for any technique with a "yes" or a "no" answer. However, for these criteria, a technique which receives a "no" may remain in the analysis. This is due to the fact that a technique may be strong enough with respect to the other criteria to offset the failure to meet one of these criteria. As an example, consider whether or not a technique is guaranteed always to yield a strongly efficient solution. One which may fail to meet this criterion will, for most applications, yield a strongly efficient solution nevertheless. If the decision maker feels that this criterion is a minor one, it is probable that the evaluation of the technique with respect to the more important criteria will offset this deficiency. Criteria included in this group are:

(1) Are the alternatives compared to a goal point?

(2) Are the alternatives compared to an aspiration level?

(3) Are the alternatives compared directly to each other?

(4) Is the solution guaranteed to be strongly efficient?

(5) Is a complete ranking obtained?

(6) Is this ranking of alternatives cardinal as opposed to ordinal?

(7) Are integer variables present in the problem formulation?

The technique dependent criteria are those which can be evaluated without any knowledge of the problem to be solved. The techniques can be evaluated once by the analyst with respect to these criteria and that evaluation can then be used for all applications. These criteria include:

(1) Computer time required relative to the other techniques.

(2) Time required for interaction with the decision maker.

(3) Time required for implementation.

(4) Level of knowledge necessary on the part of the decision maker.

(5) Consistency of results with those of other techniques.

(6) Robustness with respect to changes in problem parameters.

(7) Applicability of the technique to the case of a group decision maker.

Application dependent criteria are those for which the techniques must be evaluated independently for every new problem that is encountered. Of the four groups of criteria, this is the only one where this task must be performed for each application. This feature is desirable because it might be more advantageous to select any technique at random than to spend the time required to evaluate all of the techniques with respect to all of the criteria every time a problem is to be solved. This last group of criteria includes:

(1) Number of objectives or criteria.

(2) Number of systems to be evaluated.

(3) Number of variables in the model formulation.

(4) Number of constraints in the model formulation.

(5) Level of understanding of the techniques by the decision maker.

(6) Time available for interaction by the decision maker.

(7) Desire for interaction on the part of the decision maker.

(8) Confidence in original preference structure.

2. Alternative Multiobjective Solution Techniques

The number of solution techniques available for application to any multiobjective decision problem is limited to those with which the analyst is familiar. No one individual can be skilled in applying all of the techniques mentioned in Chapter 3, and experience in the use of a technique is a prerequisite for evaluating it with respect to a set of criteria. The set of alternative solution techniques which follows (those with which the author has at least some familiarity) is taken from those techniques listed in the classification tables of Chapter 5:

(1) Compromise Programming

(2) Goal Programming

(3) Cooperative Game Theory

(4) Multiattribute Utility Theory

(5) Surrogate Worth Trade-off

(6) ELECTRE

(7) Q-Analysis

(8) Dynamic Compromise Programming

(9) PROTRADE

(10) STEP Method

(11) Displaced Ideal

(12) Local Multiattribute Utility Theory

3. Evaluation of Technique Capabilities

In this section, the alternative solution techniques are evaluated with respect to the first three groups of criteria laying the basis for the development of the cost-effectiveness array of techniques versus criteria. It is this array which provides the information necessary to achieve the model choice.

In order to use the first group of criteria for evaluation of the techniques, it must be asked: Can this technique be applied to

(1) solve problems having qualitative criteria?

(2) choose among or rank a discrete set of alternatives?

(3) select an efficient solution from a continuous set of alternatives?

(4) solve dynamic problems?

(5) solve stochastic problems?

The answers to these questions for each of the alternative techniques are given in Table 6.1.

Another set of questions is asked for the purpose of evaluating the techniques with respect to the second group of criteria:

(1) Does this technique compare the alternatives to a goal point?

TABLE 6.1. EVALUATION FOR MANDATORY 0-1 CRITERIA

	Criteria				
Techniques	Qualitative Criteria	Discrete Set of Alternatives	Continuous Set of Alternatives	Dynamic Problems	Stochastic Problems
Compromise Programming	Yes	Yes	Yes	Yes	No
Goal Programming	No	No	Yes	No	No
Cooperative Game	Yes	Yes	Yes	No	No
Multiattribute Utility	Yes	Yes	Yes	No	No
Surrogate Worth	No	No	Yes	No	No
ELECTRE	Yes	Yes	No	No	No
Q-Analysis	Yes	Yes	No	No	No
Dynamic Comp. Prog.	No	No	Yes	Yes	No
PROTRADE	No	No	Yes	No	Yes
STEP	No	No	Yes	No	No
Displaced Ideal	Yes	No	Yes	No	No
Local Multiattribute	No	No	Yes	No	No

(2) Does this technique compare the alternatives to an aspiration level?

(3) Does this technique directly compare the alternatives to each other?

(4) Does this technique show that the solution obtained is strongly efficient?

(5) Does this technique yield a complete ranking of a discrete set of alternatives?

(6) Is the ranking of the alternatives a cardinal ranking?

(7) Can this technique be used to solve problems having integer variables?

Table 6.2 summarizes the answers to this set of questions.

The technique dependent criteria are the last ones which can be evaluated without knowledge of the application where the techniques are to be implemented. A subjective scale is used to evaluate these criteria over which each technique is assigned a value from one to ten, one representing the worst case and ten representing the best. Due to the subjective nature of these ratings, it is important that each analyst making use of multiobjective solution techniques devise his own table like that shown in Table 6.3. Having used one technique many times, an analyst is certain to consider it "easy to use" due to his familiarity with it. The ratings in Table 6.3 are based on the author's experience in applying these techniques as well as the evaluation works of Cohon and Marks (1975) and Khairullah and Zionts (1979).

TABLE 6.2. EVALUATION FOR NONMANDATORY 0-1 CRITERIA

	Criteria						
Techniques	Goal Point	Aspiration Level	Direct Comparison	Strongly Efficient	Complete Ranking	Cardinal Ranking	Integer Variables
Compromise Programming	Yes	No	No	No	Yes	Yes	Yes
Goal Programming	Yes	No	No	No	No	No	Yes
Cooperative Game	No	Yes	No	Yes	Yes	Yes	Yes
Multiattribute Utility	No	Yes	No	No	Yes	Yes	Yes
Surrogate Worth	No	No	Yes	Yes	No	No	Yes
ELECTRE	No	No	Yes	No	Yes	No	No
Q-Analysis	No	No	Yes	No	No	No	No
Dynamic Comp. Prog.	Yes	No	No	No	No	No	Yes
PROTRADE	No	Yes	No	No	No	No	No
STEP	No	No	Yes	No	No	No	No
Displaced Ideal	Yes	No	No	No	No	No	No
Local Multiattribute	No	Yes	No	No	No	No	No

TABLE 6.3. EVALUATION FOR TECHNIQUE DEPENDENT CRITERIA

Techniques	Criteria: Computer Time Required	Interaction Time Required	Implementation Time Required	Knowledge Required of Decision Maker	Consistency of Results	Robustness of Results	Group Decision Maker
Compromise Programming	6	10	9	4	8	8	8
Goal Programming	7	8	8	5	7	7	7
Cooperative Game	4	10	9	3	9	9	9
Multiattribute Utility	4	3	5	2	10	8	2
Surrogate Worth	4	6	6	3	10	6	4
ELECTRE	10	10	8	10	5	9	7
Q-Analysis	8	9	6	7	4	8	6
Dynamic Comp. Prog.	2	9	5	2	8	7	8
PROTRADE	3	5	5	2	9	7	5
STEP	7	5	7	6	8	7	5
Displaced Ideal	5	4	3	4	9	10	2
Local Multiattribute	1	1	2	2	10	10	1

The evaluation of the techniques with respect to the problem dependent criteria cannot be done at this point (a particular decision problem is needed), but the form that this array takes is shown in Table 6.4. Again, it is recommended to use a subjective scale ranging from one to ten for this evaluation. It should be noted that it is not, for example, the number of objectives which is important, but how well a technique can handle that number of objectives which is to be evaluated.

4. Model Choice Algorithm

The first step of the model choice algorithm is to formulate the model of the decision problem to be solved. The problem formulation is as important, if not more important, than the solution technique applied to it (Wymore 1976). This is true, especially for problems concerning large scale systems, partly because the manner in which the problem is formulated helps determine the most suitable solution technique to implement.

After the problem is well developed, all criteria for evaluation of the techniques are listed, and the preference structure over this set of criteria is determined for the given application. These criteria have already been listed earlier in this chapter.

The determination of the preference structure is accomplished in two phases. First, the set of criteria is reduced. Any criterion having no significance with respect to the problem at hand is assigned a weight of zero and eliminated from the analysis. For example, the ability to solve problems having integer variables is of no importance if there are

TABLE 6.4. EVALUATION FOR PROBLEM DEPENDENT CRITERIA

Techniques	Criteria							
	Number of Objectives	Number of Alternatives	Number of Variables	Number of Constraints	Level of Decision Maker's Knowledge	Time Available for Interaction	Desire for Interaction	Presence of Preference Structure
Compromise Programming								
Goal Programming								
Cooperative Game								
Multiattribute Utility								
Surrogate Worth								
ELECTRE								
Q-Analysis								
Dynamic Comp. Prog.								
PROTRADE								
STEP								
Displaced Ideal								
Local Multiattribute								

none of these variables in the problem formulation. Similarly, the ability to apply a technique to problems characterized by a discrete set of alternatives is of no use in solving a mathematical programming problem. Second, the utility functions or, more often, the weights are determined for the remaining criteria making use of one of the methods discussed in Chapter 3.

Next, all of the multiobjective solution techniques that the analyst feels capable of applying to any problem are listed. This step is the result of all of his previous experience involving multiobjective decision making and is therefore independent of the problem to be solved.

A table similar to Table 6.1 is now completed. The set of alternative solution techniques is reduced as follows:

(1) Let the value of "yes" equal one and the value of "no" equal zero. Denote these entries a(i,j) for the ith technique and the jth criteria.

(2) For each technique row i, determine the product of the entries, $\Pi a(i,j)$, for all criteria j.

(3) If $\prod_j a(i,j) = 0$, eliminate technique i from consideration.

In words, only solution techniques meeting all of the mandatory criteria remain in the analysis. These comprise the set of feasible solutions to the model choice problem.

The reduction in the number of criteria and techniques described above is necessary in order to assure that this algorithm is applicable to a wide range of problems. The analyst is spared the task of preparing these lists for every problem and allowed just to eliminate those which are not applicable. Time is saved for him to concentrate on solving the problem which, of course, is the ultimate goal.

The alternative solution techniques and evaluation criteria are now available for the construction of the techniques versus criteria array. First, tables corresponding to Tables 6.2, 6.3, and 6.4, are completed. The first two of these can be taken directly from the tables already presented by excluding the rows and columns which have been eliminated. The entries for the third table, as mentioned previously, are dependent on the particular application, so this table must be completed each time this procedure is used.

Once these tables have been prepared, they are combined into one array of techniques versus criteria. The number of techniques is the same as in these tables. If the analyst originally felt capable of applying N techniques, and n of these were eliminated, this final array will have N-n columns. The number of criteria is the sum of the numbers of criteria in the three tables. That is, if there were originally $K = K_1 + K_2 + K_3 + K_4$ criteria in all the tables, and $k_1 + k_2 + k_3 + k_4$ of them were eliminated, then this table will have $K_2 + K_3 + K_4 - (k_2 + k_3 + k_4)$ criteria. The entries, a(i,j), in this array are taken directly from Tables 6.2, 6.3, and 6.4. The objective of the model choice algorithm is to choose among alternative multiobjective solution techniques. This completed array provides the information needed to make that choice.

At this point, a multiobjective solution technique must be chosen for application to this array. Obviously, the model choice algorithm can be applied to make this choice, but this approach would just lead around in circles. Therefore, one technique must be arbitrarily chosen and applied here. Since the model choice problem is defined by a discrete set of systems, this limits the choice. However, since the analyst would usually prefer to spend his time solving his problem rather than choosing the technique by which this is accomplished, that technique which he feels is easiest to apply is recommended. That method appears to be compromise programming (McAniff et al. 1980). This method is applied with the ideal point chosen such that it is the vector consisting of the maximal values in each row of the array. This completes the model choice algorithm and the chosen technique is then applied to the multiobjective decision problem.

A summary of the steps of the algorithm follows:

(1) Formulate the model of the decision problem to be solved.

(2) (a) List the criteria for model choice.

(b) Reduce this set of criteria.

(c) Determine the preference structure over the remaining set of criteria.

(3) (a) List the alternative solution techniques.

(b) Reduce the set of alternative solution techniques.

(4) Complete the techniques versus criteria array.

(5) Select a satisficing solution from the array.

CHAPTER 7

APPLICATIONS AND CONCLUSIONS

Two applications of the model choice algorithm are presented in this chapter. The first is a problem characterized by a discrete set of alternative systems. The second problem is formulated as a mathematical programming problem (linear) with two objectives. An appropriate multiobjective solution technique is chosen for application to each problem following the procedure outlined in the preceding chapter.

1. Evaluation of Alternative River Basin Planning Strategies

The detailed formulation of this problem is developed in Appendix B leading to the systems versus criteria array of Table 7.1. This array contains the evaluations of each of 25 alternative systems with respect to each of 13 criteria. It is desired to rank these systems through the use of the data in Table 7.1.

Table 5.6 has presented a general set of 27 criteria for model choice. This set is reduced by eliminating those criteria which have no bearing on this problem. The following seven criteria are eliminated because they refer to conditions not encountered in the river basin problem:

(1) Ability to choose among continuous sets of alternatives.

(2) Ability to solve dynamic problems.

(3) Ability to solve stochastic problems.

TABLE 7.1. SYSTEMS VERSUS CRITERIA ARRAY

OBJECTIVE	CRITERIA	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
WATER SUPPLY	Aquifer Level	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	2.5
	Water Quality Urban	e	a	d	a	b	e	a	d	a	b	e	a	d	a	b	e	a	d	a	b	e	a	d	a	b
	Water Quality Agric.	a	b	b	b	b	a	b	b	b	b	a	b	b	b	b	a	b	b	b	b	a	b	b	b	b
FLOOD PROTECTION	Expect. Flood Losses	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33
	Expect. Freq.	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04	01	.01	.003	.02	.04	.01	.01	.003	.02	.04
ENVIRONMENT	Pres. Desig. Areas	d	d	c	a	a	d	d	c	a	a	d	d	c	a	a	d	d	c	a	a	d	d	c	a	a
	Effect on Wild. Veg.	c	b	d	a	a	d	c	e	c	c	c	c	d	d	b	c	c	d	b	b	c	c	d	b	b
UTILIZATION OF RESOURCE	Implem. Costs	12.7	16.8	12.3	1.9	0.2	32.5	36.6	32.1	21.8	20	28.1	32.2	27.6	17.3	15.6	12.6	16.7	12.2	1.8	.01	12.5	16.6	12.1	1.8	0
	O & M Costs	37.6	37.8	38.2	37.2	37.0	2.6	2.8	3.2	2.2	2.0	2.2	2.4	2.8	1.8	1.6	1.1	1.3	1.7	.6	.5	.6	.8	1.2	.2	0
	Indirect Costs	c	c	b	d	c	d	d	d	e	e	b	b	a	c	b	c	c	b	d	d	c	c	c	e	d
RECREATION	Natural Resource	c	c	c	b	a	d	d	d	c	c	e	e	e	d	c	c	b	e	a	a	c	b	c	b	a
	Pres. of Exist. Fac.	c	b	c	b	b	c	b	c	a	a	c	b	c	b	b	c	b	c	a	a	c	b	c	b	b
	Creation New Oppr.	b	b	a	a	c	d	d	b	c	e	d	d	b	c	e	d	d	b	c	e	d	d	b	c	e

(4) Ability to handle integer variables.

(5) Applicability to the case of a group decision maker.

(6) Number of variables in the model formulation.

(7) Number of constraints in the model formulation.

A subjective set of plausible weights describing the relative importance of each of the 20 remaining criteria is developed next. These weights are shown with the criteria in Table 7.2. The two remaining mandatory 0-1 criteria are not shown in the table because they must always receive a weight of infinity. Any technique that does not meet any one of these criteria is eliminated from consideration. The remaining weights (those shown in Table 7.2) reflect the preference structure of the author as it pertains to this problem. For example, "computer time required" receives a low weight because it is not a major factor in solving a problem of this size. As the problem size increases, however, this criterion becomes of greater significance.

Next, the set of available solution techniques is developed. These techniques, listed previously in Chapter 6, are given again here for convenience.

(1) Compromise Programming

(2) Goal Programming

(3) Cooperative Game Theory

(4) Multiattribute Utility Theory

(5) Surrogate Worth Trade-off

(6) ELECTRE

(7) Q-Analysis

TABLE 7.2. TECHNIQUES VERSUS CRITERIA ARRAY

	Weight	Comp. Prog.	Coop. Game	MAUT	ELECTRE	Q-Analysis
Goal Point	.3	1	0	0	0	0
Aspiration Level	.3	0	1	1	0	0
Direct Comparison	.3	0	0	0	1	1
Strongly Efficient	1	0	1	0	0	0
Complete Ranking	7	1	1	1	1	0
Cardinal Ranking	5	1	1	1	0	0
Computer Time Required	2	6	4	4	10	8
Interaction Time Required	3	10	10	3	10	9
Implementation Time	3	9	9	5	8	6
DM's Knowledge Required	1	4	3	2	10	7
Consistency of Results	4	8	9	10	5	4
Robustness of Results	3	8	9	8	9	8
Number of Objectives	2	6	7	4	9	8
Number of Alternatives	2	10	10	9	7	6
DM's Level of Knowledge	1	10	10	10	10	9
Availability for Interaction	3	10	10	7	9	9
Desire for Interaction	3	10	10	5	9	9
Availability of Weights	3	10	10	10	10	10

(8) Dynamic Compromise Programming

(9) PROTRADE

(10) STEP Method

(11) Displaced Ideal

(12) Local Multiattribute Utility Theory

The evaluation of these techniques with respect to the remaining mandatory 0-1 criteria (from Table 6.1) reduces the set of available techniques to the five shown in Table 7.2. Goal programming, surrogate worth trade-off, dynamic compromise programming, PROTRADE, local multiattribute utility theory, and the STEP method are eliminated because of difficulties in incorporating qualitative data. The inability of the method of the displaced ideal to analyze a discrete set of alternatives eliminates it from consideration.

Table 7.2 can now be constructed to show the evaluation of the remaining five techniques with respect to those criteria still in the analysis. The evaluation of the techniques with respect to the first two groups of criteria (nonmandatory 0-1 and technique dependent criteria) has been done in Chapter 6. The first six rows of Table 7.2 are taken from Table 6.2 and the second six rows are taken from Table 6.3. The final six rows (the problem dependent criteria) are completed at this stage of the analysis for the river basin planning problem.

As prescribed in the preceding chapter, compromise programming is applied to the array of Table 7.2 to select the technique which is closest to an ideal technique. The ideal point vector is

[1,1,1,1,1,1,10,10,9,10,10,9,9,10,10,10,10,10]

and the range over which each criterion is measured is

[1,1,1,1,1,1,10,10,10,10,10,10,10,10,10,10,10,10].

These ranges are chosen to normalize the distance metric (all distances are between zero and one). Substituting these values and the data of Table 7.2 into the L_1-distance metric (given in Chapter 3) shows that cooperative game theory is the best choice of a solution technique for this application. The complete ordering and associated distances follow:

(1) Cooperative Game Theory (3.34)

(2) Compromise Programming (4.8)

(3) ELECTRE (9.89)

(4) Multiattribute Utility Theory (11.05)

(5) Q-Analysis (13.6)

Solutions to the river basin planning problem using these techniques can be found in McAniff et al. (1980).

2. Blending of Coal at Surface Storage Bins

A detailed formulation of the coal blending example, explaining the definitions of the variables and the logic of the constraints and objectives, is given in Appendix C. There, compromise programming is applied to arrive at a solution. This example is represented by a linear programming formulation with two objectives as follows:

$$\text{maximize } 211773X_1 + 193904X_2 + 103782X_3 + 166521X_4 + 153355X_5$$

and

$$\text{minimize } 183621X_1 + 151578X_2 + 60702X_3 + 98337X_4 + 96806X_5$$

subject to the following constraints:

$$X_1 + X_2 + X_3 + X_4 + X_5 \leq 50$$

$$X_1 + X_2 + X_3 + X_4 + X_5 \geq 40$$

$$X_1 \leq 11$$

$$X_2 \leq 11$$

$$X_3 \leq 11$$

$$X_4 \leq 11$$

$$X_5 \leq 11$$

$$X_1 \geq 5$$

$$X_2 \geq 5$$

$$X_3 \geq 5$$

$$X_4 \geq 5$$

$$X_5 \geq 5$$

The reduction of the criteria set results in the elimination of the following criteria:

(1) Ability to handle qualitative criteria.

(2) Ability to choose among discrete sets of alternatives.

(3) Ability to solve dynamic problems.

(4) Ability to solve stochastic problems.

(5) Does the technique yield a complete ranking?

(6) Is a cardinal ranking obtained?

(7) Ability to handle integer variables.

(8) Applicability to the case of a group decision maker.

(9) Number of systems to be evaluated.

Only one mandatory 0-1 criterion (the ability to choose among continuous sets of alternatives) remains. The rest of the remaining criteria are shown in Table 7.3.

In determining the preference structure, the one mandatory 0-1 criterion is again assigned a weight of infinity. The remaining weights (shown in Table 7.3) are assigned in accordance with the desires of the decision maker (in this case, a mining engineer).

Of the 12 alternative techniques under consideration, ten satisfy the need to choose among continuous sets of alternatives. ELECTRE and Q-analysis do not meet this criterion. In addition, dynamic compromise programming and PROTRADE can also be (but don't necessarily have to be) eliminated due to the absence of any dynamic or stochastic properties in the problem. This leaves eight techniques to be evaluated.

The evaluation of the remaining eight techniques with respect to the reduced criteria set (17 criteria) is given in Table 7.3. Again, Tables 6.2 and 6.3 are the direct sources of the evaluations with respect to the nonmandatory 0-1 criteria and the technique dependent criteria. The problem dependent criteria are the only ones for which new evaluations must be performed.

TABLE 7.3. TECHNIQUES VERSUS CRITERIA ARRAY

	Weight	Comp. Prog.	Goal Prog.	Coop. Game	MAUT	SWT	STEP	Dis. Id.	LMAUT
Goal Point	3	1	1	0	0	0	0	1	0
Aspiration Level	.5	0	0	1	1	0	0	0	1
Direct Comparison	.2	0	0	0	0	1	1	0	0
Strongly Efficient	.1	0	0	1	0	1	0	0	0
Computer Time Req.	2	6	7	4	4	4	7	5	1
Interaction Time Req.	1	10	8	10	3	6	5	4	1
Implementation Time	3	9	8	9	5	6	7	3	2
DM's Knowledge Req.	1	4	5	3	2	3	6	4	2
Consistency of Results	1	8	7	9	10	10	8	9	10
Robustness of Results	1.5	8	7	9	8	6	7	10	10
Number of Objectives	1	10	9	7	6	8	8	7	5
Number of Variables	1	10	10	10	10	9	8	9	8
DM's Level of Know.	1.5	10	10	9	6	7	9	10	6
Avail. for Interaction	1	9	9	10	7	8	8	8	7
Desire for Interaction	1.2	9	8	10	7	5	5	5	5
Avail. of Weights	1.2	10	10	10	8	8	8	9	8
Number of Constraints	1	10	10	8	8	8	9	9	7

The L_1-distance metric of compromise programming is evaluated for each of the techniques using the data of Table 7.3. The ideal point vector is

[1,1,1,1,7,10,9,6,10,10,10,10,10,10,10,10,10]

and the range over which each criterion is measured is

[1,1,1,1,10,10,10,10,10,10,10,10,10,10,10,10,10].

The technique which is closest to the ideal solution is compromise programming. The complete ordering and associated distances follow:

(1) Compromise Programming (1.77)

(2) Goal Programming (2.59)

(3) Cooperative Game Theory (5.0)

(4) Displaced Ideal (5.32)

(5) STEP Method (7.04)

(6) Surrogate Worth Trade-off (8.29)

(7) Multiattribute Utility Theory (8.6)

(8) Local Multiattribute Utility Theory (11.54)

Solutions obtained through the application of compromise programming to this problem are given in Kim, Knudsen, and Baafi (1981).

3. Conclusions

An algorithm has been developed which allows a systems analyst, when confronted with a multiobjective decision problem, to be able to select the best multiobjective solution technique for application to that problem. The algorithm is based on the evaluation of the techniques with respect to a set of 27 criteria. To ease the implementation of the algorithm, the criteria are broken down into four groups as follows:

(1) Mandatory 0-1 criteria.

(2) Nonmandatory 0-1 criteria.

(3) Technique dependent criteria.

(4) Problem dependent criteria.

The first three groups can be evaluated without knowledge of the specific problem under consideration. Thus, only the last group need be evaluated each time the algorithm is applied.

These evaluations of the methods are meant to be subjective. It should be emphasized that only techniques with which the analyst is familiar are included. Providing information for him to be able to evaluate other techniques would serve no purpose because he would still be unable to apply those techniques. Further, the evaluations should reflect the degree of familiarity he has with each technique.

The following conclusions can be drawn from this research.

(1) Due to inconsistencies in usage between various well-known authors, the terminology found in the literature of multiobjective decision making is confusing and, at times, misleading.

(2) Most of the available multiobjective techniques have only been applied by the same people who developed them.

(3) Several gaps, most notably in the areas of dynamic and stochastic decision making, exist in the literature of multiobjective decision making.

(4) One of these gaps may be filled by the dynamic compromise programming approach proposed herein which thus appears as an important contribution to the application of multiobjective decision making to dynamic decision problems.

(5) Computational problems still exist, limiting the applicability of dynamic compromise programming.

(6) A set of criteria has been developed through which the multiobjective solution techniques can be evaluated with respect to the problem that must be solved.

(7) An algorithm easily applicable by the systems analyst has been developed to use these criteria to select the best solution technique for that problem.

(8) The model choice algorithm is applicable to both systems analysis (rank the alternative systems) and systems design (mathematical programming) problems, as illustrated by two examples.

In addition to applying this model choice algorithm to the problem of multiobjective model choice, it can be applied to other model choice problems as well. The most notable of these is in the field of statistics where the choice of the "best" regression model for a given set of data must be made. For example, Akaike (1974) and Weber and Monarchi (1978) discuss this problem of alternative statistical models.

As mathematical systems models come into increased usage in the earth sciences, model choice problems arise here as well. Kisiel and Duckstein (1972) discussed this problem in the context of hydrology while Hipel (1981) presents a similar discussion in terms of geophysical models.

4. Recommendations for Further Research

Four areas are identified for further research: dynamic multiobjective problems, stochastic multiobjective problems, application of the model choice algorithm to multiobjective problems, and its application to other fields.

As mentioned in Chapter 4, the use of classical dynamic programming places limitations on the size of the problems that can be solved by the dynamic compromise programming technique. Research is needed in applying differential dynamic programming to multiobjective problems. This would ease, but not eliminate, the problem. In addition, there is room for research leading to other approaches to solving dynamic multiobjective problems as well as defining the efficient solutions.

Any research leading to an approach to solve stochastic multiobjective problems is desirable. This could involve either a new technique or the extension of a presently available technique. Another interesting topic in this area would be to investigate the relationships between multivariate statistical analysis and multiobjective analysis. Possibly, this would lead to a good multiobjective stochastic approach.

The model choice algorithm must be applied to more problems to check how the bias of the analyst effects the choice. This would require that it be applied by a larger number of analysts. It would also be desirable to apply it to cases where a multiobjective problem has already been solved to see, first, if a "better" technique than that used is available, and second, if applying that "better" technique really is easier or provides better results.

Finally, the model choice algorithm should be applied to other model choice problems than the one discussed here. Research should focus on its applicability to other areas and on tailoring it for use to these areas.

APPENDIX A

APPLICATION OF COOPERATIVE GAME THEORY TO A REGIONAL WATER PROBLEM

1. Introduction

In Hungary, large-scale mining development (coal and bauxite) is being planned in the region of the Transdanubian Mountains. However, mineral resources are located under the aquifer (karstic) water level. This karstic aquifer is the source of water supply in the region as both municipal and industrial water demands increase. Any sinkage of the aquifer level is environmentally adverse because the thermal waters of Budapest receive their recharge from the aquifer.

Control of the water supply is expressed in terms of three objectives. First, the cost of controlling the mine water to allow the extraction of the minerals is to be minimized. Second, the water demands in the region must be met at the least possible cost. Third, the aquifer must be maintained at a level such that the natural recharge to the thermal baths continues.

2. Game-theoretical Concept for Solving Multiobjective Problems (Szidarovszky et al. 1978)

Multiobjective programming problems can be formulated as

$$\underline{u} \in U \qquad (1)$$

$$\phi_k(\underline{u}) \rightarrow \max \; (k = 1,2,\ldots,n)$$

where $\underline{u}$ is the decision vector, ϕ_k is the goal function for objective k, and n is the number of objectives.

Let the set $L = \{\phi | \phi = (\phi_1(\underline{u}),\ldots,\phi_n(\underline{u})),\ u \in U\}$ denote the feasible payoff set; then any optimal solution must be a function of L, which can be denoted by $\psi(L)$. A function $\underline{\psi}(L)$ is a generalized Nash solution (Nash 1953) of the problem, if it satisfies the following axioms:

(1) The domain and range of $\underline{\psi}$ are given by

$$D(\underline{\psi}) = \{L | L \subset R^n \text{ convex, closed and bounded}\}$$
$$R(\underline{\psi}) \subset R^n. \tag{2}$$

(2) $\underline{\psi}(L) \in L$ for $\forall L$.

(3) $\underline{\psi}(L)$ is nondominated with respect to L for $\forall L$.

(4) If $(\phi_1,\ldots,\phi_i,\ldots,\phi_j,\ldots,\phi_n) \in L$ if and only if $(\phi_1\ldots,\phi_j\ldots,\phi_i,\ldots,\phi_n) \in L$, then for the vector $\underline{\psi}(L) = (\psi_k)$, we must have $\psi_i = \psi_j$.

(5) Let t be a strictly increasing linear transformation defined on the set of n-tuples,

$$t(\phi_1,\ldots,\phi_n) = (\alpha_1\phi_1+\beta_1,\ldots,\alpha_n\phi_n+\beta_n) \text{ with } \alpha_k > 0 \ (1 \leq \varepsilon \leq n),$$

then

$$\underline{\psi}(t(L)) = t(\underline{\psi}(L)).$$

(6) Let L_1, $L \varepsilon D(\underline{\psi})$ and that $L_1 \subset L$ and for $k = 1,2,\ldots,n$, $\min\{\phi_k | (\phi_1,\ldots,\phi_n) \varepsilon L\} = \min\{\phi_k | (\phi_1,\ldots,\phi_n) \varepsilon L_1\}$, and $\underline{\psi}(L) \varepsilon L_1$, then $\underline{\psi}(L_1) = \underline{\psi}(L)$.

The first axiom defines the set of multiobjective problems discussed. Axioms 2 and 3 imply the feasibility and the nondominated property of the solution. Axiom 4 can be represented in the following way. Assume that the objectives i and j are equivalent in the decision problem; then they should also be equivalent in the solution. Axiom 5 implies that after exchanging the dimensions and units of the objectives, the solution is invariant with respect to strictly increasing linear transformations. The last axiom is illustrated next. Assume that with additional constraints the payoff set is decreased in a special way such that the worst values for each of the objectives and also the multiobjective solution remain in the feasible payoff set. Then the solution of the problem with the restricted payoff set must be the same as that of the original problem.

It can be proven that the generalized Nash solution exists and is unique, and that it can be obtained by the solution of a certain nonlinear programming problem of the form:

$$\underline{\phi} \varepsilon L$$

$$\text{maximize } g = \prod_{i=1}^{n} (\phi_i - \phi_i^*). \qquad (3)$$

The plausibility of axioms 1-6 should be checked prior to applying the method.

3. Regional Multiobjective Model

The model (Duckstein et al. 1979) is described in terms of n water intake points and r natural recharge sites. Of the n intake points, n_1 are mines and n_2 are other intakes.

The decision variables are:

x_i = yield of withdrawal from mine i.

xm_i = yield of inrush allowed mine i.

xd_i = yield inrush allowed into the inner drainage system of mine i.

xg_i = yield of water prevented from entering mine i by sealing or grouting.

d_j = water demands at demand site.

v_{ik} = yield of water conveyed from mine i to recharge point k.

The economic variables are, in annual values:

$C_i(x_i)$ = costs of withdrawal from mine i.

$L_i(xm_i)$ = economic loss due to the occurrence of inrushes xm_i.

$D_i(xd_i)$ = costs of inner drainage.

$G_i(xg_i)$ = sealing cost.

$B_{ik}(v_{ik})$ = conveyance cost.

With this notation, the cost function of a mine is:

$$f_{1i} = C_i(x_i) + L_i(xm_i) + D_i(xd_i) + G_i(xg_i) + \sum_{i=1}^{r} B_{ik}(v_{ik}) \tag{4}$$

The regional mining objective is to minimize $f_1 = f_{1i}$, which is a traditional objective in the industry. Constraints are, for $i = 1,\dots,n$,

$$xm_i + xd_i = x_i$$

$$x_i + xg_i = A_i \quad (5)$$

where A = average yield of inrush into mine i in the absence of input control.

Thus, the mining objective is

$$\text{minimize } f_1 = \sum_i f_i \quad (6)$$

or

$$\text{maximize } -f_1.$$

Regional water management aims at satisfying water demands at the least possible costs. Possible groundwater intakes (including mines) and water requirement sites are considered at grid points over the region. Then the water-supply objective can be expressed as

$$f_2 = \sum_{i=1}^{n} \sum_{j=1}^{m} S_{ij}(y_{ij}) \rightarrow \min \quad (7)$$

or

$$-f_2 \rightarrow \max$$

where

m = the number of water demand grid points

$j = 1,2,\ldots,m$

y_{ij} = decision variable, annual amount of water supplied from mine or other intake i to water demand grid point j

S_{ij} = annual cost of supply including capital, operation, treatment, and conveyance costs.

The following constraints are to be observed:

$$\sum_{i=1}^{n} y_{ij} = d_j \quad \text{for every } j = 1,2,\ldots,m.$$

$$\sum_{j=1}^{m} y_{ij} + \sum_{k=1}^{r} v_{ik} \leq x_i \quad \text{for every mine } i = 1,2,\ldots,n_1. \tag{8}$$

<u>The environmental objective</u> is satisfied if the decisions $\{x_i, y_{ij}, v_{ik}\}$ are realized such that the amount of system outflow q necessary for the recharge of thermal waters is maximized. Thus, the environmental objective can be formulated as

$$\text{maximize } f_3 = q(\underline{x},\underline{y},\underline{v}). \tag{9}$$

However, the recharge may not exceed the present value of q_0 = 30 m^3/min; thus, the following constraint is added:

$$q \leq q_0 \tag{10}$$

To estimate relationship (q), sample sets of values of $\underline{x}$, $\underline{y}$, and $\underline{v}$ are selected at random, and for each set of values, the system model is used to calculate the discharge q. These calculated values have been fitted by least squares to a linear function of the decision variables to yield the function:

$$q = 30.5 - 0.021x_1 - 0.012x_2 - 0.014x_3 - 0.006y_4 - 0.0021y_5 - 0.0042y_6 + 0.07v_1 + 0.14v_2 \tag{11}$$

The standard error of estimate of the fit is 0.15 m^3/min. Using Eq. (11), q can be estimated with reasonable accuracy as a function of withdrawals and artificial recharge.

The game theoretical goal function, g, of the problem is

$$\text{maximize } g = (-f_1)(-f_2)f_3$$

subject to constraints (5), (8), (10), and (11).

As noted at the end of the previous section, plausibility of axioms 1-6 must now be checked. Axioms 1, 2, and 3 essentially consist of a formulation of the multiobjective problem. Axiom 4 means that equal importance is given to the three objectives. Axiom 5 (the solution is invariant in a linear transformation) expresses the possible use of different units in which the objective functions are measured (U.S. dollars or Hungarian Forint, m^3/day or ft^3/min). Axiom 6 may be interpreted as follows: If the decision maker imposes additional constraints on the problem, such as further cost limitations, then any optimal solution that stays feasible with the new constraints remains an optimal solution.

Since the programming problem has linear constraints, and the objective function is quasiconcave, a gradient type iterative process is used for solution.

Results show that mine 1 is partially sealed against a water inrush of 12.4 m^3/min. and mining is conducted under karstic water inflow of 47.6 m^3/min. to be pumped out of the mine. A volume v_{12} = 14.6 m^3/min. is conveyed from mine 1 to recharge site 2. Mines 2 and

3 are protected by inner drainage systems, and the whole amount of karstic water inflow, 150 and 100 m^3/min., respectively, is pumped to the surface.

Regional water supply is based exclusively on pumped water from mines 1, 2, and 3. The game theoretical solution satisfies the objective of environmental protection f_3 = 30 m^3/min., as a result of artificial recharge. These results are presented in Table A.1.

4. Conclusions

Results of this study lead to the following conclusions:

(1) Game theoretical concepts can be used to develop a solution methodology for a certain class of multiobjective problems.

(2) The plausibility of six axioms should be checked before the game theoretical approach is used.

(3) In the case of regional natural resource development where a balanced preference between economic and environmental objectives is sought, as implied by axiom 4, the game theoretical approach seems to be realistic.

(4) It has been proven that under the axioms mentioned, the multiobjective problem can be formulated as a nonlinear programming problem leading to a unique solution.

(5) The development of this approach broadens the kinds of approaches available for solving multiobjective problems.

TABLE A.1. GAME THEORETICAL SOLUTION

i Withdrawal Site	j Requirement Site 1	2	3	4	5	6	7
1	33	0	0	0	0	0	0
2	0	0	25	14	0	83	2
3	0	14	58	0	14	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0

Water Supply, m^3/min.

Lencsehegy 1		Nagyegyháza 2		Mányi 3	
Satorkö	Budai	Satorkö	Budai	Satorkö	Budai
1	2	1	2	1	2
0	14.6	0	0	0	0

Artificial Recharge, m^3/min.

	Lencsehegy 1	Nagyegyháza 2	Mányi 3
Strategy pumping	47.6	150	100
Inner drainage	0	150	100
Sealing	12.4	0	0
Mining under water	47.6	0	0

Mine Water Control, m^3/min.

APPENDIX B

FORMULATION OF RIVER BASIN PLANNING PROBLEM

The model formulation presented below is taken from Gershon et al. (1980).

1. Regional Overview

River basin planning in the Upper Santa Cruz Basin is a complex issue involving urban, agricultural, and mining interests. These users account for 29, 41, and 27 percent, respectively, of total water depletion in 1975. Competition for water is extremely intense, and the basin tends to be one of the most critically overdrafted areas in the state. Roughly 236,000 acre ft. of water were used in 1975 compared with only 74,000 acre ft. of water supplied by imports or natural recharge. This represents a depletion-supply ratio of 3.3 to 1. Agricultural water demands are projected to decline slightly over the next 30 years, but both urban and mining demands are expected to more than offset any savings from agriculture.

2. Objectives of the System

In developing the river basin study within a multiobjective context, the following system objectives are given.

(1) Water Requirements: The region's supply of water (both quality and quantity) should be regulated in order to meet both present

and future demands without seriously affecting the region's present ground water level.

(2) Flood Protection: Flood protection should be provided along the main stem of the Upper Santa Cruz River while maintaining environmental quality.

(3) Enhancement of Environment: The effects on wildlife, vegetation, and historical or anthropological sites should be kept to a minimum.

(4) Utilization of Resources: The physical and socio-economic resources required to implement the alternatives should be kept to a minimum.

(5) Recreation Enhancement: Recreation opportunities should be provided wherever possible along the Santa Cruz.

3. Specifications and Criteria

To be useful to the decision maker (DM), the objectives of the research should be quantified to the fullest extent possible. A method to achieve this quantification is to express each objective in terms of a set of specifications. For example, the specifications for water supply might include aquifer level and water quality. Each specification in turn can be measured according to a set of criteria. One criterion is provided for each specification. For example, the aquifer level might be measured with respect to the net increase or decrease in feet per year. The specifications and criteria are presented in Table B.1.

TABLE B.1. OBJECTIVES, SPECIFICATIONS AND CRITERIA

Objective	Specifications	Criteria
Water Supply	Aquifer Level	Net change in ft./year
	Water Quality--Urban	a,b,c,d,e scale
	Water Quality--Agriculture	a,b,c,d,e scale
Flood Protection	Expected Flood Losses	Expected dollars
	Expected Frequency	Expected number of years between floods (from annual prob.)
Environmental	Preservation of Designated Areas	a,b,c,d,e scale
	Effect on Wildlife and Vegetation	a,b,c,d,e scale
Utilization of Resources	Implementation	Present dollars
	Operation and Maintenance	Present dollars
	Indirect Costs	a,b,c,d,e scale
	Natural Resources	a,b,c,d,e scale
Recreation	Preservation of Existing Facilities	a,b,c,d,e scale
	Creation of New Opportunities	a,b,c,d,e scale

Note: For the subjective (a,b,c,d,e) scale, "a" is the best, and "e" is the worst.

4. Alternative Systems

River basin plans are generally developed for the purpose of reducing the risk of floods or increasing water supplies. Keeping in this spirit, several alternative systems will be evaluated. Although these systems are designed with flood protection and water requirements in mind, they will be evaluated in terms of all objectives. In an MCDM problem, the decision maker must, in some manner, accept trade-offs among the objectives. A number of alternative development strategies have been defined which embody these trade-offs.

Alternative actions for the flood control objective include:

(1) levee construction

(2) channelization

(3) construction of dams and multipurpose reservoirs

(4) flood plain management including floodproofing of existing structures

(5) no action

Alternatives for water supply include:

(1) wastewater reclamation

(2) new groundwater development

(3) the Central Arizona Project

(4) conservation and education program

(5) no action

Since there are five actions under flood control and five for water supply, there are at most 25 different alternative systems to be evaluated, since some of the alternatives obtained from the cartesian product may be infeasible. Table B.2 defines these objectives in tabular form.

5. Alternative Systems Performance Capabilities

The information contained in this section is the basis for the completion of the alternative systems versus criteria array. This array is a display of the performance capabilities of the systems.

Water Supply

Aquifer level. The affect of the water supply alternatives on the aquifer level is calculated in feet decrease per year. The no action alternative represents the worst case where the drawdown rate is approximately 3.5 feet/year. The system which will provide the best results is new groundwater development, where the corresponding figure is estimated at 1.6 feet/year.

Water quality. As the water is pumped from deeper levels, the amount of solids in the water increases, thus reducing the quality of the water. If no action is taken, the quality of the water will deteriorate with time since the aquifer level will drop. Additional pumping will increase the rate of deterioration. However, water pumped for the first time from new areas will be of a high quality.

Flood Protection

Flood losses. The number of structures in the flood plain is given in Table B.3. Table B.4 illustrates total flood losses based upon an average flood loss per structure at $5,000, $15,000, and $20,000

TABLE B.2. ALTERNATIVE SYSTEMS

Alternatives	Levee Construction	Channelization	Reservoirs and Dams	Flood Plain Management	No Action
1. Wastewater Reclamation	1-(1,1)	2-(1,2)	3-(1,3)	4-(1,4)	5-(1,5)
2. Groundwater Development	6-(2,1)	7-(2,2)	8-(2,3)	9-(2,4)	10-(2,5)
3. Central Arizona Project	11-(3,1)	12-(3,2)	13-(3,3)	14-(3,4)	15-(3,5)
4. Conservation and Education	16-(4,1)	17-(4,2)	18-(4,3)	19-(4,4)	20-(4,5)
5. No action	21-(5,1)	22-(5,2)	23-(5,3)	24-(5,4)	25-(5,5)

TABLE B.3. FLOOD PLAIN AFFECTED AREAS

Area	Structures	Residents
Green Valley	60	120
Marana	20	50
Canada del Oro	1,000	4,000
Rillito	1,500	6,000
Tanque Verde	70	180
Aqua Caliente	130	400
Pantano	130	650
Rodeo	520	2,100
Airport	800	3,500

TABLE B.4. FLOOD LOSSES

Area	25	50	100
Santa Cruz	.5	10.3	12.4
Tributaries	21	63	84

(in millions)

for the 25, 50, and 100 year flood, respectively, for the affected areas given in Table B.3. In addition, agricultural losses along the Santa Cruz based upon actual figures of the 1977 year flood are included.

Expected frequency of floods. Since operation of this system is not considered at this stage, none of the alternative actions suggested for the water supply objective will have an effect on this criterion. The expected frequency of floods is just a function of which flood control action is selected.

Environment

For both criteria under the environmental objective (preservation of designated areas and effect on native wildlife and vegetation), subjective ratings are assigned as follows:

Criteria for Environmental Objective	
Preservation of designated areas of value	a,b,c,d,e
Effect on native wildlife and vegetation	a,b,c,d,e

Utilization of Resources

Implementation and operation and maintenance costs. Cost estimates for the flood control actions were obtained from discussions with the Army Corps of Engineers (Tucson urban study group). Costs for the water supply alternatives were obtained from discussions with members of the Tucson City Planning Department. These costs are given as gross figures and will be combined and converted into present values in the systems versus criteria array. Costs are given in Table B.5.

TABLE B.5. COSTS OF ALTERNATIVES

System	Capital Costs	Operation and Maintenance
Wastewater	.18	2.7
Groundwater Development	20	.2
Central Arizona Project	15.6	.16
Conservation and Education	.05	.05
Levee Construction	12.5	.06
Channelization	16.6	.08
Dams, Reservoirs	12.1	.12
Flood Plain Management	1.8	.02

Note: Costs are in millions of dollars.

Indirect costs and natural resources. Subjective evaluations are again developed for each of the alternative actions and combined to evaluate the alternative systems for the systems versus criteria array.

Recreation

Subjective evaluations are performed as described previously for the preservation of existing facilities and creation of new facilities criteria.

6. Systems Versus Criteria Array

Each system has been evaluated with respect to each of the criteria. These evaluations are incorporated into a systems versus criteria array in Table B.6.

TABLE B.6. SYSTEMS VERSUS CRITERIA ARRAY

OBJECTIVE	CRITERIA	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
WATER SUPPLY	Aquifer Level	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	2.5
	Water Quality Urban	e	a	d	a	b	e	a	d	a	b	e	a	d	a	b	e	a	d	a	b	e	a	d	a	b
	Water Quality Agric.	a	b	b	b	b	a	b	b	b	b	a	b	b	b	b	a	b	b	b	b	a	b	b	b	b
FLOOD PROTECTION	Expect. Flood Losses	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33
	Expect. Freq.	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04
ENVIRONMENT	Pres. Desig. Areas	d	d	c	a	a	d	d	c	a	a	d	d	c	a	a	d	o	c	a	a	d	d	c	a	a
	Effect on Wild. Veg.	c	b	d	a	a	d	c	e	c	c	c	c	d	d	b	c	c	d	b	b	c	c	d	b	b
UTILIZATION OF RESOURCE	Implem. Costs	12.7	16.8	12.3	1.9	0.2	32.5	36.6	32.1	21.8	20	28.1	32.2	27.6	17.3	15.6	12.6	16.7	12.2	1.8	.01	12.5	16.6	12.1	1.8	0
	O & M Costs	37.6	37.8	38.2	37.2	37.0	2.6	2.8	3.2	2.2	2.0	2.2	2.4	2.8	1.8	1.6	1.1	1.3	1.7	.6	.5	.6	.8	1.2	.2	0
	Indirect Costs	c	c	b	d	c	d	d	d	e	e	b	b	a	c	b	c	c	b	d	d	c	c	c	e	d
RECREATION	Natural Resource	c	c	c	b	a	d	d	d	c	c	e	e	a	d	c	c	b	e	a	a	c	b	c	b	a
	Pres. of Exist. Fac.	c	b	c	b	b	c	b	c	a	a	c	b	c	b	b	c	b	c	e	e	c	b	c	b	b
	Creation New Oppr.	b	b	a	a	c	d	d	b	c	e	d	d	b	c	e	d	d	b	c	e	d	d	b	c	e

APPENDIX C

BLENDING OF COAL AT SURFACE STORAGE BINS

This problem formulation is taken from Kim et al. (1981).

1. Introduction

Low-grade coal is usually cleaned by a coal cleaning plant (CCP) to remove some of the unwanted ash and pyritic sulfur content. An immediate problem which some CCP operators may have to solve in the day-to-day operations of the plant is how much coal should be cleaned from various stockpiled coals feeding the plant. The specific purpose here is to combine stockpiled coals to feed a CCP with the objective of minimizing sulfur content of clean coal per million BTU.

In order to decide on the optimum stockpile combination which will yield minimum sulfur per million BTU of clean coal, for example, one should have the capability to predict clean coal product at any desired raw ash level. The quality of clean coal product, in general, depends on the raw coal characteristics and both the type and mode of operation of the cleaning unit. A common approach usually taken is to develop empirical models which can predict from a wide range of raw coal qualities.

Using deterministic models of similar empirical nature, an optimization model is developed to yield minimum total sulfur and maximum total BTU, i.e., minimum sulfur per clean coal million BTU.

Both sulfur and BTU contents of coal are given top priorities in the modeling. This is because BTU content relates to the heat balance in burning the coal while the sulfur content produces corrosive combustion products which pollute the air.

2. Deterministic Models of Coal Cleaning Plant Performance

BTU content and sulfur content of clean coal are computed from raw coal ash and sulfur sample values of the stockpiled coal using deterministic models of CCP performance. This information is then used to compute the "optimum" stockpile combination which gives the minimum total sulfur and maximum total BTU. The raw coal sulfur and ash sample values are assumed to be representative of the amount of coal which is cleaned during a convenient time interval, e.g., an hour interval.

Three basic deterministic models are used to compute the end product of the coal cleaning plant from raw coal ash and sulfur sample values.

For the ith stockpiled coal, if the representative ash and sulfur sample values are A_i (%) and S_i (%), respectively, then the calorific value (BTU/lb.) of the raw coal, B_i, is given by equation (1).

$$B_i = 15769 - 175.1A_i \qquad (1)$$

The constants given in equation (1) may vary slightly from one coal deposit to the other.

Pilot plant studies of coal cleaning plants owned by the Homer City Owners, Homer City, Pennsylvania, also suggest that the following models hold:

For the number 3 coal cleaning plant,

$$R_3 = 87.72 + 34.34\ln[2.84/A_i] \tag{2}$$

and

$$SB_3 = 0.86 + 0.28\ln[2.84/(A_i/S_i)] \tag{3}$$

where R_3 is the predicted percent BTU recovery of the CCP and SB_3 is the predicted pounds of sulfur per million BTU of cleaned coal. Similarly, for numbers 1 and 2 coal cleaning plants,

$$R_{1-2} = 94.95 - R_3 \tag{4}$$

$$SB_{1-2} = 0.38 + 1.63\ln[17.75/(A_i/S_i] \tag{5}$$

where definitions of R_{1-2}, SB_{1-2} are the same as above with subscript (1-2) referring to number 1 and 2 CCP. In practice, the constant terms in equations (2) through (5) above will be dictated by both the type and the performance of the coal cleaning plant.

3. Compromise Programming Formulation of Blending Problem

In arriving at the "optimum" solution, the program recognizes two conflicting variables which dictate pounds of sulfur per million BTU. Minimum pounds of sulfur per million BTU can be guaranteed when the sulfur content of clean coal is minimized and BTU content is maximized. Since these cannot be done simultaneously, the model reconciles the two objectives in the best manner so as to arrive at a compromise solution.

Let X_i represent the tonnage of coal cleaned from the ith stockpile of coal during a time interval, e.g., one hour. Mathematically, the objective function to maximize total BTU content of clean coal is given by equation (6).

$$\text{Maximize} \sum_{i=1}^{M} H_i X_i \tag{6}$$

where H_i is BTU content of a ton of cleaned coal from ith stockpiled coal, and M is the total number of stockpiles.

Also, the objective function which minimizes total sulfur content of clean coal is given by equation (7).

$$\text{Minimize} \sum_{i=1}^{M} H_i SB_i X_i \tag{7}$$

where SB_i is the predicted pound of sulfur per million BTU of cleaned coal from ith stockpiled coal. The constant terms, H_i and SB_i, in equations (6) and (7) can be computed from equations (1) through (5).

There are other practical constraints which must be met in operating a coal cleaning plant. First, the nominal plant capacity must be met.

$$C_{min} \leq \sum_{i=1}^{M} X_i \leq C_{max} \tag{8}$$

where C_{min} and C_{max} are, respectively, minimum and maximum capacities of the cleaning plant. Also, coal should not be stockpiled and unused for a long period of time so as to prevent a possible oxidation and spontaneous combustion.

$$F_{min_i} \leq X_i \leq F_{max_i}, \quad i = 1,2,\ldots,M \qquad (9)$$

where F_{min_i} and F_{max_i} are the minimum and maximum feed, respectively, from the ith stockpiled coal.

In dealing with the multiple objectives given by equations (6) and (7) above, compromise programming is used to obtain the solution which is the "best" for the two objectives under the given constraints. The term "best" is measured in terms of being closest to an ideal solution.

4. Hypothetical Example of Stockpiled Coal Blending

The use of this approach can best be illustrated by the following simple problem. It is required to determine the tonnages of coal to be cleaned from each of the five stockpiles for the next one hour. Additional restrictions considered are listed below.

(i) Minimum of five tons to be cleaned from each of the five stockpiles.

(ii) Not more than eleven tons of coal must be supplied from each stockpile.

(iii) The coal cleaning plant capacity is 40-50 tons per hour.

(iv) Representative sample assay values of raw coal ash and sulfur for the next one hour are as listed below:

	Stockpile Number 1	2	3	4	5
Ash (%)	14.4	15.4	22.0	17.1	18.0
Sulfur (%)	5.2	4.1	2.9	2.3	2.8

(v) 80% weight is assigned to minimization of sulfur while 20% weight is assigned to maximization of BTU content of clean coal.

The complete model formulation is:

$$\text{maximize} \sum_{i=1}^{M} H_i X_i$$

$$\text{minimize} \sum_{i=1}^{M} H_i SB_i X_i$$

subject to:

$$\sum_{i=1}^{M} X_i \geq C_{min}$$

$$\sum_{i=1}^{M} X_i \leq C_{max}$$

$$X_i \geq F_{min_i} \quad i = 1,\ldots,M$$

$$X_i \leq F_{max_i} \quad i = 1,\ldots,M$$

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