THE EFFECTS OF TAPE DROPOUTS ON PULSE COMPRESSION RECORDING

W. N. Waggener
Sangamo Weston, Inc.
Data Systems Division
P.O. Box 3041
Sarasota, FL 33578

ABSTRACT

It is widely recognized that tape dropouts are the major impediment to error-free recording in high density digital recorders (HDDR). Conceptually, the effects of tape dropouts on error performance can be combatted by error correcting codes, signal design or a combination of the two. In this paper the effect of tape dropouts on wide time-bandwidth signals is considered. Wide time-bandwidth signaling techniques, commonly refered to as pulse compression, would appear to be capable of combatting the effects of short tape dropouts. Although the wide time-bandwidth signals are, to a certain degree, immune to short signal dropouts, an excessive performance penalty is paid when dropouts exceed about 10% of the signal duration. The effects of tape dropouts are shown to effectively reduce noise margin by decreasing the signal detection filter output and by introducing intersymbol interference through increased sidelobe levels.

INTRODUCTION

A major goal in the design of high density digital tape recorders is the minimization of the numbers of bit errors. Typical high density digital instrumentation recorders are currently capable of achieving bit error rates in the order of 10^{-6} to 10^{-7} at record densities of 25 to 40 kilobits per inch. When an analysis is made of the instrumentation tape recorder signal-to-noise ratio it is found that the actual bit error rate is typically much higher than the theoretical rate for the average signal-to-noise ratio. The discrepancy between the experienced error rate and theoretical error rate is explained by the occurence of signal dropouts which cause bursts of bit errors.

Various means for combating burst errors have been proposed. One method of combating burst errors is to provide a powerful burst error correcting code. Various types of codes can be used to specifically combat errors and very recently an integrated circuit has been announced (1) using a Fire code to correct burst errors for magnetic disk recording applications. A second method of combating burst errors which has been proposed uses wide time-bandwidth signals which are relatively immune to short dropouts. This paper specifically examines the effects of tape dropouts on wide time-bandwidth signals.

SIGNAL DESIGN TO COMBAT TAPE DROPOUTS

Both radar and communication applications for wide time-bandwidth signals for pulse compression systems has led to extensive research into the design of these types of signals. No attempt will be made in this paper to review this literature and the interested reader is directed toward the references (3, 4) provided at the end of this report for more specific information. In general, however, the signal design problem concentrates on the design of two classes of signals; (1) baseband signals, and (2) carrier signals. Baseband signals are considered to be pulse sequences whose frequency spectrum extends from roughly DC up to a bandwidth of roughly N times the reciprocal of the signal duration. Carrier signal designs are, on the other hand, based upon the use of modulated carriers whose frequency spectrum is concentrated around some nominal center frequency. Although both types of signals are applicable to the magnetic recorder problem the results presented here focus on baseband signals.

For a magnetic tape recorder the use of baseband signals is particularly desirable since the equalized response of the recorder channel extends nearly to DC and optimum utilization of the recorder frequency response is desired. The goal of the signal design problem for wide time-bandwidth product signals is to create a signal waveform such that when it is passed through its matched filter the output is ideally an impulse. Thus with the optimum receiver, as far as the outside world is concerned, the intermediate communication path is no different than if a single pulse of shorter duration had been transmitted. The goal of the signal design problem is, therefore, to find sequences of signals which have this property[*].

With a high density data recorder, the search for signals rather naturally begins with the search for binary sequences having the impulse equivalent property. Barker (6) was one of the first to investigate binary sequences which have approximately impulse equivalent properties. The output of the matched filter for the Barker code is of the form shown in Figure 1.The Barker codes have the property that the matched filter output has a peak response equal to the length of the codes while maintaining sidelobes which are less than or equal to unity. Barker developed codes which satisfy this property for lengths up to 13. Unfortunately, no Barker codes for lengths greater than 13 have yet been found. It has been proved that no odd Barker codes greater than 13 exist and no even sequences between 4 and 6,084 have been found. While the Barker sequence approximates the impulse equivalent property, the finite sidelobes will interfere, with adjacent signals to cause a form of inter-symbol interference which will ultimately degrade performance.

[*] Huffman (5) has termed sequences with this property, “Impulse Equivalent Sequences”.

Signal designs of baseband signals can be greatly improved if the constraint to using binary level signals is removed. Huffman (8) has shown that there is a class of signals which can be designed which closely approximate the ideal impulse equivalent sequence. The Huffman designed sequences have the property that the sidelobes are all zero except for relatively small sidelobes at each end of the impulse response as illustrated in Figure 1. The Huffman sequences are generated and the matched filters implemented using finite impulse response filters. The Huffman sequences are designed by selecting the zeros of a finite impulse response filter from one or the other of a constant radius circle in the Z-plane. Ackroyd (7, 8) has described a means for selecting the zeros so as to minimize the peak to RMS waveform ratio thereby improving the overall performance of the Huffman sequences.

Signal Processing Implementation

A block diagram of the implementation of a baseband wide time-bandwidth product signaling system for tape recorder applications is shown in Figure 2. The input digital data is fed to the signal waveform generator which generates the wide time-bandwidth product signal. Even though the basic signal may have only binary levels, symbols from suceeding data bits are overlapped such that the output of the waveform generator is a composite analog signal. Thus this signal must be recorded in the direct record mode on the analog tape recorder. On the reproduce side of the tape recorder, after equalization, the composite signal must be passed through a matched filter to recover the digital data. The complexity of both the signal waveform generator and the matched filter depends upon the types of waveforms utilized. The baseband waveforms are the simplest to both generate the signal and realize the matched filter.

The signal waveform generator for baseband signals such as the Barker sequence or the Huffman sequences can be implemented using a digital shift register with weighting resistors at the output of each stage. The matched filter for the baseband signals is implemented using a transversal filter. The charge coupled device (CCD) is ideally suited as the matched filter for the baseband sequences.

PERFORMANCE ANALYSIS

The analysis of the performance of wide time-bandwidth signaling techniques must consider two cases. The first case is the performance in the absence of signal dropouts and considers only the effect of white, gaussian noise. The second case considers the peformance in the presence of signal dropouts such as those encountered in the tape recorder. Ultimately the measure of performance desired is an assessment of bit error probability both with and without dropouts. Two generic types of signals have been considered for this analysis: (1) The Barker sequences and, (2) Huffman sequences of lengths 16, 32 and 128.

It is well known that the signal-to-noise ratio at the output of a matched filter is dependent only on the ratio of the signal energy to the noise spectral density. With NRZ PCM signals the average bit error probability is given by:

$$P_e = \frac{1}{2} \cdot \mathrm{Erfc} \sqrt{\frac{E_s}{N_o}} \tag{1}$$

E_s = signal energy

N_o = one sided noise spectral density

Of all possible waveforms the NRZ PCM signal has the maximum energy for a given constraint on peak amplitude. Consequently, the performance of NRZ PCM forms a lower bound to the best achievable bit error probability in white, gaussian noise. To compare a wide time-bandwidth signal to NRZ PCM, consider the case of a baseband signal such as the Barker sequence which consists of only binary levels. Since the composite waveform for the wide time-bandwidth signal consists of a number of overlapping symbol waveforms the average power of the composite waveform is N times the power of a given symbol waveform where N is the length of the waveform in bit periods. Thus to accomodate the composite signal in the same dynamic range as an NRZ PCM signal requires that individual symbol waveforms be reduced in power by a factor of approximatley N. In reproducing the composite waveform the matched filter output is proportional to the energy of the transmitted signal and, consequently, the signal-to-noise ratio for the coded sequence can equal that of the NRZ PCM signal and can, therefore, achieve the same average bit error probability.

This assumes, however, that an impulse equivalent coded sequence can be designed which has zero response except at the correlation peak. As indicated previously none of the wide time-bandwidth product signals have this property. The Huffman sequences approach this condition but still exhibit finite level sidelobes at the extreme ends of the impulse response. The effects of finite side lobe levels on the bit error probability can be numerically computed by treating these levels as sources of intersymbol interference. The existence of finite level sidelobes guarantees that the wide time-bandwidth signals will have a poorer bit error probablilty than the ideal N RZ PCM signal . This is in the absence of any further distortions in the recording process. The effects of imperfect channel equalization will, in turn, degrade the performance of both NRZ PCMsignaling and wide time-bandwidth response signaling.

The main thrust of this paper is to examine the effect of signal dropouts on the performance of wide time-bandwidth signals. Two types of signal dropouts or fading models are considered as illustrated in Figure 3. The first fading model is termed the “cookie cutter” model in which the signal level is assumed to abruptly drop to zero for the duration of the fade. The fade is described by the starting time of the fade and the fade duration. The second fading model which is probably a reasonable good approximation to the typical tape recorder dropout is a gaussian fade in which the signal level drops according to a gaussian function. For this fade model the fade is described by the time of occurrence of the center of the fade and the RMS duration of the fade. The depth of the fade can further be described by the amplitude of the gaussian function.

There are some intuitive ideas which can provide insight into the effects of these types of dropouts on performance. The matched filter output at the optimum sampling time is proportional to the energy of the transmitted signal. If a dropout occurs during the transmission of this signal, one would expect that the matched filter output amplitude at the sampling time would be reduced in approximate proportion to the reduction in the received signal energy. Thus, one effect of the dropout will be to reduce the noise margin at the optimum sampling time. Noise margin will be reduced in more or less the same proportion as the reduction in apparent energy of the symbol. A dropout will also cause a mismatch between the received signal and the matched filter impulse response which will not only reduce the peak amplitude at the output of the matched filter but will tend to cause spurious outputs at other sampling times. In other words, the sidelobe levels will be apparently increased.

For the case of the Huffman types of signals, Dent and Schneider (2) have given an approximate means for calculating error probability. The analysis of Dent and Schneider assumes that a temporary signal loss of length M bits may be viewed as an unwanted burst of additive gaussian noise whose values are of equal magnitude but opposite sign to the data which was lost. Based upon this type of analysis Dent-Schneider compute the probability of bit error in terms of the length of the signal and the dropout period as:

$$P_e \simeq \frac{1}{2} \cdot \mathrm{Erfc} \sqrt{\frac{N}{2\ (M-1)}} \tag{2}$$

N = length of signal

M = length of dropout

The results of Dent and Schneider can be extended to the case in which additive gaussian noise is also present in which case the average bit error probability as a function of the dropout length, the sequence length, and the signal-to-noise ratio is given by:

$$P_e \simeq \frac{1}{2} \cdot \mathrm{Erfc} \sqrt{\frac{1}{\frac{(M-1)}{N} + \left(\frac{N_0}{E_s}\right)}} \qquad (3)$$

Figure 4 plots the bit error probability for the case of additive gaussian noise in addition to a dropout with the relative dropout duration (M-1)/N, as a parameter. Even when the sequence length is 20 times the dropout duration a signal-to-noise degradation of greater than 3 dB is incurred. In any case the sequence length must be typically in excess of 10 times the dropout length in order that an excessive error rate is not incurred.

Baseband Signal Analysis

Two types of baseband signals have been analyzed. The first signal is the longest known Barker sequence of 13 bits. Secondly, three Huffman sequences have been designed and analyzed. The three sequences chosen were a 16 bit sequence, 32 bit sequence, and a 128 bit sequence. The sequences were designed using the method of Ackroyd and real sequences were obtained by using only upper half plane transmission zeros together with their complex conjugates.

The effects of tape dropouts were simulated for each signal using a computer model. First, the system was simulated with no noise and no signal dropouts. Next, “cookie cutter” and gaussian signal dropouts were introduced and the system analyzed without random noise. Finally, random noise was added and the bit error probability measured for a variety of signal dropout types and durations. Prior to running the simulation the model was validated by simulating the performance of an NRZ PCM signal and comparing the simulated results with the theoretical performance.

Numerous combinations of dropout type, duration and time of occurance were simulated for the Barker and Huffman sequences. The results presented here are typical of most of the simulations. The 16 bit Huffman sequence and the matched filter output in the absence of noise and dropouts is shown in Figure 5. Figure 6 shows the effects of a two (2) bit dropout on the matched filter output for both a “cookie cutter” and a gaussian fade. As anticipated, the peak response is decreased and the level of sidelobes increased thereby decreasing the effective noise margin. Similar results are shown in Figures 7 and 8 for a 32 bit Huffman sequence with a five (5) bit dropout.

Using a Monte Carlo simulation the bit error performance for the pulse compression technique was estimated. Figures 9 and 10 compare the simulated bit error performance with the performance predicted by the approximation of equation (3). In general, the simulations for a wide variety of conditions are consistent with the predicted effect of dropouts.

Excessive bit error rates are incurred whenever the dropout duration exceeds 10% to 20% of the signal duration. Thus to combat the dropouts typical of an instrumentation recorder, signal sequences of hundreds to thousands of bits are required.

CONCLUSIONS

Although there are many parameters which can be varied in the analysis of the wide time-bandwidth product signals, the cursory investigation presented here indicates several trends which are indicative of general principles. First, relatively long sequences are required to closely approximate the ideal impulse equivalent response. Short sequences tend to have sidelobe levels which create a substantial amount of intersymbol interference even in the absence of fading or signal dropouts. Secondly, the performance of the wide time-bandwidth product signals have been compared with NRZ PCM signaling on the basis of equal signal powers. This is an optimistic assumption and does not include consideration of the effects of recorder dynamic range or the difference between saturation and linear recording on overall performance. Consequently, even the most optimistic assumptions probably indicate that wide time-bandwidth signaling will incur several dB performance penalty in the absence of dropouts even with nearly perfect channel equalization. Although the wide time-bandwidth product signals are, to a cerain degree, immune to short signal dropouts, an excessive performance penalty is paid when the drop-outs exceed about 10% of the signal length. If typical tape recorder drop-outs have durations from 30 to 300 but periods, exceptionally long signals are required to effectively combat the signal dropouts. Even within this constraint a significant signal-to-noise penalty is paid in the presence of random noise.

The implementation of the wide time-bandwidth signaling technique is also complex. In addition to the necessity for designing relatively long transversal filters, the receiver must also contain a good automatic level control circuit and a good symbol synchronization circuit in order to provide optimum bit decisions. At the present time the CCD probably represents the best technology to implement the matched filter. Unfortunately, current commercial devices are limited in clock rates to about 5 Mbps and are relatively expensive.

While the analysis and simulations do show that this technique can provide some protection against relatively short signal dropouts, in general the technique would appear to be very cumbersome and expensive to implement when attempting to cope with long dropouts. There are other techniques using error correcting coding which appear to have a great deal more potential and are considerably easier to implement.

REFERENCES

(1) AMZ 8065, AMD Inc.

(2) Dent, R. F. and Schneider, R. C., “Pulse Compression Recording”, IEEE Trans. on Magnetics, Vol. MAG-12, No. 6, November 1976.

(3) Fowle, E. N., “The Design of Radar Signals”, The Mitre Corp. Report SR-98, November 1963.

(4) Van Trees, H., <u>“Detection, Estimation and Modulation Theory”</u>, Vol. II, J. Wiley, Inc., 1968.

(5) Huffman, D.A., “The Generation of Inpulse-Equivalont Pulse Trains”, IRE Trans. Info Theory, September 1962.

(6) Barker, R. H., Group Synchronizing of Binary Digital Systems”, <u>Communication Theory</u>, London.

(7) Ackroyd, M. H., “The Design of Huffman Sequences”, IEEE Trans. Aeorspace and Elect. Sys., November 1970.

(8) ibid, “Synthesis of Efficient Huffman Sequences”, IEEE Trans. Aerospace and Elect. Sys., January 1972.

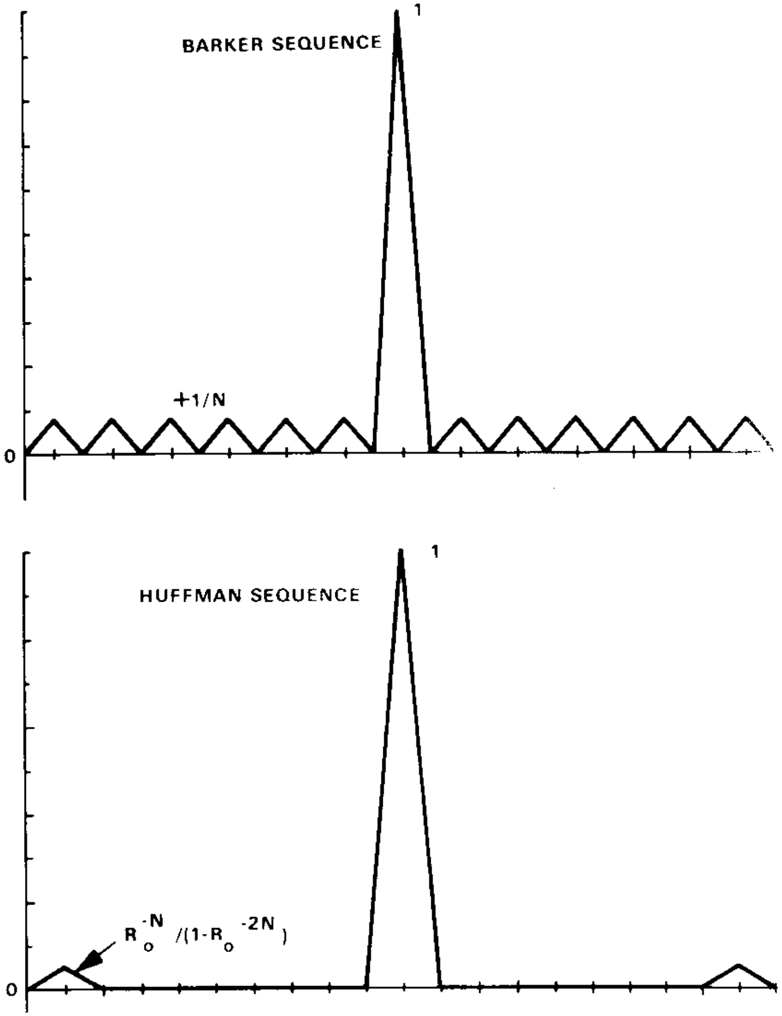
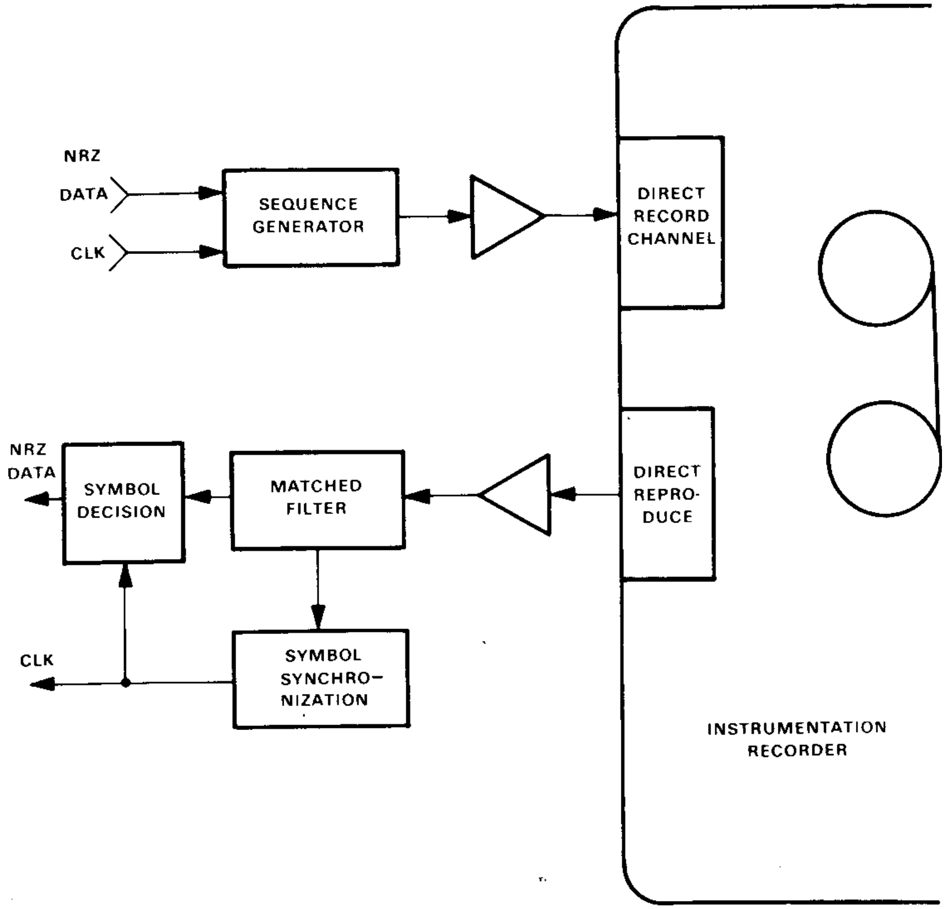
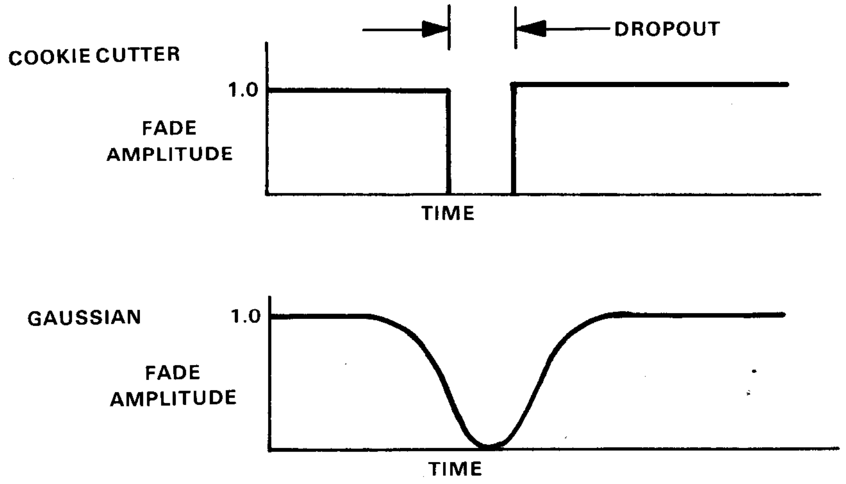


FIGURE 1
MATCHED FILTER OUTPUTS FOR
BARKER AND HUFFMAN SEQUENCES



WIDE TIME-BANDWIDTH
RECORDING SYSTEM
FIGURE 2



$$A(t) = 1 - \frac{\mathrm{EXP}\left(\frac{(t-t_o)^2}{2\sigma^2}\right)}{\sqrt{2\pi}\ \sigma}$$

FIGURE 3
DROPOUT MODELS

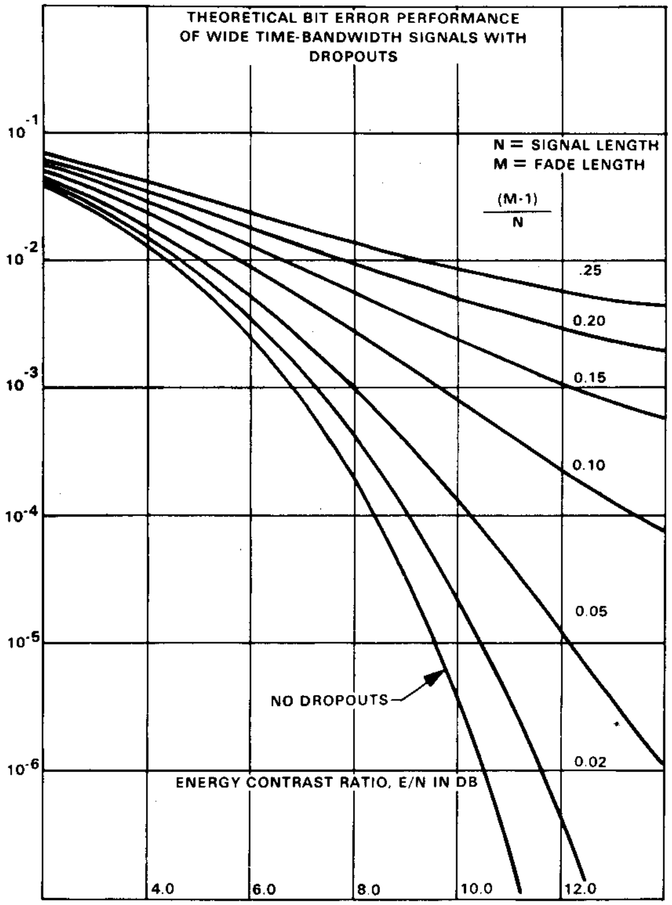


FIGURE 4

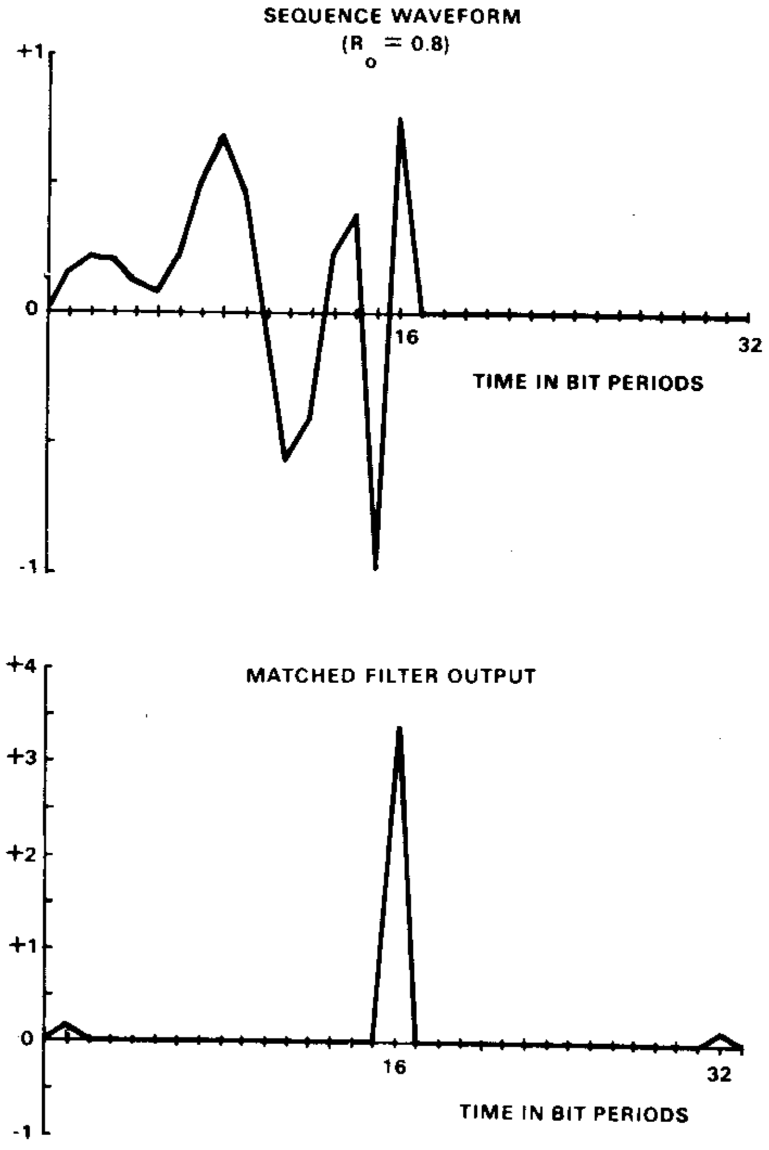


FIGURE 5
16 BIT HUFFMAN SEQUENCE

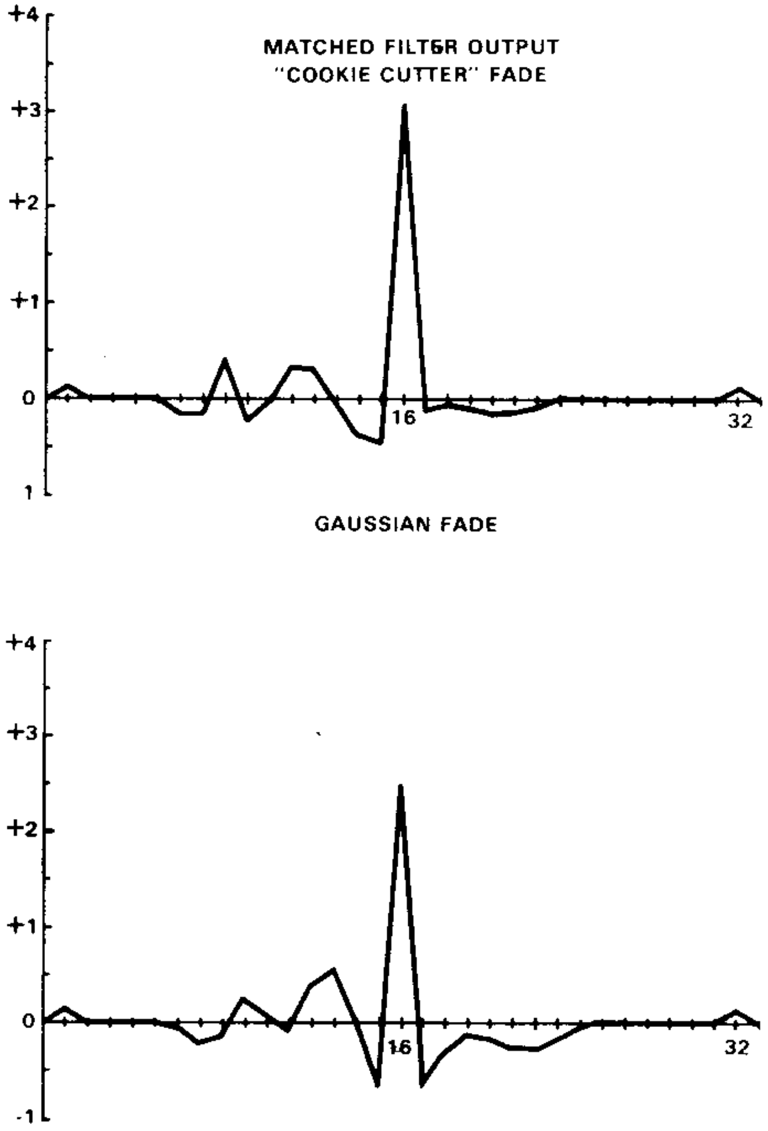


FIGURE 6
EFFECT OF 2 BIT DURATION
DROPOUT ON 16 BIT HUFFMAN SEQUENCE

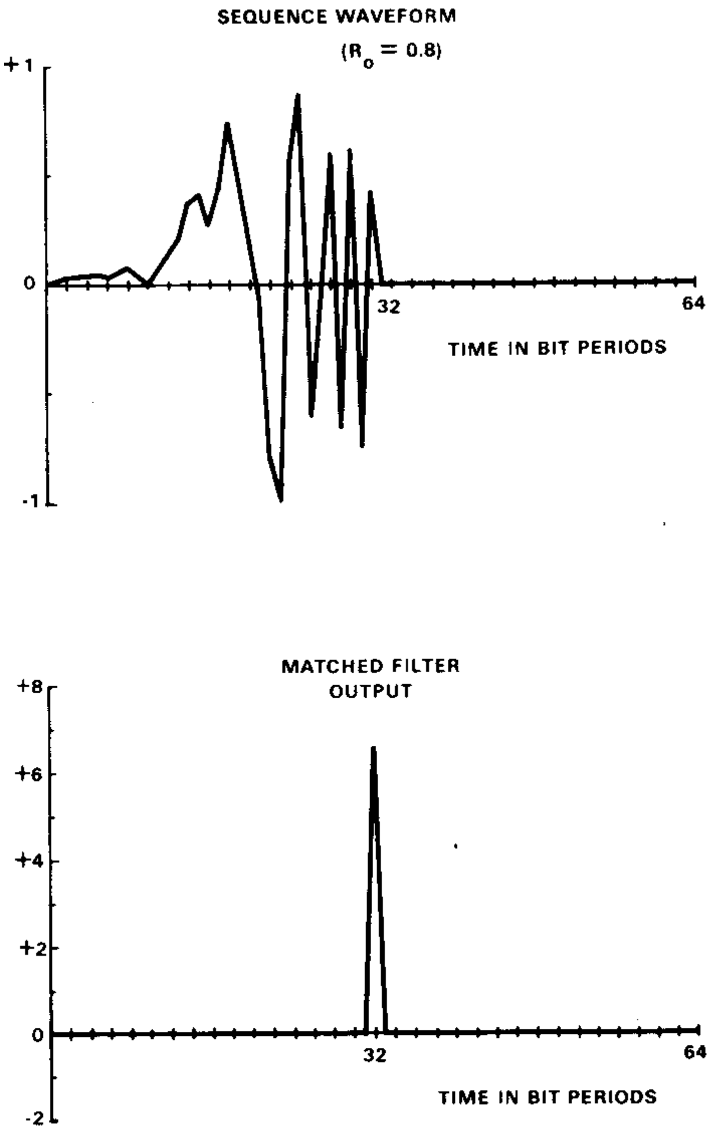


FIGURE 7
32 BIT HUFFMAN SEQUENCE

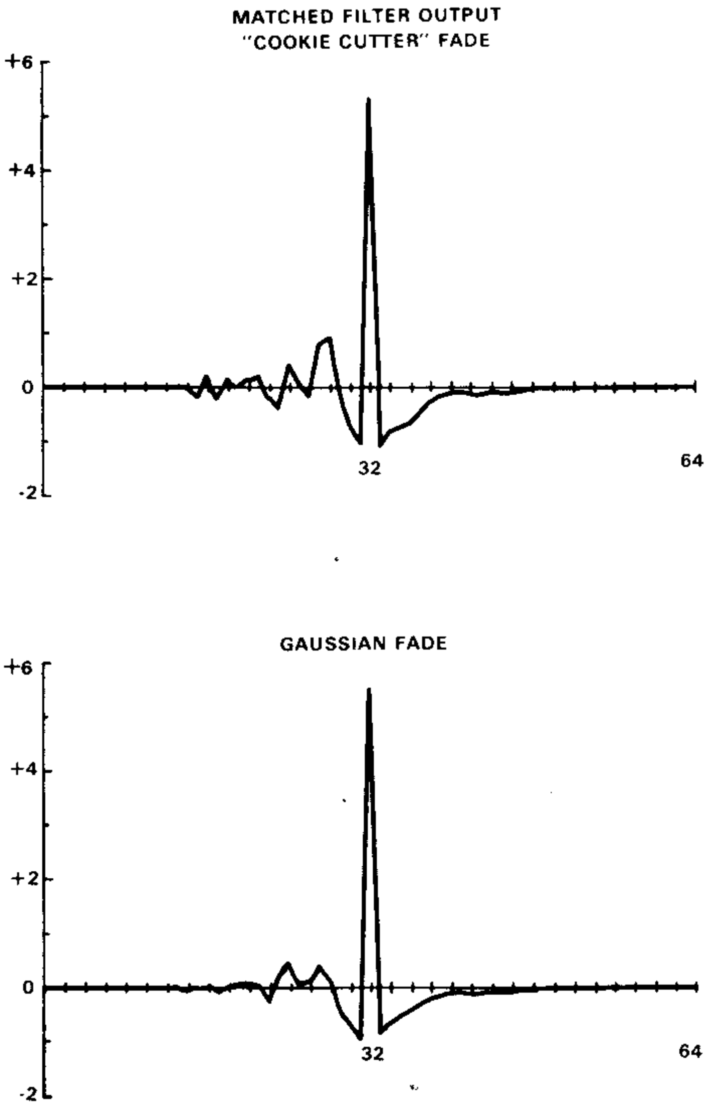


FIGURE 8
EFFECT ON 5 BIT DURATION
DROPOUT ON 32 SIT HUFFMAN SEQUENCE

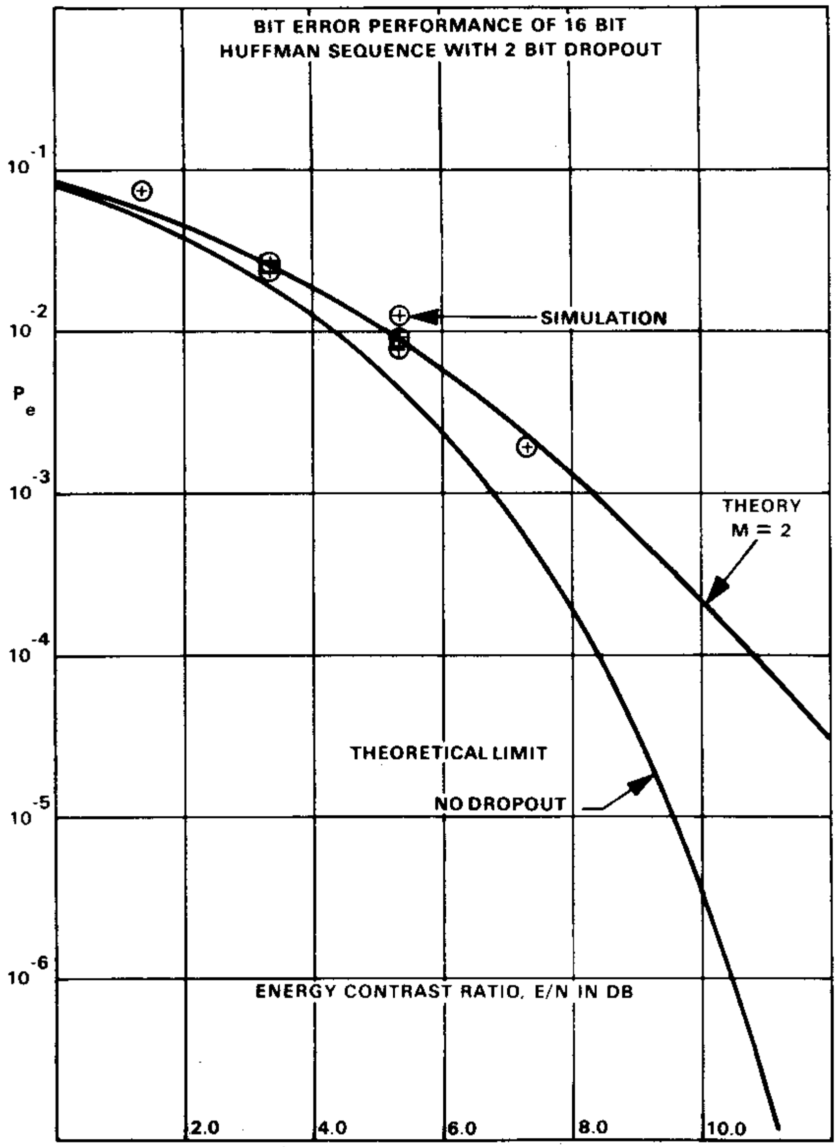


FIGURE 9

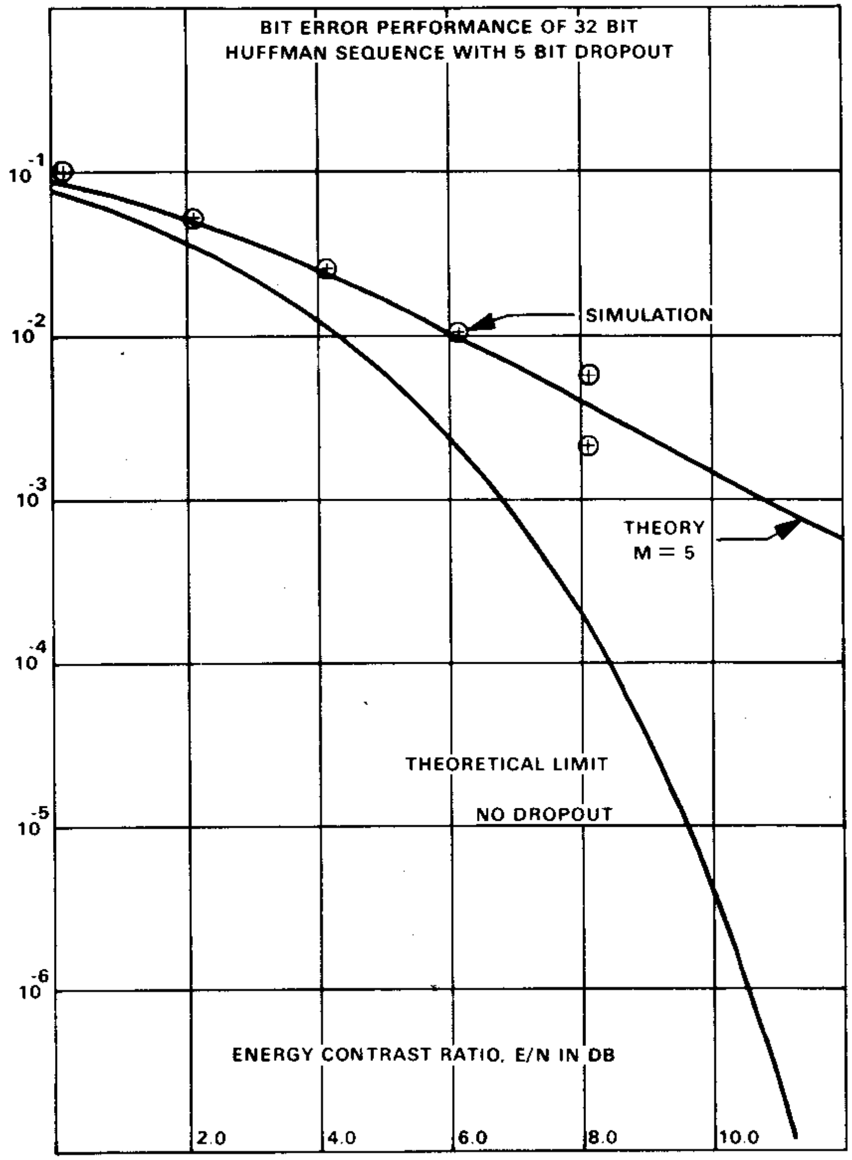


FIGURE 10