A QUANTIZED EUCLIDEAN SOFT-DECISION MAXIMUM LIKELIHOOD SEQUENCE DECODER: A CONCEPT FOR SPECTRALLY EFFICIENT TM SYSTEMS

Frank Carden, Professor
Electrical Engineering Department
New Mexico State University
Las Cruces, New Mexico 88003

ABSTRACT

Trellis-Coded Modulation, TCM, combines the processes of modulation and encoding to achieve an overall coding gain that is usually greater than that achieved by performing the operations independently. This paper is concerned with utilizing TCM and 8-PSK to achieve a spectrally efficient modulation scheme with a constant envelope. Since TCM involves Euclidean distance as a metric, the concept of a quantized decoder is developed to decrease the decoding time.

INTRODUCTION

The function of modulation and error-correction coding in traditional communication systems are separate operations. In order to improve the bit error rate conventional encoders add redundant check symbols to a fixed number of information bits. This decreases the information rate. In order to compensate for this, a higher order modulation scheme is used with an increased power requirement or an increased modulation rate is employed with an attendant increased bandwidth. Using these two operations separately gives marginally improved performance. For example, going from 4-PSK to 8-PSK and using a 2/3 binary convolutional code and a complex 64 state Viterbi decoder, the performance is similar to that of an uncoded 4-PSK system (1).

Ungerbock (2) (3) and others have developed a Trellis-Coded Modulation, TCM, technique that combines the processes of modulation and encoding. The coding gain varies between 3 and 6 dbs. depending upon the complexity of the system. There has been substantial work in this area during the past six years. The CCITT has adopted a TCM format for the use in high data rate modems over voice band channels and uses a QAM constellation. For many telemetry systems this is unsatisfactory because the envelope of the modulation is not constant, the equipment necessary is fairly large and the data rate may not be adequate.

This work is concerned with the utilization of TCM in telemetering systems. Specifically, the concept of a quantized Euclidean soft-decision maximum-likelihood sequence decoder (Viterbi decoder) is developed which increases the speed of demodulating and decoding.

TCM ENCODING PROCEDURE

In trellis coded modulation, m information bits are encoded by means of a R = m/m+l convolutional encoder into m+1 bits which are mapped into one of the 2^{m+1} signal set. Although the 2^{m+1} signal set has a reduced minimum distance, compared to the original 2^{m} signal set this is no longer the limiting factor. The dependency created by the convolutional encoder between the transmitted symbols make the Euclidean distance between the allowed sequences the determining factor.

Specifically, whenever m information bits are to be transmitted per symbol of the expanded signal set, $\hat{m}$ of the in bits are expanded into $\hat{m}$+1 bits by the $\frac{\hat{m}}{\hat{m}+1}$ rate convolutional encoder.

The remaining uncoded m-$\hat{m}$ bits are fed directly into the mapping process as shown in Figure 1.

The mapping is achieved according to the following rules (2):

Rule 1: All parallel transitions in the trellis receive maximum Euclidean distance.
Rule 2: All transitions diverging or merging into a trellis state receive the next maximum Euclidean distance.

The above rules are implemented by using “set partitioning”.

That is, a conventional trellis is established by the convolutional encoder. This trellis is then expanded by adding parallel paths. For example, in expanding a 2^{m} (when m=2) or 4-PSK signal set into a 2^{m+1} or 8-PSK signal set $\hat{m}$=1 and m-$\hat{m}$=1.

DECODING PROCEDURE

The Viterbi algorithm is a maximum-likelihood sequence estimation that determines the allowed sequence with the shortest distance from the received sequence. This is achieved in a recursive manner by computing the distance between the received symbol and the allowed transitional symbol between node k and node k+l for each branch. This metric is then attached to the respective branch. The branch metrics for the branches entering each node are compared and only the smallest is retained as a “survivor” at that node. This process is repeated for the k+l to the k+2 node and so on. The metrics for the extended paths are computed until a sequence of transmitted zeros drive the paths to the all zero path. At this point the shortest path is selected.

It should be noted, however, that the Viterbi Decoder utilized in TCM uses Euclidean distance as a metric and path lengths will no longer be determined by simply binary addition. In order to reduce the longer computational time required for Euclidean operations, a quantized Euclidean model is developed and studied but not simulated; hence refinements are anticipated in this development.

8-PSK TRELLIS CODED MODULATION UTILIZING A QUANTIZED EUCLIDEAN DECODER

The 8-PSK TCM format seems attractive because of its simplicity and the 3db coding gain over 4-PSK. A block diagram for the encoded 8-PSK system is shown in Figure 2. An expanded version of the $R = \frac{1}{2}$ convolutional encoder is shown in Figure 3-a and a trellis diagram strictly for the convolutional encoder is shown in Figure 3-b. Once the encoder is in one state there can only be two outputs from the given state depending upon whether X_1 is a 1 or a zero. The insertion of X_0 the uncoded bit now requires four outputs from each state depending upon whether X_0 X_1 are (0,0) (0,1) (1,0) (1,1). Parallel transitions are introduced and the resulting trellis is shown in Figure 4-a.

Set partitioning is now used according to the above two rules to map the binary transition outputs into the expanded symbol set. Set partitioning also aids in understanding the transitions from state to state in the TCM trellis.

The set partitioning is shown in Figure 5-a. Figure 5-b shows an eight PSK signal set with the “set partitioning” assignments achieved by using Figure 5-a and Figure 4 and according to Rule 2. The four signals assigned to the four branches entering or diverging from a node are either B0 or B1 and have the next largest Euclidean distance. The two parallel paths entering or leaving a node are selected from C_0, C_1, C_2 or C_3 since these signals have the greatest distance. The impact of these assignments will be seen shortly in the Viterbi decoding.

8-PSK TCM DECODING

In order to gain insight into the decoding procedure an example will be given. Whenever one of the eight PSK signals is received, it is demodulated resulting in an I and Q coordinate. Say cos 270° was transmitted and with demodulated coordinates indicated as a star on the unit circle of Figure 6. The initial decoding need only determine if 001 (cos 270°) or 000 (cos 90°) was transmitted. This decision is made outboard of the decoder and compares the two signals with the maximum distance giving a gain of d_3/d_0 or

3db when compared with deciding which of two nearest neighbors was transmitted. The decoder then operates on the four nearest neighbors only. In this case 001, 011, 101, and 110. This decision concerning 000 or 001 immediately eliminates 4 of the 8 paths between nodes allowing only 1 of each of the paths in parallel. Hence the trellis of Figure 4-a becomes the trellis of Figure 4-b. The distance from the received coordinates to the four signals is attached to each branch as a metric. This process is repeated for each received signal until the process is terminated and the shortest path is chosen.

Several observations are in order.

i). The outboard decision eliminates half the paths that would have to be added and compared in the Viterbi Algorithm.

ii). In the Sig to noise range of practical operation only single error events or nearest neighbor errors have a high expectation of occurring.

The implications of ii and Rule 1 and 2 is that whenever the decoder exists in one state, and since no nearest neighbors are allowed on exit branches by Rule 1 and 2 and even if a nearest neighbor detection error is made (and although a non-zero metric will be added to that branch,) the error will be corrected at the end of the Viterbi algorithm. Nearest neighbor type errors are the primary ones to be concerned with.

An example of a sequence with a correctable or nearest neighbor error is given next. Assume the decoder is in the zero state and a sequence is transmitted as follows:

TABLE I

Binary Code	Symbol	Time Increment
111	$\cos(T_o t + 180)$	1
010	$\cos(T_o t + 45°)$	2
001	$\cos(T_o t + 270)$	3
101	$\cos(T_o t + 315)$	4
011	$\cos(T_o t + 225)$	5
100	$\cos(T_o t + 135)$	6
111	$\cos(T_o t + 180)$	7

The transmitted sequence is shown in Figure 6 with continuous lines and the received sequence is shown with a dotted line. A fairly sever 15° phase error due to fitter, say, is built in each received signal as a worst case. The fourth symbol transmitted is $\cos(T_o t+315)$ and is detected as a nearest neighbor error, $\cos T_o t$. The correct path is shown in Figure 7. This path was selected by the Viterbi algorithm as shown in Figure 8 since it had the smallest total path link between it and the detected sequence.

Note the non-zero metric that was added to the correct path during the fourth time period increases the length of the correct path but it is correctable because the detected symbol is not allowed. That is, the decoder, on the correct path is in state #1 and the allowed exit symbol from that state once the received coordinates determine which 180° region to work is cos 225. This is the result of Rule #2 that prohibits nearest neighbors from being in the same exit set.

THE QUANTIZED EUCLIDEAN DECODER, QED

In order to decrease the demodulating time a Quantized Euclidean decoder model was developed. Since TCM decoding requires Euclidean operations instead of simple binary operations, quantization of the Euclidean space will reduce the processing time.

The concept of the quantized model is developed next. For 8-PSK the signal constellation is shown in Figire 9. The 45° sector between each symbol phasor is quantized into 15° segments. The dotted fines in Figure 9 shows the quantization. Whenever a set of cartisian coordinates is obtained by demodulation and falls into one of the quantized sectors, it is stored as the closest quantization sector coordinates. Since the distance from the quantized sector to the four nearest neighbors is in memory only 4 look ups are necessary to determine the metric to attach to each branch in the Viterbi decoder. The decoder will be making soft decisions as required. That is, without qunatization and in order to make soft decisions the demodulated I_i and Q_q coordinate must be used to compute $d_j^2 = (Q_q - y_j)^2 + (I_i - X_j)^2$ to the four nearest neighbors where Y_j and X_j are the cartesian coordinates of the four nearst neighbors for j = 1, 2, 3, 4. This metric, d_j, from a continum range of values is the metric that would be attached to the 4 paths. The quantization process reduces the above Euclidean operations to a look up process.

The TCM sequence given above in Table 1 was processed by a Quantized Euclidean decoder. With the exception of the error inserted in the sequence in the fourth time frame all signals were assumed decoded correctly but with a phase jitter that resulting in being quantized to the sector 15° fromt the correct symbol. The resulting paths and the associated metrics are shown in Figure 8. The paths eliminated due to having the largest metric of the two paths going into a node is shown with an X. The box attached to each path has the cumulative incremental d_0's and d_1's in the upper part and their sum in the lower. The distance between the demodulated quantized signal and the allowed signal is given in terms of d_1 and d_0 on the branches. The correct path determined by the shortest distance is the heavy line.

CONCLUSIONS

The overall system studied has an asymptotic coding gain of 3db when compared to an uncoded 4-PSK system and a spectral efficiency of 2 information bits/sec/Hz. Since 8-PSK has a constant amplitude format, it is amendable to transmission using the S-Band. Further, the $R = \frac{1}{2}$ convolutional encoder is simple and small and is on the transmission end of the TM link whereas the more complex decoder is located at the receiver. For communication links where the high data rates are one-way such as in flight evaluation TM systems, TCM with a quantized decoder to decrease decoding time appears to have extremely desirable characteristics.

REFERENCES

1. Clark, G. C. and J. B. Cain, “Error-Correction Coding for Digital Communications, Plenum Press, New York and London, 1981.

2. Ungerbock, G., “Trellis-Coded Modulation with Redundant Signal Sets - Part I: Introduction,” IEEE Communication Magazine, Vol. 25, No. 2, pp. 5-11, Feb. 1987.

3. Ungerbock, G., “Trellis-Coded Modulation with Redundant Signal Sets - Part II: State-of-the-art,” IEEE Communication Magazine, Vol. 25, No. 2, pp. 12-21, Feb. 1987.

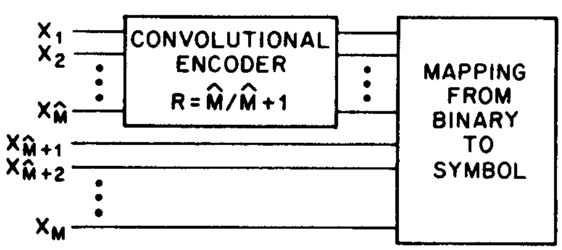


FIGURE 1. TCM ENCODER

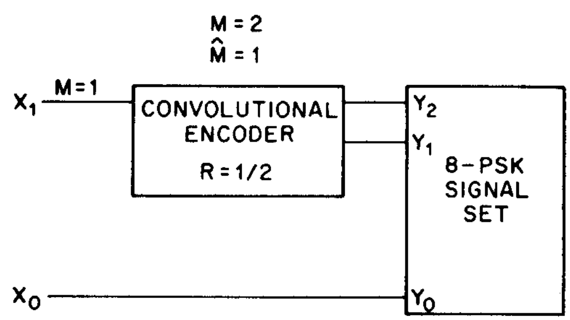


FIGURE 2. 8-PSK TCM ENCODER

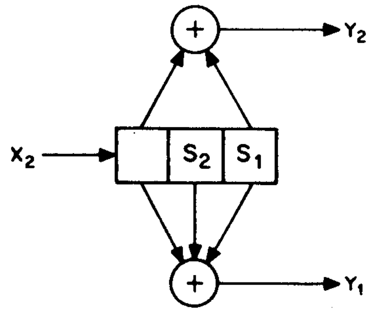


FIGURE 3-a. R=1/2 CONVOLUTIONAL ENCODER

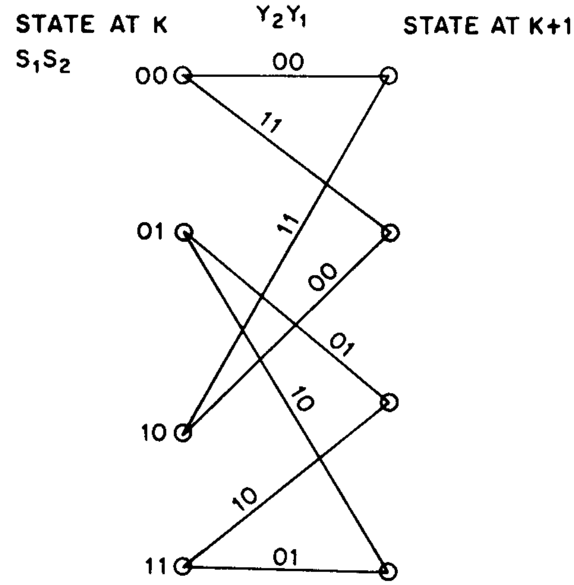


FIGURE 3-b. TRELLIS FOR CONVOLUTIONAL ENCODER OF FIG. 3-a

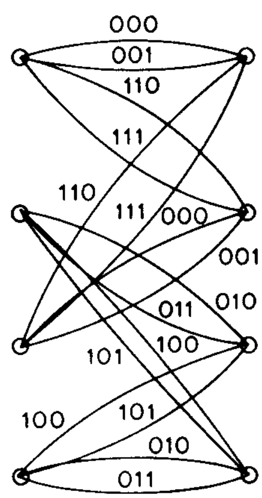


FIGURE 4-a. 8-PSK TCM TRELLIS WITH PARALLEL PATHS

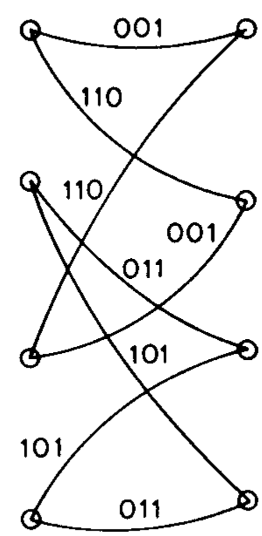


FIGURE 4-b. 4 NEAREST NEIGHBORS TO RECEIVE 285° SIGNAL

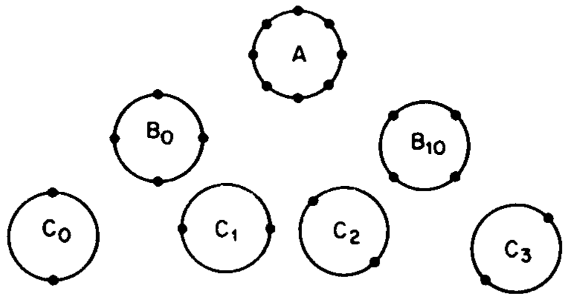


FIGURE 5-a. SET PARTITIONING

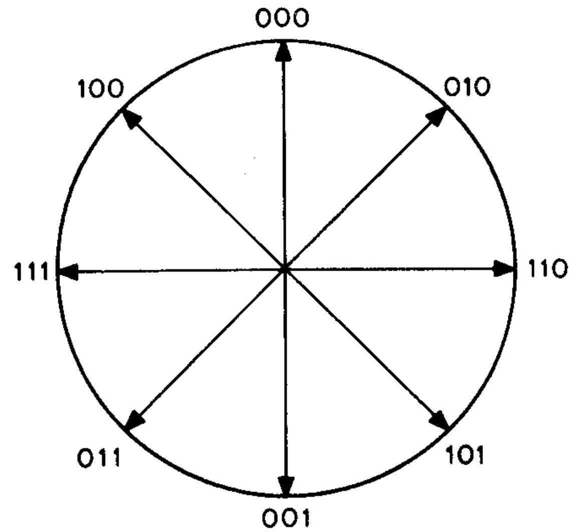
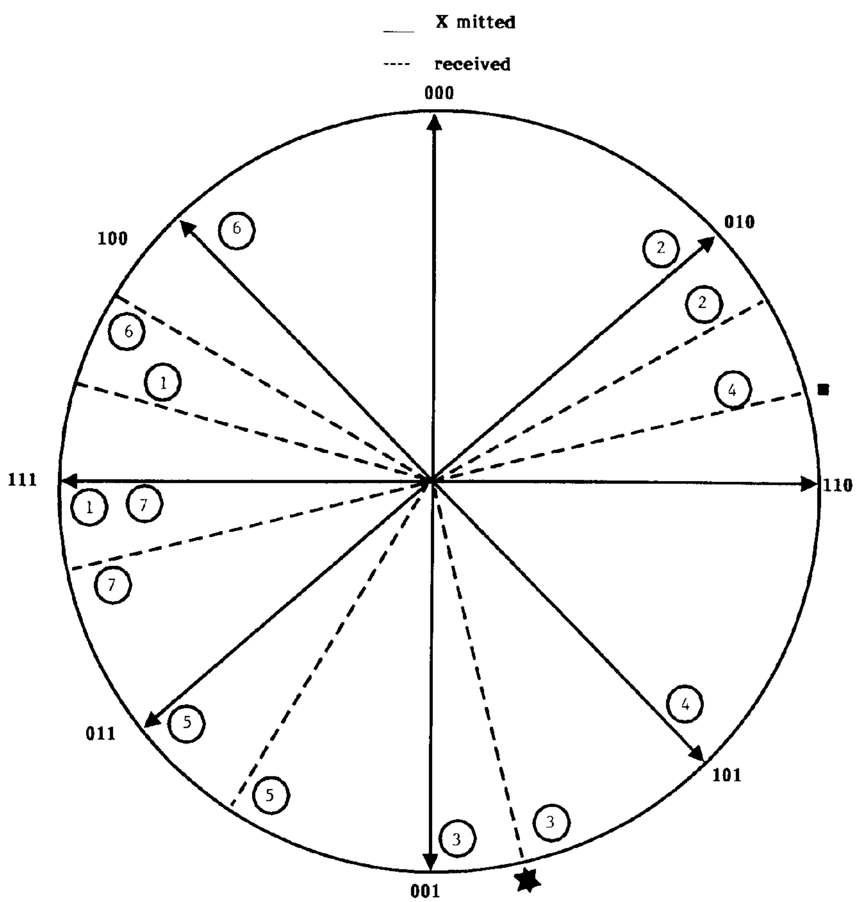


FIGURE 5-b. PSK SIGNAL ASSIGNMENT



Transmittal sequence
Continuous lines represent transmitted signals
Dashed lines represent signals
■indicates error in detected signal

FIGURE 6 8-PSK TRANSMITTED AND RECEIVED SEQUENCE

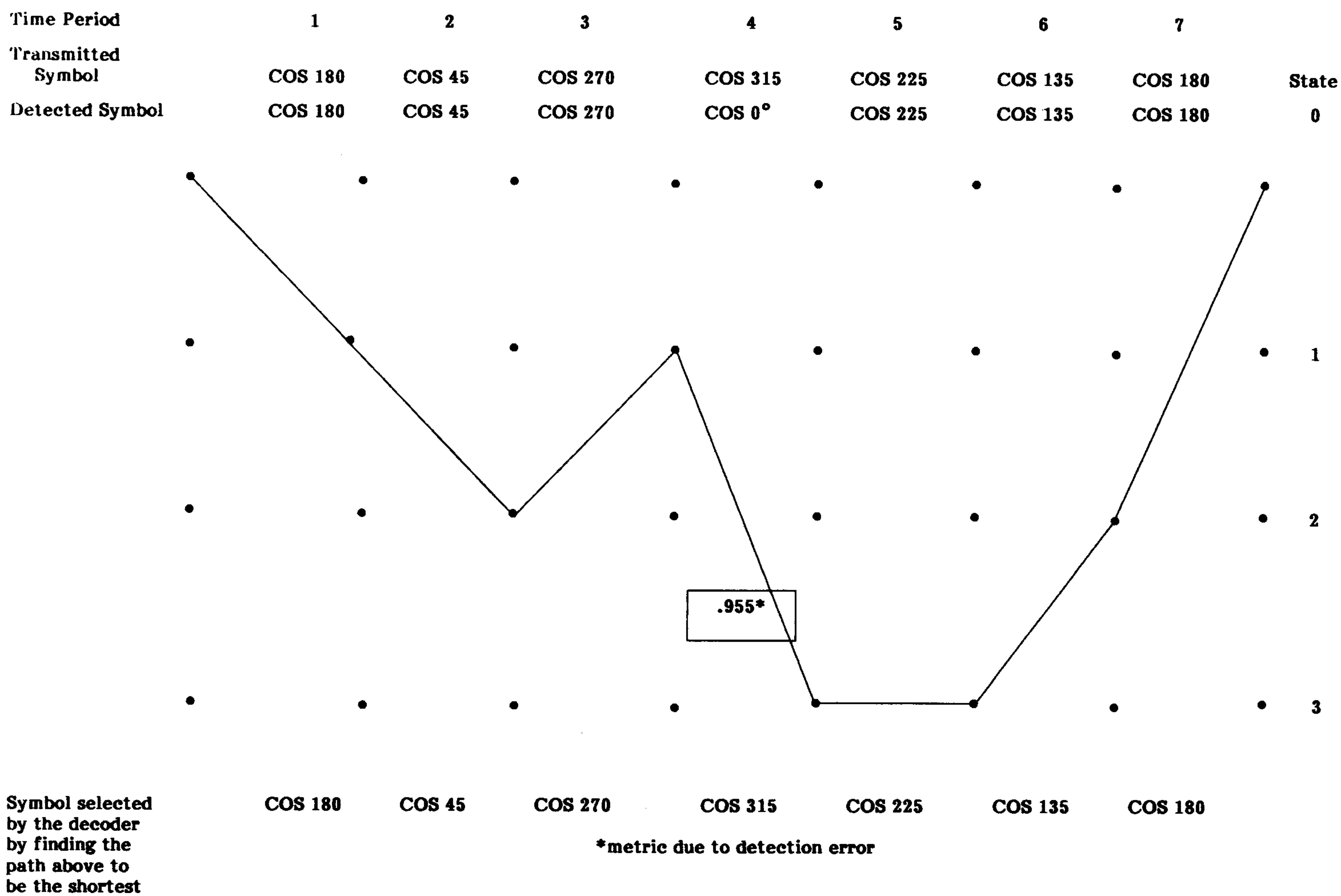


FIGURE 7 PATH CHOSEN BY THE QUANTIZED DECODER AS THE CORRECT SEQUENCE

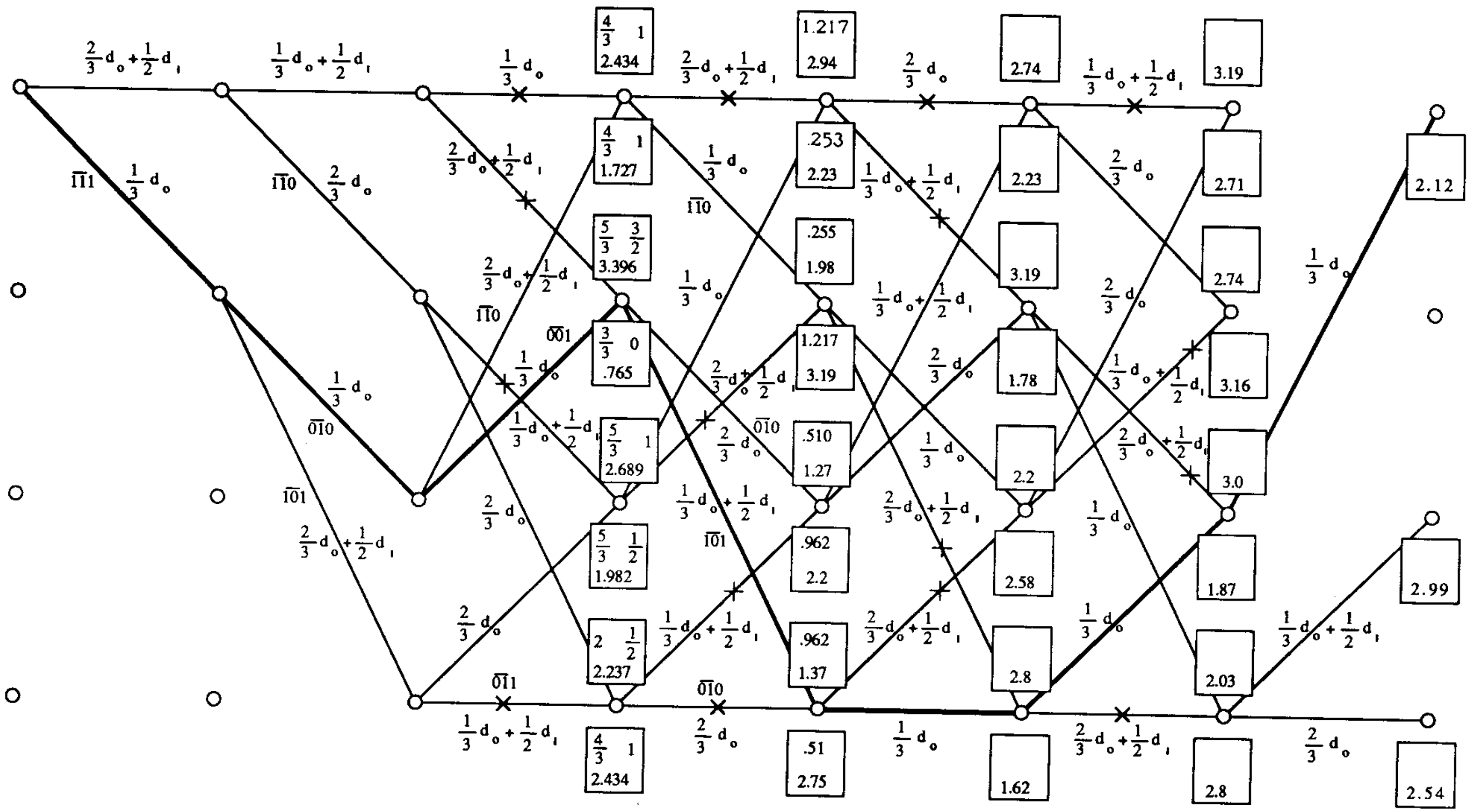
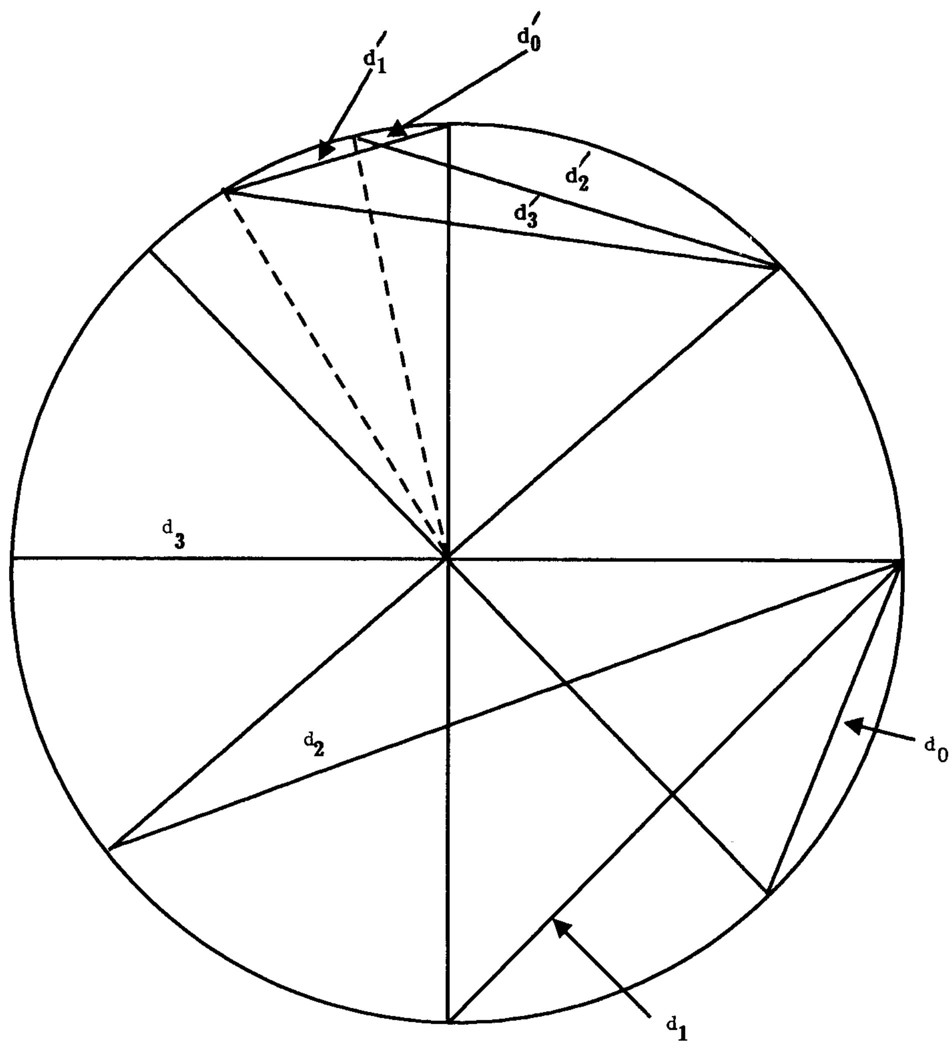


FIGURE 8 QUANTIZED EUCLIDEAN DECODER TRELLIS



Continuous sector lines represent 8 PSK symbols
Dashed lines represent quantized sectors.
Each quantized sector is 15°

FIGURE 9 QUANTIZATION SEGMENTS FOR 8-PSK