RESEARCH AND RECOMMENDATION OF OPTIMUM GROUP SYNCHRONIZATION CODES FOR N = 7 -- 32

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ABSTRACT

In this paper, based on a series of research achievements [2,3,4,5,6], are examined the "The Optimum Frame Synchronization Codes" provided by J. L. Maury Jr. and F. J. Styles for IRIG Telemetry Standards USA, and furthermore, recommended a set of Optimum Group Synchronization Codes for China Telemetry Standards.

KEY WORDS: Optimum Code, Group Synchronization Code, False Synchronization Probability.

INTRODUCTION

In Telemetry, Remote-control and Digital Communication Systems, group synchronization plays a important role.

Since PCM Telemetry System came into being, many scholars have contributed themselves to the research on the theory and the searching method of optimum group synchronization codes. Among them, being the most representative, are Maury and Styles. They presented a set of Optimum Frame Synchronization Codes for N = 7 -- 30 for GODDARD SPACE PLIGHT CENTER using the classical exhaustion technique.[1] Afterwards, that table was brought into the American IRIG Standards as a recommendation of Optimum Frame Synchronization Codes. In 1984, as establishing the China Military Telemetry Standards, we also adapted that table in temporarily.

Twenty-five years have passed since paper [1] was published. Progress for searching Optimum Group Synchronization Codes was not great. In 1978, ESA presented Satellite Telemetry Standards [7], recommended the Optimum Frame Synchronization Codes of length N = 16, 24, 32. Among those codes, the codes of length 16 & 24 are the same as which in IRIG, only the code of length 32 is a new result. In about 1985, American Metraplex Corp. contributed a table of Optimum Frame Synchronization codes for N = 7 -- 33, but the first 24 codes of length 7 through 30 originated from IRIG.

NEED NEW DEVELOPMENT

Our work began at such a background:

(1) Some problems exist in the table presented by IRIG Telemetry Standards;
* the optimality of some codes is equivocal ;
* the application condition of optimum codes did not expound;
* optimum codes of length greater than 30 were not provided.

(2) The codes of length 31, 32, 33 recommended by ESA and Metraplex are actually not optimal;

(3) As the increase of word capacity and the transmission capacity in PCM communication systems, those optimum codes of length greater than 30 (such as 32, 40, 48, 50, 56, 60, 64 and even longer) are needed;

(4) Now, under the way of Classical Exhaustion Technique and the present computer speed, it's difficult to search out the optimum codes of length greater than 32.

Considering the facts mentioned above, it is imperative to develop new methods for searching optimum group synchronization codes and search out longer optimum codes.

OUR EFFORT

In the last ten years, we devoted ourselves to study new theory for construction optimum group synchronization codes, to develop new method for synthesizing and searching for optimum codes, to improve traditional exhaustion technique, to raise searching speed, looking forward to finding out the optimum codes of length greater than 30 and even more. In recent years, several methods have been developed, such as the "Subpeak Logic Method"[3,4], "Method of Synthetic Optimum Criterior"[5] and "Improved Exhaustion Technique". The theory has been put into practice, and much effect achieved.

(1) The Tables have been examined of Optimum Frame Synchronization Codes recommended by IRIG Telemetry Standards and ESA. Some defects in them have been pointed out. After that, those Tables have been revised, replenished, and expanded. And a set of new optimum codes have been recommended for China Telemetry Standards (provided codes of length 7 to 32 on the first stage, and shall provid codes of length 33 to 64 on the second stage.).

(2) The application conditions of optimum codes have been clarified.

(3) Some optimum codes have been provided for certain new Remote-control Telemetry and other PCM Systems. For example, a set of (four) conditional Optimum Group Synchronization Codes were provided for CK-8102 Multi-target Remote-control/ Telemetry/Direction Integrated System in 1981. In 1987, the group synchronization codes were examined of length 60 in American Meteorological Satellite TIROS, and discovered not optimal. Furthermore, we searched out a set of quasi-optimum codes (N = 60) [9] for the second generation China Meteorological Satellite being designed.

This paper is a final report for the first stage's work on optimum codes.

TABLES AND COMPARISON

A set of tables of new Optimum Group Synchronization Codes of length N = 7 -- 32 have been edited. In Table 1 to 4, the bit error rate is P = 0.10, and the error tolerance E is respectively 0, 1, 2, 3.

For comparison, the optimum codes originated form IRIG Telemetry Standards USA are listed in Table 5. This table is produced under the condition of P = 0.10 & E = 2.

After the comparison of the five tables, the following facts can be found out:

(1) The optimum codes in table 1 and table 2 with code-length N from 7 to 32 are in the same pattern as long as in the same length. That is, those 26 codes, under the error tolerance E = 0 & 1, resist in the optimum status.

(2) As error tolerance increased to 2, these codes marked with stars in table 3 (N = 7, 8, 9, 11, 15, 16) are different to those in table 1 & 2.

(3) As error tolerance being 3, the ten codes in table 4 marked with dark triangles (N = 7, 8, 9, 10, 12, 13, 14, 16, 18, 20) are different as the corresponding codes in table 1.

From the above comparisons, it could be seen that the optimality of codes is conditional and changes with error tolerance. In general, however, the optimality of a optimum code doesn't change until the error tolerance exceeding 10% of the code length. As error tolerance E exceeds that limit, the optimality of the optimum codes may be changed. (The examination of some optimum codes at error tolerance E = 4, 5, 6 confirmed the above conclusion. But to shorten this paper, those tables were omitted)

This conclusion is very important. It can be formularized as following:

$$E < 10\% * N \qquad (1)$$

As comparing Table 1 -- 4 with 5, we can find out:

(1) In Table 5, the optimum codes recommended by IRIG Standards are the same as those first 24 of 26 optimum codes in Table 2 searched out by our program. In Table 5 however, the first 10 codes (N = 7 -- 16) marked with white triangles have different probability of false synchronization (more accurately, the expectation value of the number of false synchronization occured during the overlap region) with the corresponding value in Table 3, while the rest 14 codes have the agreeable probability. The reason may be that Styles and his friend used different programs to calculate the first 10 optimum codes and the rest 14 and there would be some defects in the program calculating the first 10 codes.

(2) In Table 5, the six codes (N = 7, 8, 9, 11, 15, 16) marked with stars disagree with the corresponding codes in Table 1 & 2.

To make sure of our results, we applied several programs and different computers to search the optimum codes with N = 7 -- 32. As a result, we made the same outputs (with the same code pattern and same probability).

During our original searching work, because certain programing technique was not adapted, integer overflow once occured. After program revised, overflow solved, the output of different programs in different computers are completely the same. So, we would surmise that Styles's program for searching the optimum codes of length 7 -- 15 should have such problems as overflow.

There are no recommended codes of length 31 & 32 in IRIG Standards. To confirm the optimality of the codes in Table 1 to 4 of length 31 & 32, the calculation has been made for comparison with the codes provided by ESA and Metraplex. The results are listed in Table 6 & 7.

In Table 8 is presented the optimum code of length 33 searched out by our program. Compared with the code provided by Metraplex, this code is acturally optimal in both probability and autocorrelation. The codes of length exceeding 32 are beyond the discussing in this paper. We shall publish the optimum codes of length more than 33 in another paper in the near future.

A BRIEF TABLE

For briefness and convenience, we would recommend the brief Table of Optimum Group Synchronization Codes in Table 9.

REFERENCE

[1]. T. L. Maury Jr. and F. J. Styles, 1964, "Development of Optimum Frame Synchronization Codes for GODDARD SPACE FLIGHT CENTER". PCM Telemetry Standards. NTC'64, pp. 3.1/1-9, USA.

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[4]. Xie Qiucheng, 1986, "A New Searching Method for PCM Telemetry Optimum Frame Synchronization Codes". ITC'86, October, 86, USA.

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[8]. "Optimum Frame Synchronization Patterns for PCM Telemetry", Metraplex Corp., 1985, USA.

[9]. Cao Jie and Xie Qiucheng, 1987, "Analysis on the Optimum Group Synchronization Code of TIROS Satellite", Remote-control and Telemetry, No: 5, Vol.8, Oct., 1987, China.

Table 1. Bit error rate P = 0.10 Error tolerance E = 0

	N	Sequence / counts	P
*	N = 7	1101 000	P = 0.54370E-02
		7 3 3 2 1 0 0	
*	N = 8	1101 1000	P = 0.29846E-02
		8 4 2 3 2 1 0 0	
*	N = 9	1110 0100 0	P = 0.15977E-02
		9 5 3 3 3 2 0 0 0	
	N = 10	1101 1100 00	P = 0.68086E-03
		10 6 4 3 3 2 1 1 0 0	
*	N = 11	1110 1001 000	P = 0.31587E-03
		11 5 5 5 3 3 2 2 0 0 0	
	N = 12	1101 0110 0000	P = 0.14169E-03
		12 6 6 5 3 4 3 2 1 1 0 0	
	N = 13	1110 1011 0000 0	P = 0.65877E-04
		13 7 7 5 4 4 4 3 1 1 0 0 0	
	N = 14	1110 0110 1000 00	P = 0.33765E-04
		14 8 6 6 5 4 5 3 3 2 1 0 0 0	
*	N = 15	1110 0111 0100 000	P = 0.13703E-04
		15 9 7 5 6 5 4 4 3 3 2 1 0 0 0	
*	N = 16	1111 0100 1000 1000	P = 0.62532E-05
		16 8 8 8 7 6 4 5 3 3 2 2 1 0 0 0	
	N = 17	1111 0011 0101 0000 0	P = 0.29691E-05
		17 9 9 7 7 6 6 4 5 4 3 3 1 0 0 0 0	
	N = 18	1111 0011 0101 0000 00	P = 0.13288E-05
		18 10 10 8 8 7 6 5 5 5 3 3 2 1 0 0 0 0	
	N = 19	1111 1001 1001 0100 000	P = 0.61549E-06
		19 11 9 9 9 7 7 6 5 5 4 4 3 2 0 0 0 0 0	
	N = 20	1110 1101 1110 0010 0000	P = 0.31499E-06
		20 12 10 10 9 9 8 7 5 5 4 1 3 3 2 1 1 0 0 0	
	N = 21	1110 1110 1001 0110 0000 0	P = 0.15505E-06
		21 11 11 11 8 8 8 7 7 7 5 4 4 4 2 1 1 1 0 0 0	
	N = 22	1111 0011 0110 1010 0000 00	P = 0.70123E-07
		22 12 12 12 10 9 8 8 7 6 5 5 5 4 3 2 2 1 0 0 0 0	
	N = 23	1111 0101 1100 1101 0000 000	P = 0.34553E-07
		23 13 13 11 11 10 10 9 8 4 7 5 6 4 3 3 2 1 1 0 0 0 0	
	N = 24	1111 1011 0011 0000 1010 0000	P = 0.16910E-07
		24 14 12 12 12 10 10 10 9 8 7 5 5 3 5 4 3 2 2 0 0 0 0 0	
	N = 25	1111 1001 0110 1110 0010 0000 0	P = 0.84263E-08
		25 15 13 13 11 11 10 8 8 9 7 8 6 5 5 4 4 3 3 1 0 0 0 0 0	
	N = 26	1111 1010 0110 1011 0001 0000 00	P = 0.40548E-08
		26 14 14 14 12 12 11 8 9 10 8 7 7 7 5 5 4 4 3 2 1 0 0 0 0 0	
	N = 27	1111 1010 1101 0011 0011 0000 000	P = 0.19913E-08
		27 15 13 15 13 13 10 10 10 9 9 8 7 7 6 6 5 4 4 3 1 1 0 0 0 0 0	
	N = 28	1111 0101 1110 0101 1001 1000 0000	P = 0.99050E-09
		28 16 14 14 12 13 13 12 11 8 10 8 8 8 6 6 4 4 4 4 3 2 1 1 0 0 0 0	
	N = 29	1111 0101 1110 0110 0110 1000 0000 0	P = 0.48773E-09
		29 17 15 15 15 14 12 13 11 10 10 9 9 8 4 7 5 6 4 3 3 2 2 1 1 0 0 0 0	
	N = 30	1111 1010 1111 0011 0011 0100 0000 00	P = 0.23938E-09
		30 18 16 16 16 14 13 13 12 11 11 10 9 8 5 8 5 6 5 4 3 3 2 1 1 0 0 0 0 0	
	N = 31	1111 1110 1100 0110 1001 0001 0100 000	P = 0.12017E-09
		31 17 17 17 15 15 14 12 12 12 12 11 9 8 8 7 6 7 5 6 4 3 3 3 1 1 0 0 0 0 0	
	N = 32	1111 1101 1011 0001 1000 0101 0100 0000	P = 0.59621E-10
		32 18 18 16 16 16 14 14 14 11 12 11 9 10 9 8 7 6 7 4 5 5 4 3 2 2 0 0 0 0 0 0	

Table 2. Bit error rate P = 0.10 Error tolerance E = 1

	N	Code / Distribution	P
*	N = 7	1101 000	
		7 3 3 2 1 0 0	P = 0.11757E+00
*	N = 8	1101 1000	
		8 4 2 3 2 1 0 0	P = 0.66530E-01
*	N = 9	1110 0100 0	
		9 5 3 3 3 2 0 0 0	P = 0.38654E-01
	N = 10	1101 1100 00	
		10 6 4 3 3 2 1 1 0 0	P = 0.18293E-01
*	N = 11	1110 1001 000	
		11 5 5 5 3 3 2 2 0 0 0	P = 0.87343E-02
	N = 12	1101 0110 0000	
		12 6 6 5 3 4 3 2 1 1 0 0	P = 0.42053E-02
	N = 13	1110 1011 0000 0	
		13 7 7 5 4 4 4 3 1 1 0 0 0	P = 0.20903E-02
	N = 14	1110 0110 1000 00	
		14 8 6 6 5 4 5 3 3 2 1 0 0 0	P = 0.10883E-02
*	N = 15	1110 0111 0100 000	
		15 9 7 5 6 5 4 4 3 3 2 1 0 0 0	P = 0.46369E-03
*	N = 16	1111 0100 1000 1000	
		16 8 8 8 7 6 4 5 3 3 2 2 1 0 0 0	P = 0.21708E-03
	N = 17	1111 0011 0101 0000 0	
		17 9 9 7 7 6 6 4 5 4 3 3 1 0 0 0 0	P = 0.10292E-03
	N = 18	1111 0011 0101 0000 00	
		18 10 10 8 8 7 6 5 5 5 3 3 2 1 0 0 0 0	P = 0.47740E-04
	N = 19	1111 1001 1001 0100 000	
		19 11 9 9 9 7 7 6 5 5 4 4 3 2 0 0 0 0 0	P = 0.22091E-04
	N = 20	1110 1101 1110 0010 0000	
		20 12 10 10 9 9 8 7 5 5 4 1 3 3 2 1 1 0 0 0	P = 0.11830E-04
	N = 21	1110 1110 1001 0110 0000 0	
		21 11 11 11 8 8 8 7 7 7 5 4 4 4 2 1 1 1 0 0 0	P = 0.58156E-05
	N = 22	1111 0011 0110 1010 0000 00	
		22 12 12 12 10 9 8 8 7 6 5 5 5 4 3 2 2 1 0 0 0 0	P = 0.26456E-05
	N = 23	1111 0101 1100 1101 0000 000	
		23 13 13 11 11 10 10 9 8 4 7 5 6 4 3 3 2 1 1 0 0 0 0	P = 0.13318E-05
	N = 24	1111 1011 0011 0000 1010 0000	
		24 14 12 12 12 10 10 10 9 8 7 5 5 3 5 4 3 2 2 0 0 0 0 0	P = 0.65757E-06
	N = 25	1111 1001 0110 1110 0010 0000 0	
		25 15 13 13 11 11 10 8 8 9 7 8 6 5 5 4 4 3 3 1 0 0 0 0 0	P = 0.33363E-06
	N = 26	1111 1010 0110 1011 0001 0000 00	
		26 14 14 14 12 12 11 8 9 10 8 7 7 7 5 5 4 4 3 2 1 0 0 0 0 0	P = 0.16173E-06
	N = 27	1111 1010 1101 0011 0011 0000 000	
		27 15 13 15 13 13 10 10 10 9 9 8 7 7 6 6 5 4 4 3 1 1 0 0 0 0 0	P = 0.80493E-07
	N = 28	1111 0101 1110 0101 1001 1000 0000	
		28 16 14 14 12 13 13 12 11 8 10 8 8 8 6 6 4 4 4 4 3 2 1 1 0 0 0 0	P = 0.40622E-07
	N = 29	1111 0101 1110 0110 0110 1000 0000 0	
		29 17 15 15 15 14 12 13 11 10 10 9 9 8 4 7 5 6 4 3 3 2 2 1 1 0 0 0 0	P = 0.20332E-07
	N = 30	1111 1010 1111 0011 0011 0100 0000 00	
		30 18 16 16 16 14 13 13 12 11 11 10 9 8 5 8 5 6 5 4 3 3 2 1 1 0 0 0 0 0	P = 0.10128E-07
	N = 31	1111 1110 1100 0110 1001 0001 0100 000	
		31 17 17 17 15 15 14 12 12 12 12 11 9 8 8 7 6 7 5 6 4 3 3 3 1 1 0 0 0 0 0	P = 0.51925E-08
	N = 32	1111 1101 1011 0001 1000 0101 0100 0000	
		32 18 18 16 16 16 14 14 14 11 12 11 9 10 9 8 7 6 7 4 5 5 4 3 2 2 0 0 0 0 0 0	P = 0.26217E-08

Table 3. Bit error rate P = 0.10 Error tolerance E = 2

	N	Code	Distance profile	P
*	N = 7	1011 000	7 3 2 2 1 1 0	P = 0.78544E+00
*	N = 8	1011 1000	8 4 3 1 2 1 1 0	P = 0.52524E+00
*	N = 9	1011 1000 0	9 5 4 2 2 1 1 1 0	P = 0.33864E+00
	N = 10	1101 1100 00	10 6 4 3 3 2 1 1 0 0	P = 0.19171E+00
*	N = 11	1011 0111 000	11 5 4 4 3 3 2 2 1 1 0	P = 0.94337E-01
	N = 12	1101 0110 0000	12 6 6 5 3 4 3 2 1 1 0 0	P = 0.52617E-01
	N = 13	1110 1011 0000 0	13 7 7 5 4 4 4 3 1 1 0 0 0	P = 0.28556E-01
	N = 14	1110 0110 1000 00	14 8 6 6 5 4 5 3 3 2 1 0 0 0	P = 0.15242E-01
*	N = 15	1110 1100 1010 000	15 7 7 7 6 5 5 3 4 2 2 1 0 0 0	P = 0.66302E-02
*	N = 16	1111 0110 0010 1000	16 8 8 6 7 6 5 5 2 3 3 2 1 0 0 0	P = 0.33562E-02
	N = 17	1111 0011 0101 0000 0	17 9 9 7 7 6 6 4 5 4 3 3 1 0 0 0 0	P = 0.16579E-02
	N = 18	1111 0011 0101 0000 00	18 10 10 8 8 7 6 5 5 5 3 3 2 1 0 0 0 0	P = 0.82293E-03
	N = 19	1111 1001 1001 0100 000	19 11 9 9 9 7 7 6 5 5 4 4 3 2 0 0 0 0 0	P = 0.38368E-03
	N = 20	1110 1101 1110 0010 0000	20 12 10 10 9 9 8 7 5 5 4 1 3 3 2 1 1 0 0 0	P = 0.21751E-03
	N = 21	1110 1110 1001 0110 0000 0	21 11 11 11 8 8 8 7 7 7 5 4 4 4 2 1 1 1 0 0 0	P = 0.10506E-03
	N = 22	1111 0011 0110 1010 0000 00	22 12 12 12 10 9 8 8 7 6 5 5 5 4 3 2 2 1 0 0 0 0	P = 0.49058E-04
	N = 23	1111 0101 1100 1101 0000 000	23 13 13 11 11 10 10 9 8 4 7 5 6 4 3 3 2 1 1 0 0 0 0	P = 0.25326E-04
	N = 24	1111 1011 0011 0000 1010 0000	24 14 12 12 12 10 10 10 9 8 7 5 5 3 5 4 3 2 2 0 0 0 0 0	P = 0.12547E-04
	N = 25	1111 1001 0110 1110 0010 0000 0	25 15 13 13 11 11 10 8 8 9 7 8 6 5 5 4 4 3 3 1 0 0 0 0 0	P = 0.64491E-05
	N = 26	1111 1010 0110 1011 0001 0000 00	26 14 14 14 12 12 11 8 9 10 8 7 7 7 5 5 4 4 3 2 1 0 0 0 0 0	P = 0.31438E-05
	N = 27	1111 1010 1101 0011 0011 0000 000	27 15 13 15 13 13 10 10 10 9 9 8 7 7 6 6 5 4 4 3 1 1 0 0 0 0 0	P = 0.15830E-05
	N = 28	1111 0101 1110 0101 1001 1000 0000	28 16 14 14 12 13 13 12 11 8 10 8 8 8 6 6 4 4 4 4 3 2 1 1 0 0 0 0	P = 0.80360E-06
	N = 29	1111 0101 1110 0110 0110 1000 0000 0	29 17 15 15 15 14 12 13 11 10 10 9 9 8 4 7 5 6 4 3 3 2 2 1 1 0 0 0 0	P = 0.40926E-06
	N = 30	1111 1010 1111 0011 0011 0100 0000 00	30 18 16 16 16 14 13 13 12 11 11 10 9 8 5 8 5 6 5 4 3 3 2 1 1 0 0 0 0 0	P = 0.20700E-06
	N = 31	1111 1110 1100 0110 1001 0001 0100 000	31 17 17 17 15 15 14 12 12 12 12 11 9 8 8 7 6 7 5 6 4 3 3 3 1 1 0 0 0 0 0	P = 0.10771E-06
	N = .32	1111 1101 1011 0001 1000 0101 0100 0000	32 18 18 16 16 16 14 14 14 11 12 11 9 10 9 8 7 6 7 4 5 5 4 3 2 2 0 0 0 0 0 0	P = 0.55268E-07

Table 4. Bit error rate P = 0.10 Error tolerance E = 3

	N	Sequence / profile	P
▲	N = 7	1001 001	P = 0.24388E+01
		7 2 1 4 1 0 1	
▲	N = 8	1011 0001	P = 0.18791E+01
		8 3 2 2 2 2 0 1	
▲	N = 9	1011 0000 1	P = 0.14209E+01
		9 4 3 3 1 2 2 0 1	
▲	N = 10	1110 0010 00	P = 0.89607E+00
		10 5 3 3 3 2 1 1 1 1	
	N = 11	1011 0111 000	P = 0.51996E+00
		11 5 4 4 3 3 2 2 1 1 0	
▲	N = 12	1011 0111 0000	P = 0.33460E+00
		12 6 5 5 3 3 2 3 1 1 1 0	
▲	N = 13	1110 0110 1000 0	P = 0.20244E+00
		13 7 5 5 4 4 4 3 3 1 0 0 0	
▲	N = 14	1011 1011 1100 00	P = 0.11795E+00
		14 8 7 5 5 4 4 3 2 2 1 1 1 0	
	N = 15	1110 1100 1010 000	P = 0.57748E-01
		15 7 7 7 6 5 5 3 4 2 2 1 0 0 0	
▲	N = 16	1111 0011 0101 0000	P = 0.31124E-01
		16 8 8 6 6 6 5 4 4 4 3 2 0 0 0 0	
	N = 17	1111 0011 0101 0000 0	P = 0.16508E-01
		17 9 9 7 7 6 6 4 5 4 3 3 1 0 0 0 0	
▲	N = 18	1111 0101 1001 1000 00	P = 0.86685E-02
		18 10 8 8 8 7 6 6 4 4 4 4 2 1 0 0 0 0	
	N = 19	1111 1001 1001 0100 000	P = 0.43219E-02
		19 11 9 9 9 7 7 6 5 5 4 4 3 2 0 0 0 0 0	
▲	N = 20	1111 0011 0110 1010 0000	P = 0.24162E-02
		20 10 10 10 8 7 7 7 6 5 4 3 4 3 3 1 0 0 0 0	
	N = 21	1110 1110 1001 0110 0000 0	P = 0.12305E-02
		21 11 11 11 8 8 8 7 7 7 5 4 4 4 2 1 1 1 0 0 0	
	N = 22	1111 0011 0110 1010 0000 00	P = 0.60759E-03
		22 12 12 12 10 9 8 8 7 6 5 5 5 4 3 2 2 1 0 0 0 0	
	N = 23	1111 0101 1100 1101 0000 000	P = 0.32410E-03
		23 13 13 11 11 10 10 9 8 4 7 5 6 4 3 3 2 1 1 0 0 0 0	
	N = 24	1111 1011 0011 0000 1010 0000	P = 0.16098E-03
		24 14 12 12 12 10 10 10 9 8 7 5 5 3 5 4 3 2 2 0 0 0 0 0	
	N = 25	1111 1001 0110 1110 0010 0000 0	P = 0.83377E-04
		25 15 13 13 11 11 10 8 8 9 7 8 6 5 5 4 4 3 3 1 0 0 0 0 0	
	N = 26	1111 1010 0110 1011 0001 0000 00	P = 0.40933E-04
		26 14 14 14 12 12 11 8 9 10 8 7 7 7 5 5 4 4 3 2 1 0 0 0 0 0	
	N = 27	1111 1010 1101 0011 0011 0000 000	P = 0.20874E-04
		27 15 13 15 13 13 10 10 10 9 9 8 7 7 6 6 5 4 4 3 1 1 0 0 0 0 0	
	N = 28	1111 0101 1110 0101 1001 1000 0000	P = 0.10529E-04
		28 16 14 14 12 13 13 12 11 8 10 8 8 8 6 6 4 4 4 4 3 2 1 1 0 0 0 0	
	N = 29	1111 0101 1110 0110 0110 1000 0000 0	P = 0.54775E-05
		29 17 15 15 15 14 12 13 11 10 10 9 9 8 4 7 5 6 4 3 3 2 2 1 1 0 0 0 0	
	N = 30	1111 1010 1111 0011 0011 0100 0000 00	P = 0.28225E-05
		30 18 16 16 16 14 13 13 12 11 11 10 9 8 5 8 5 6 5 4 3 3 2 1 1 0 0 0 0 0	
	N = 31	1111 1110 1100 0110 1001 0001 0100 000	P = 0.14675E-05
		31 17 17 17 15 15 14 12 12 12 12 11 9 8 8 7 6 7 5 6 4 3 3 3 1 1 0 0 0 0 0	
	N = 32	1111 1101 1011 0001 1000 0101 0100 0000	P = 0.76277E-06
		32 18 18 16 16 16 14 14 14 11 12 11 9 10 9 8 7 6 7 4 5 5 4 3 2 2 0 0 0 0 0 0	

Table 5. From IRIG Telemetry Standards P = 0.10 E = 2

	N	Pattern / Distribution	P	
*	N = 7	1011 000 7 3 2 2 1 1 0	P = 5.723E-01	△
*	N = 8	1011 1000 8 4 3 1 2 1 1 0	P = 4.235E-01	△
*	N = 9	1011 1000 0 9 5 4 2 2 1 1 1 0	P = 2.950E-01	△
	N = 10	1101 1100 00 10 6 4 3 3 2 1 1 0 0	P = 1.783E-01	△
*	N = 11	1011 0111 000 11 5 4 4 3 3 2 2 1 1 0	P = 9.065E-02	△
	N = 12	1101 0110 0000 12 6 6 5 3 4 3 2 1 1 0 0	P = 5.142E-02	△
	N = 13	1110 1011 0000 0 13 7 7 5 4 4 4 3 1 1 0 0 0	P = 2.821E-02	△
	N = 14	1110 0110 1000 00 14 8 6 6 5 4 5 3 3 2 1 0 0 0	P = 1.514E-02	△
*	N = 15	1110 1100 1010 000 15 7 7 7 6 5 5 3 4 2 2 1 0 0 0	P = 6.611E-03	△
*	N = 16	1111 0110 0010 1000 16 8 8 6 7 6 5 5 2 3 3 2 1 0 0 0	P = 3.460E-03	△
	N = 17	1111 0011 0101 0000 0 17 9 9 7 7 6 6 4 5 4 3 3 1 0 0 0 0	P = 1.657E-03	
	N = 18	1111 0011 0101 0000 00 18 10 10 8 8 7 6 5 5 5 3 3 2 1 0 0 0 0	P = 8.228E-04	
	N = 19	1111 1001 1001 0100 000 19 11 9 9 9 7 7 6 5 5 4 4 3 2 0 0 0 0 0	P = 3.837E-04	
	N = 20	1110 1101 1110 0010 0000 20 12 10 10 9 9 8 7 5 5 4 1 3 3 2 1 1 0 0 0	P = 2.175E-04	
	N = 21	1110 1110 1001 0110 0000 0 21 11 11 11 8 8 8 7 7 7 5 4 4 4 2 1 1 1 0 0 0	P = 1.051E-04	
	N = 22	1111 0011 0110 1010 0000 00 22 12 12 12 10 9 8 8 7 6 5 5 5 4 3 2 2 1 0 0 0 0	P = 4.906E-05	
	N = 23	1111 0101 1100 1101 0000 000 23 13 13 11 11 10 10 9 8 4 7 5 6 4 3 3 2 1 1 0 0 0 0	P = 2.533E-05	
	N = 24	1111 1011 0011 0000 1010 0000 24 14 12 12 12 10 10 10 9 8 7 5 5 3 5 4 3 2 2 0 0 0 0 0	P = 1.255E-05	
	N = 25	1111 1001 0110 1110 0010 0000 0 25 15 13 13 11 11 10 8 8 9 7 8 6 5 5 4 4 3 3 1 0 0 0 0 0	P = 6.449E-06	
	N = 26	1111 1010 0110 1011 0001 0000 00 26 14 14 14 12 12 11 8 9 10 8 7 7 7 5 5 4 4 3 2 1 0 0 0 0 0	P = 3.144E-06	
	N = 27	1111 1010 1101 0011 0011 0000 000 27 15 13 15 13 13 10 10 10 9 9 8 7 7 6 6 5 4 4 3 1 1 0 0 0 0 0	P = 1.583E-06	
	N = 28	1111 0101 1110 0101 1001 1000 0000 28 16 14 14 12 13 13 12 11 8 10 8 8 8 6 6 4 4 4 4 3 2 1 1 0 0 0 0	P = 8.036E-07	
	N = 29	1111 0101 1110 0110 0110 1000 0000 0 29 17 15 15 15 14 12 13 11 10 10 9 9 8 4 7 5 6 4 3 3 2 2 1 1 0 0 0 0	P = 4.093E-07	
	N = 30	1111 1010 1111 0011 0011 0100 0000 00 30 18 16 16 16 14 13 13 12 11 11 10 9 8 5 8 5 6 5 4 3 3 2 1 1 0 0 0 0 0	P = 2.070E-07	

Table 6. Three code: of length 31 (P = 0.10)

CODE SOURCE	PROBABILITY OF FALSE SYNC E = 0	E = 1	E = 2	E = 3	E = 4	CODE PATTERN
Optmum code	(1) 0.12017E-9	(1) 0.51925E-8	(1) 0.10771E-6	(1) 0.14675E-5	(1) 0.15118E-4	1111,1110,1100,0110,1001,0001,0100,000
Metra-plex	(2) 0.12941E-9	(2) 0.55504E-8	(2) 0.12494E-6	(2) 0.19532E-5	(2) 0.24138E-4	1111,1110,0110,1111,1010,1000,0100,000
IRIG	(3) 0.83417E-8	(3) 0.29006E-6	(3) 0.46329E-5	(3) 0.46367E-4	(3) 0.33244E-3	1010,1001,0101,1010,0101,1001,0101,000

Table 7. Four Codes of Length 32 (P = 0.10)

CODE SOURCE	PROBABILITY OF FALSE SYNC E = 0	E = 1	E = 2	E = 3	E = 4	CODE PATTERN
Optmum code	(1) 0.59621E-10	(1) 0.26217E-08	(1) 0.55268E-07	(1) 0.76277E-06	(1) 0.79372E-05	1111,1101,1011,0001,1000,0101,0100,0000
ESA	(2) 0.60928E-10	(2) 0.27664E-08	(2) 0.62372E-07	(2) 0.96964E-06	(3) 0.11948E-04	1111,1010,1111,0011,0011,0100,0000,0000
Metra-plex	(3) 0.64239E-10	(3) 0.30485E-08	(4) 0.73105E-07	(4) 0.12094E-06	(4) 0.15412E-04	1111,1110,0110,1011,0010,1000,0100,0000
289 P.R.C	(4) 0.66779E-10	(4) 0.31021E-08	(3) 0.69890E-07	(3) 0.10304E-06	(2) 0.11222E-04	1111,0010,1101,0101,0011,0011,0000,0000

Table 8. Two Codes of Length 33 (P = 0.10)

CODE SOURCE	PROBABILITY OF FALSE SYNC E = 0	E = 1	E = 2	E = 3	E = 4	CODE PATTERN
Optmum code	(1) 0.29611E-10	(1) 0.13273E-08	(1) 0.28565E-07	(1) 0.40371E-06	(1) 0.43308E-05	1111,1110,1101,1000,1100,0010,1010,0000,0
Metra-plex	(3) 0.32751E-10	(3) 0.15768E-08	(4) 0.37801E-07	(4) 0.61408E-06	(4) 0.76121E-05	1111,1011,1010,0111,0100,1010,0100,1000,0

Table 9. A Brief Table for Optimum Group Synchronization Codes (N = 8, 10, 12, 16, 20, 24, 28, 30, 32)

CODE LENGTH	CODE PATTERN	PROBABILITY OF FALSE SYNCHRONIZATION E = 0	E = 1	E = 2	E = 3	E = 4
N=8	1101,1000	P = 0.29846E-02	P = 0.66530E-01			
N=8	1011,1000			P = 0.52524E+00		
N=12	1101,0110,0000	P = 0.14169E-03	P = 0.42053E-02	P = 0.52617E-01		
N=16	1111,0100,1000,1000	P = 0.33765E-06	P = 0.10883E-02			
N=16	1111,0110,0010,1000			P = 0.33562E-02		
N=20	1110,1101,1110,0010,0000	P = 0.31499E-02	P = 0.11830E-01	P = 0.21751E-02		
N=20	1111,0011,0110,1010,0000				P = 0.24162E-02	
N=24	1111,1011,0011,0000,1010,0000	P = 0.16910E-07	P = 0.65757E-06	P = 0.12547E-04	P = 0.16098E-03	
N=28	1111,0101,1110,0101,1001,1000,0000	P = 0.99050E-09	P = 0.40622E-07	P = 0.80360E-04	P = 0.10529E-04	
N=30	1111,1010,1111,0011,0011,0100,0000,00	P = 0.23938E-09	P = 0.10128E-07	P = 0.20700E-06	P = 0.28255E-05	
N=32	1111,1101,1011,0001,1000,0101,0100,0000	P = 0.59621E-10	P = 0.26217E-08	P = 0.55268E-07	P = 0.76277E-06	P = 0.79372E-05