# INFORMATION RATES IN FIBEROPTIC LINKS WITH MULTI-LEVEL PULSE INTENSITY ENCODING (MPIM)

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In fiberoptic communication links using digital transmission, fiber dispersion limits the transmitted data rate, the rate at which data bits are transmitted, while the receiver noise characteristics determine the ability to decide. To incorporate both of these effects into a singel design parameter, it is convenient to deal with the link mutual information rate. The latter is defined as the data rate minus the effective loss in rate due to decoding errors. In this paper, information rates of some typical fiber links are developed and used to compare possible design trade-offs under various signaling formats. Mutual information is particularly effective for evaluating the advantages of using multi-level pulse amplitude encoding in an attempt to increase data rates.

Consider the digital fiber link in Figure 1. An optical source (LED or injection laser) is digitally modulated and feeds the resulting optical field into the fiber. After propagating over the fiber length L, the optical field is photo-detected and converted to an electronic signal for bit decoding. Digital modulation is typically accomplished by noncoherent methods in which data bits are sent as optical pulses whose intensity have been modulated. The common noncoherent amplitude modulation formats [1] are two level pulse intensity on-off keying (OOK) modulation, and multilevel pulse intensity modulation (MPIM). In the MPIM case, a block of source bits is encoded onto one of the M possible intensity levels for data transmission. For decoding, the electronic signal produced from the photo-detection is integrated over the pulse time and compared to a voltage threshold, or a set of thresholds, for detection.

The important parameters characterizing the fiber as a communication channel are its attenuation loss and its dispersion. The attenuation loss (including both absorptive and scattering loss) specifies the amount of power loss during fiber propagation. Fiber dispersion causes an inherent spreading of the time width of the propagated intensity pulse. In a digital data transmission, this pulse spreading limits the rate at which light pulses can be sent, and therefore limits the pulse rate of the link. The total amount of pulse spreading that will occur in a fiber depends directly on the fiber dispersion coefficient, measured in seconds of pulse spreading per unit fiber length. Values of this coefficient typically vary from 50 ns /km for a uniform core to 1 ns/km for a quadratic core tapering, to about several picoseconds for a single mode fiber.

The data rate of the fiber (or MPIM), is dependent on the number of intensity levels transmitted into the fiber. For on-off keyed formats, each optical pulse carries one data bit, and the fiber data rate simply equals its pulse rate. In MPIM formats, each pulse carries $\log_2 M$ bits, and the data rate R is related to pulse rate $R_p$ by

$$R = R_p \log_2 M \text{ bits/sec} \qquad (1)$$

The mutual information rate of a fiber system is the rate at which mutual information [2-4] is transferred. In terms of multi-level signaling with a single optical pulse, the mutual information per pulse is given by (2).

$$I = \sum_{j=1}^{M} \sum_{i=1}^{M} \left[ \log \left( \frac{P_{ij}}{P_i} \right) \right] P_{ij} \quad P_i \qquad (2)$$

$$= \sum_{j=1}^{M} \sum_{i=1}^{M} P_i + \sum_{j=1}^{M} \sum_{i=1}^{M} \log ()_{ij)} P_{ij} P_i$$

where

$P_{ij}$ = probability intensity level i was decoded, given level j was sent

$P_i$ = probability intensity level i was transmitted

M = number of pulse levels

If we assume equally likely data bits, $P_i$ = 1/M, then (2) becomes

$$I = \mathrm{LOG}\ M + \frac{1}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} P_{ij} \log P_{ij} \qquad (3)$$

The information rate H of the link is then the product of the mutual information per pulse times the pulse rate $R_p$ of the channel. Thus,

$$H = I R_p$$
$$= R_p \log_2 M - \frac{R_p}{M} \sum_{i=1}^{M} \sum_{j=1}^{M} P_{ij} \log P_{ij} \qquad (4)$$

The first term is the fiber data rate in (1), while the second term accounts for information loss due to incorrect decoding. Note as $P_{ij} \rightarrow 1/M$, $H \rightarrow o$. In the latter case, the receiver is, in essence, guessing at the transmitted pulse level and no effective information is, in fact, being transmitted.

The probability $P_{ij}$ of erring in decoding a particular intensity level depends on the characteristics of the receiver. The basic photo detection model is shown in Figure 2. The encoded optical pulse is photodetected at the receiver and converted to an electron flow that is amplified by avalanche regeneration [5-6]. The resulting electron stream flows through a load resistance R generating a detector signal used for decoding. Receiver thermal noise is added to this detected signal and forms the pulse signal value which is integrated over the pulse width T. This generates the integrate-and-dump voltage.

$$v = \frac{emR}{T} + n_T \tag{5}$$

where e is the electron charge, m is the number of electrons through the resistor R in time T, and $n_T$ is the integrated thermal noise. The latter is taken as a zero mean Gaussian variable, with variance 4 $kT^{o}BR$, where k is Boltzmann's constant, $B = 1/T$, and $T^{o}$ is the receiver noise temperature in degrees Kelvin. The electron count m generated over the T sec pulse time is a random variable governed by the randomness of the field photodetection and avalanche amplification. The probability distribution of m electrons occurring was derived by Conradi [6], using results of MacIntyre [5]. Webb [7] showed that an accurate approximation to the somewhat complicated distribution of Conradi is given by

$$P(m|n) = \frac{1}{(2\pi n G^2 F)^{1/2}} \cdot \frac{1}{\left(1 + \frac{m-ng}{\sigma\lambda}\right)^{3/2}} \cdot \exp\left[ - \frac{(m-nG)^2}{2\sigma^2 \left[1 + \frac{m-nG}{\sigma\lambda}\right]} \right] \tag{6}$$

where G is the avalanche gain, and

$$F = kG + \left(2 - \frac{1}{G}\right)(1-k)$$

$$= (nF)^{1/2} / (F-1)$$

$\sigma^2 = nG^2F$

n = injected electrons into avalanche region.

The probability density of the receiver voltage v follows from (6) by first conditioning on the number of photo-detected counts m, then averaging over the avalanche statistics. Since $n_T$ is Gaussian, the conditioned variable v is also Gaussian with mean 23BRm and variance $4KT^oBP$. The probability density on the detected receiver voltage v is then

$$P(v \mid n) = \sum_{m=1}^{\infty} \phi \quad (v,\ 2eBRm,\ 4kT^{o}BR)\ P(m|n) \tag{7}$$

where

$$\phi(v, a, b) = \frac{1}{\sqrt{2\pi b}} \quad \exp\left[-\frac{(v-a)^2}{2b}\right]$$

Thus, the receiver noise converts the discrete counting variable m to the continuous observation variable v. Note that the density in (7) depends on the total received count n, and therefore requires specification of the particular signalling format used at the transmitter.

When the receiver temperature is negligible (i. e., $4kT^oBR << 2eBRm$), the density in (7) can be considered a discrete density and the receiver is said to be shot noise limited. In this case, statistical performance based on observations of v can be equivalently stated directly in terms of counting statistics on m.

For MPIM, a pulse of one of M levels is transmitted into the fiber, producing an integrated count intensity $n_i$ in (7) depending on the level. Decoding is achieved by comparing the observed voltage v to a set of thresholds ($v$, $v_2$ - -) and decoding the level based upon which pair of thresholds the observable v falls between. The corresponding decoding error probability is obtained by computing

$$P_{ij} = \int_{v_{j-1}}^{v_j} P(v/n_i)dv \tag{8}$$

where ($v_j$, $v_{j-1}$ ) are the thresholds for intensity level $n_i$. Equation (8) was examined in the work of Sorensen (8, 9) in which the selection of the thresholds was related to the intensity level set $|n_i|$ and the APD gain G, and later were chosen to minimize total error probability for a given set of receiver parameters. Figure 3 shows a typical plot of the level error probability $P_{ij}$ in (3) for a specific set of parameters and several values of M. The abscissa plots the peak pulse energy associated with the highest level. The remaining intensity levels were set according to a non-linear level separation [9] and the thresholds and APD gain adjusted to uield the minimal average value of $P_{ij}$.

When the curves in Figure 3 are substituted into (3), the mutual information plot in Figure 4 results. For the given set of receiver parameters, the amount of mutual information per opticalpulse is shown for various numbers of intensity levels M, as a function of the parameter $n_M / \log_2 M$ which is the peak signal count per data bit. Both shot noise and receiver noise cases are shown. At high count values, the information per pulse approaches the data rate per pulse allowed by the pulse encoding. At the lower values; however, the increased difficulty in decoding decreases the amount of information conveyed. As a result, distinct crossovers exist where a given value of M produces the highest pulse information. Note that at low count values, per pulse, the binary system (M=2) is the most effective, while increasing count values favor increasing values of M. In essence, high levels of pulse encoding should not be used unless the peak pulse energy is available to support it. Curves such as Figure 4 are helpful for assessing this trade-off.

**Effect of Fiber on Mutual Information**

While I in Figure 4 indicates the information content per pulse, the achievable information rate H in (4) depends on the rate at which optical pulses can be transmitted. As stated earlier, this depends on the fiber dispersion which causes pulse overlap and intersymbol distortion if pulses are transmitted at too fast a rate. As the length of the fiber increases, its dispersion increases, and the pulse rate must be slowed to prevent distortion. At the same time, fiber attenuation reduces the available source pulse energy and decreases the information per pulse. If we assume minimum pulse width is limited to the pulse dispersion, then the maximum possible pulse rate is given by the reciprocal of the expected fiber dispersion. The received pulse energy is reduced from that at the source by the accumulated fiber attenuation. Figures 5 and 6 show the resulting mutual information rate as a function of the fiber length, assuming fixed dispersion and attenuation values of 10 nsec/km and 10 dB/km, respectively. A source rate of $10^8$ pulse/sec is used, and a source power level producing an average signalling count of 1000 and 10, 000 counts/pulse is considered. The ordinate plots directly the achievable information rate in bits/sec for the link. At short fiber lengths, the fiber attenuation is negligible and pulse encoding is advantageous as the pulse counts indicated. The information rate is approximately the data rate, and the use of more pulse levels increases the available link bit rate. As the fiber length is increased, the available information rate at any M is reduced from the source data rate due to the decoding errors caused by fiber attenuation. This degradation is more severe at the higher values of M (due to the increased decoding difficulty) causing an apparent crossover in achievable information rates. As the fiber length is increased, the smaller M systems become more advantageous, and for very long fibers the binary link (M=2) produces the highest information rate.

**REFERENCES**


1.) Gagliardi, R., and Karp, S., Optical Communication, Optical Communication, John Wiley and Sons, Inc. , New York, 1975.

2.) Ash, R., Information Theory, John Wiley and Sons, Inc., New York, 1967.

3.) Wozen Craft, J. , and Jacobs, I., Principles of Communication Engineering, John Wiley and Sons, Inc., New York,1967.

4.) Gregg, W., Analog and Digital Communication, John Wiley and Sons, Inc., New York, 1977.

5.) McIntyre, R. , “The Distribution of Gains in Uniformly Multiplying Avalanche Photodiodes: Theory”, IEEE Trans. on Electron Devices, Vol. 6, June 1972 pp. 713-718.

7.) Webb, D., McIntyre, R., and Conradi, J., “Properties of Avalanche Photodiodes,” RCA Review, Vol. 35, June 1974, pp. 234-278.

8.) Sorensen, N., and Gagliardi, R., “Performance of Optical Receivers With Avalanche Photodetection”, IEEE Trans. on Communications, Vol. Com-27, No. 9 September 1979, pp. 1315 -1321.

9.) Sorensen, A. N. , “Optical System Design With Avalanche Gain Detection”, Dissertation (University of Southern California), September 1978.

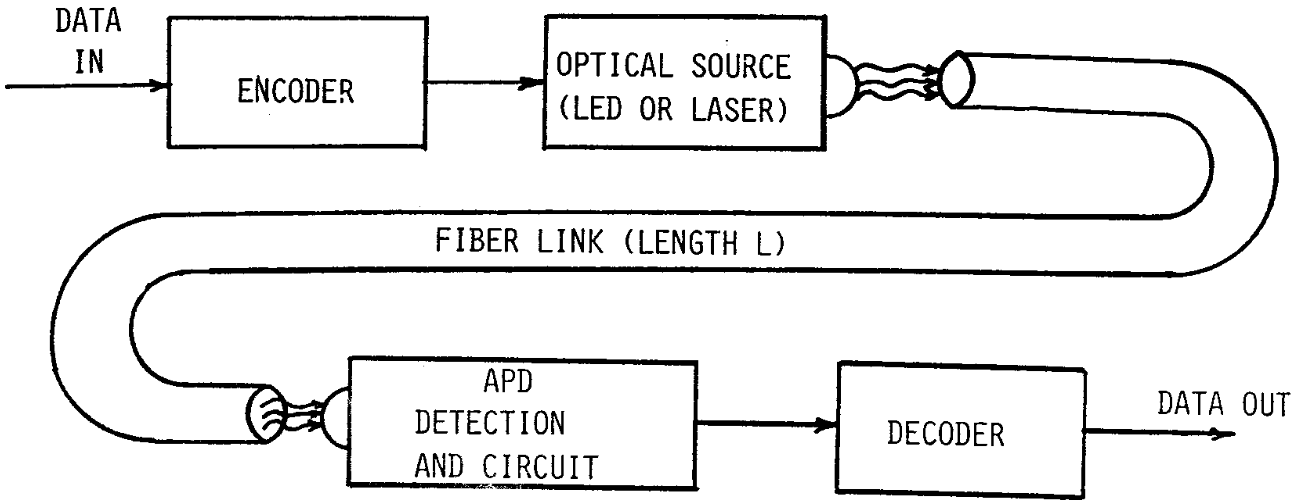


**FIGURE 1 -- ENCODED DIGITAL FIBER LINK WITH AVALANCHE PHOTO-DETECTIOM (APD)**

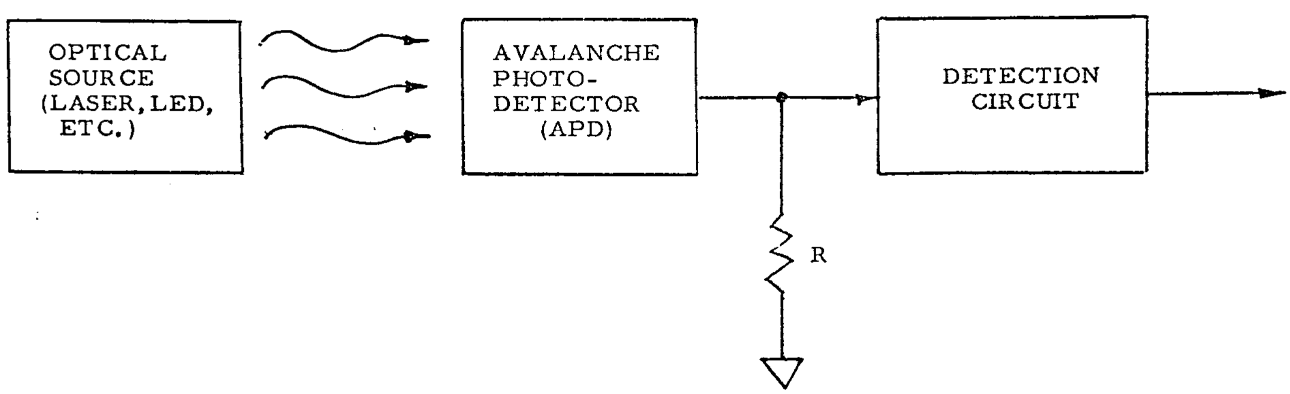


**FIGURE 2 --AVALANCHE PHOTO-DETECTION MODEL (APD)**

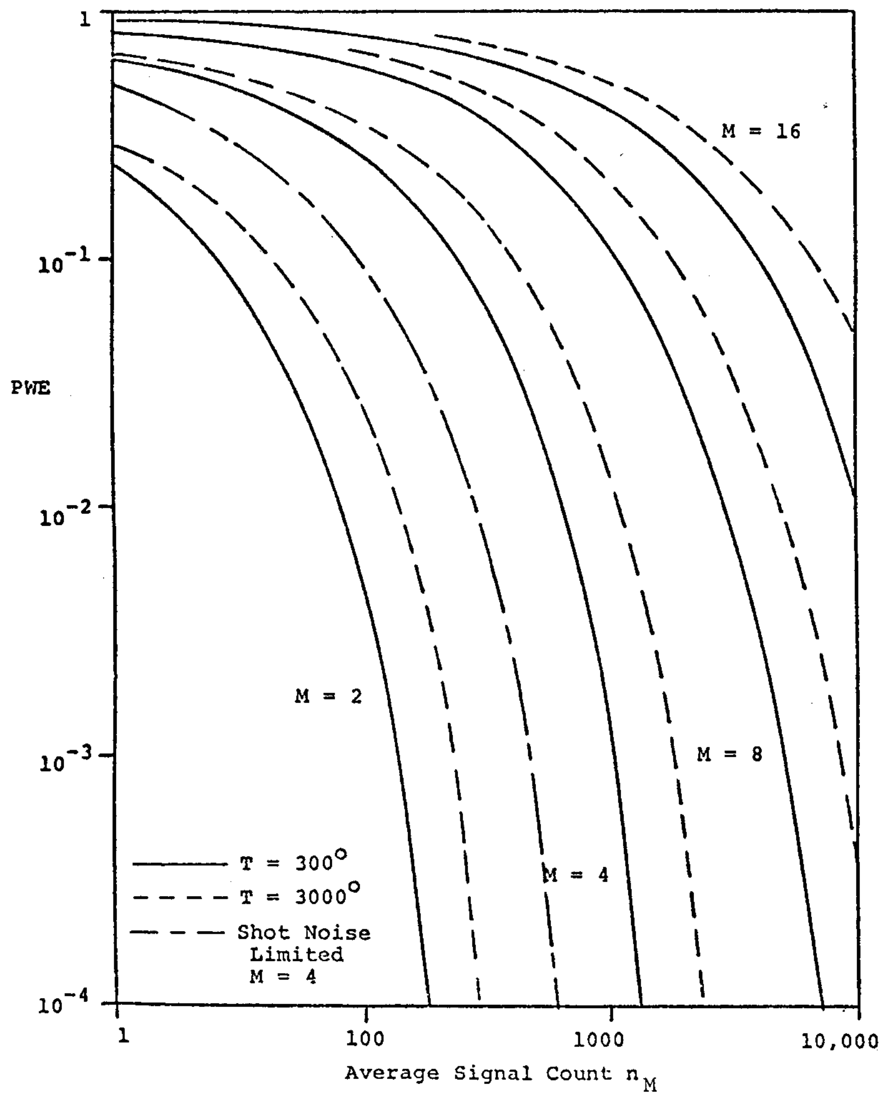


**Figure 3. M-ary PIM Performance (Additive Thermal Noise, Non-Linear Intensity Levels, Optimum Gain G Selected) k = .028, R = 50 , B = 1000 MHz.**
**$\bar{n}_b$ (background noise) = 1**

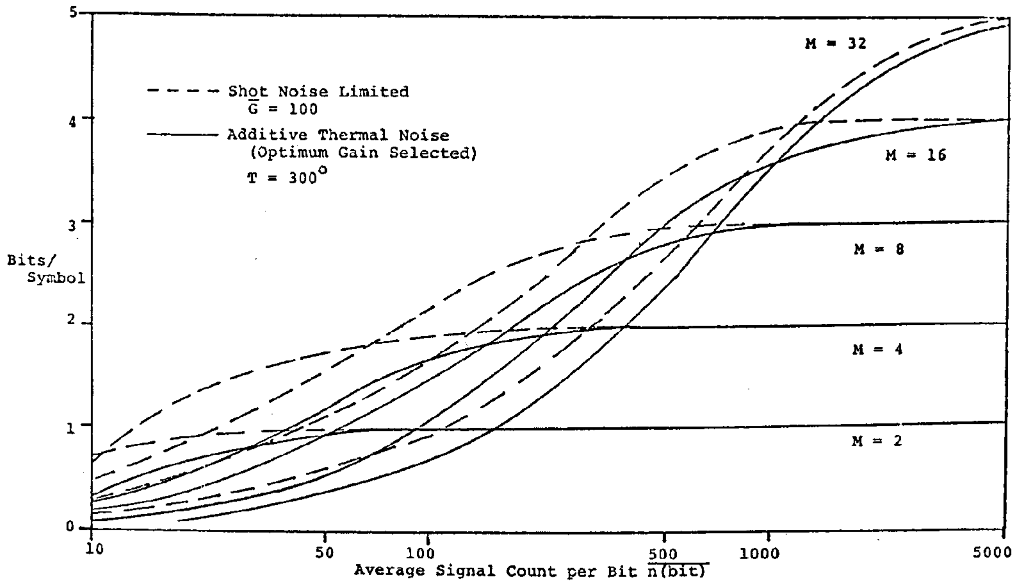


**Figure 4. M-ary PIM Mutual Information as a Function of Average Signal Count per Bit. k = .028, R = 50 , B = 1000 MHz.**
**$\bar{n}_b$ (back round noise = 1**

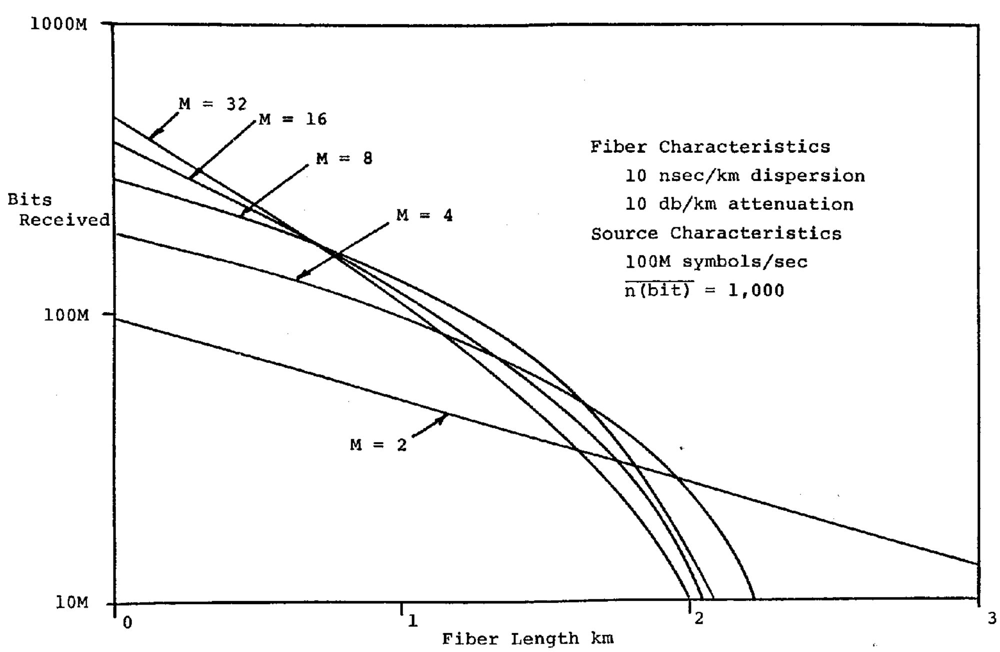


**Figure 5. M-ary PIN Fiber Link (Bits Received per Fiber Length, Shot Noise Limited) $\bar{n}_b$ = 1, k = .028, Non-Linear Spacing, Optimum Gain Selected.**

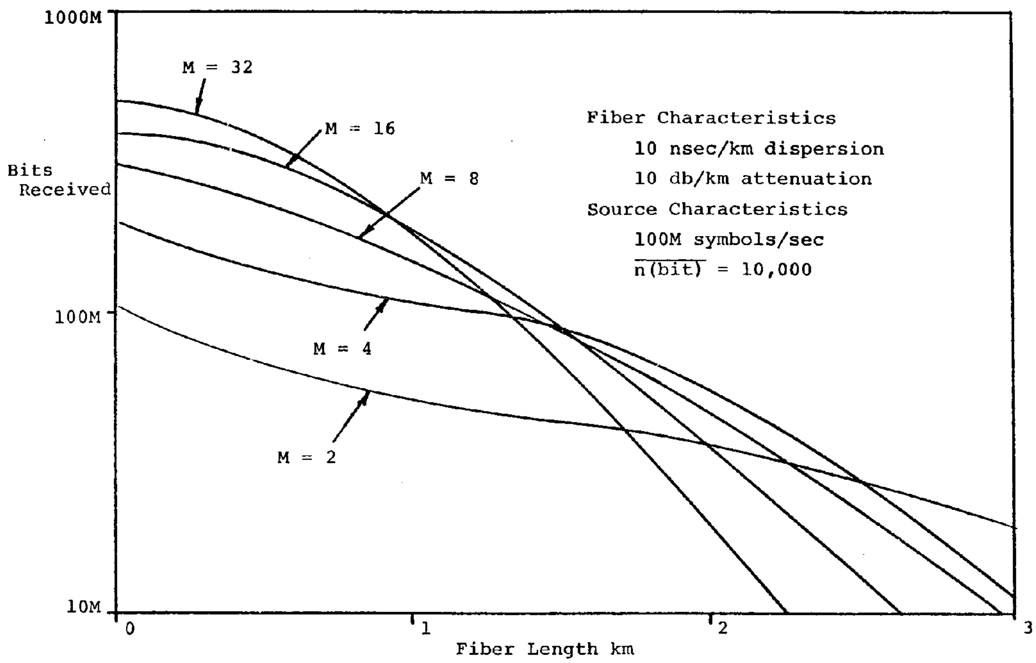


**Figure 6. M-ary PIM Fiber Link (Bits Received per Fiber Length, Additive Thermal Noise) $\bar{n}_b = 1$, k = .028, T = 300°, R = 50 , B = 100 MHz, Non-Linear Intensity Spacing, Optimum Gain Selected.**