COHERENT DETECTION OF FREQUENCY-HOPPED QPSK AND QASK SYSTEMS WITH TONE AND NOISE JAMMING

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ABSTRACT

Perfectly coherent demodulation provides a lower bound on the bit error probability (BEP) of any spread spectrum system. Here the performance of coherent QPSK and QASK systems combined with frequency hopping (FH) or frequency-hopping direct-sequence (FH/PN) spread spectrum techniques in the presence of a multitone or noise jammer is shown. The worst-case jammer and worst-case performance are determined as functions of the signal-to-noise ratio (SNR) and signal-to-jammer power ratio (SJR). Asymptotic results for high SNR show a linear dependence between the jammers' optimal power allocation and the system performance.

INTRODUCTION

Quadriphase-shift-keying (QPSK) and quadrature amplitude-shift-keying (QASK) are bandwidth-efficient modulation-techniques whose performance over the additive "white" Gaussian noise channel is well documented in the literature(1,2). The purpose of this paper is to present the comparable results when a frequency-hopping (FH) spread spectrum modulation is superimposed on these conventional techniques in order to combat the intentional interference introduced by a jammer.

In general, two possibilities exist with regard to the manner in which the FH modulation is implemented. If the individual hop pulse phases bear no relation to each other, the spread spectrum technique is referred to as <u>noncoherent FH</u>. If, on the other hand, phase continuity is maintained from one hop pulse to another, <u>coherent FH</u> is obtained. In either case, one usually makes the assumption that the hop pulse phase is constant over a single hop interval. We shall consider the coherent FH case only; thus, the receiver structures analyzed will be those normally employed for coherent detection of QPSK and QASK in an additive white Gaussian noise background.

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Pseudonoise (PN) direct-sequence balanced modulation of the individual FH frequencies may also be employed to obtain larger spreading ratios. This hybrid technique is known simply as FH/PN modulation, which can be combined with the QPSK and QASK modulations as FH/PN-QPSK and FH/PN-QASK.

In designing any spread spectrum system to be jam-resistant, one must propose a scenario under which the jammer is assumed to operate. As such, one normally assumes an intelligent jammer in that he has knowledge of the form of data and spread spectrum modulations including such items as data rate, spreading bandwidth and hop rate, but no knowledge of the code selected for determining the spectrum spreading hop frequencies. Thus, the strategy normally employed is to design for the worst case in the sense that, given the modulation form of the communicator, the jammer is assumed to employ the type of jamming which is most deleterious to the communication receiver.

While many possible jammer models can be proposed, the most common are partial-band (PB) noise jamming and partial-band multitone jamming. In this paper, we shall treat these two jamming types separately and develop, in each case, the performance corresponding to a worst-case jammer strategy for the four modulation forms: FH-QPSK, FH-QASK, FH/PN-QPSK and FH/PNQASK.

PERFORMANCE OF FH-QPSK IN THE PRESENCE OF PARTIAL-BAND MULTITONE JAMMING

An FH-QPSK signal is characterized by transmitting

$$s^{(i)}(t) = \sqrt{S}\left(a_i \cos\omega_H^{(i)}t - b_i \sin\omega_H^{(i)}t\right) \tag{1}$$

in the ith signaling interval $(i-1)T_s \le t \le iT_s$, where $\omega_H^{(i)}$ is the particular carrier frequency selected by the frequency hopper for this interval*, and a_i, b_i are independent data streams of ±1's for the in-phase and quadrature components, respectively. The received signal r(t) consists of $s^{(i)}(t)$ plus some additive white Gaussian noise n(t) and the jammer i(t).

The bandpass noise n(t) has the usual narrowband representation

$$n(t) = \sqrt{2}\left[N_c(t)\cos\omega_H^{(i)}t - N_s(t)\sin\omega_H^{(i)}t\right] \tag{2}$$

* We assume for simplicity that the hop rate is equal to the information symbol rate and that the frequency hopper and symbol clock are synchronous.

where $N_c(t)$ and $N_s(t)$ are statistically independent lowpass white Gaussian noise processes with single-sided noise spectral density N_0 [W/Hz]. The PB multitone jammer j(t) is assumed to have a total power J which is evenly divided among q jammer tones. Thus, each tone has power

$$J_0 = \frac{J}{q} \tag{3}$$

Furthermore, since the jammer is assumed to have knowledge of the hop frequencies or, equivalently, the exact location of the spreading bandwidth W and the number of hops N in this bandwidth, it is safe to assume that he will randomly locate each of his q tones coincident with q of the N hop frequencies. Thus,

$$\alpha \triangleq \frac{q}{N} \tag{4}$$

represents the fraction of the total band which is continuously jammed with tones, each having power J_0. Although, by definition, α is a rational number, it will be treated here as a real number for purposes of analysis. Such an assumption is legitimate when N is a large integer. Once again, the jammer's strategy is to distribute his total power J (i.e., choose α and J_0) in such a way as to cause the communicator to have maximum probability of error.

The jammer j(t) assumes the form

$$j(t) = \sqrt{2J_0}\cos\left(\omega_H{}^{(i)}t + \theta_j\right) \tag{5}$$

with θ_j uniformly distributed within the interval (0,2π) and independent of the data bits and noise.

In view of the foregoing assumptions, the total received signal in the signaling interval (hop interval) which contains a jamming tone at the hop frequency is given by

$$r^{(i)}(t) = s^{(i)}(t) + n(t) + j(t) \tag{6}$$

Over an integral number of hop bands, the fraction α of the total number of signaling intervals will be characterized by (6). In the remaining fraction (1-α) of the signaling intervals, the received signal is simply characterized by

$$r^{(i)}(t) = s^{(i)}(t) + n(t)\ . \tag{7}$$

After ideal coherent demodulation by the frequency hopper, the in-phase and quadrature components of the received signal, ignoring double-harmonic terms, become

$$\varepsilon_c(t) \triangleq r(t)\sqrt{2}\cos\omega_H^{(i)}t\Big|_{LP} \quad ; \quad \varepsilon_s(t) \triangleq -r(t)\sqrt{2}\sin\omega_H^{(i)}t\Big|_{LP}$$

These signals are then passed through integrate-and-dump filters of duration equal to the information symbol interval T_s to produce the in-phase and quadrature decision variables (see Figure 1)

$$y_c \triangleq \frac{1}{T_s}\int_{(i-1)T_s}^{iT_s} \varepsilon_c(t)\,dt = \sqrt{\frac{S}{2}}\,a_i + \sqrt{J_0}\cos\theta_j + N_c \tag{8}$$

$$y_s \triangleq \frac{1}{T_s}\int_{(i-1)T_s}^{iT_s} \varepsilon_s(t)\,dt = \sqrt{\frac{S}{2}}\,b_i + \sqrt{J_0}\sin\theta_j + N_s \tag{9}$$

where

$$N_c \triangleq \frac{1}{T_s}\int_{(i-1)T_s}^{iT_s} N_c(t)\,dt \quad ; \quad N_Q \triangleq \frac{1}{T_s}\int_{(i-1)T_s}^{iT_s} N_s(t)\,dt$$

are zero-mean independent Gaussian random variables with variance $N_0/2T_s$.

The receiver estimates of a_i and b_i are obtained by passing y_c and y_s through hard limiters, giving

$$\hat{a}_i = \mathrm{sgn}\{y_c\} \quad ; \quad \hat{b}_i = \mathrm{sgn}\{y_s\} \tag{10}$$

Hence, given $\hat{a}_i$, $\hat{b}_i$ and θ_j, the probability that the ith symbol is in error is the probability that either $\hat{a}_i$, or $\hat{b}_i$ is in error, i.e.,

$$P_{E_i}(\theta_j) = \mathrm{Prob}\{\hat{a}_i\neq a_i \text{ or } \hat{b}_i\neq b_i\} = \mathrm{Prob}\{\hat{a}_i\neq a_i\} + \mathrm{Prob}\{\hat{b}_i\neq b_i\} - \mathrm{Prob}\{\hat{a}_i\neq a_i\}\,\mathrm{Prob}\{\hat{b}_i\neq b_i\} \tag{11}$$

Since the signal set is symmetric, we can compute (11) for any of the four possible signal points and obtain the average probability of symbol error conditioned on the jammer phase $\overline{P}_E(\theta j)$. The unconditional average probability of symbol error $\overline{P}_{EJ}$ for symbol intervals which are jammed is obtained by averaging $P_E(\theta j)$ over the uniform distribution of θj. The result is

$$\overline{P}_{EJ} = \frac{1}{2\pi}\int_0^{2\pi}\left[P_s(\theta_j) + P_c(\theta_j) - P_s(\theta_j)\,P_c(\theta_j)\right] d\theta_j \tag{12}$$

where

$$P_s(\theta_j) = Q\left[\sqrt{\frac{ST_s}{N_0}}\left(1 + \sqrt{\frac{2J_0}{S}}\sin\theta_j\right)\right] \quad ; \quad P_c(\theta_j) = Q\left[\sqrt{\frac{ST_s}{N_0}}\left(1 + \sqrt{\frac{2J_0}{S}}\cos\theta_j\right)\right] \tag{13}$$

and the function

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{y^2}{2}\right) dy$$

is the Gaussian probability integral.

Recognizing that, for a QPSK signal, the symbol time T_s is twice the bit time T_b and letting $E_b = ST_b$ denote the bit energy, we then have

$$\frac{ST_s}{N_0} = \frac{2ST_b}{N_0} = \frac{2E_b}{N_0} \tag{14}$$

Furthermore, from (3) and (4),

$$\frac{2J_0}{S} = \frac{2J}{\alpha NS} \tag{15}$$

Now, if the hop frequency slots are $1/T_s$ wide in terms of the total hop frequency W and the number of hop slots N in that band, we then have

$$N = \frac{W}{1/T_s} = WT_s = 2WT_b \tag{16}$$

Substituting (16) into (15) gives

$$\frac{2J_0}{S} = \frac{J/W}{\alpha ST_b} = \frac{J/W}{\alpha E_b} \triangleq \frac{N_J}{\alpha E_b} \tag{17}$$

The quantity J/W represents the effective jammer power spectral density in the hop band; thus, we have introduced the notation N_J to represent this quantity. Rewriting (13) using (14) and (17) gives

$$P_s(\theta_j) = Q\left[\sqrt{\frac{2E_b}{N_0}}\left(1 + \sqrt{\frac{N_J}{\alpha E_b}} \sin \theta_j\right)\right] \quad ; \quad P_c(\theta_j) = Q\left[\sqrt{\frac{2E_b}{N_0}}\left(1 + \sqrt{\frac{N_J}{\alpha E_b}} \cos \theta_j\right)\right] \tag{18}$$

For the fraction (1-α) of symbol (hop) intervals where the jammer is absent, the average symbol error (SEP) is given by the well known result [1]

$$\overline{P}_{E_0} = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right) - Q^2\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{19}$$

Thus, the average error probability over all symbols (jammed and unjammed) is simply

$$\overline{P}_E = \alpha \overline{P}_{E_J} + (1-\alpha)\, \overline{P}_{E_0} \,, \tag{20}$$

where $\overline{P}_{E_J}$ is given by (12), together with (18), and $\overline{P}_{E_0}$ is given in (19).

The above discussion dealt with the SEP $\overline{P}_E$ Given a specific code, one can proceed to evaluate the average bit error probability (BEP) $\overline{P}_b$. Assuming a Gray code, which has the property that adjacent symbols differ by only a single bit, $\overline{P}_b$ for QPSK can be evaluated exactly and is given by

$$\overline{P}_b = \alpha\left[\frac{1}{2\pi}\int_0^{2\pi} Q\left[\sqrt{\frac{2E_b}{N_0}}\left(1 + \sqrt{\frac{N_J}{\alpha E_b}} \sin\theta_j\right)\right] d\theta_j\right] + (1-\alpha)\, Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{21}$$

In fact, $\overline{P}_b$ for QPSK is identical to $\overline{P}_b$ for binary PSK (BPSK). Figure 2 is a plot of $\overline{P}_b$ versus α, with E_b/N_J as a parameter for the case E_b/N_0 = 20-dB. It is seen that, for fixed E_b/N_0 and E_b/N_J, there exists a value of α which maximizes $\overline{P}_b$ and thus represents the worst-case multitone jammer situation. The worst-case α versus E_b/N_J with E_b/N_0 as a parameter is shown in Figure 3. Figure 4 is a plot of the corresponding $(\overline{P}_b)_{worst}$ versus E_b/N_J.

It is of interest to examine the limiting behavior of $(\overline{P}_b)_{worst}$ as a function of E_b/N_J when E_b/N_0 approaches infinity. From (21), it follows that

$$\lim_{\frac{E_b}{N_0}\to\infty} \overline{P}_b = \alpha \lim_{\frac{E_b}{N_0}\to\infty} \overline{P}_{b_J} \tag{22}$$

where $\overline{P}E_J$ denotes the BEP when the jammer is present. Because of the symmetry between the two channels, the limit in (22) can be determined from either (8) or (9) when the noise component is eliminated. It follows from (8) that

$$\begin{aligned}\lim_{\frac{E_b}{N_0}\to\infty} \overline{P}_{b_J} &= P_r\left\{y_c < 0 \,\middle|\, a_i = 1;\ N_c = 0\right\} = P_r\left\{\sqrt{\frac{S}{2}} + \sqrt{J_0}\cos\theta_j < 0\right\} \\ &= P_r\left\{\cos\theta_j < -\sqrt{\frac{S}{J_0}}\right\} = P_r\left\{\cos\theta_j < -\sqrt{\frac{\alpha E_b}{N_J}}\right\} \\ &= \begin{cases} 0 & ; \quad \dfrac{\alpha E_b}{N_J} > 1 \\ \dfrac{1}{\pi}\cos^{-1}\sqrt{\dfrac{\alpha E_b}{N_J}} & ; \quad 0 < \dfrac{\alpha E_b}{N_J} < 1 \end{cases}\end{aligned} \tag{23}$$

The last step in deriving (23) is based on the uniform assumption about θ_j, Combining (22) and (23) results in

$$\lim_{\frac{E_b}{N_0}\to\infty} \overline{P}_b = \frac{\alpha}{\pi}\cos^{-1}\sqrt{\frac{\alpha E_b}{N_J}} \tag{24}$$

which can be differentiated with respect to α and equated to zero to yield the worst-case α and $\overline{P}_b$. Differentiation of (24) results in an equation of the form

$$\cos^{-1}\sqrt{\frac{\alpha E_b}{N_J}} = \frac{1}{2}\sqrt{\frac{\frac{\alpha E_b}{N_J}}{1 - \frac{\alpha E_b}{N_J}}}$$

whose numerical solution yields

$$\alpha_{worst} = \frac{0.6306}{E_b/N_J} \tag{25}$$

with corresponding worst-case $\overline{P}_b$ equal to

$$\left(\overline{P}_b\right)_{worst} \triangleq \max_{\alpha} \lim_{\frac{E_b}{N_0} \to \infty} \overline{P}_b = \frac{0.13112}{E_b/N_J} \tag{26}$$

The straight line implied by (26) is shown in Figure 4 for $E_b/N_0 \to \infty$.

PERFORMANCE OF FH-QPSK IN THE PRESENCE OF PARTIAL-BAND NOISE JAMMING

Now assume that the jammer j(t) spreads his total power J uniformly across a fraction α of the total hop frequency band W. Then, insofar as the data demodulation process is concerned, the jammer appears as an additional additive noise source of power spectral density

$$N_J' = \frac{J}{\alpha W} = \frac{N_J}{\alpha} \tag{27}$$

Note that the power spectral density of the noise jammer defined in (27) is identical to the effective power spectral density defined for the multitone jammer in (17).

Since the jammer noise can be assumed to be independent of the background additive Gaussian noise, one can add their power spectral densities and use this sum to represent the total noise perturbing the receiver. Thus, the error probability performance of FH-QPSK in the presence of partial-band noise jamming is characterized by taking the well known results for just an additive white Gaussian noise background and replacing N_0 by $N_0 + N_J'$.

Without going into great detail, we then see that the SEP is once again given by (21), with $\overline{P}_{E_0}$ as in (19) but, however,

$$\overline{P}_{E_J} = 2Q\left(\left[\left(\frac{2E_b}{N_0}\right)^{-1} + \left(\frac{2\alpha E_b}{N_J}\right)^{-1}\right]^{-1/2}\right) - Q^2\left(\left[\left(\frac{2E_b}{N_0}\right)^{-1} + \left(\frac{2\alpha E_b}{N_J}\right)^{-1}\right]^{-1/2}\right) \tag{28}$$

Also, the average BEP is now

$$\overline{P}_b = \alpha Q\left(\left[\left(\frac{2E_b}{N_0}\right)^{-1} + \left(\frac{2\alpha E_b}{N_J}\right)^{-1}\right]^{-1/2}\right) + (1-\alpha)\, Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{29}$$

which is identical to the result for noise jamming of FH-BPSK.

The limiting behavior of (29) as $E_b/N_0 \rightarrow \infty$ is seen to be

$$\lim_{\frac{E_b}{N_0} \rightarrow \infty} P_b = \alpha Q\left(\sqrt{\frac{2\alpha E_b}{N_J}}\right) \tag{30}$$

Differentiating (30) with respect to a and equating to zero yields

$$Q\left(\sqrt{\frac{2\alpha E_b}{N_J}}\right) = \sqrt{\frac{\alpha E_b}{N_J}}\, \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{\alpha E_b}{N_J}\right)$$

whose solution yields

$$\alpha = \frac{0.709}{E_b/N_J} \tag{31}$$

with corresponding $(\overline{P}_b)_{worst}$ equal to

$$(\overline{P}_b)_{worst} = \frac{0.082865}{E_b/N_J} \tag{32}$$

Figures 5 through 7 illustrate the performance of an FH-QPSK system in the presence of noise jamming.

PERFORMANCE OF FH-QASK IN THE PRESENCE OF PARTIAL-BAND MULTITONE JAMMING

An FH-QASK M-ary signal is characterized by transmitting

$$s^{(i)}(t) = \sqrt{2}\, \delta\left[a_i \cos \omega_H^{(i)} t - b_i \sin \omega_H^{(i)} t\right] \tag{33}$$

in the ith signaling interval. The total number M of possible signals is typically the square of an even number K, and the quadrature amplitudes a_i and b_i take on equally likely values ±1, ±3,...±(K-1). Also, δ is a parameter which is related to the average power S of the signal set by

$$S = \frac{2}{3}\left(K^2 - 1\right) \delta^2 \tag{34}$$

Analogous to the steps leading to (8) and (9), we can arrive at expressions for the in-phase and quadrature decision variables, namely,

$$y_c = a_i \delta + \sqrt{J_0} \cos\theta_j + N_c \quad ; \quad y_s = b_i \delta + \sqrt{J_0} \sin\theta_j + N_s$$

The QASK receiver estimates of a_i and b_i are obtained by passing y_c and y_s through K-level quantizers

$$\hat{a}_i = Q_K\left(y_c\right) \quad ; \quad \hat{b}_i = Q_K\left(y_s\right) \tag{35}$$

Hence, given a_i, b_i and θ_j, the probability that the ith symbol is in error is the probability that $\hat{a}_i$ or b_i is in error. Thus, once again, (17) is valid. Here, however, we must compute (17) for the $K^2/4$ points in any quadrant in order to obtain the SEP conditioned on the jammer phase.

For K= 4 (i.e., 16-QASK), the average SEP is once again given by (21), where now the average symbol error probability $\overline{P}_{E_0}$ for the symbol intervals which are not jammed is given by [2]

$$\overline{P}_{E_0} = 3Q\left(\sqrt{\frac{4E_b}{5N_0}}\right) - \frac{9}{4}Q^2\left(\sqrt{\frac{4E_b}{5N_0}}\right) \tag{36}$$

and the average symbol error probability $\overline{P}_{E_J}$ for symbol intervals which are jammed is given by (12) with

$$P_s(\theta_j) = Q\left[\sqrt{\frac{4E_b}{5N_0}}\left(1 + \sqrt{\frac{5N_J}{2\alpha E_b}} \sin\theta_j\right)\right] + \frac{1}{2} Q\left[\sqrt{\frac{4E_b}{5N_0}}\left(1 - \sqrt{\frac{5N_J}{2\alpha E_b}} \sin\theta_j\right)\right]$$

$$P_c(\theta_j) = Q\left[\sqrt{\frac{4E_b}{5N_0}}\left(1 + \sqrt{\frac{5N_J}{2\alpha E_b}} \cos\theta_j\right)\right] + \frac{1}{2} Q\left[\sqrt{\frac{4E_b}{5N_0}}\left(1 - \sqrt{\frac{5N_J}{2\alpha E_b}} \cos\theta_j\right)\right] \tag{37}$$

The average BEP $\overline{P}_b$ depends on the specific encoding scheme employed and is not easily obtained in closed form. If one encodes the 16-QASK symbols with a perfect Gray code, then, accounting only for adjacent symbol errors, the average BEP for large E_b/N_0 and E_b/N_0 is related to the average SEP by

$$P_b \cong \frac{1}{4}\overline{P}_E \qquad \left(\text{for } E_b/N_0 \gg 1 \quad ; \quad \alpha E_b/N_J \gg 1\right) \tag{38}$$

If one wishes to improve upon the approximation in (38) by accounting for symbol errors resulting in two bit errors, one can then obtain the following result:

$$\overline{P}_b = \alpha \overline{P}_{b_j} + (1-\alpha)\,\overline{P}_{b_0} \tag{39}$$

where

$$\begin{aligned} \overline{P}_{b_j} \cong\ & \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} P_{s,3}^{-}(\theta_j)\left[P_{c,3}^{-}(\theta_j) - P_{c,1}^{-}(\theta_j)\right] d\theta_j \\ & + \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{8}\left[P_{s,3}^{-}(\theta_j)\ P_{c,1}^{+}(\theta_j) + P_{c,3}^{-}(\theta_j)\ P_{s,1}^{+}(\theta_j)\right] d\theta_j \end{aligned} \tag{40}$$

with

$$P_{s,i}^{\pm}(\theta_j) = Q\left[\sqrt{\frac{4E_b}{5N_0}}\left(\pm i - \sqrt{\frac{5N_J}{2\alpha E_b}}\sin\theta_j\right)\right] \quad ; \quad P_{ci}^{\pm}(\theta_j) = Q\left[\sqrt{\frac{4E_b}{5N_0}}\left(\pm i - \sqrt{\frac{5N_J}{2\alpha E_b}}\cos\theta_j\right)\right] \tag{41}$$

and

$$\overline{P}_{b_0} = \frac{3}{4}Q\left(\sqrt{\frac{4E_b}{5N_0}}\right) - \frac{1}{2}Q\left(3\sqrt{\frac{4E_b}{5N_0}}\right) + \frac{1}{2}Q^2\left(3\sqrt{\frac{4E_b}{5N_0}}\right) - \frac{3}{4}Q\left(\sqrt{\frac{4E_b}{5N_0}}\right)Q\left(3\sqrt{\frac{4E_b}{5N_0}}\right) \tag{42}$$

Note that the above relationship is not an exact expression, however, since symbol errors resulting in more than two bit errors are not accounted for. Then E_b/N_0 and $\alpha E_b/N_J$ must still be large in order for the approximation to be accurate.

In a jamming environment, when the signal is hit by the jammer, the instantaneous $\alpha E_b/N_J$ is small (i.e., the jammer power is large). Thus, the approximation for $\overline{P}_b$ with large E_b/N_0 and $\alpha E_b/N_J$ is not very accurate. Alternately, in the limit as E_b/N_0 and $\alpha E_b/N_J$ approach zero, when an error is made, any one of the 15 incorrect symbols could be equally likely chosen. Thus, if an error is made, the probability that i of the four bits are in error is

$$P_e(i) = \frac{\binom{4}{i}}{15}$$

since each symbol has four bits. Hence, the conditional average probability that a specific bit is in error, given that a symbol is in error, is

$$\left(\overline{P}_b\right)_{cond} = \sum_{i=1}^{4} P_e(i)\left(\frac{i}{4}\right) = \frac{1}{60}\sum_{i=1}^{4} i\binom{4}{i} = \frac{8}{15}$$

which means that the unconditional bit error probability is

$$\left(\overline{P}_b\right) = \frac{8}{15}\,\overline{P}_E \qquad \left(\text{for } E_b/N_0 \to 0\ ;\ \ \alpha E_b/N_J \to 0\right) \tag{43}$$

The relationship between $\overline{P}_b$ and $\overline{P}_E$ in (43) is exactly true only at $E_b/N_0 = 0$ and $\alpha E_b/N_J = 0$; however, it represents a maximum value of $\overline{P}_b$ for all values of E_b/N_0 and $\alpha E_b/N_J$. The relationship between $\overline{P}_b$ and $\overline{P}_E$ in (38) represents the minimum value P_b for all values of E_b/N_0 and $\alpha E_b/N_J$. In comparing the performance of various modulation schemes in jamming, it is important to determine the maximum value that $\overline{P}_b$ can be for a given E_b/N_0 and $\alpha E_b/N_J$ when α is optimized. Therefore, to make sure that the maximum value of $\overline{P}_b$ is found, the relationship in (43) is used for all performance comparisons made in a jamming environment.

As was done for FH-QPSK, one can compute the limiting performance of FH-16-QPSK as E_b/N_0 approaches infinity. The result is

$$\left(\overline{P}_b\right)_{worst} \triangleq \max_{\alpha} \lim_{\frac{E_b}{N_0}\to\infty} \overline{P}_b = \frac{0.5245}{E_b/N_J} \tag{44}$$

where use of (43) has been made and

$$(\alpha)_{worst} = \frac{1.577}{E_b/N_J} \tag{45}$$

Equation (44) is shown in Figure 8 as a function of E_b/N_J.

PERFORMANCE OF FH-QASK IN THE PRESENCE OF NOISE JAMMING

Assuming the same noise jammer model as that discussed in the section for FH-QPSK, the average symbol error probability of FH-QASK-16 is then given by (21), with $\overline{P}_{E_0}$ as in (36), but now

$$\overline{P}_{E_J} = 3Q\left(\left[\left(\frac{4E_b}{5N_0}\right)^{-1} + \left(\frac{4\alpha E_b}{5N_J}\right)^{-1}\right]^{-1/2}\right) - \frac{9}{4}Q^2\left(\left[\left(\frac{4E_b}{5N_0}\right)^{-1} + \left(\frac{4\alpha E_b}{5N_J}\right)^{-1}\right]^{-1/2}\right) \quad (46)$$

An analysis of the limiting behavior of $\overline{P}_E$ yields

$$\lim_{\frac{E_b}{N_0} \to \infty} \overline{P}_E = \frac{0.57}{E_b/N_J} \quad (47)$$

The worst-case average bit error probability in the limit is obtained by multiplying (47) by 8/15. Figure 9 shows $(BEP)_{worst}$ versus E_b/N_J.

PERFORMANCE OF FH/PN-QPSK AND FH/PN-QASK IN THE PRESENCE OF PARTIAL-BAND MULTITONE JAMMING

When a direct-sequence pseudonoise (PN) balanced modulation is superimposed on an FH-QPSK signal, each jammer tone of power J_0 is then spread over a bandwidth equal to the PN chip rate R_c. As such, the tone jammer is now caused to behave like a noise jammer of spectral density

$$N_J' \doteq \frac{J_0}{R_c} \quad (48)$$

where N is again the number of slots available to the communicator. However, each of them should be R_c wide to accommodate the PN modulation*. Hence, $N = W/R_c$ which, when combined with (49) and substituted in (48) gives

$$N_J' = \frac{J}{\alpha W} \quad (49)$$

Equation (50) is independent of the chip rate R_c. Hence, the tone jamming performances of FH/PN-QPSK and FH/PN-QASK are identical to the noise jamming performances of FH-QPSK and FH-QASK.

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* This implicity assumes contiguous hop slots within the total hop band.

REFERENCES

1. W. C. Lindsey and M. K. Simon, Telecommunication Systems Engineering, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1973,Chapter 5.

2. M. K. Simon and J. G. Smith, "Carrier Synchronization and Detection of QASK Signal Sets," IEEE Transactions on Communications, Vol. COM-22, No. 2, February 1974.

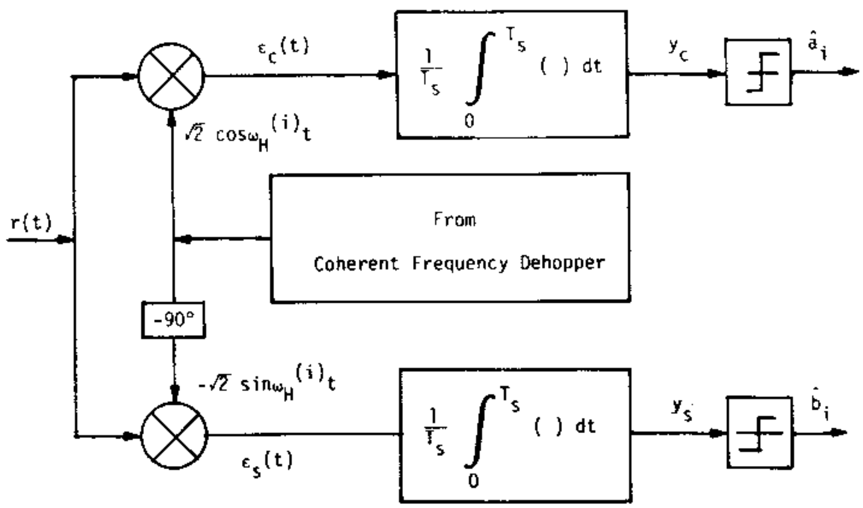

Figure 1. Coherent FH-QPSK Demodulator

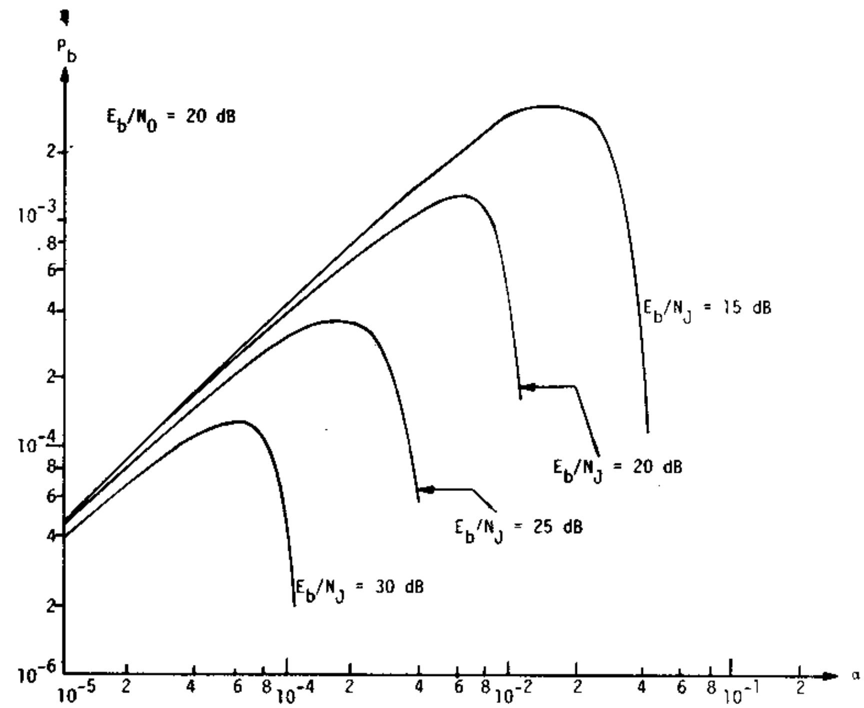

Figure 2. $\bar{P}_b$ versus α for FH-QPSK (Tone Jamming)

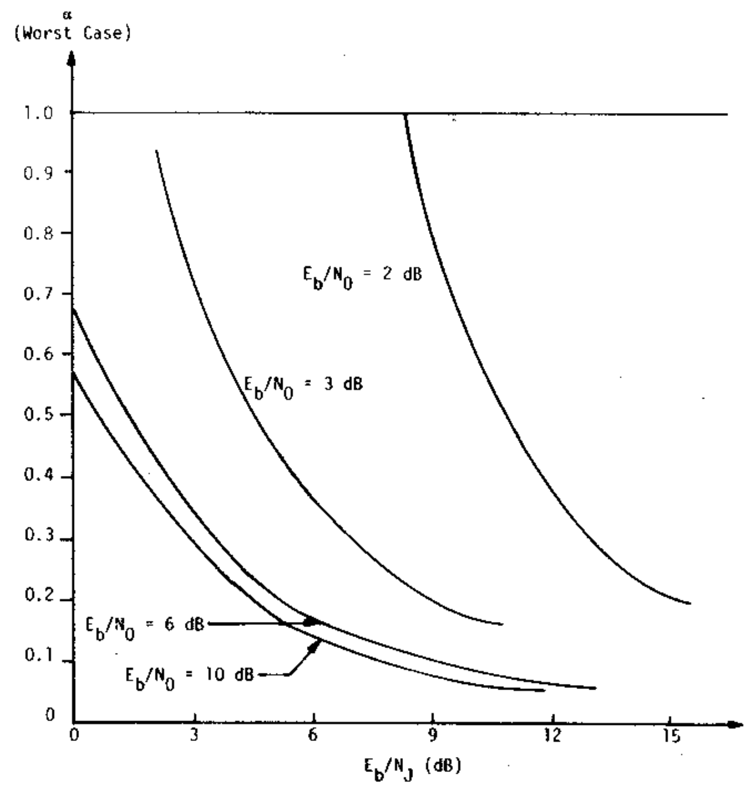

Figure 3. Worst-Case α versus E_b/N_J for FH-QPSK (Tone Jamming)

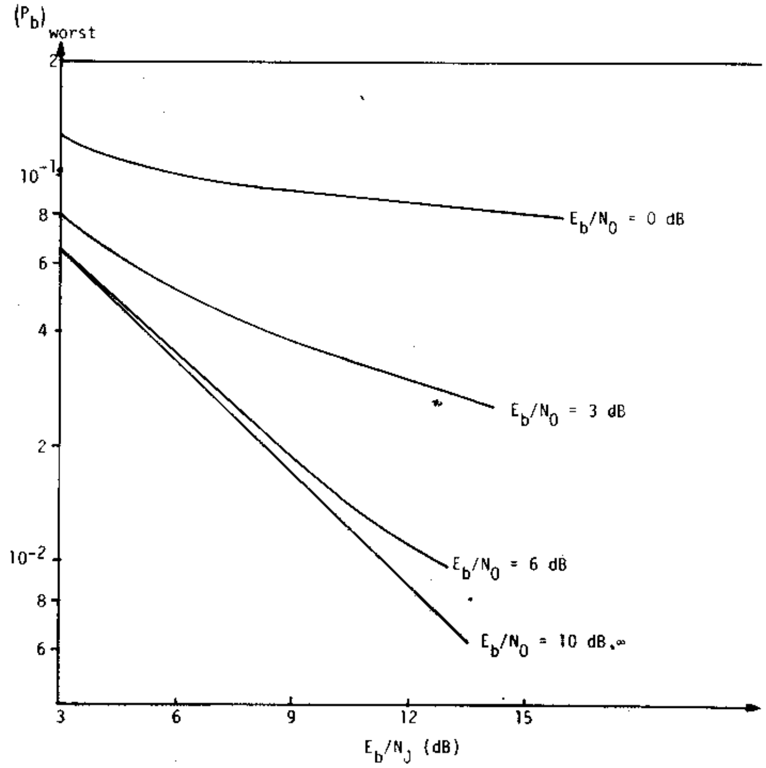

Figure 4. Worst-Case $\overline{P}_b$ versus E_b/N_J for FH-QPSK (Tone Jamming)

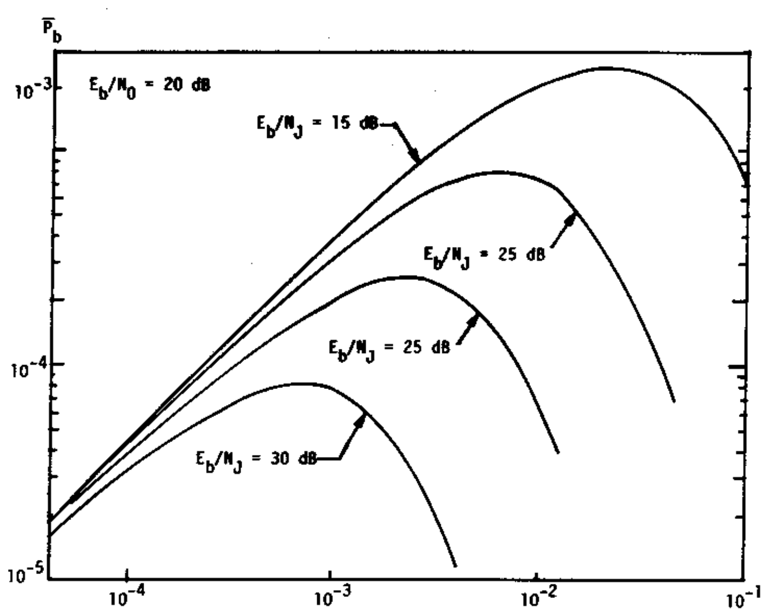

Figure 5. $\overline{P}_b$ versus α for FH-QPSK (Noise Jamming) with E_b/N_0 = 20 dB

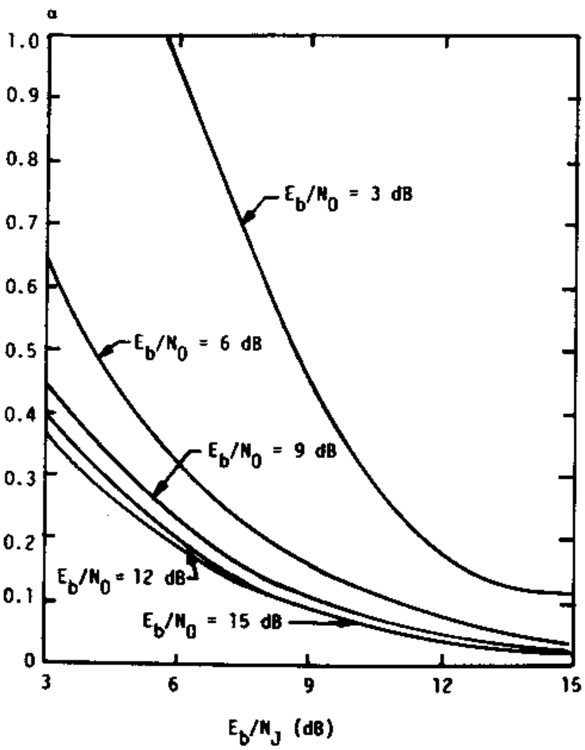

Figure 6. Worst-Case α versus E_b/N_J for FH-QPSK (Noise Jamming)

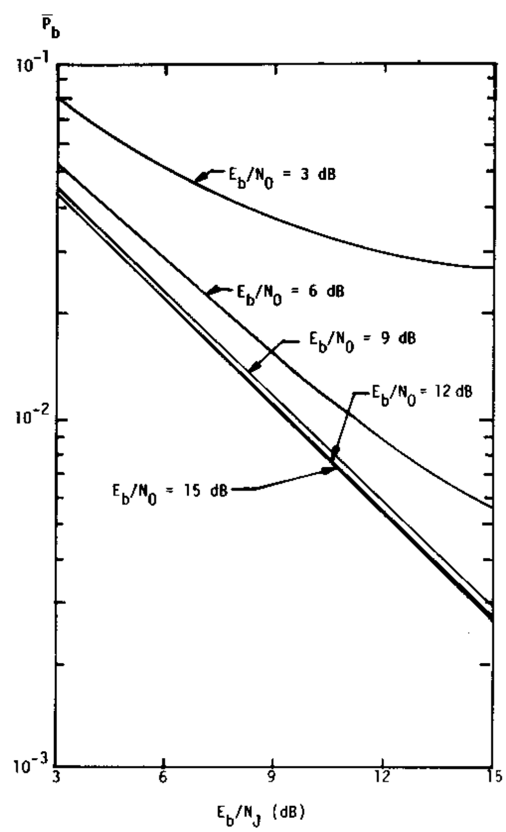

Figure 7. Worst-Case $\overline{P}_b$ versus E_v/N_J for FH-QPSK (Noise Jamming)

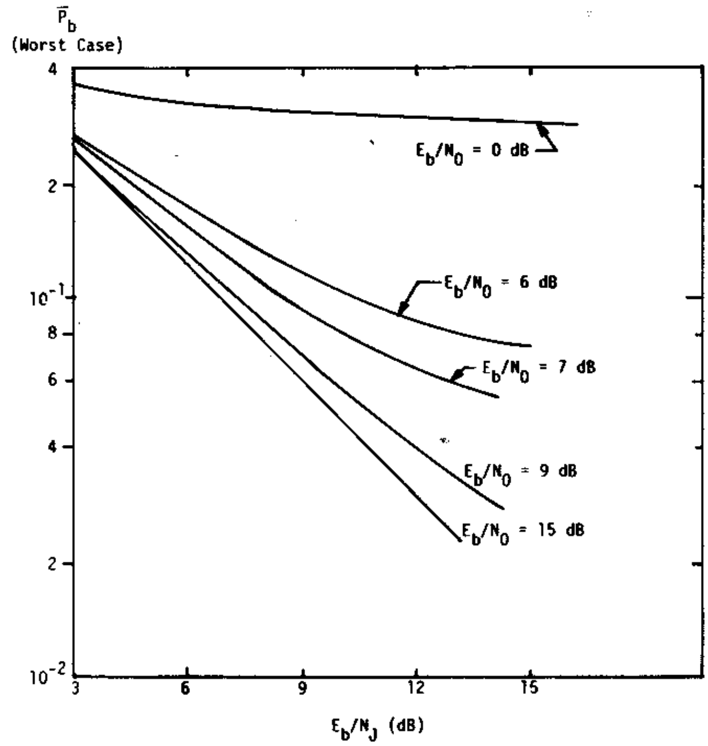

Figure 8. Worst-Case $\overline{P}_b$ for E_b/N_J for FH-QASK-16 (Tome Jamming)

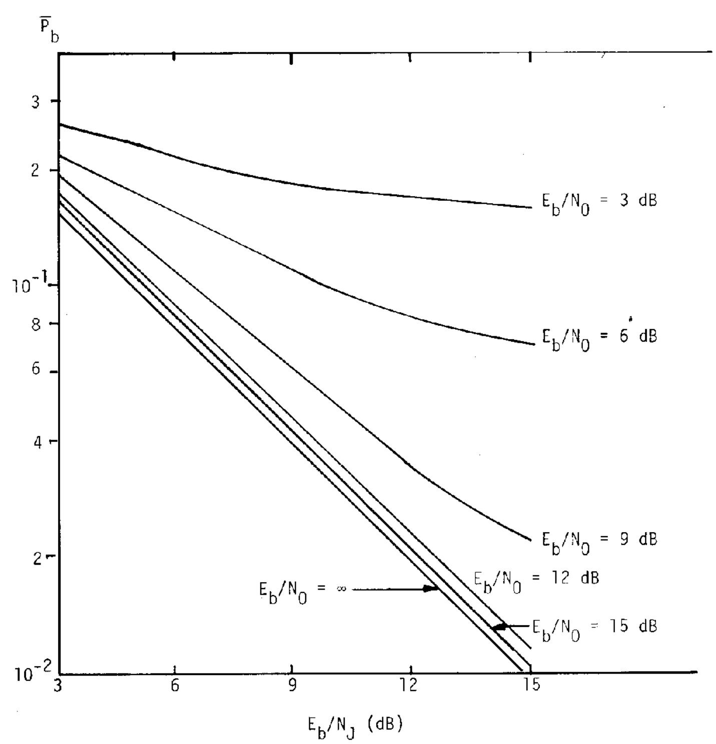

Figure 9. Worst-Case $\overline{P}_b$ versus E_b/N_J for FH-QASK-16 (Noise Jamming)