TELEMETRY SYSTEM
BASED ON WALSH FUNCTIONS

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ABSTRACT

In this paper a telemetry system based on Walsh functions is described. After a brief introduction of Walsh functions a sequency division multiplex system is introduced. Synchronization problem is discussed in some detail. Finally, experimental results are given to justify the design consideration.

INTRODUCTION

Walsh functions were introduced by an American mathematician J.L.Walsh in 1923. Walsh functions make up a double-valued function system, only taking values of ±1. They are defined on unit interval, 0 to 1, and they are orthogonal, normalized, and complete. The first 16 Walsh functions are shown in figure 1. Sequency is defined as one half the average number of zero-crossings in the interval. Some properties of Walsh functions are valuable not only mathematically but also in engineering applications.

Why should Walsh functions be used? Both practice and theory have shown that sine-cosine functions are very useful. However, they are not quite perpect. For example, theoretically they can be used only in R-C circuits which contain only time-invariant linear elements. In Fourier analysis, a signal is either supposed to be applied for an infinite period of time, or has to be represented by an infinite frequency spectrum. As a consequence, the bandwidth of the signal has to be taken as limited one when sampling theorem is applied. Also in an ideal filter, the output signal would appear before the input voltage. All these and other inconveniences will be avoided when Walsh functions are used.

As commonly used so far, there are two kinds of telemetry system: the frequency division multiplex (FDM), and the time division multiplex (TDM). Recently the sequency division multiplex (SDM) has been introduced.

PRINCIPLES OF SDM

As long as the cross-interference between the channels is minimum, the multiplex system can be formed. Expressing in mathematical terms, the condition of orthogonality should be

$$\int_{-T/2}^{T/2} f(k,\Theta)\, f(j,\Theta)\, d\,\Theta = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$$

where the function f can be sine or cosine, and can also be Walsh or any other orthogonal functions. Of course, some other factors must be taken into consideration in practical applications.

For sine and cosine functions, amplitude, frequency and phase modulations are often used. Walsh functions can be used as subcarriers for the transmission of signals. The, information may be contained in the amplitude, in the time base and in the time position of the Walsh subcarrier. Amplitude modulation is comparatively more important, because it enables several independent signals to transmit through one common channel. The equipment needed for the amplitude modulation is similar to that of FDM.

Figure 2 shows a block diagram of SDM, which is suitable for the transmission of the N signals. At the transmitter, before the signals arrive at modulators M, they first pass through the sequency low pass filter TP. The amplitude of signal is determined in the way that the mean square error keeps as minimum as possible. This can be done by expanding a signal into a set of Walsh series and keeping only the first several terms. In fact it is performed by making a signal passing the sequency low pass filter. Then the amplitude modulated signals are added in a SUM stage.

After the transmission the receiver must separate the induvidual signals. In the sequency techique the channel separation is affected by correlation. The input signal at receiver simultaneously arrives at all channel demodulators M. They are multiplied by the same Walsh subcarriers which were fed to the multipliers M in the transmitter. Then they pass through sequency low pass filter. If these filters are identical with those provided in the transmitter, the signals obtained at their outputs will be the signals which are of the same shape as original signals but are delayed with reseet to time.

CONSIDERATION OF SYNCHRONIZATION

Synchronization problem in a key factor of SDM. In the transmitter, the synchronization and clock signal needed by sequency low pass filter is produced by a generator. At the receiver, they are needed too, and must synchronize with those in the transmitter. Otherwise the distortion and cross-interference will be produced. In this system, period and bit synchronitions are required. The method is that a cross correlation function is used to produce an error signal, and track the signal by digital PLL, then reduce the error signal to minimum to get synchronization.

In mathematics there are some special rules for Walsh cross correlation function. Let

$$F_{i,j}(t_\theta) = \int_0^T Wal(i,t)Wal(j,t-t_\theta)dt,$$

when i is corresponding odd sequency and j is corresponding even sequency, $F_{i,j} \equiv 0$. When the sequency of corresponding i ans j both are odd, $F_{i,j} \neq 0$. According to this rule, we may select, for example, i = 1, j = 2, their sequencies both equal to 1 and select the Walsh subcarrier as even sequency, say Wal(3,t), Wal(4,t), Wal(7,t), Wal(8,t) and so on. In the Walsh generator of transmitter, besides the subcarriers, the WaL(2,t) is also produced. At the generator of receiver, besides the subcarriers, the Wal(1,t) is also produced. Using digital PLL, set the $F_{1,2}(t_\theta)$ to minimum (very close to zero). it means that $t\theta = 0$, the synchronization is achieved.
At the same time, since subcarriers are taken as even sequency, so $F_{i,j}(t_\theta) \equiv 0$ (j is corresponding even sequency), thus Wal(1,t) does not produce non-zero value of cross correlation with any other subcarriers. It does not effect the accuracy of synchronization. In the same way, the Wal(7,t) may be used instead of Wal(1,t), and the Wal(8,t) may be used instead of Wal(2,t), the synchronization can also be obtained. Thus we can use the Wal(k,t) as subcarriers, where k = 0,1,2,3,4,5,6,9,10,11,12,13,14,15(see figure 3). Therefore by properly selecting the Walsh functions used in synchronization, one may get more subcarriers, that is, for a given generator, the number of channels can be increased. For example, according to figure 3, if we take Wal(15,t) for synchronization, since $F_{15,15}(t_\theta) \neq 0$ and $F_{i,15}(t_\theta) \neq 0$ (i = 0, 1,...,14.), thus the synchronizationcan be achieved, provided the tracking fiter keeps the $F_{15,15}(t\theta)$ maximum. Generally speaking, this method may extend to $Wal(2^n\text{-}l,t)$, more subcarriers can be used. To summary, synchronization problem can be solved by using cross correlation functions of Walsh functions and digital phase locked loop.

CONCLUTION

Experimental results are given are given to justify the design consideration. A laboratory model with eight channels has been built. The accuracy of synchronization signal signal is about 1µs. The system has been tested by using eight signals. The accuracy of transfer is about 2%, for example, a signal of 3 volt DC is put to the sourse end, the resultant signal at the receiver end is 3,000 ± 40 mv. It has been used to demostrate the principle of SDM, which is regarded as the third method other than FDM and TDM.

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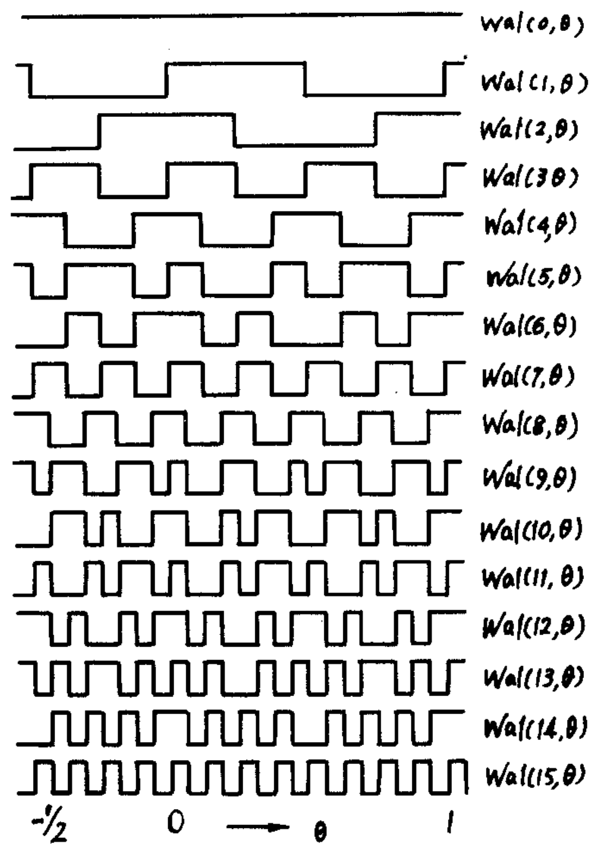

Figure 1. Walsh function

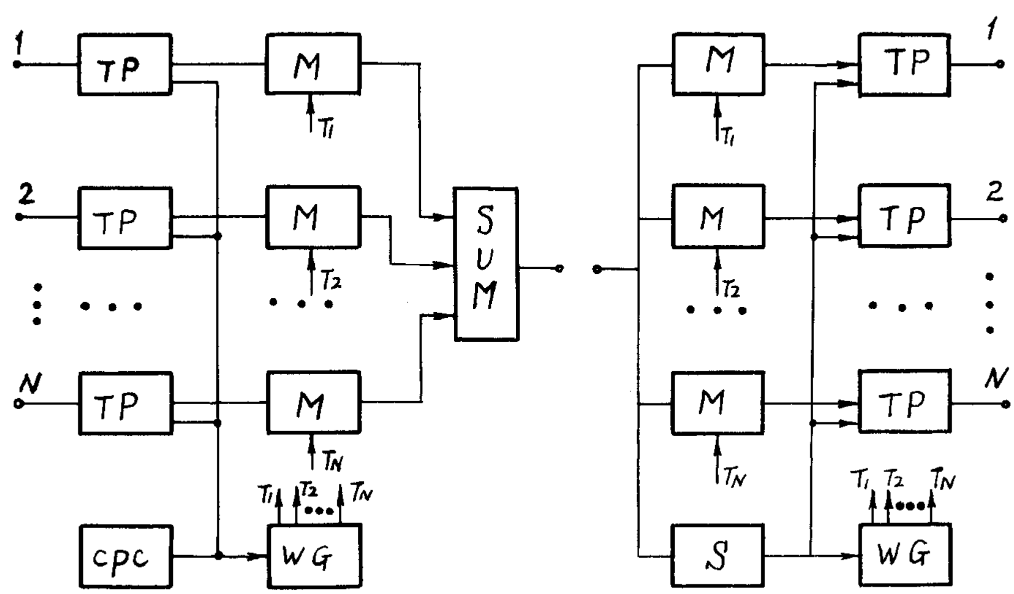

Figure 2. block diagram

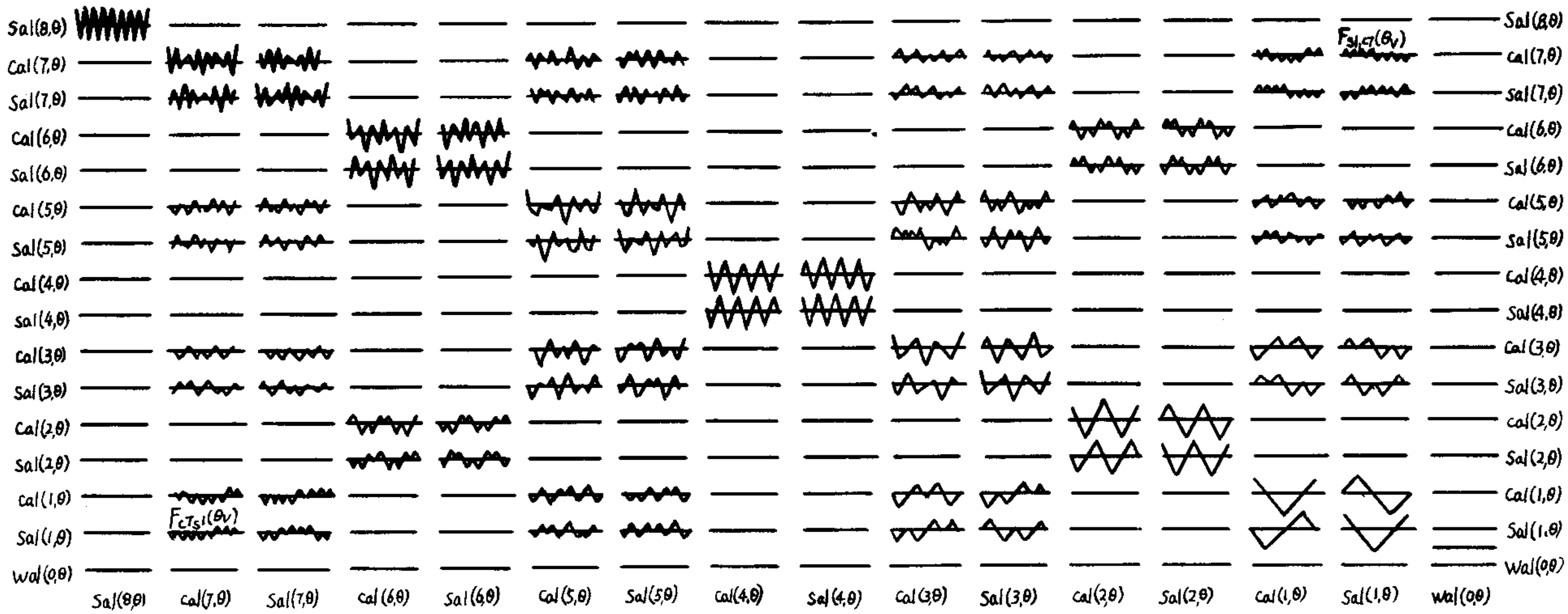

Figure 3 Correlation function for periodic Walsh function