ON THE CHARACTERIZATION OF NOISE IN THE SIMULATION OF NONLINEAR WIDEBAND SATELLITE DIGITAL LINKS

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ABSTRACT

The prediction of performance (BER) for a digital satellite link possessing typical impairments is a difficult analytical undertaking. When, further, the repeater is nonlinear and the uplink noise is nonnegligible, the difficulty of an analytical approach is greatly compounded. Consequently simulation is becoming increasingly common as a tool for solving the BER-prediction problem. It is often assumed, however, that a link is characterized by a single Gaussian noise source at the receiver. This assumption can lead to considerable error in the weak uplink nonlinear system. For the latter situation we consider in this paper the problem of characterizing an equivalent single noise source at the receiver, which will better account for actual system behavior than the Gaussian source. For this equivalent source we postulate that the pdf is drawn from a family of the form const x exp $[- | x |^{\nu}]$ and obtain through simulation the BER performance as a function of ν. Comparing these results with Monte Carlo simulation runs shows the possibility of approximately characterizing the complex system's behavior by a single non-Gaussian noise source.

1.0 INTRODUCTION

The work reported here originated in the search for means to reduce computer time on Monte Carlo simulations of satellite nonlinear digital links. The simulation of such links has typically been done in either of two ways (1): (a) an "analytical" approach in which all of the noise is presumed to arise at the input to the earth-terminal receiver and, consequently, can be assumed to be Gaussian, and (b) a Monte Carlo approach. When the uplink noise is not negligible the analytical approach can lead to considerable error, and a requirement for accuracy would seem to dictate the Monte Carlo approach.[*] The major drawback of the latter, of course, is the use of relatively large amounts of computer time. The analytical method is very fast because it is not simulation in the pure sense, but a computation based on an a priori Gaussian assumption. But it leads one to ask whether another kind of (simple) assumption can be made, which will permit something like the speed of analytical simulation but yield an accuracy nearer to that of Monte Carlo simulation. While this question has a number of ramifications, it is possible to answer it in the positive in the following sense. One can find a noise characterization equally simple as the Gaussian form whose use yields more accurate results. This much we can ascertain by trial-and-error after the fact, that is,after having the “true” (i.e., Monte Carlo) BER curve. The difficulty, of course, is to determine the proper noise characterization before the fact. Although there appears to be no definite way of solving this problem it is possible, as the following simulation results and analysis will show, to bring a measure of insight to the problem, and as a by-product reveal the sensitivity of the BER to the shape of the noise density.

2.0 ANALYSIS

In accordance with the analytical approach we assume a single noise source at the receiver and seek a simple characterization of its probability density function (pdf) that will account for the behavior of the actual system. The simplicity requirement leads us to a natural generalization of the Gaussian density referred to as the generalized exponential (3), which has the following form:

$$p_{\nu}(x) = \frac{\nu}{2\sqrt{2}\,\sigma\,\Gamma(1/\nu)} \exp\left[-\left|(x-\mu)/\sqrt{2}\,\sigma\right|^{\nu}\right] \tag{1}$$

where $\Gamma(\bullet)$ is the Gamma function, μ is the mean, and the variance V_{ν} is related to the parameter a through the formula

$$V_{\nu} = 2\sigma^{2}\,\Gamma(3/\nu)/\Gamma(1/\nu). \tag{2}$$

Consider now the repetitive transmission of a sequence of symbols

$$\{a_k\}_{k=1}^{N}$$

through a satellite link. Let the (noiseless) voltage at the instant of sampling symbol k be denoted V_k. Assuming the noise is in the family (1) and has zero mean, and given that a_k = “one”, the probability of making a decision error is

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[*] Although it is possible to bring analytical techniques to bear on this problem (2), we feel these are not as flexible as simulation in respect of the representation of practical impairments or of the effect of “post-link” devices such as phase-lock-loops, bit synchronizers, and equalizers.

$$P(k, 1) = C \int_{-\infty}^{0} \exp\left[- \left| (x - v_k)/\sqrt{2}\,\sigma \right|^{\nu} \right] dx$$

where

$$C \triangleq \left(2\sqrt{2}\,\sigma\,\Gamma(1/\nu)/\nu\right)^{-1}.$$

If we assume that the polarity of a noiseless sample is the same as that of the original symbol, a safe assumption in virtually any system, then we can remove the conditioning on the symbol and set

$$P(k) = C \int_{-\infty}^{0} \exp\left[- \left| (x - m_k)/\sqrt{2}\,\sigma \right|^{\nu} \right] dx$$

where $m_k \triangleq |v_k|$. With the substitution $y = (x - m_k)/\sqrt{2}\,\sigma$, a series of manipulations leads to

$$P(k) = \frac{1}{2\,\Gamma(1/\nu)} \int_{\alpha}^{\infty} e^{-z} z^{[(1/\nu)-1]} dx$$

where $\alpha \triangleq (m_k/\sqrt{2}\,\sigma)^{\nu}$, and the integral is recognized as the incomplete Garnma function. Thus

$$P(k) = \left[2\,\Gamma(1/\nu)\right]^{-1} \Gamma(\nu^{-1}, \alpha). \tag{3}$$

The average probability of error over the sequence is evidently the average over k; for a two-channel (quadrature) system, with which we shall deal, we assume sequences

$$\{a_k\}_{k=1}^{N}, \quad \{b_k\}_{k=1}^{N}$$

for the two channels and attach subscripts 1 and 2 to denote them. Thus, the system BER can be put in the simple form

$$P = \frac{1}{2N} \sum_{k=1}^{N} \left[P_1(k) + P_2(k) \right]. \tag{4}$$

in evaluating (4) for a fixed carrier-to-noise ratio, it should be remembered that σ will be a function of ν as prescribed by (2).

3.0 NUMERICAL RESULTS

The system shown in Figure 1was simulated both by analytical and Monte Carlo methods, the former incorporating Equation (4). This was done for two sets of design characteristics and the results are shown in Figures 2 and 3. These results show, among other things, the significant influence on BER exerted by the noise statistics, and the relative unimportance of signal-to-noise ratio alone in assessing performance. The dashed curve labeled MC on the figures is that resulting from Monte Carlo runs using equal uplink and downlink CNR. We can conclude that the degradation resulting from that mode of operation (relative to the Gaussian noise case) is much more attributable to the alteration of the noise pdf due to the uplink nonlinearity than to power sharing.

3.1 REMARKS

We note from the results (and others not shown here) that the actual system cannot be precisely characterized as one having a single noise source describable by a density in the family (1). Although it would be possible to further generalize such a family, for example by setting $\sigma = \sigma(x)$, such complications would defeat the simplicity we are seeking. Thus we cannot reliably extrapolate (at least to within finely prescribed accuracy) to higher carrier-to-noise ratios the results obtained at lower ones, for example by estimating the exponent ν . But for some purposes we may get an adequate extrapolation by doing just that, although the extrapolation cannot be for a region much beyond that whence the estimate of ν comes. (One order of magnitude, perhaps two, might be a reasonable extrapolation range.) Certainly, a much more realistic assessment will surface than that arising from the Gaussian assumption. The extrapolation technique, which is due to Weinstein (3), was originally intended to work for a fixed carrier-to-noise ratio. Under that constraint we have successfully applied the extrapolation for the cases discussed. This highlights the fact that, due to the nonlinearity, the equivalent (single-source) noise pdf is a function of carrier-to-noise ratio; that is, changing the latter is not the same as changing the threshold with a fixed CNR, as would be the case in a linear system.

As was mentioned, we can still get a rough idea of the expected performance (BER vs E_b/N_o curve) of an actual nonlinear system by using an appropriate value of ν, which perhaps can be estimated by some curve-fit technique applied to Monte Carlo points obtained at low CNR. As the figures show, for the cases considered the equivalent source has an exponent $1.1 \lesssim \nu \lesssim 1.2$ depending on the specific system and the BER of interest. Results of other runs not shown here, applied to still other systems, tend to somewhat enlarge the range of the equivalent source exponent; but most results can be grouped in the Interval $1.0 \lesssim \nu \lesssim 1.4$. These results apply to a related class of systems, each one being a variation of the other. Naturally, for a different class of systems (for example one with a different ratio of uplink-to-downlink CNR) we can expect the characteristic range of ν to shift. Whatever class of system one deals with, the characterization of the link as one with a single noise source of exponent approximately ν is a useful tool in helping to visualize the BER performance, inasmuch as the complicated interaction of distorted signal + noise in the uplink nonlinearity and subsequent addition of downlink noise are all captured by an easily visualizable parameter variation on the noise density. It would therefore be of obvious value if the equivalent exponent ν could be derived analytically, even if only approximately, but this must await the result of future studies.

4.0 REFERENCES

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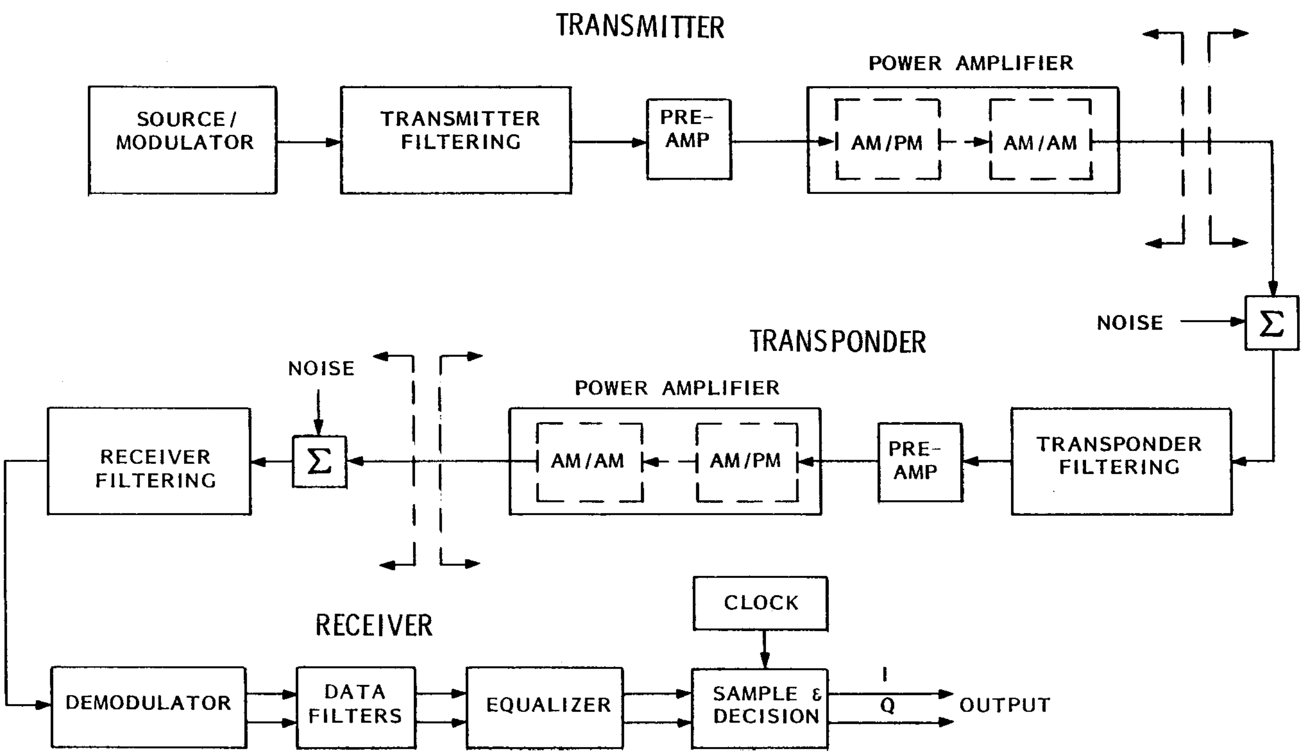

Figure 1. Simulation Block Diagram

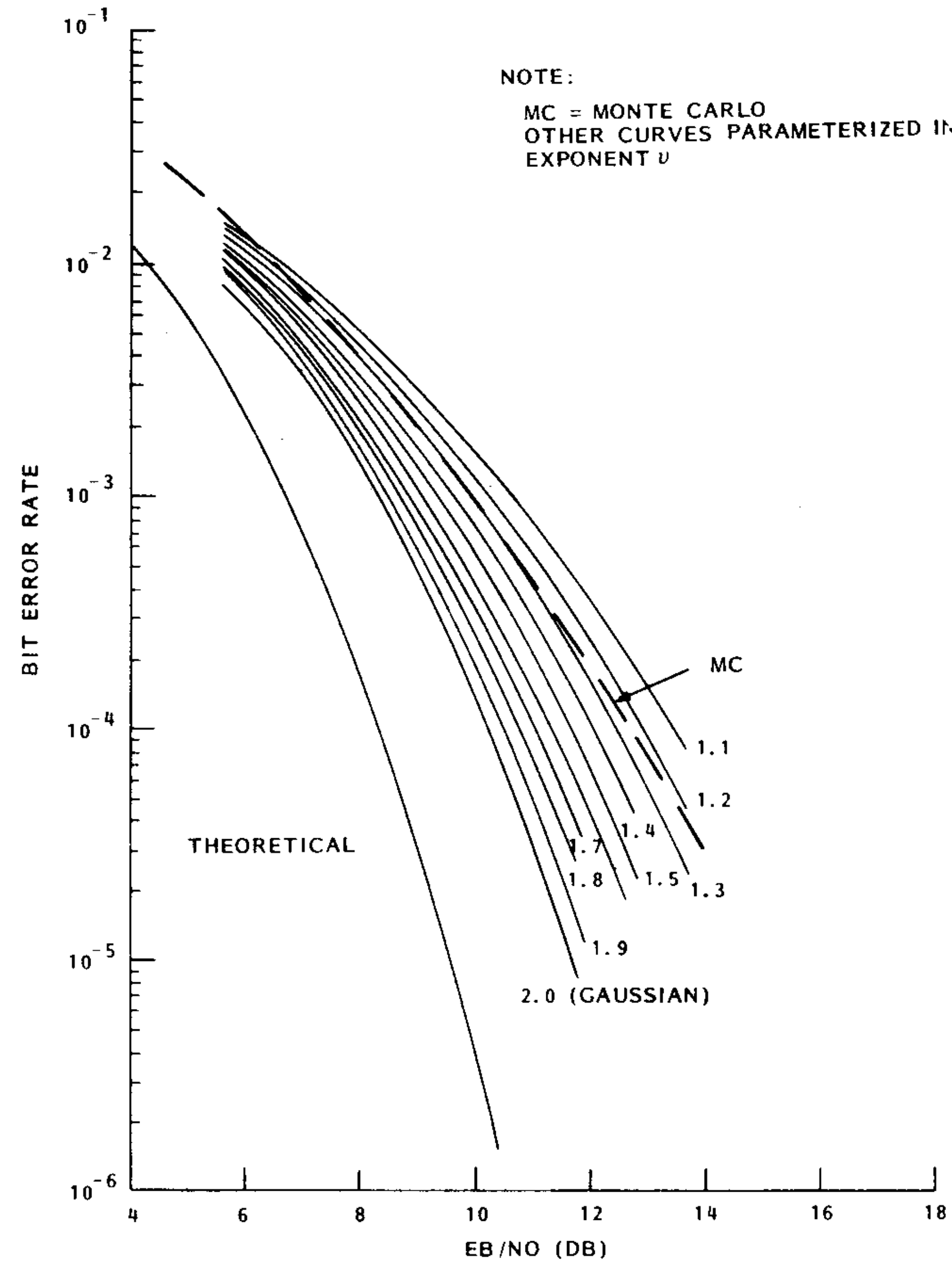

Figure 2. BER Performance as a Function of Noise-Source Exponent (System 1)

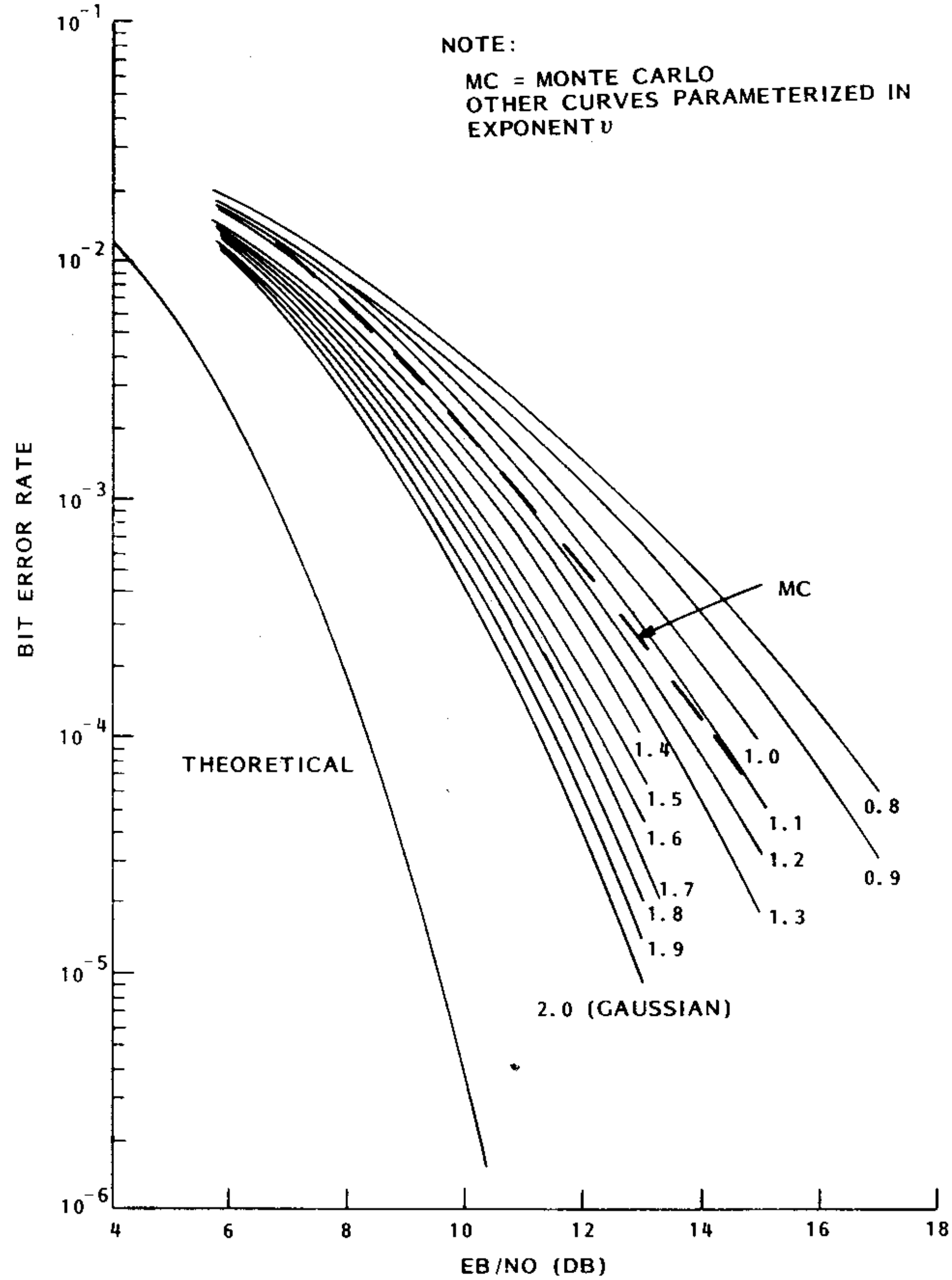

Figure 3. BER Performance as a Function of Noise-Source Exponent (System 2)