Characterization of Self-Focusing and Self-Defocusing of Light in Sodium Vapor

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Abstract - Self-focusing of light in sodium vapor was first observed on a cw basis in 1974. Recently at Northwestern University, efforts to develop a quantum optical communications network employing squeezed states of light have required quantitative characterization of the self-action effects. It has been determined that self-focusing and self-defocusing change the spatial structure of the output beams of the forward four-wave mixer used in the experiments, thus worsening the homodyne-detection efficiency by creating a mismatch between the squeezed output beam and the local-oscillator beam. Consequently, the need to characterize the self-action effects in sodium vapor has arisen. By characterizing the self-action effects as a function of the sodium cell temperature, input beam intensity, and the dye laser frequency, it will be possible to modify the local-oscillator wavefront to compensate for the spatial mismatch, and thus improve the homodyne-detection efficiency.

This paper reports the results of an experiment carried out in the Fall and Winter Quarters of the 1987/1988 school year as an Honors Project in Electrical Engineering. The theories of self-focusing of optical beams and Gaussian beam propagation are developed early in the paper in order to lay the groundwork for the presentation and interpretation of the experimental results. A general description of the laboratory setup is given, and the experimental procedure is described in detail. Finally, the paper concludes with a presentation and interpretation of the experimental findings.

I. Introduction

The possibility of self-focusing near an atomic resonance was first proposed in 1966 by Javan and Kelley.[1] In their paper, they proposed a model in which an intensity dependent index of refraction produced a decreased phase velocity where the beam was most intense, leading to the self-focusing of an incident beam.

Just seven years later at Bell Telephone Laboratories, Bjorkholm and Ashkin, observed cw self-focusing in sodium vapor for the first time.[2] They attributed the self-action effect they observed to the intensity dependent saturation of the anomalous dispersion, which Javan and Kelley had proposed in 1966. In their experiments, Bjorkholm and Ashkin measured the self-focusing effect for two different temperatures and two incident beam power levels at a single frequency. A recent literature search has revealed that no further characterization of the self-action effects in sodium vapor has occurred since 1974.

Recently in the department of Electrical Engineering and Computer Science at Northwestern University, Dr. Prem Kumar has been developing a quantum optical communications network employing squeezed light.[3] Included in the ongoing work is an experiment being conducted in order to make theoretical comparison with his Quantum Theory of Nondegenerate Multiwave Mixing, which includes propagation effects in a nonlinear medium.[4] It has been determined that self-focusing and self-defocusing change the spatial structure of the output beams of the forward four-wave mixer, thus worsening the homodyne-detection efficiency by creating a mismatch between the squeezed output beam and the local-oscillator beam. Consequently, the need to characterize the self-action effects in sodium vapor have arisen. By characterizing the self-action effects as a function of the sodium cell temperature, input beam intensity, and the dye laser frequency, it will be possible to modify the local-oscillator wavefront to compensate for the spatial mismatch, and thus improve the homodyne-detection efficiency.

In order to improve the homodyne-detection efficiency and the efficiency of the entire quantum communications network, an experiment was conducted to accomplish the goal of characterizing the self-action effects in sodium vapor for various input beam frequencies, sodium cell temperatures, and input beam power levels. This experiment was performed during the Fall and Winter quarters of the 1987/1988 school year as an Honors Project in Electrical Engineering. This paper reports the results of the aforementioned experiment.

In order to present the experimental conclusions in an understandable fashion, the theories of self-focusing of optical beams and Gaussian beam propagation are developed early in the paper. After a general description of the laboratory setup, the experimental procedure is described in detail. Finally, the paper concludes with a presentation and interpretation of the experimental findings.

II. Self-focusing of Optical Beans

In the introduction, the self-focusing effects of interest were said to have resulted from an intensity dependent index of refraction. However, in order to understand how an intensity dependent index of refraction will lead to self-focusing of light, one needs to first understand the process by which a monochromatic beam is focused by a simple lens.

Fig. 1(a) illustrates a lens, which consists of a medium characterized by an index of refraction n_1, surrounded by a medium with index of refraction n_0. It is common knowledge that when n_1 is greater than n_0, the lens will focus an incoming monochromatic beam. The process by which the focusing action occurs is easily understood when one examines the optical path length that an incoming beam traverses.

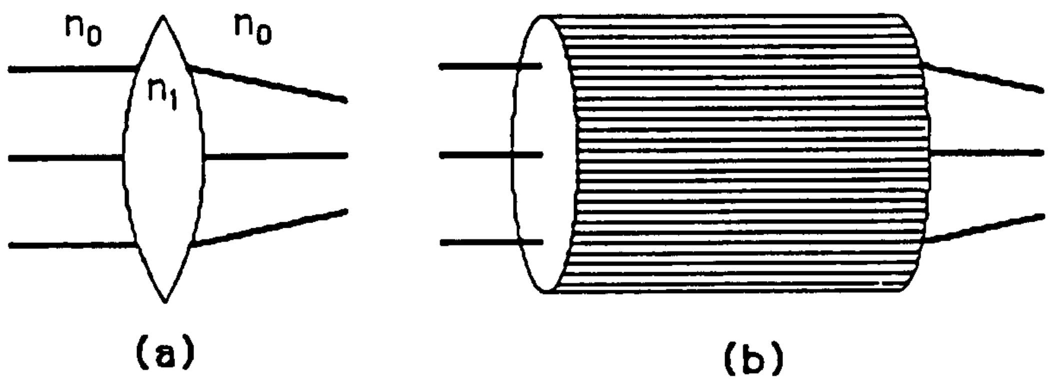


Figure 1

Mathematically, the optical path length is calculated as the index of refraction of the medium times the length of the medium (n x 1). Physically, an increased optical path length (higher index of refraction or greater length) results in a decrease in phase velocity for a beam of light traversing the medium. For the positive lens pictured in Fig. 1(a), the index of refraction is continuous throughout the lens, but the length of the medium is greater on the axis than off the axis. Consequently, the optical path length is greater on the axis of the lens than near the edges. As a result, as the beam propagates through the lens, the on-axis light experiences a decrease in phase velocity with respect to the off-axis light. Due to the decreased phase velocity at the center of the lens, the original equiphase surface becomes more and more distorted as the beam propagates through the lens. Finally, Huygen’s principle tells us that the rays propagate perpendicular to the wavefront, and thus, the beam is focused by the lens.

The situation in which the focusing effect occurs due to an intensity dependent index of refraction is easily understood if one considers the medium which possesses the intensity dependent index of refraction as an analogue of the simple lens discussed above. Fig. 1(b) illustrates a medium characterized by an intensity dependent index of refraction. For a Gaussian input beam, the typical output of a coherent source, the intensity of the field is greatest on the axis of the beam. Therefore, as a Gaussian beam propagates through the pictured cell, the beam will see an increased index of refraction and an increased optical path length where the beam is most intense. Just as in the case of the positive lens in Fig.1(a) , the optical path length is greater at the center of the focusing medium than at the edges and focusing occurs.

In the cases discussed above, the optical path length varies in the transverse direction. Although in the simple lens, the length varies, and in the material characterized by an intensity dependent index of refraction, the index of refraction varies, the resulting optical path lengths vary in the same fashion, leading to similar focusing behavior.

In summary, Fig. 2 is a plot of intensity I on the beam axis versus the distance z that the beam propagates. The three cases of normal dispersion (a), self-focusing (b), and self-defocusing (c) are all illustrated.

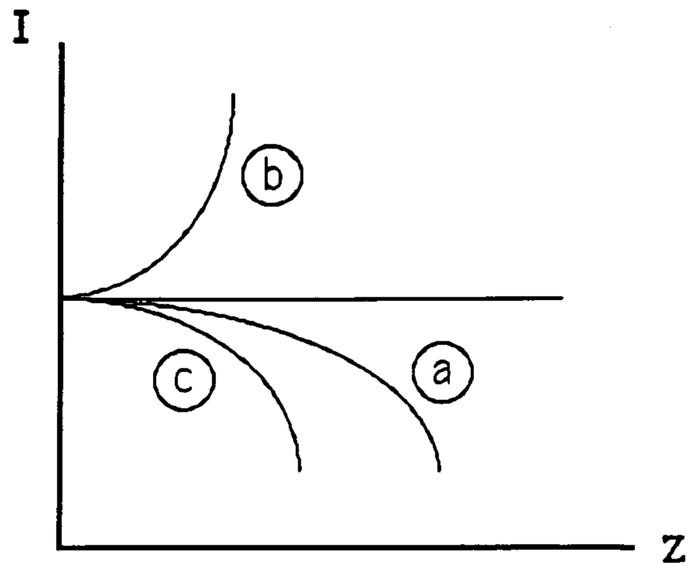


Figure 2

The self-focusing effect explained above results from the influence of intensity dependent anomalous dispersion. Close to an atomic resonance, the real part of the susceptibility experiences a strong dependence on the intensity of the light passing through the medium. As the intensity increases, the saturation of level population results in the nonlinear effect observed. To express this nonlinear effect mathematically, the electric permeability may be written as

$$\varepsilon = \varepsilon_0 + \varepsilon' \langle E \cdot E \rangle \tag{1}$$

The expression for the index of refraction also contains a nonlinear term and can be written as

$$n = n_0 + n' \langle E \cdot E \rangle \tag{2}$$

According to equation (2), as the intensity increases, the index of refraction also increases, producing the nonlinear self-focusing effect of interest. One should note that in general, the nonlinear term can either add to or subtract from the non-resonant term, leading to self-focusing or self-defocusing, respectively.

III. Gaussian Beam Propagation

Having developed a model which leads to self-focusing and self-defocusing, the question arises as to how to measure this effect. Since the output beam of the dye laser utilized in the experiment is Gaussian, an understanding of the effect that a lens has on the propagation of a Gaussian beam is necessary. For any Gaussian beam, there are two parameters that characterize the given beam, namely the beam waist and the radius of curvature. In free space, these quantities are given by the following equations:

$$\omega^2(z) = \omega_0^2(z)\left[1+\left(\frac{\lambda z}{\pi \omega_0^2}\right)\right] \tag{3}$$

$$R(z) = z\left[1+\left(\frac{\pi \omega_0^2}{\lambda z}\right)\right] \tag{4}$$

Given a Gaussian beam characterized by w(z) and R(z), the complex number q(z) , which completely characterizes the given beam is calculated as follows:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi \omega^2(z)} \tag{5}$$

Once the complex number q has been calculated, the Gaussian beam will propagate through free space and a thin lens according to equations (6) and (7), which are illustrated in Fig. 3(a) and 3(b), respectively.

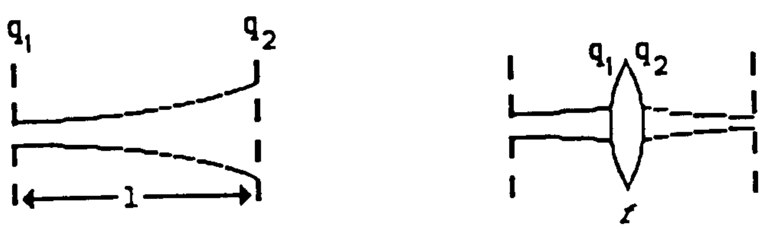


Figure 3

$$q_2 = q_1 + 1 \tag{6}$$

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} \tag{7}$$

IV. Laboratory Setup

The optical table used for the experiment was configured as depicted in Fig. 4. A Coherent Radiation model CR8 argon ion laser was used to pump a Coherent Radiation model 699-21 dye laser. By use of intercavity elements and active stabilization, single frequency coherent radiation was delivered at the output of the dye laser. Using plane mirrors, the beam was directed across the table and into the sodium cell, which was surrounded by a heat pipe oven. Passing through the sodium vapor, the beam was brought to a focus by a simple lens having a focal length of 0.10 meters. This lens was utilized in the setup for two reasons. First, the focusing effect of the sodium vapor was not great enough to bring the beam to a focus on the table without using additional mirrors. The second, and most important reason will become evident shortly. Suffice it to say that the defocusing effects could not have been measured without the beam being brought to a focus.

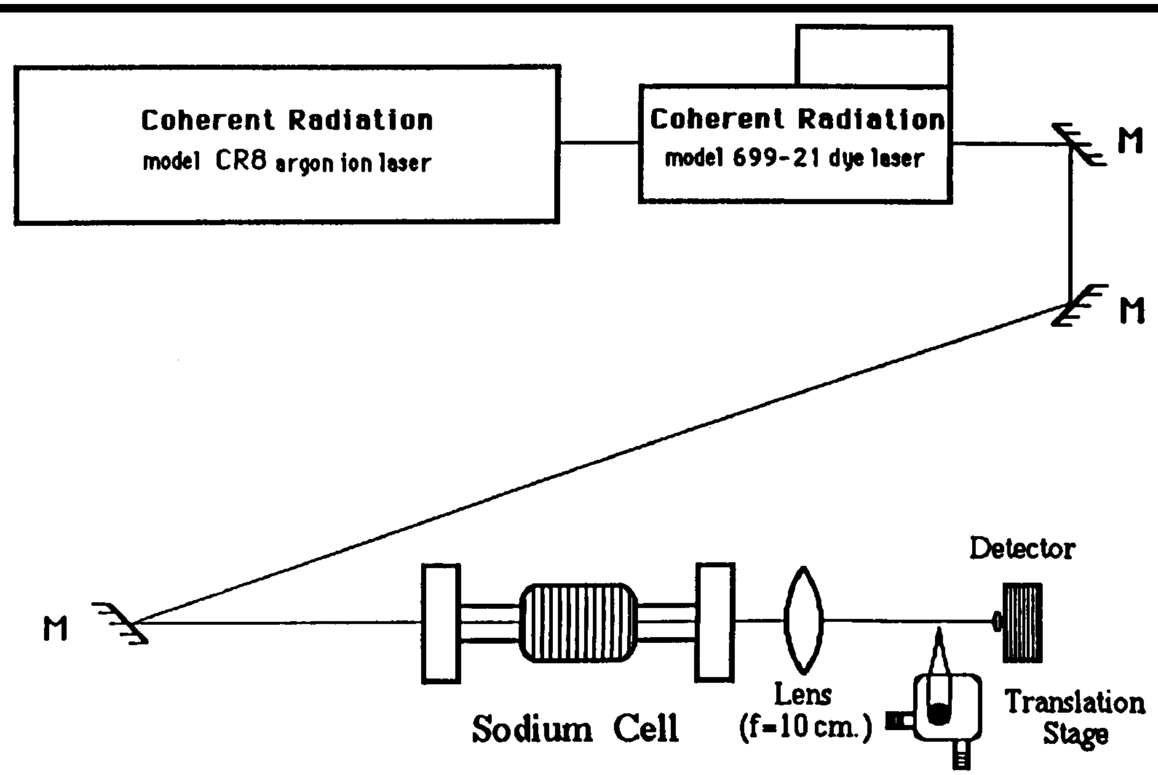


Figure 4

Approximately 10 cm. beyond the thin lens, a razor blade oriented in a plane parallel to the transverse beam profile was mounted on a two-axis translation stage. The axes of the translation stage were oriented in a manner that allowed the razor blade to be moved in a plane perpendicular to the direction of propagation of the beam (referred to as the x direction) and in a direction parallel to the direction of propagation of the beam (referred to as the y direction) . A potentiometer was attached to the rotating arm of the translation stage that drove the razor blade in the x direction. The voltage from this potentiometer, which was proportional to the position of the razor blade in the x direction was fed to an analog x-y plotter. Finally,-light was collected onto the receiver of a high speed detector place just beyond the focal point. The output of this detector was fed to the unused port of the x-y plotter. Thus, the position of the recorder pen was a function of the position of the razor blade in the x direction and the intensity of the light that was incident on the detector.

V. Experimental Procedure

Having developed the fundamentals of Gaussian beam propagation and explained the experimental setup, the experimental procedure can be easily described and understood.

In order to establish a “baseline” from which to start, consider the case in which the sodium cell has no focusing effect on the light beam. If the beam waist and radius of curvature of the beam, and consequently q, are known as the beam leaves the laser, one can use equations (6) and (7) to calculate the value of q after the beam has propagated across the table and through the lens of f = 10 cm., finally coming to a focus in front of the detector. An examination of equation (5) reveals that at the focal point, since the radius of curvature is equal to infinity, the real part of q drops out and that for a given wavelength, q becomes a function of the beam waist alone. This realization is the key to devising a simple method of determining the focusing power of the sodium cell. Given the value of q at the output coupler of the laser, q can be calculated at the input side of the sodium cell by application of equations (6) and (7). Measuring the beam waist, and easily calculating q, at the focal point near the translation stage, it is possible to calculate the value of q at the output side of the sodium cell by working backwards using equations (6) and (7). Finally, equation (7) can be utilized to calculate the focal length of the sodium cell “lens.” The only assumption that has been made is that the sodium cell is a “thin” lens. This assumption is valid as long as the axial thickness of the lens is small in comparison with the distances generally associated with its optical properties. It was sound that this assumption was valid for the sodium cell.

The only item remaining to be explained is the method of determining the position and magnitude of the beam waist. For this task, the translation stage, detector, and x-y plotter were utilized.

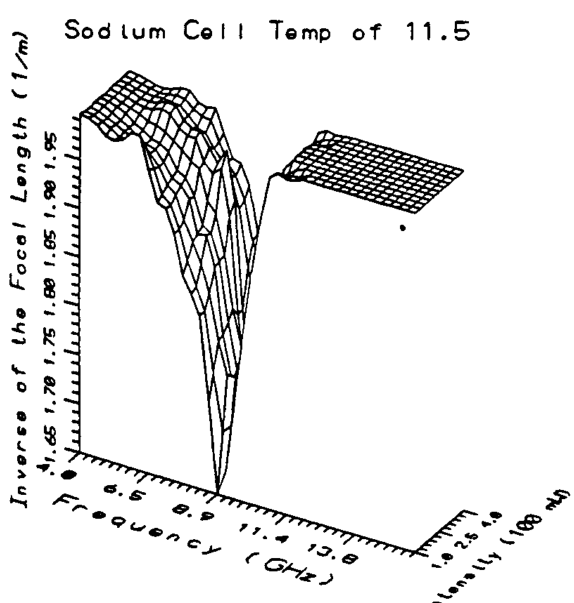
In accordance with the discussion in section II, the self-action effect does not occur when the frequency of the light beam is far from resonance. In this case, the beam waist will be found at a position previously referred to as the "baseline." It is obvious that if the sodium cell acts on the light in a manner that leads to self-focusing, the beam waist will be found at a position between the sodium cell and the baseline. Conversely, if the sodium cell acts on the light beam in a manner that leads to self-defocusing, then the beam waist will occur at a position between the baseline and the detector. In order to determine the exact location and value of the beam waist, the razor blade mounted on the translation stage was positioned near the beam waist location predicted by equations (6) and (7).

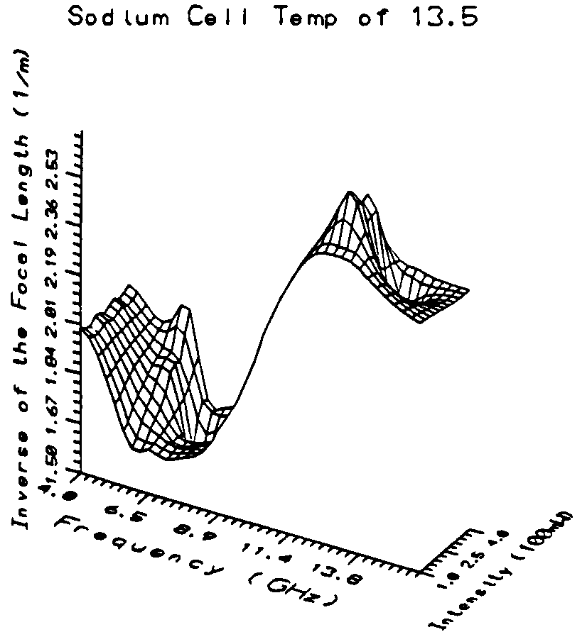
To begin the measurement process, the razor blade was translated in the y direction until it was on one side of the beam waist. Next, the razor blade was fully retracted in the x direction, allowing the entire beam to impinge on the detector. As the razor blade was slowly translated across the beam in the x direction, the amount of light hitting the detector dropped, causing the intensity to decrease. Recalling that the x-y plotter received voltages proportional to the intensity of the light impinging on the detector and the position of the razor blade in the x direction, one can see that the plotter pen traced the quantity, one minus the integral of the Gaussian curve. By referring to a table of values for the Gaussian integral, the beam waist, defined as the 1/e point in the field amplitude, was calculated by measuring the distance between the $1/e^2$ points (recall that the intensity and not the amplitude is the measured quantity) of the plotted curve. Retracting the translation stage in the x direction, and subsequently moving the razor blade along the beam in the y direction, the process was repeated until several values of the beam waist were measured on either side of the beam waist. By interpolating between y position values, exact values for the position and magnitude of the beam waist were ascertained. As explained above, once the position and magnitude of the beam waist has been determined, the problem of calculating the focal length of the sodium cell lens reduces to algebraic manipulation. This process was repeated for the intended input beam frequencies, sodium cell temperatures, and input beam power levels.

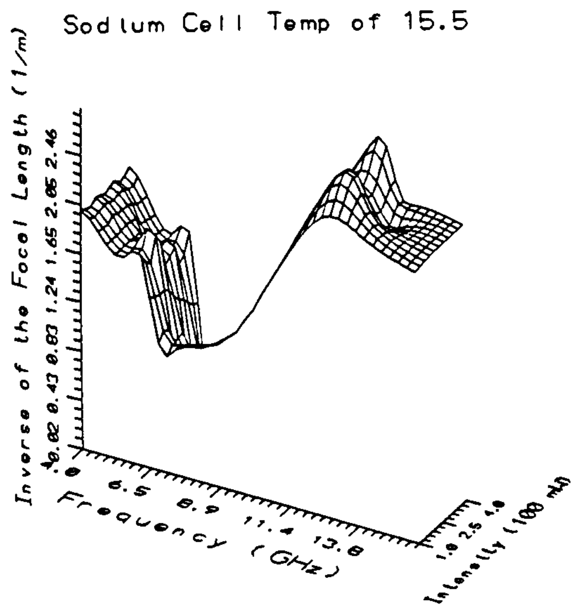
VI. Experimental Results

The results of the experiment are presented in the form of the three-dimensional graphs that appear below. They present the focal length as a function of the input beam frequency, intensity, and the temperature of the sodium cell. Before the plots are examined, some remarks concerning the labelling of the axes are in order. The software used to generate the three-dimensional graphs that follow, only plots in one quadrant of the three-dimensional Cartesian coordinate system. Consequently, a shift of variable was made in order to translate the graphs up into the first quadrant. For example, the x-axis, which represents the magnitude of the detuning off of resonance (in GHz) , is shifted up so that the value of 10 corresponds to the center of the atomic resonance. In order to obtain the actual value in GHz, simply subtract the number 10 from the value shown on the axis. The y-axis, which represents the intensity of the input light beam in hundreds of mW is self-explanatory. The focusing effect of the sodium vapor is represented by the z-axis, which plots the inverse of the focal length in inverse meters. The inverse of the focal length, and not the focal length itself is plotted, so that a value of infinity for the focal length (no self-focusing) will be plotted as a zero on the graph. In this way, defocusing will yield a negative value, and focusing will yield a positive value. In a fashion similar to the x-axis, the z-axis was shifted by a value of two, so that all the experimental values would be found in the first quadrant. Consequently, values less than two represent defocusing and values greater than two represent focusing. Finally, three plot appear, for three different sodium cell temperatures. The values given correspond to a voltage proportional to the temperature of the cell. Presently, the sodium cell and heat pipe oven are being rewired to provide a temperature in Celsius, but until this work is completed, arbitrary units will be used.

In examining the graphs, one can see that defocusing occurs for detuning below resonance and focusing above resonance, as predicted by the theory. As the temperature of the cell is increased, the self-action effects increase as expected. In general, the results that Bjorkholm and Ashkin produced are reproduced here, but in greater detail and in numbers applicable to our laboratory setup. By utilizing this data, it will be possible to modify the local-oscillator wavefront to compensate for the spatial mismatch, and thus improve the homodyne-detection efficiency as planned. Since the goal of the experiment was to provide data for this modification, the experiment was quite successful.







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ENDNOTES

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