EFFECTS OF BASEBAND PULSE SHAPE AND DEVIATION RATIO SELECTIONS ON THE SPECTRUM OCCUPANCY OF M-ARY PCM/FM

Timothy F. Cox
Space, Telecommunications, and Radioscience Laboratory
Stanford Electronics Laboratory
Electrical Engineering Department
Stanford University
Stanford, CA 94305-4055

ABSTRACT

Power spectra of M-ary, or multi-level, PCM/FM are calculated for a limited selection of baseband pulse shapes and deviation ratios. Parameter values, such as number levels, symbol rates, and deviation ratios, are determined for those cases in which the corresponding spectra fit into 1, 3, and 10 MHz bandwidth channels. The IRIG specifications are used to produce the spectrum occupancy values from the normailized power spectra densities.

INTRODUCTION

In many RF telemetry applications the system is constrained in transmitted bandwidth or power or both. When the telemetry data is in digital form it is often the case that bandwidth constraints are more critical than power limitations. In this case M-ary, or multi-level, PCM/FM may be considered as a modulation scheme in which the data throughput rate of a channel can be increased at a cost of higher signal-to-noise requirements. This paper will concentrate on the spectrum occupancy characteristics of M-ary PCM/FM without regard to the error performance resulting from certain parameter selections.

The spectrum occupancy requirements used in this study are given in the Telemetry Standards, IRIG Standard 106-80 [1]. Specific details of these requirements are presented later. Power spectra are computed for a limited selection of system parameters to determine the maximum rate of data throughputs relative to the occupancy constraints of narrowband, mediumband and wideband channel. The three system parameters which are considered free are the baseband pulse shape, the deviation ratio, and the number of

levels (M). A set of these three parameters is chosen and the occupancy constraints are applied which results in a maximum data rate for that system. These results are presented in the conclusion of the paper.

SPECTRAL OCCUPANCY REQUIREMENTS

IRIG-106 defines the allowable bandwidths of the narrowband, mediumband, and wideband channels as 1, 3, 10 MHz, respectively. The first corner frequency occurs at 1 MHz away from center frequency for the narrowband channel, 2 MHz for mediumband, and 5.5 MHz for the wideband. The specification corresponding to this corner is that there shall be no power measured in any 3 kHz bandwidth between this corner and the second corner frequency more than -60 dBc or -25 dBm, whichever is greater. The second corner frequency occurs at 1.5 MHz away from center frequency for the narrowband channel, 2.5 MHz for the mediumband, and 6 MHz for the wideband. Power measured in any 3 kHz bandwidth beyond the second corner shall be no greater than - [55+10log(carrier power)] dBc, which is equal to -25dBm.

The specification of allowable levels in terms of absolute carrier power is somewhat inconvenient for this generalized study. Having a single corner for each bandwidth with power levels specified relative to the carrier power would be much more convenient. This is possible if a moderate carrier power around 3 watts is considered nominal. Then the first corner specified at -60 dBc should be sufficient for spectra that are asymptotically decreasing beyond that corner.

The particular goal of this study is to determine the maximum data throughput rate of M-ary PCM/FM in narrowband, mediumband, and wideband channels given a nomial carrier power and a set of system parameters. The specific spectrum occupancy constraint is that the power measured in any 3 kHz bandwidth beyond the corner frequency shall not be greater that -60 dBc. Here the corner frequency is that 1 MHz away from the center frequency for a narrowband channel, 2 MHz for a mediumband, and 5.5 MHz for a wideband.

M-ARY PCM/FM FUNDAMENTALS

The name PCM/FM implies a transmission system in which the carrier is frequency modulated in some manner by a digital signal. Furthermore, this digital signal is derived by digitally encoding an analog signal, or possibly a multiplex of analog signals. The manner of modulation by the digital signal is also implied as a frequency modulated carrier with the amount of frequency deviation as a linear function of the baseband input voltage. The baseband input can be the digital signal itself comprised of an infinite sequence of square pulses, each with a duration of exactly one symbol period, or a filtered or “shaped”

version of the square pulse sequence. In any case it is possible to express this PCM/FM signal as follows:

$$s(t) = A \cos\left[2\pi f_c t + \int_0^t f_d(\mu)d\mu + \phi_0\right] \quad (1)$$

where

f_c ≡ carrier or center frequency

$f_d(t)$ ≡ baseband frequency deviation function

ϕ_0 ≡ initial carrier phase.

For digital data the baseband function is comprised of the superposition of all pulses corresponding to the infinite sequence of data symbols. This superposition is expressed as

$$f_d(t) = \sum_{k=-\infty}^{\infty} x_{a(k)}(t - kT_s) , \qquad a(k) = 1, 2, \ldots , M \quad (2)$$

where

Ts ≡ data symbol period

a(k) ≡ M-ary value corresponding to the data symbol occuring at kT_s

$x_{a(k)}$ ≡ baseband frequency deviation function corresponding to a(k).

Inspection of (1) and (2) suggests to the system designer that $x_{a(k)}$, T_s, and M can be chosen to satisfy certain requirements. However, due to the complex nature of the M-ary PCM/FM signal, s(t), effects on it from various choises of $x_{a(k)}$, T_s, and M are not intuitively obvious. To determine these effects the power spectrum of s(t) was calculated by computer with the computer program allowing a choice of input variables over a selected range.

CALCULATION OF POWER SPECTRA

The calculation of the power spectrum of s(t) is a complex chore and the background mathematical development is beyond the scope of this paper. Fortunately, this development can be found in two excellent papers, one bt Anderson and Salz [2] in 1965,

and the other by Rowe and Prabhu [3] in 1975. The reader is strongly recommended to refer to these papers not only for specific solutions, but also for a greater general understanding of the problem.

At this point some parts of the problem need to be specified so that a particular solution can be obtained. First of all this paper considers the baseband pulse sequence to consist of overlapping or nonoverlapping pulses generated from a common shape. With this $x_{a(k)}$ may be expressed as,

$$X_{a(k)}(t) = \pi\Delta[2a(k) - M - 1]\, f(t) \tag{3}$$

where

$f(t)$ ≡ common, normalized, baseband pulse

Δ ≡ frequency separation corresponding to the interval of the equally spaced baseband levels.

A definition of the deviation ratio, h, can be presented here as,

$$h \equiv \Delta T_s. \tag{4}$$

Now (2) can be rewritten as,

$$f_d(t) = \frac{\pi h}{T_s} \sum_{k=-\infty}^{\infty} [2a(k) - M - 1]\, f(t - kT_s)\,, \qquad a(k) = 1, 2, \ldots, M. \tag{5}$$

If nonoverlapping pulses are uses then f(t) exists only over the interval (0, T], but if overlapping pulses are used then the existence interval is given by Egn. 31 of [3]. This paper is limited to the use of two specific pulse shapes. They are,

1) Square pulse, no overlap, thus

$$f(t) = \begin{cases} 1 & \text{for } 0 < t \le T_s \\ 0 & \text{otherwise} \end{cases}$$

2) Raised-cosine pulse, overlap of one symbol period, thus

$$f(t) = \begin{cases} \frac{1}{2}\left(1+\cos\frac{\pi t}{T_s}\right) & \text{for } -T_s < t \le T_s \\ 0 & \text{otherwise.} \end{cases}$$

Assuming that each symbol in the M-ary set, {1, 2, ... , M}, is equiprobable then Egn. 54 of [2] yields the power spectral density (PSD) of M-ary PCM/FM signal with a square pulse input, while the approach given in section 7.2 of [3] can be used to find the PSD of the signal with the raised-cosine pulse input. This study is further limited to values of M of 2, 4, and 8, and to values of h of 0.715 and 1/M.

MAXIMUM DATA THROUGHPUT RATE FOR EACH CASE

Given the choices above which specify $x_{a(k)}$ and M for 12 cases, the power spectrum of s(t) for each case can be calculated to determine the maximum allowable data throughput rate for each of the three channel spectrum occupancy requirements. The data throughput rate is measured as the equivalent binary data rate through the system. For M-ary systems this rate is defined in terms of T_s and M, as

$$f_{tp} = \frac{1}{T_s} \log_2 M. \tag{6}$$

Use of normalized PSD curves is quite helpful in this application. Assuming that the PSD functions are nearly linear over any 3 kHz detection bandwith then the power spectrum, S(f), can be approximated in terms of the normalized PSD, symbol rate, and detection bandwith, as

$$S(\beta) \approx B_d T_s \overline{P(\beta)} \tag{7}$$

where

B_d ≡ detection bandwidth, 3kHz in this case

$\overline{P(\beta)}$ ≡ PSD of s(t) normalized to a symbol period of one second

β ≡ frequency difference from the carrier, normalized to T_s.

The normalized frequency difference can be expressed in terms of the absolute frequency, f, as

$$\beta = (f-f_c)T_s. \tag{8}$$

Since $\overline{P(\beta)}$ is easily computed by itself and T_s can be expressed in terms of β and other known factors, then (7) can be arranged to yield,

$$\overline{P(\beta)} = \frac{S(\beta)}{B_d T_s} \tag{9}$$

$$= S(\beta) \frac{(f - f_c)}{B_d \beta} \tag{10}$$

To achieve the spectrum occupancy requirement of -60 dBc at the corner frequency, $\Delta f = f - f_c$, then (10) becomes

$$P(\beta') = -60 \text{ dBc} + 10 \log \frac{\Delta f}{B_d \beta'} \tag{11}$$

where

$\beta' \equiv$ values of β for which (11) is true.

If the l.h.s. of (11) is plotted as one curve and the r.h.s. of (11) is plotted as a second curve, then the curves will intersect at the β' points. The smallest β' which satifies the inequality,

$$\overline{P(\beta')} > \overline{P(\beta)} \qquad \text{for } \beta > \beta', \tag{12}$$

yields a value of T_s which corresponds to the maximum throughput rate relative to that particular corner frequency. Using (6) and (8) under conditions of (12) yields,

$$\left(f_{tp}\right)_{max} = \frac{\Delta f}{\beta'} \log_2 M \,. \tag{13}$$

Note that, since there are three corner frequencies corresponding to the three channel bandwidth specifications, there are three curves generated by the r.h.s. of (11) for each one generated by the l.h.s. of (11). As a result there is a β' produced for each channel.

CONCLUSION

The procedure outlined above produced a maximum f_{tp} for each of the 12 cases specified. The results are given in Table I, II, III, for the narrowband, mediumband, and wideband channels, respectively. According to these results the 4-ary and 8-ary systems with h = 1/M generally provide transmission efficiencies in excess of 1 bit / hertz based only on spectrum occupancy requirements and channel bandwidth definitions. This observation can be quite misleading, though, since no performance analysis has been done and there is no guarantee that these “high-efficiency” systems will work properly within the allocated bandwidths. With that thought aside, it can be concluded that properly chosen baseband pulse shapes and deviation ratios can be favorably affect the spectrum occupancy of M-ary PCM/FM.

REFERENCES

1. IRIG STANDARDS 106-80: TELEMETRY STANDARDS, Secretariat, Range Commanders Council, White Sands Missile Range, New Mexico, September 1980.

2. Anderson, R.R. and SaIz, J., "Spectra of Digital FM," BELL SYSTEMS TECHNICAL JOURNAL, Volume 44, Number 6, July-August, 1965, p.p. 1165-1189.

3. Rowe, H.E. and Prabhu, V.K., "Power Spectrum of a Digital, Frequency-Modulation Signal," BELL SYSTEM TECHNICAL JOURNAL, Volume 54, Number 6, July-August, 1975, p.p. 1095-1125.

Table I - Maximum data throughput rate for narrowband (1 MHz) channel.

Pulse Shape	Throughput rate in Mbits per second					
	Number of bits per symbol					
	1		2		3	
	Deviation ratio					
	0.715	0.500	0.715	0.250	0.715	0.125
Square	0.320	0.391	0.358	0.908	0.366	1.350
Raised-cosine	0.799	0.885	0.904	1.638	0.768	2.397

Table II - Maximum data throughput rate for mediumband (3 MHz) channel.

Pulse Shape	Throughput rate in Mbits per second					
	Number of bits per symbol					
	1		2		3	
	Deviation ratio					
	0.715	0.500	0.715	0.250	0.715	0.125
Square	0.792	0.975	1.070	1.888	0.846	2.814
Raised-cosine	1.678	1.849	1.892	3.450	1.596	5.061

Table III - Maximum data throughput rate for wideband (10 MHz) channel.

Pulse Shape	Throughput rate in Mbits per second					
	Number of bits per symbol					
	1		2		3	
	Deviation ratio					
	0.715	0.500	0.715	0.250	0.715	0.125
Square	3.544	4.823	3.972	8.990	3.174	13.221
Raised-cosine	5.021	8.088	5.528	10.508	4.641	15.408