THROUGHPUT DELAY FOR MIXED-LENGTH, MIXED-PERIOD PACKETS WITH BUSY-SENSE MULTIPLE-ACCESS PROTOCOL

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ABSTRACT

Packet TM of asynchronous multi-instrumented spacecraft requires framing of packets having different lengths and different arrival intervals. A similar situation would occur in a general message networking application with messages of arbitrary length and arrival rate at a network node. Use of a bus-system for packet transfer, or an order wire for message transfer, requires a minimum rate of status inquiry, and of data read rate, to insure stability and non-overloading.

Our paper offers a rather general, but compact, throughput delay analysis suited to packet characteristics from uniform to source-dependent period and length. Arrival probability may vary from “soft” or uniform to “hard” arrival induced by slotted delay. The arrival probability is modular in source packet-period. A closed solution form, though expressible, is not tractable, and a recursive solution was used to obtain numerical results.

Computed throughput delays for various combinations of identical sources and mixed sources are illustrated. For identical sources, under “hard” periodic arrival probability, channel slotting is desirable to maintain channel capacity. Some comparative results are offered. For mixed sources with source-dependent packet length and rate, slotting may not be feasible or efficient. Only read-rate controls the expected throughput.

The analysis and results shown should prove helpful both to further study and present application of packet TM and message networking.

INTRODUCTION

Multiple sensor satellites may operate with each sensor delivering data at periodic intervals. The periods are different, independent, and asynchronous; and the data per period may be of different quantity from each sensor. The data is to be packetized, framed, and time-multiplexed into a single stream as for packet TM.

The arrival probability at the channel node of the data from each sensor may be accurately periodic, or an impulse probability. This is a special case of more general arrival probability. No mutual arrival probability is assumable between sources, so each source must be identifiable. It is of interest to have an estimate of the delay and throughput for such a mixed system.

Our formulation assumes a busy-sense protocol, which is modifiable to other protocols by suitable redefinition. Arrival probability is assumed periodic, with one arrival per period, but of probability from uniform to impulse, and with variable delay.

Formulation development begins with observing source “i” in the channel at time t, which leads to a channel availability probability as a definite function of source arrival probability. Calculation of the channel availability probability requires solution of a set of quadratic equations, which is done by first obtaining a separation constant by recursion.

Waiting time is then the convolution of arrival probability with the channel availability probability.

The arrival probability is modular in arrival period, normed over a period, and defined to permit “soft” or uniform, to “hard” or impulse arrival probability.

A set of three identical sources is first evaluated and some characteristics shown, culminating in delay and throughput for uniform arrival. Some curves of channel capacity for synchronous impulse arrival probability with an unslotted channel are also shown, followed by examples for a slotted channel with “soft”-to-“hard” arrival probabilities.

A set of three mixed sources then is considered, illustrated with curves of delay versus channel loads. A delay curve and a throughput curve also are shown. Performance for the mixed set depends on the relative load per source.

FORMULATION

Protocol and Waiting Time

Figure I illustrates and defines the busy-sense protocol. The next ready source "i" interrogates the channel; if busy with source "j", "i" waits and enters after "j" is read. "i" may be the first in queue when it interrogates the channel, or the next ready if on an interrogation bus-system or message order wire. Alternatively, the sources may arbitrarily deliver to the channel node. The cumulative source waiting times permit computing the probability of total channel delay.

Channel Availability

We observe the channel at time t_m = m units. If source "i" is observed in the channel, no other source is in the channel and a queue condition exists in the interval:

$$\left[m - (d_i - 1)\ ,\ m\right]$$

where d_i is the read duration of source "i." We can express this as:

$$s_i = f_i \prod_{j \neq i}^{N} (1 - s_j) = \frac{f_i \prod_{j=1}^{N} (1 - s_j)}{1 - s_i} \tag{1}$$

where

$s_i = s_i$ (m), is the probability source "i" is observed, or being served

$f_i = f_i$ (m), is probability of required queue condition during interval [m - (d_i -1), m]

Define

$$k_i = k_i(m) = \frac{\prod_{j=i}^{N} (1 - s_j)}{1 - s_i}\ , \tag{2}$$

as the channel availability to "i," N is number of sources.

Then $s_i = f_i\, k_i$,whence

$$k_i = \frac{\prod_{j=1}^{N} (1 - f_j\, k_j)}{1 - f_i\, k_i} = \frac{h(m)}{1 - f_i\, k_i} \ , \tag{3}$$

where

$$h(m) = \prod_{j=1}^{N} (1 - f_j\, k_j) \tag{4}$$

is a constant for all sources. Given f_j, k_j can in principle be found, j = 1,2,... N. A very difficult set of simultaneous quadradic equations results, of form

$$k_i\, (1 - f_i\, k_i) = k_j\, (1 - f_j\, k_j\,) = h\ . \tag{5}$$

However, if h can be found then k_i may be found directly. Note the complication of the two branches of the quadratic equations.

To find h by recursion, solve Equation 5 to find:

$$1 - f_j\, k_j = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4\, f_j\, h} \tag{6}$$

Since

$$h = \prod_{j=1}^{N} (1 - f_j\, k_j) \ , \tag{7}$$

a constrained choice of h is made for use in Equation 6, checked against Equation 7, and h is adjusted until h used in Equation 6 reproduces h in Equation 7. This recursive solution (of Equations 6 and 7) is discussed in the appendix, including the use of the two branches in the solution.

For identical sources, the process can be simplified. If written N times and multiplied,

$$(\Pi\, k_j)\ \Pi\, (1 - f_j\, k_j) = h^N \ ,$$

or

$$(\Pi k_j)\ h = h^N\ ;\ \prod_{j=1}^{N} k_j = h^{N-1} \tag{8}$$

If all k_j alike, $k_j = k_i$, then

$$k_1^N = h^{n-1}\ ;\ k_1 = h^{\frac{N-1}{N}} = (1 - f_1\ k_1)^{N-1} \tag{9}$$

This equation can also be solved recursively, directly for k_1 . By solving for

$$f_1 = \frac{1 - k_1^{\frac{1}{N-1}}}{k_1} \tag{10}$$

the inverse k_1 (f_1, N) can be plotted. Figure 2 shows a set of curves for this special condition.

A simple example for two sources is shown below:

$$k_1\ (1 - f_1\ k_1) = h = (1 - f_1\ k_1)(1 - f_2\ k_2)$$

$$k_2\ (1 - f_2\ k_2) = h = (1 - f_1\ k_1)(1 - f_2\ k_2)\ ;$$

$$k_1 = 1 - f_2\ k_2\ ;\ k_2 = 1 - f_1\ k_1$$

whence

$$k_1 = \frac{1 - f_1}{1 - f_1\ f_2}\ ;\ k_2 = \frac{1 - f_1}{1 - f_1\ f_2}\ . \tag{11}$$

If

$$f_1 = f_2\ ;\ k_1 = k_2 = \frac{1}{1 + f_1}$$

Having obtained k_i, the channel availability to source “i,” we obtain the waiting time by convolving $k_i(m)$ with the arrival probability $b_i(m)$.

Waiting Time

Suppose source "i" found the channel available at time m, with probability $k_i(m)$; and had arrived earlier with probability $b_i(m')$. In the z-domain,

$$D_k(z) = \sum_m k_i(m)\, z^m \;;\; D^*(z) = \sum_{m'} b_i(m')\, z^{-m'} \qquad (12)$$

The joint function becomes

$$D_k\, D_b^* = \sum_m \sum_{m'} b_i(m')\, k_i(m)\, z^{m-m'} \qquad (13)$$

Letting

$$m-m' = w_i ; D_k\, D_b^* = \sum_m b_i(m - w_i)\, k_i(m)\, z^{w_i} \qquad (14)$$

$b_i(m - w_i)\, k_i(m)$ is just the joint probability that the waiting time is w_i. We take the exclusive or sequential accumulation of these joint probabilities defined as

$$q_i(w_i) = \int_{m=0}^{T_i-1} b_i(m - w_i)\, k_i(m) \;. \qquad (150$$

Note the sequential cumulant is taken only over the period T_i, for which probability b_i is normed. The cumulative probability is

$$Pr\,(w_i \le g) = \int_0^g q_i(w_i) \;,$$

or

$$Pr\,(w_i \le g) = Pr\,(w_i - 1) + q_i(w_i)(1 - Pr\,(w_i - 1)),$$
$$w_i = 0, 1, \ldots g \;. \qquad (16)$$

To obtain total waiting time for the full set of N source packets, we take each $q_i\,(w_i)$ into the z-domain. We find

$$\prod_{i=1}^{N} q_i(w_i)$$

is the joint probability of

$$\sum_{i=1}^{N} w_i = w \ .$$

Again, we take the sequential cumulant as

$$q(w) = \frac{1}{N} \int_{\Sigma w_i = w} \prod_{i=1}^{N} q_i(w_i) \tag{17}$$

Normalization by l/N is required because of the N loops over w_1, w_2 ... w_N .

Calling the cumulative probability over q(w) as Q, we accumulate as

$$Q + q(1 - Q) = Q \ ; \ Q = Q(w); w = 0, 1, \ldots g \tag{18}$$

In summary,

$$Pr\ (\Sigma w_i \leq g) = \int_{w=0}^{g} \frac{1}{N} \int_{0}^{W} \prod_{i=1}^{N} q_i(w_i) \qquad \Sigma w_i = w \tag{19}$$

The upper range w is chosen to encompass the needed span for the probability functions q_i (w_i), set by the source periods.

Extended Convolution

Because the arrival probability is normed to 1 over a period, should the read durations exceed a period, the probability remains at 1, resulting in compression of the waiting time. From Equation 17,

$$q\ (w + n) = \frac{1}{N} \int_{\Sigma w_i = w+n} \Pi q_i(w_i)$$

$$= \frac{1}{N} \int \prod_i q_i(w_i) \quad q_i\left(\frac{n_i}{N}\right) \tag{20}$$

which results from writing

$$w + n = \left(w_1 + \frac{n_1}{N}\right) + \left(w_2 + \frac{n_2}{N}\right) + \cdots + \left(w_N + \frac{n_N}{N}\right) .$$

If we assume for the moment that all sources are quasisimilar, then,

$$\prod_i q_i(w_i)\ q_i\left(\frac{n_i}{N}\right) = q_1^{\,N}(w_1)\ q_1^{\,N}\left(\frac{n_1}{N}\right) \tag{21}$$

Now suppose we obtain the equivalent by an effective extension of N, such that

$$\prod_{i=1}^{N\left(1+\frac{n}{N}\right)} q_i(w_i) = q_1^{\,N}(w_1) \prod_{i=1}^{N} q_i^{\frac{n_i}{N}}(w_i) . \tag{22}$$

Comparing Equations 21 and 22,

$$q_1^{\frac{n}{N}}(w_1) = q_1^{\,N}\left(\frac{n_1}{N}\right) \tag{23}$$

Thus from Equation, 23 Equation 21 becomes

$$\prod_i q_i(w_i)\ q_i\left(\frac{n_i}{N}\right) = \prod_{i=1}^{N} q_i^{\left(1+\frac{n_i}{N}\right)}(w_i) \tag{24}$$

which is then used in Equation 20.

$$n_i\ (w_i) = \left(\left(\prod_{j\neq i}^{N} \sum_{x=u_j}^{T_i-1+u_j+w_i} p_j\ (x)\right) (D-d_i) + d_i + h\ (w_i-1)\ D\right) - T_i \geq 0; D = \sum_{j=1}^{N} d_j$$

otherwise $n_i = 0$.

Now we give further attention to the arrival probabilities.

ARRIVAL AND QUEUE STATE PROBABILITIES

Arrival Probability

This is made modular in the period and normed. The basic arrival probability is defined as:

$$b_i(x) = \frac{(T_i - x \bmod T_i)^\alpha}{\sum_{x=0}^{T_i-1} (T_i - x \bmod T_i)^\alpha} \tag{25}$$

T_i is the arrival period of source "i" data. In general, $\alpha = \alpha_j$, but we use the same α for all sources. Examples of the probability of arrival are shown in Figures 3 and 4 for negative and positive α's. When using negative α, we shift the function by -1 to produce arrival coincident with reference t = 0.

Arrival Gating or Slotting

Though b_i (x) is called the arrival probability, source "i" may find further constraint in the region of the channel node, such as gating. If the channel queue is gated, then "i" arrives at queue with joint probability,

$$p_i(x) = b_i(x)\, y_i(x) \tag{26}$$

gating function y_i (x) is defined as:

$$\text{if: } -0.5\,(i - 1)\,(T_i/N) < x \bmod T_i < i\,T_i/N + 0.5\,,\ y_i = 1\ ; \tag{27}$$

otherwise $y_i = 0$.

N slots are generated in the interval [0, T_i].

Interval Arrival Probability

The interval arrival probability is defined as:

$$a_i(t_1, t_2) = \sum_{x=t_1}^{t_2} p_i(x);\ t_1 \leq t_2\ ; \tag{28}$$

if $t_1 < 0$; t_1 and t_2 are shifted upward by T_i until $0 \leq t_1 + n\,T_i$. If $|t_2 - t_1| \geq T_i - 1$ $a_i = 1$.

Queue State Probability f_i

We visualize an arrival queue state associated with observing source “i” in the channel. Given “i” in the channel, then only operationally permitted combinations of the remaining sources have next arrived at queue during the interval [m - (d_i - 1), m]. If they had arrived earlier, they would have preceded “i” into the channel. Conditions on the queue arrival state may vary from allowing all possible combinations, to restriction to a single source arrival during the interval.

We have chosen the condition allowing the interval arrival of any new sources, together with arrival of held over sources from time t - 1. These have an “or” probability of

$$1 - \prod_{j=1}^{N} (1 - f_j k_j) = 1 - h$$

(see Equation 3). “Or”-ing this with the new interval arrivals results in

$$f_i(m) = 1 - h(m-1) \prod_{j \neq i}^{N} (1 - a_{i,j}(m)) ; \quad (29)$$

$$a_{i,j}(m) = \sum_{c=0}^{d_i - 1} \left(\sum_{x=m-(d_i-1)+c+u_i}^{m+u_i} p_j(x) \right) b_i(m-(d_i-1)+c+u_i) \quad (30)$$

DELAY AND THROUGHPUT

We first define the rate offered by the sources to the channel a

$$R = \sum_{j=1}^{N} \frac{d_j}{T_j} \quad (31)$$

The delay

$$D = D(R) \quad (32)$$

We estimate channel capacity at that point where $\delta^2 D/\delta R^2 > 0$, significantly. Throughput S at rate R for channel capacity C is:

$$S = S(R, C) = Pr(D(R) \geq D(C)) R \tag{33}$$

We estimate D as three times the value where $Pr(D \leq g = 0.9)$, giving ≥ 0.999 probability that delay $D \leq 3g$.

ILLUSTRATIVE RESULTS

Results of limited numerical computations are presented in Figures 5 through 11. Figures 5 and 6 give delay curves for N = 3 identical and mixed sources under different loads. The identical sources were run with uniform arrival probability, $\alpha = 0$; while the mixed source curves were run with $\alpha = -2$, giving them a partial impulse arrival probability.

Figure 7 shows the individual waiting times for mixed sources, and illustrates how the burden of delay is placed on the shortest period source as load approaches channel capacity.

Figures 8 and 9 illustrate the effect of channel gating on undelayed and delayed identical sources from “soft”-to-“hard” arrival probability. Figure 9, compared with Figure 8, shows that “hard” arrival sources benefit from a slotted channel, which eliminates the burden of held-over sources.

Figures 10 and 11 show delay and throughput for the example identical and mixed sources of Figures 5 and 6. The mixed sources have a flatter delay characteristic because over-loading builds up more slowly, beginning with the shortest period source. However, the channel capacity is apparently lower for mixed sources.

CONCLUSION

We have presented a rather general approach to waiting time or delay, for arbitrary combinations of source packets based on busy-sense multiple access protocol. Delay for each of the individual sources is available as well as the total delay. The definition of channel queue state at time t is a combination of new sources and those held-over from t - 1. Actual choice will depend on the buffering and control structure to be utilized. We simply allowed any of the possible new sources or the held-over sources, nonexclusively.

Arrival probability was modular and normed per period, from “soft” or uniform, to “hard” or impulse. Examples for N = 3 identical, and a set of three mixed sources, were given. Total delay for varying load levels was shown. Delay and throughput were then plotted as functions of offered rate. The mixed sources apparently have a somewhat lower channel capacity than the identical sources.

Though not illustrated, the order of mixed source assignment affects the calculated delay, but not the channel capacity. For example, with the mixed sources reversed in order, the delay is about 30 percent greater with increased slope and with a sharper break at capacity "C."

We also showed the individual source delays for mixed sources at near channel capacity rates. These curves portray how the burden of delay is placed mainly on the shortest period source.

Other curves illustrated how identical sources having "hard" arrival probability benefit from having arrival delays synchronized with channel slots. The improvement results from eliminating held-over sources.

REFERENCES

1. Pippenger, Nicholas; "Bounds on the Performance of Protocols for a Multiple-Access Broadcase Channel;" IEEE Trans. Vol. 1T-27, No. 2, March 1981, pp. 145-151.

2. Jenq, Y. C.; "Optimal Retransmission Control of Slotted ALOHA Systems;" IEEE Trans. Vol. COM-29, No. 6, June 1981, pp. 891-895.

3. Sant, Deepak; "Throughput of Unslotted ALOHA Channels With Arbitrary Packet Interarrival Time Distributions," IEEE Trans Vol COM-28, No. 8, August 1980; pp. 1422-1425.

4. Tobagi, F. A.; "Multiaccess Protocols in Packet Communication Systems," IEEE Trans Vol COM-28, No. 4, April 1980, pp. 468-488.

5. Bellini, Sandro and Flamino Borgonovo, "On The Throughput of an ALOHA Channel With Variable Length Packets;" IEEE Trans Vol COM-28, No. 11, November 1980; pp. 1932-1935.

6. Jenq, Y. C.; "On The Stability of Slotted ALOHA Systems," IEEE Trans Vol COM-28, No. 11, November 1980; pp. 1936-1939.

APPENDIX

Here we consider briefly the matter of recursive solution for h of Equations 6 and 7.

First we show some plots of k versus h in Figure A-1; and of (1 - fk) versus h in Figure A-2, where

$$1 - fk = \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4 f h} \qquad \text{(a-1)}$$

Recall we recurse such that

$$\prod_{j=1}^{N} (1 - f_j k_j) = h ,$$

and we must use the proper branches of the $\sqrt{\ }$. The bound 1 - f, shown on Figure A-2, constrains $k \leq 1$; the shaded region is excluded.

A list of observations to aid in the recursion is as follows:

Let

$$w = h_{max}$$

The f-vector is first reindexed so that $f(1) \geq f(2) \geq \ldots \geq f(N)$.
If f(1) < 0.25, w = 1
If f(l) ≥ 0.25, w = 1/(4 f(1))
If w > 0.5, use + $\sqrt{\ }$ only.
If h^+ (w) < w, the solution is in + $\sqrt{\ }$ branch only; decreasing h from w will increase h^+ (h) until h and h^+ (h) cross.

If w < 0.5
If h^+ (w) < w; solution in + $\sqrt{\ }$.
If h^+ (w) > w; some part of the - $\sqrt{\ }$ is required.
If f(1) > 0.25, w = 1/(4 f(1))
If h^+ (w) > w; again some portion of - $\sqrt{\ }$ is required
Set radical sign to -1
Set h = 1 f(1) < w
If h_1 (h I - f(1)) < w, there must be a solution; increase h < w until found
----

If h_1 (1 - f(1)) > W
If 1 - f(2) < w; set radical sign to -1
Set h = 1 - f(2) < w
If h_2 (1 - f(2)) < w, must be a solution; increase h < w until found
----
If h_2 (1 - f(2)) > w
If 1 - f(3) < w; set radical sign to -1
If h_3 (1 - f(3)) < w, increase h to find the solution
------------ etc.
There are certain sensitive regions where special computation care must be taken.

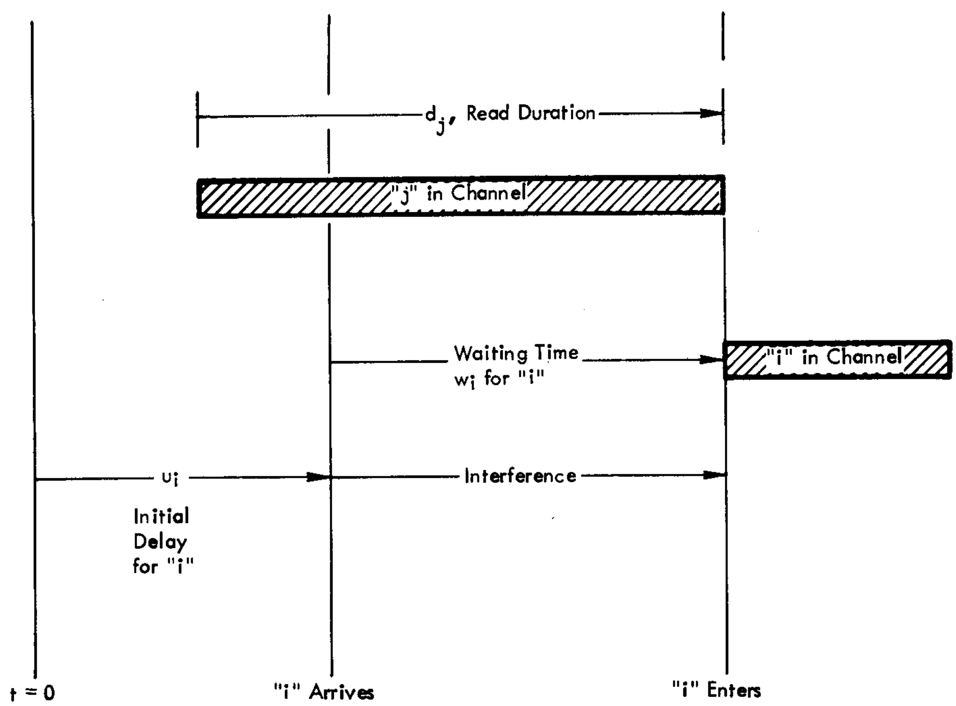

FIGURE 1. WAITING TIME FOR SOURCE 'i" RESULTS FROM INTERFERENCE BY SOURCE "j"; j = 1, 2.......N; j i

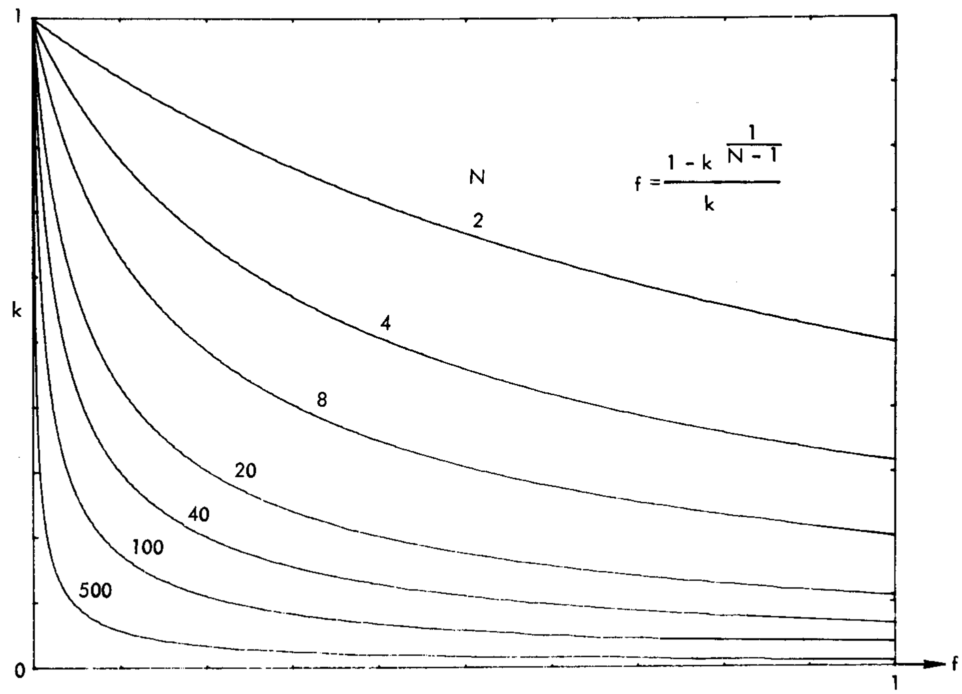

FIGURE 2. CHANNEL AVAILABILITY PROBABILITY k (f, N) FOR N IDENTICAL SOURCES; N = 2, 4, 8, 20, 40, 100, 500

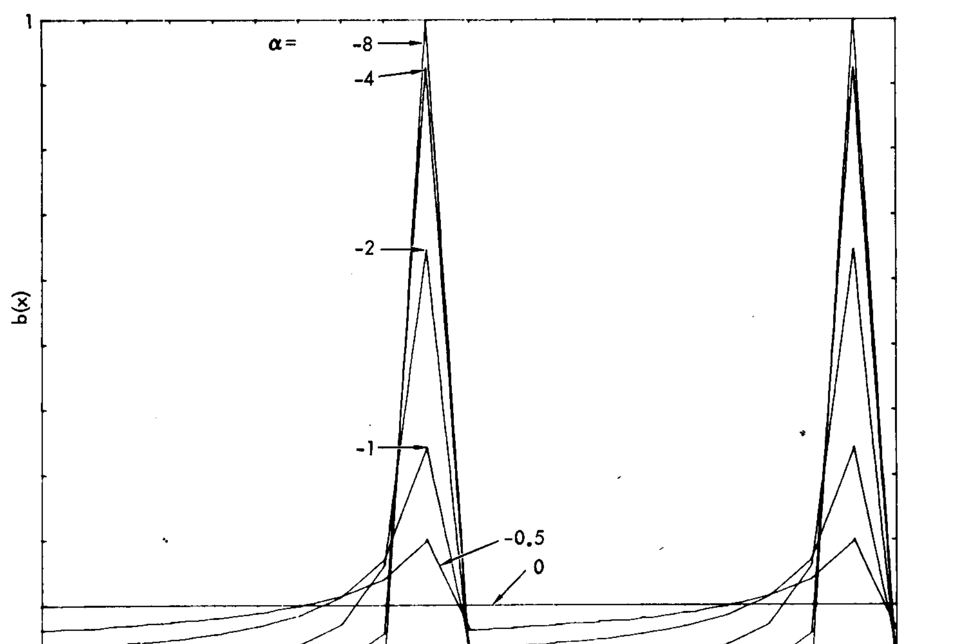

FIGURE 3. MODULAR SOURCE ARRIVAL PROBABILITY FOR α = 0, -0. 5, -1, -2, -4, -8; PERIOD T = 10

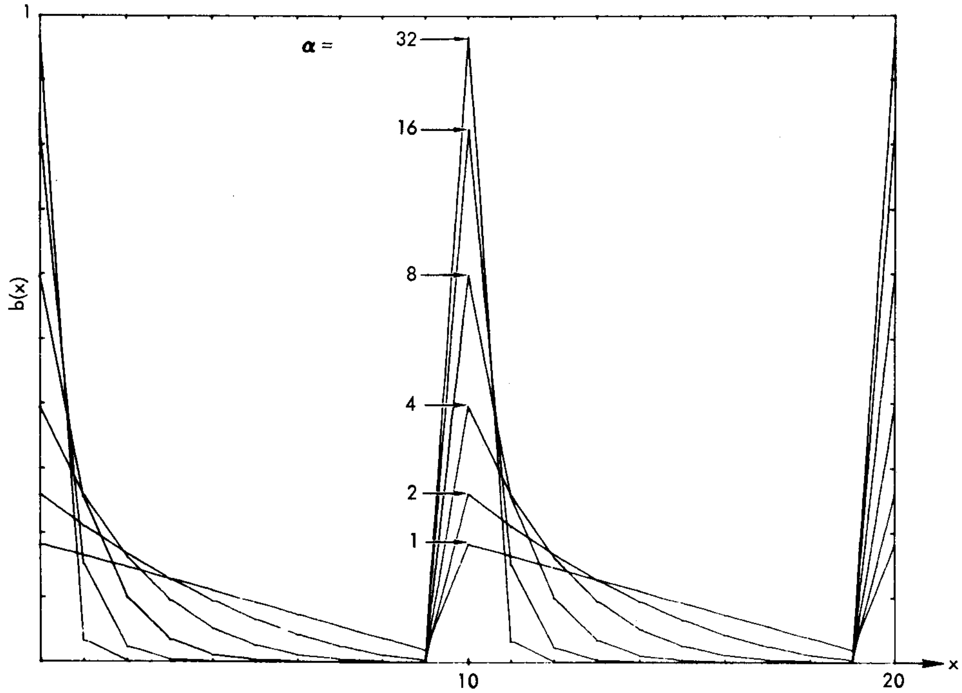

FIGURE 4. MODULAR SOURCE AR RIVAL PROBABILITY FOR α = 1, 2, 4, 8, 16, 32; PERIOD T = 10

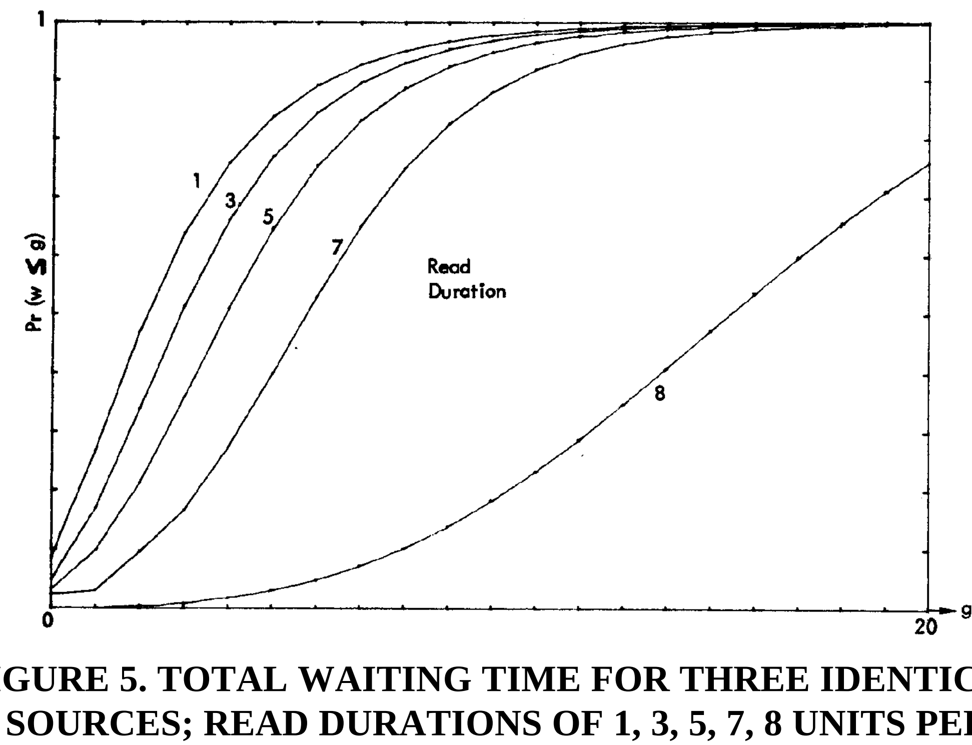

FIGURE 5. TOTAL WAITING TIME FOR THREE IDENTICAL SOURCES; READ DURATIONS OF 1, 3, 5, 7, 8 UNITS PER SOURCE. PERIOD T = 21. α = 0.

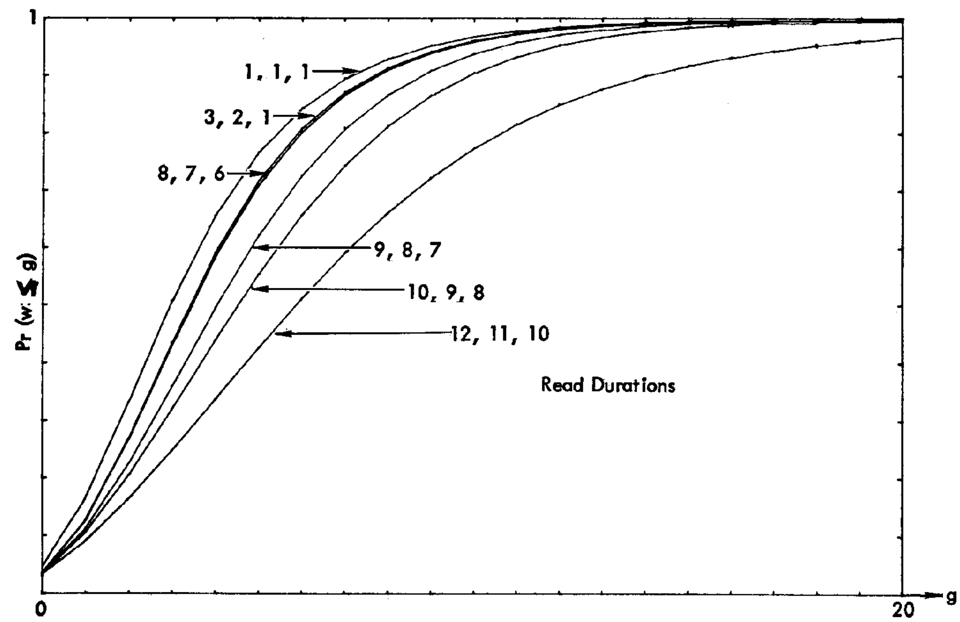

FIGURE 6. TOTAL WAITING TIME FOR THREE MIXED SOURCES. PERIODS T = 41, 31, 21; FOR ORDERED READ DURATIONS SHOWN; α = -2.

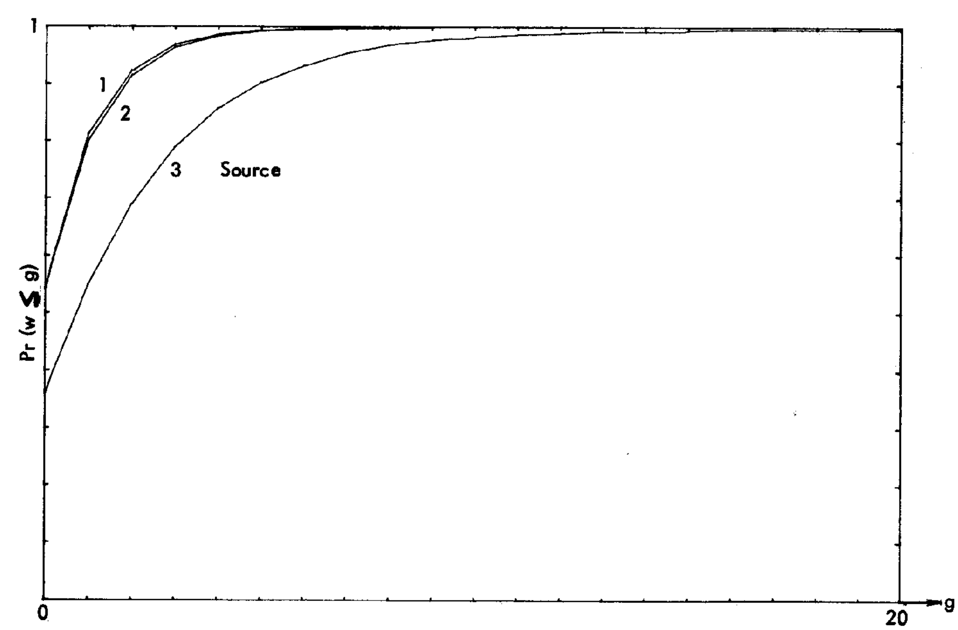

FIGURE 7. INDIVIDUAL SOURCE WAITING TIMES FOR MIXED SOURCES NEAR CHANNEL CAPACITY; T = 41, 31, 21; READ DURATION = 8, 7, 6: α = -2.

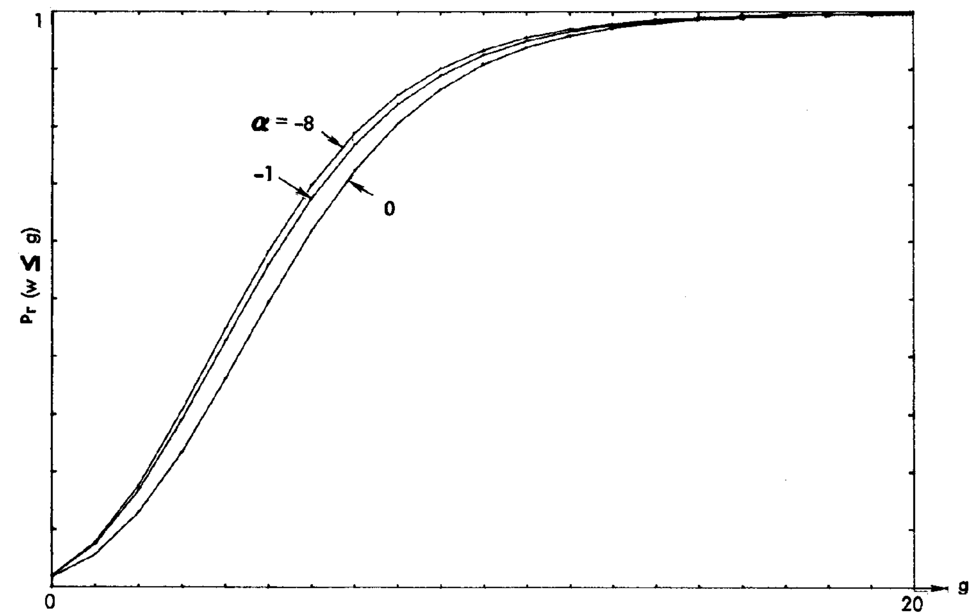

FIGURE 8. TOTAL WAITING TIME FOR THREE IDENTICAL UNDELAYED SOURCES, 3-SLOT CHANNEL; α = 0, -1, -8; T = 21; READ DURATION = 7 (SOURCE DELAY = -1)

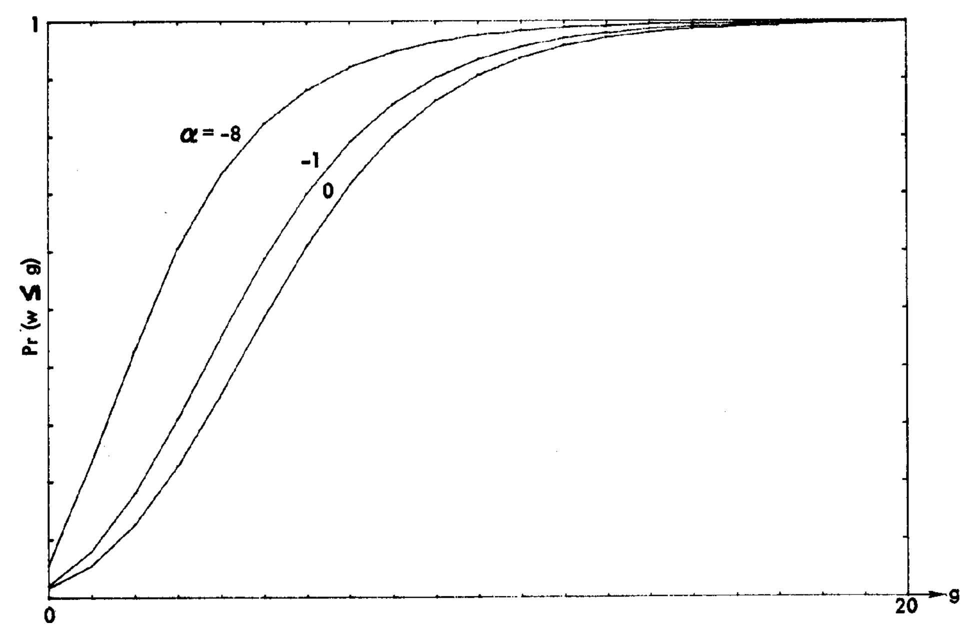

FIGURE 9. TOTAL WAITING TIME FOR THREE IDENTICAL DELAYED SOURCES; 3-SLOT CHANNEL; α = 0, -1, -8; T = 21; READ DURATION = 7; SOURCE DELAYS -1, -8, -15.

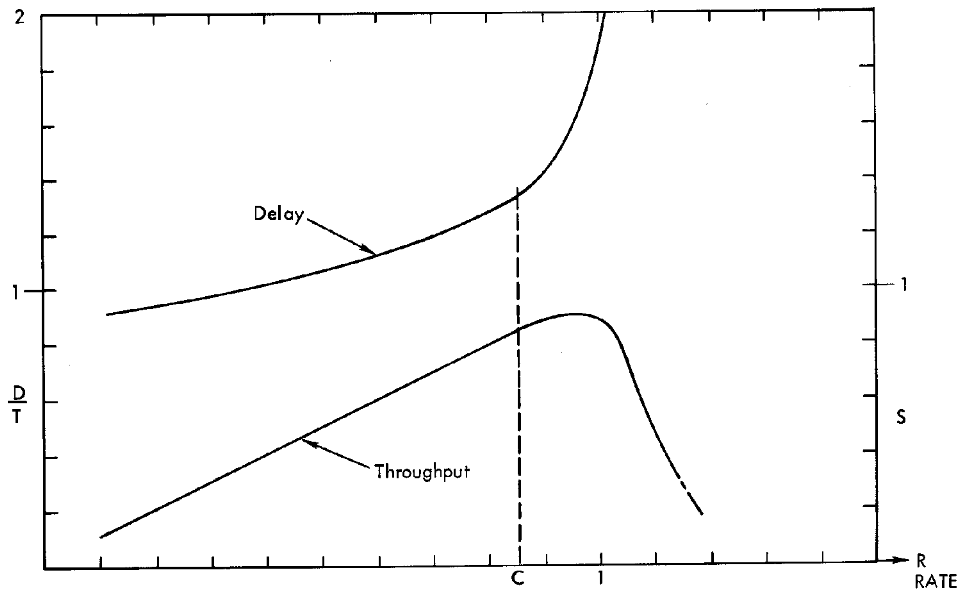

FIGURE 10. DELAY D(R) AND THROUGHPUT S(R) FOR THREE IDENTICAL SOURCES; α = 0

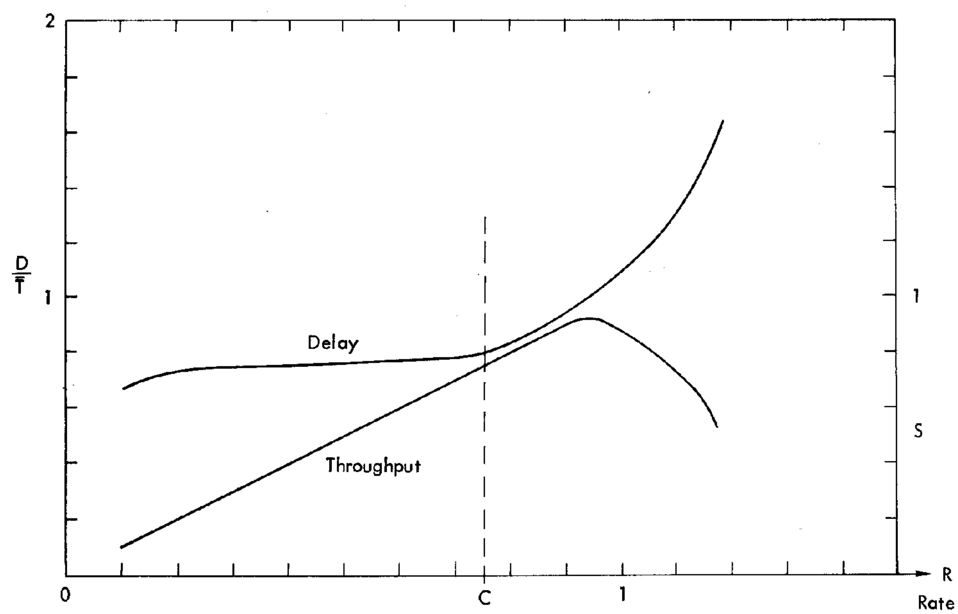

FIGURE 11. DELAY D(R) AND THROUGHPUT S(R) FOR THREE MIXED SOURCES; T = 41, 31, 21; $\overline{T}$ = 31; α = -2.

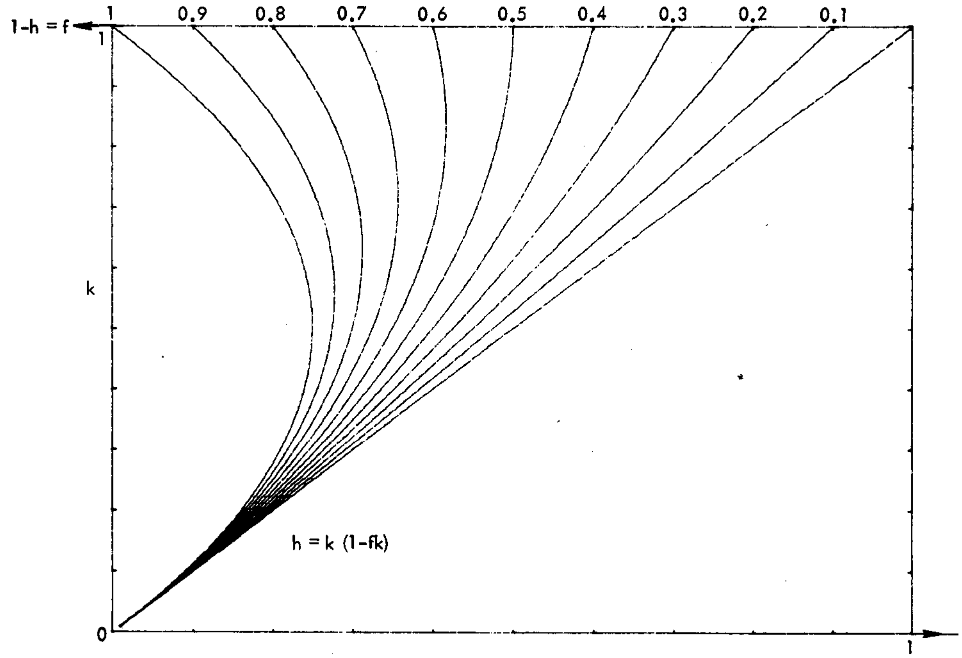

FIGURE A -1. CHANNEL AVAILABILITY PROBABILITY k(h); f = 0, 1; (0.1)

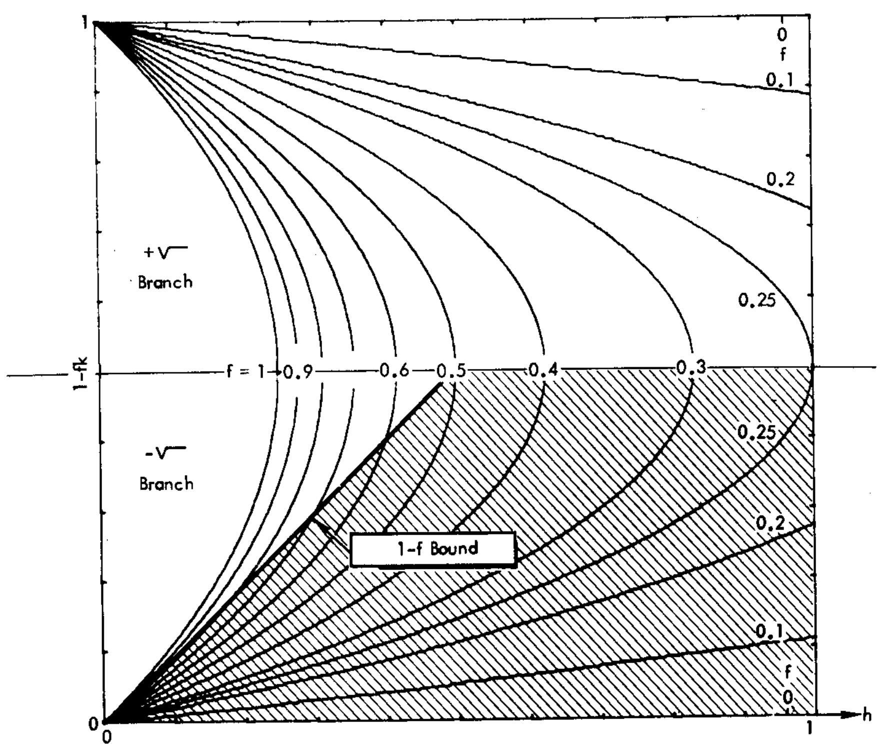

FIGURE A -2. FUNCTION 1-fk VERSUS h, SHOWING USED AND EXCLUDED REGIONS FOR RECURSIVE SOLUTION FOR h