EFFECTS OF CO-CHANNEL INTERFERENCE WITH FREQUENCY OFFSET ON PSK SIGNALS

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ABSTRACT

The sharing of C-band between microwave terrestrial and satellite communication systems invariably introduces interference from one system into the other. Such co-channel interference becomes even more important in satellite receive stations with smaller antennas and must be minimized to achieve system performance objectives. In this paper, co-channel interference due to two TD2 (FDM/FM) carriers into a satellite receive system, receiving binary phase-shift keyed (BPSK) signal, is considered. It is shown that the frequency offset of the TD2 carriers from the BPSK carrier can be used to minimize co-channel interference effects. Equations are given which compute the bit error rate (BER) of BPSK signals in the presence of an interfering unmodulated carrier. They are followed by some results due to TD2 carrier interference.

1. INTRODUCTION

Phase-shift keyed (PSK) signals that are received from a satellite are subject to co-channel interference from other satellites, terrestrial microwave systems, cross-polarized signals in an adjacent transponder, and intermodulation products within the same transponder. A model showing the various sources of interference is illustrated in Fig. 1. In a single carrier digital transmission system, the dominant interference sources are adjacent satellites and terrestrial microwave systems. Interference due to these two sources is even more pronounced if the satellite receive earth station antenna sizes are small, such as 3 m or less. If the desired signal is wideband, such as the one considered here, an 8 megabits per second (MBPS) BPSK interference from wideband sources will degrade the performance more than a narrowband interference such as from a single channel per carrier (SCPC). In this paper, we consider the interference from two FDM/FM carriers (TD2) which are frequency offset from the center of the BPSK carrier. The desired BPSK spectrum and the relative locations of both narrowband and wideband interference carriers are given in Fig. 2. In the following sections, equations are developed to compute BER in the presence of co-channel interference for an M-ary PSK (MPSK) system; these equations are then applied to evaluate the 8 MBPS system when interfered with by two TD2 carriers. Also

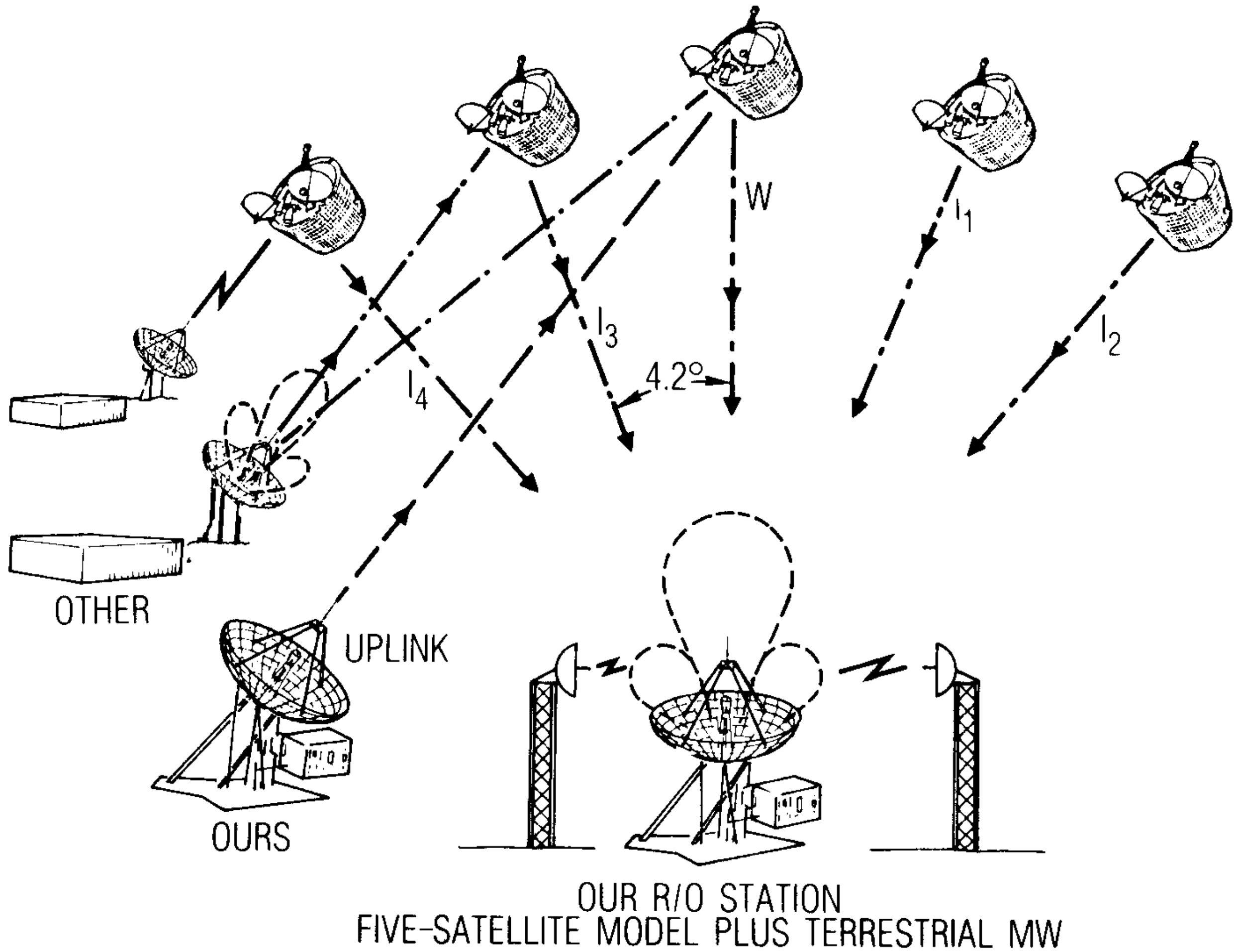

Fig. 1. Interference <u>from</u> Model

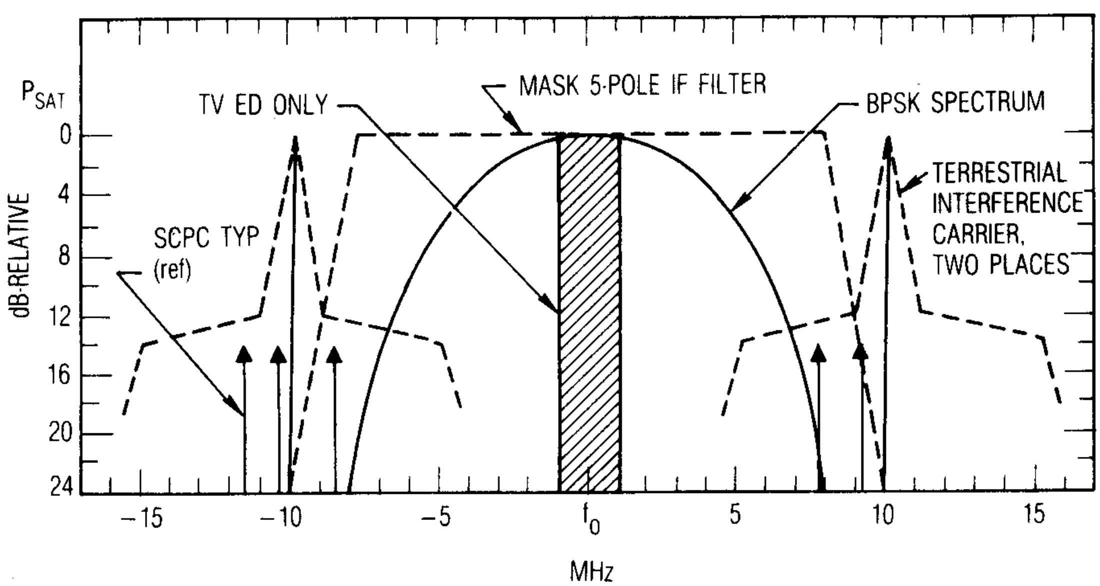

Fig. 2. Interference Protection of BPSK System

presented are the effect of varying the receiver IF filter bandwidth on the amount of interference and the frequency offset advantage as a function of the amount of frequency offset.

2. DESCRIPTION

In this section, equations are developed to compute the symbol error probability of MPSK signals in the presence of co-channel interference occurring at a frequency offset from the desired signal frequency. Frequency offset advantage (F_{ad}) in decibels is calculated as the difference between the C/I ratios in dB required to achieve a given probability of symbol error with and without frequency offset. Results are given to show the variation of F_{ad} with frequency offset (f_Δ) for various signal-to-noise ratios.

The degradation in BER caused by a single sinusoid co-channel interference has been calculated by Spilker[1], where it is assumed that the interfering signal has the same frequency as the desired one. These results are extended to include frequency offset; for ease in following, the notation is kept the same as in Ref. 1.

Let the MPSK signal with symbol duration T second and received power level P_s be represented by

$$S_N(t) = \sqrt{2P_s}\cos(\omega_0 t + \theta),\ NT \le t \le (N+1)T \tag{1}$$

where the phase modulation $\theta = 2\pi k/_M$; K = 1, 2, ..., M; $M = 2^n$; and n is the number of bits per symbol. Let the interference sinusoid of power P_I at a frequency offset f_Δ Hz ($\omega_\Delta = 2\pi f_\Delta$) from the carrier frequency f_0 be denoted by

$$I_N(t) = \sqrt{2P_I}\cos(\omega_0 t + \omega_\Delta t + \eta),\ NT \le t \le (N+1)T \tag{2}$$

where η is the random reference phase variable which is uniformly distributed over the interval $(0, 2\pi)$. The total received signal during the N-th time interval is

$$r_N(t) = S_N(t) + I_N(t) + n(t) \tag{3}$$

where n(t) is white Gaussian noise of density N_0 (one sided):

$$n(t) = n_c(t)\cos(\omega_0 t + \phi) + n_s(t)\sin(\omega_0 t + \phi) \tag{4}$$

where the in-phase and quadrature noise components $n_c(t)$, $n_s(t)$ are also Gaussian with density N_0 and ϕ is unknown phase offset of the received signal from the quadrature reference signals of the demodulator.

The received signal is demodulated and passed through the in-phase and quadrature integrate-and-dump filters as shown in Fig. 3. The sampled output, which gives a phase measurement, is compared with the phase decision boundaries for the value of M being used. Without any loss in generality, we can set $\phi = 0$ in the analysis.

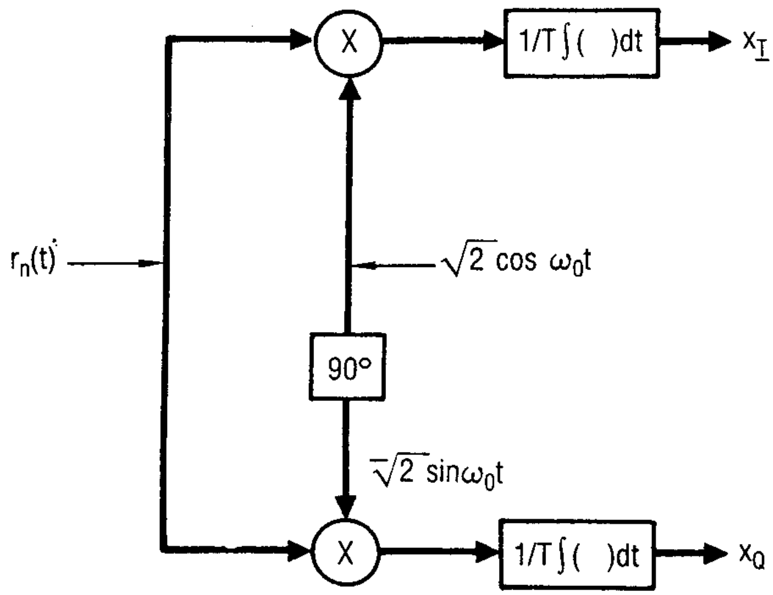

Fig. 3. Block Diagram of QPSK Receiver

The in-phase and quadrature outputs X_I and X_Q are

$$X_I(t) = \sqrt{P_s} + \sqrt{P_J}\ \cos\ (\omega_\Delta t + \eta)\ \Big|ID + n_c(t) \tag{5}$$

and

$$X_Q t) = \sqrt{P_s}\ \sin\ (\omega_\Delta t + \eta)\ \Big|ID + n_s(t) \tag{6}$$

where ID signifies the integrate and dump operation. Now define

$$\begin{aligned} I_1 &= \sqrt{P_I}\ \cos\ (\omega_\Delta t + \eta)\ \Big|ID \\ &= \frac{1}{T}\int_T \sqrt{P_I}\ \cos\ (\omega_\Delta t + \eta)dt \end{aligned} \tag{7}$$

and

$$I_2 = \sqrt{P_I}\ \sin(\omega_\Delta t + \eta)\ |ID$$

$$= \frac{1}{T}\int_T \sqrt{P_I}\ \sin(\omega_\Delta t + \eta)dt \tag{8}$$

By defining $A_I = \int_T \cos\omega_\Delta t\ dt$, and $A_2 = \int_T \sin\omega_\Delta t\ dt$, we can write Eqs. (7) and (8) as

$$I_1 = \frac{\sqrt{P_I}}{T}\ [A_1 \cos\eta - A_2 \sin\eta] \tag{9}$$

and

$$I_2 = \frac{\sqrt{P_I}}{T}\ [A_2 \cos\eta + A_1 \sin\eta] \tag{10}$$

In Eqs. (5) and (6), $n_c(t)$ and $n_s(t)$ are Gaussian with variance σ_2 such that

$$\sigma^2 = \frac{N_o}{T} \tag{11}$$

Symbol energy-to-noise density ratio is defined as

$$\rho^2 = \frac{P_s}{\sigma^2} = \frac{P_s T}{N_0} = \frac{E_s}{N_0} \tag{12}$$

and the ratio of interference-to-signal power is

$$R^2 \triangleq \frac{P_I}{P_s} \tag{13}$$

The phasor representation of X_I and X_Q and the decision region for an MPSK signal are shown in Fig. 4. The minimum distances to the error-decision thresholds A, B, are DA and D_B, respectively

$$D_A = \sqrt{P_s}\ \sin\pi/M + \frac{\sqrt{P_I}}{T} A_1 \sin\left(\frac{\pi}{M} - \eta\right) - \frac{\sqrt{P_I}}{T} A_2 \cos\left(\frac{\pi}{M} - \eta\right)$$

$$D_B = \sqrt{P_s} \sin \pi/M + \frac{\sqrt{P_I}}{T} A_1 \sin\left(\frac{\pi}{M} + \eta\right) + \frac{\sqrt{P_I}}{T} A_2 \cos\left(\frac{\pi}{M} + \eta\right)$$

or

$$D_A = \sigma\rho \left[\sin \pi/M + \frac{R}{T}\left(A_1 \sin\left(\frac{\pi}{M} - \eta\right) - A_2 \cos\left(\frac{\pi}{M} - \eta\right)\right)\right] \tag{14}$$

and

$$D_B = \sigma\rho \left[\sin \pi/M + \frac{R}{T}\left(A_1 \sin\left(\frac{\pi}{M} + \eta\right) + A_2 \cos\left(\frac{\pi}{M} + \eta\right)\right)\right] \tag{15}$$

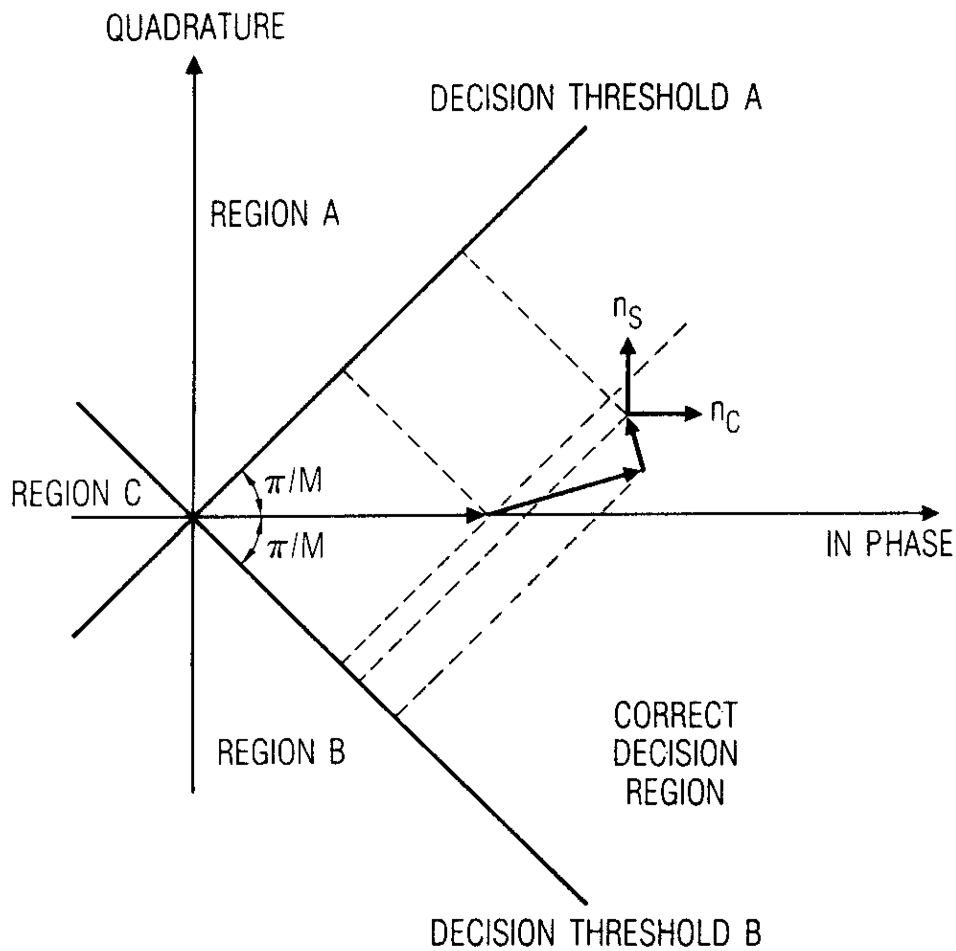

Fig. 4. Phasor Representation of MPSK Signal with Interference and Additive Noise

where σ and ρ are given by Eqs. (11) and (12). If we denote the probabilities of a noise component causing region A, B errors as π_A π_B (respectively) for a given value of η, then

$$\pi_A(\eta) = P_r\,(D_A < n_s < \infty) = \frac{1}{2}\,\mathrm{erfc}\,\frac{D_A}{\eta}$$

$$= \frac{1}{2}\,\mathrm{erfc}\,\rho\{\sin\frac{\pi}{M} + \frac{R}{T}\,[A_1\,\sin(\frac{\pi}{M} - \eta) - A_2\,\cos(\frac{\pi}{M} - \eta)]\} \qquad (16)$$

and

$$\pi_B(\eta) = \frac{1}{2}\,\mathrm{erfc}\,\rho\{\sin\frac{\pi}{M} + \frac{R}{T}[A_1\,\sin(\frac{\pi}{M} + \eta) + A_2\,\cos(\frac{\pi}{M} + \eta)]\} \qquad (17)$$

where

$$\mathrm{erfc}\,\alpha \triangleq \frac{2}{\sqrt{\pi}}\int_{\alpha}^{\infty} e^{-u^2}\,du$$

The probabilities of region A and region B decision errors P_A and P_B are obtained by taking expected value of π_A and π_B averaged over all values of the co-channel interference phase η.

$$P_A = E[\pi_A(\eta)],\; P_B = E[\pi_B(\eta)],\; \text{and}\; P_C = P_A \cap P_B$$

The probability of symbol error P_s is

$$P_S = P_A + P_B - P_C \qquad (18)$$

From Eqs. (16) and (17) we see that, unlike the case with no frequency offset, the probabilities P_A and P_B are not equal when the interference carrier is offset from the desired signal carrier. By using the same developments as in Eqs. (11-48) and (11-49)[1], we can express P_A and P_B as

$$P_A = \frac{1}{2}\,\mathrm{erfc}\,(\rho\sin\frac{\pi}{M}) + \frac{1}{\sqrt{\pi}}\,\exp\,(-\rho^2\,\sin^2\frac{\pi}{M})$$

$$\sum_{i=1}^{\infty}(-1)^i\,H_{i-1}\,(\rho\sin\frac{\pi}{M})\,\frac{(\rho R_1)^i}{(i!)} \qquad (19)$$

where H_i is the Hermite polynomial of order i and (i !) denotes factorial i.

Also,

$$P_B = \frac{1}{2}\,\mathrm{erfc}\,(\rho \sin \frac{\pi}{M}) + \frac{1}{\sqrt{\pi}} \exp\,(-\rho^2 \sin^2 \frac{\pi}{M})$$

$$\sum_{i=1}^{\infty} (-1)^i\, H_{i-1}\,(\rho \sin \frac{\pi}{M}) \quad \frac{(\rho R_2)^i}{(i!)^2} \tag{20}$$

where

$$R_1 = \frac{R}{T}\,[A_1 \sin(\frac{\pi}{M} - \eta) - A_2 \cos(\frac{\pi}{M} - \eta)] \tag{21}$$

and

$$R_2 = \frac{R}{T}\,[A_1 \sin(\frac{\pi}{M} + \eta) + A_2 \cos(\frac{\pi}{M} + \eta)] \tag{22}$$

For binary PSK, M = 2 and $R_1 = R_2$. Then

$$P_A = P_B = \frac{1}{2}\,\mathrm{erfc}(\rho) + \frac{1}{\sqrt{\pi}} \exp\,(-\rho^2)$$

$$\sum_{i=1}^{\infty} (-1)^i\, H_{2i-1}(\rho)\, \frac{(\rho R_1)^{2^i}}{2^{2^i}\,(i!)^2} \tag{23}$$

Probability of symbol error $P_S = P_A$. This result is the same as the results obtained by Spilker[1], the only difference being the weighting of R by A_1/T. Since A_1/T depends upon the amount of frequency offset and filter response, knowing these two parameters we can easily compute the frequency offset advantage.

3. RESULTS

The BER as computed from Eq. (23) for 8 MBPS BPSK interfered by an unmodulated carrier as a function of carrier-to-interference ratio (C/I) is plotted in Fig. 5 with E_b/N_o as a parameter. Solid lines are for zero frequency offset interference, and dashed lines indicate an offset of 10 MHz. In each case, it can be seen that 10 MHz frequency offset has considerably reduced BER. For example, an ideal BPSK has a BER of 10^{-5} at E_b/N_o = 9.628. The same BER is maintained until C/I is less than 17 dB. That is, at each E_b/N_o, for a given channel error probability rate, interference power with 10 MHz offset can be increased by about 15 dB. This is approximately equal to the attenuation provided by the integrate-and-dump (I&D) filter at 10 MHz to a monotone input. The difference in decibels

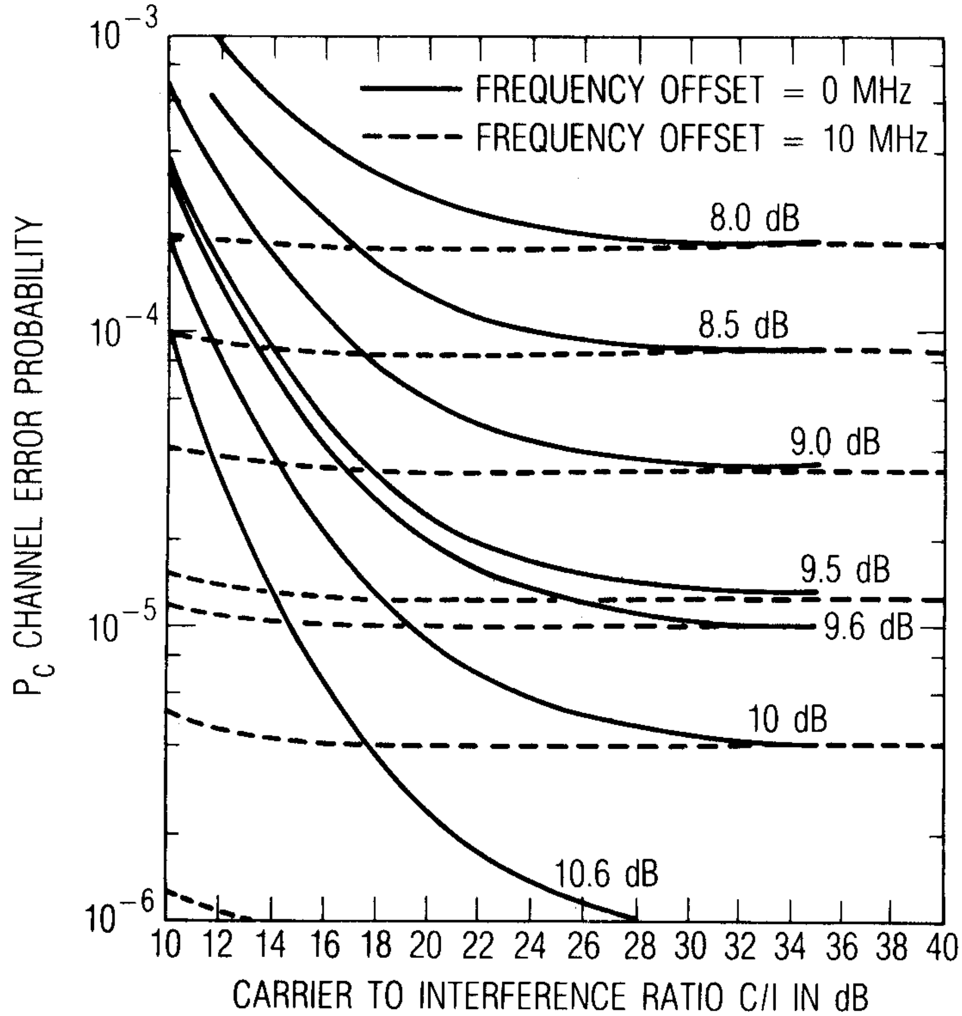

Fig. 5. PC versus Carrier to Interface Ratio

of C/I ratios to maintain a given BER, at a given E_b/N_o = 9.6 dB, is plotted in Fig. 6. Since the I&D filter is matched to 8 MBPS BPSK, the nulls at 8 MHz and multiples of 8 MHz show up as infinite offset advantage as illustrated in Fig. 6.

So far, only unmodulated carrier interference was assumed to keep the analysis tractable. It is quite complex to modify the equations to include interference from modulated carriers such as TD2 under consideration. Therefore, instead of using Eq. (23), we simply compute the amount of interference power at the detector output using the TD2 spectrum with 10 MHz offset.

The interference receive power objective is -140 dBW/MHz; the interference rejection provided by the I&D filter alone is not sufficient to achieve this objective. To obtain further interference rejection, a 0.1 dB ripple, 5-pole Chebyshev filter is used. A block diagram of the computer simulation used to compute the interference rejection is given in Fig. 7, and the spectrum at various outputs in the block diagram is given in Fig. 8. The

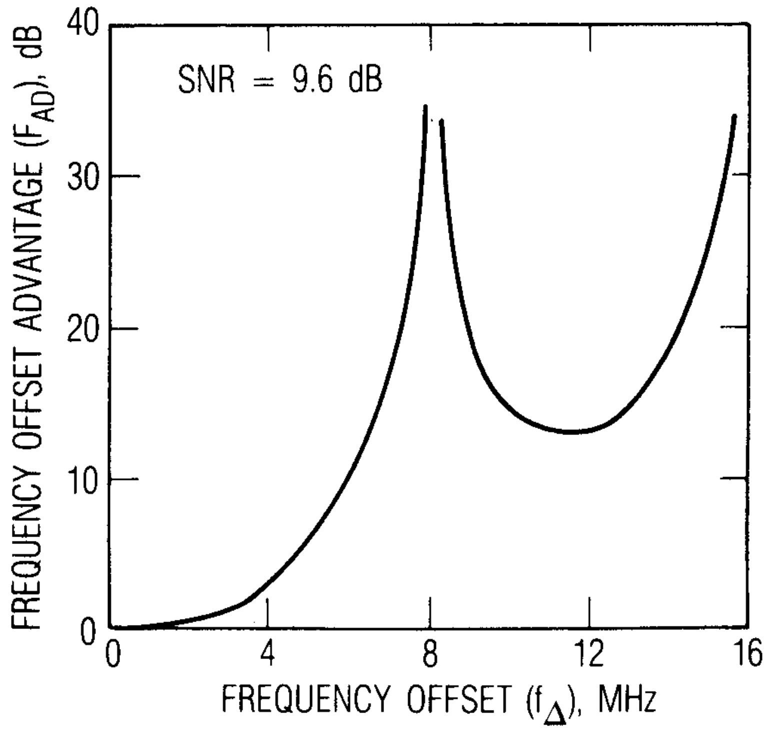

Fig. 6. Frequency Offset Advantage versus Frequency Offset

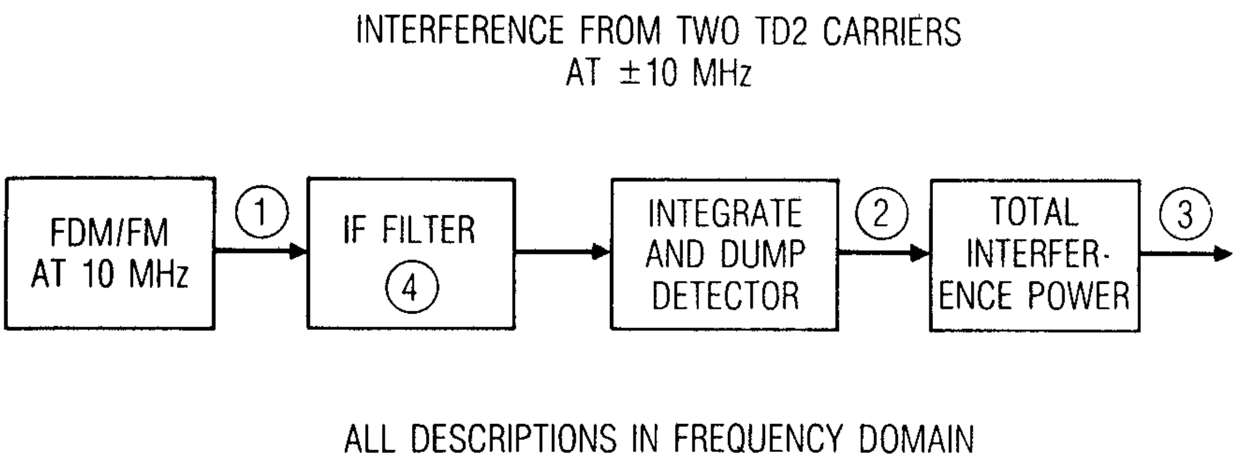

Fig. 7. Computer Simulation Block Diagram

choice of IF filter was made by taking into consideration the intersymbol interference and also interference from TD2 (see Table 1). As can be seen from Fig. 8, the interference rejection due to the IF filter and I&D filter is 29.2 dB which is sufficient to satisfy the -140 dBW/MHz coordination level.

To summarize the results, the frequency offset of terrestrial TD2 carriers can be used to advantage by employing IF filters in conjunction with I&D filters. This will achieve the required interference rejection to satisfy the overall performance.

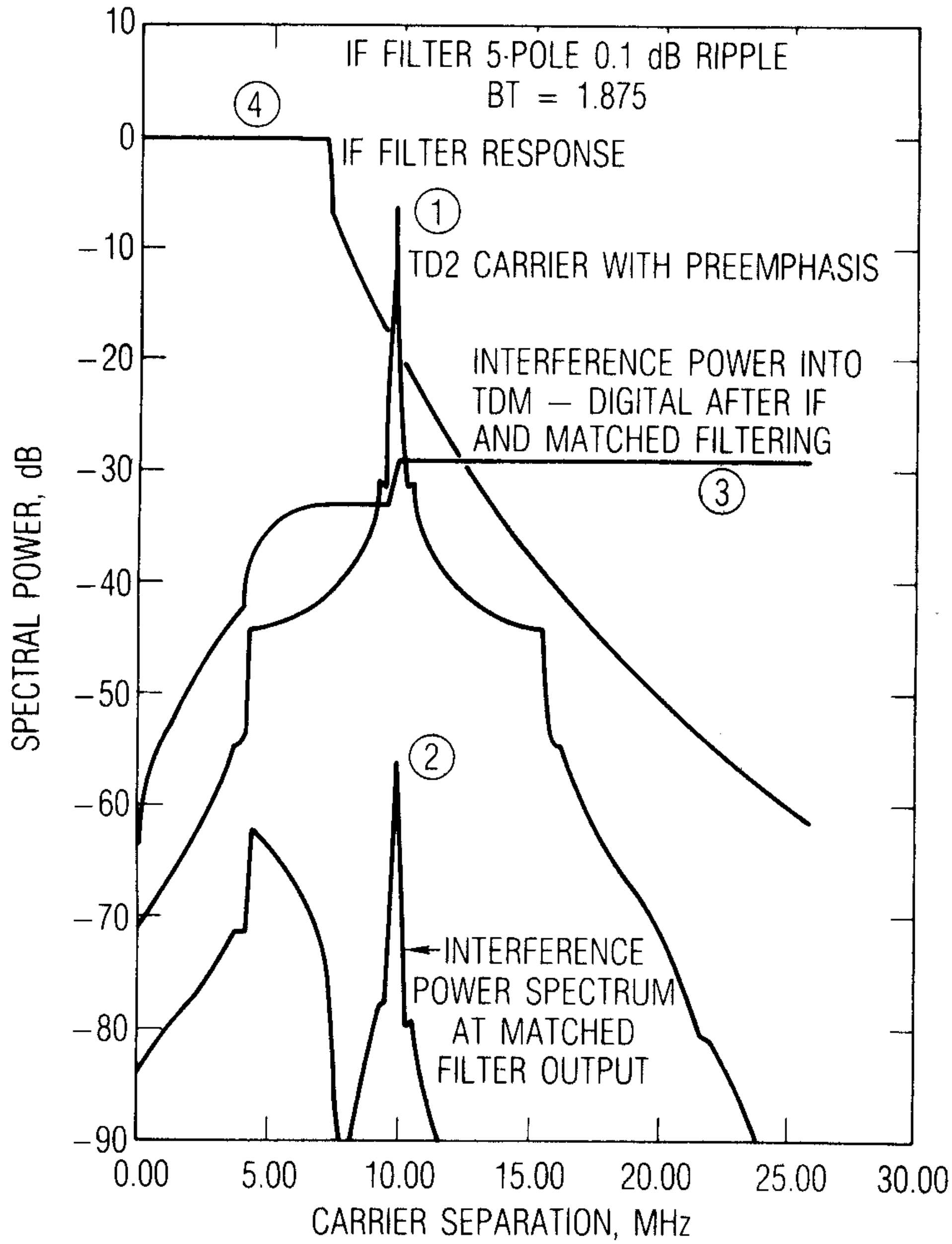

Fig. 8. Interference into TDM-Digital due to TD2 Carriers at ± 10 MHz

REFERENCE

1. Spilker, J. J., “Digital Communications by Satellite,” Prentice-Hall Electrical Engineering Series, 1977.

Table 1. Interference Rejection Due to IF Filtering

Filter Bandwidth (MHz)	BT*	Interference from Terrestrial TD2 at ± 10 MHz**
8	1.0	-41.339
10	1.25	-35.1
12	1.5	-32.93
14	1.75	-30.75
15	1.875	-29.2
16	2.0	-27.1
18	2.25	-22.63
20	2.5	-12.82
22	2.75	-12.69

*I/T = R Data rate
**5-pole, 0.1 -dB ripple Chebyshev IF filter