ANTIJAM PERFORMANCE OF CODED FH/MFSK

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SUMMARY

Frequency hopping (FH) is a powerful antijam spread spectrum modulation when combined with multiple-frequency-shift-keying (MFSK) and error correction coding. This paper determines the antijam message error rate performance of various formats of FH/MFSK with convolutional error correction coding techniques. The worst case jammer is determined for each signalling format and coding technique and the antijam performance for all configurations are compared.

FH/MFSK SPREAD SPECTRUM MODULATION

Using MFSK modulation $\ell = \log_2 M$ bits determine which one of M frequencies is to be transmitted. The center frequency of the M-ary signal set is shifted pseudo-randomly by the frequency hopper. There may be only one frequency hop for several MFSK signals (slow hopping) or there may be several (N) frequency hops for each MFSK symbol (fast hopping). A block diagram of a FH/MFSK demodulator is shown in Figure 1. The received signal is frequency dehopped resulting in a baseband signal. Actually the frequency dehopping is followed by an IF chain with the final output a baseband signal consisting of one of the signalling M frequencies plus noise or tone jamming. Receiver noise is considered to be negligible compared to the jammer and signal powers. To detect which of the M frequencies was transmitted, a bank of M envelope detectors is used. To prevent a large tone jammer from dominating the detector, the resulting envelopes after each hop (chip) are clipped at the expected signal level. Each of the clipped envelopes is summed over N hops for fast frequency hopping to form the statistic R_i for the i^{th} frequency. If there is no error correction coding or if the error correction decoder uses only hard decisions, then the maximum of the R_i determines the demodulated frequency and the ℓ bits corresponding to that frequency are output to the decoder or to the user. Alternately, some error correction decoders use soft decisions. For soft decision decoders, all M statistics (probably quantized) are output to the decoder.

UNCODED SYMBOL ERROR RATE PERFORMANCE

For hard decisions made at the output of the FH/MFSK demodulator, the probability of the FH/MFSK demodulator, the probability of error can be calculated as a function of the type of jamming. For N = 1, that is slow hopping, the optimum receiver[1] for full band noise jamming computes the statistic

$$R_i = X_i^2 + Y_i^2 \tag{1}$$

where

$$\begin{aligned} X_i &= \sqrt{2S} \int_0^{T_s} y(t) \cos \omega_i t \, dt \\ Y_i &= \sqrt{2S} \int_0^{T_s} y(t) \sin \omega_i t \, dt \end{aligned} \tag{2}$$

where S is the signal power; the symbol rate, R_s, defines the symbol period, $T_s = 1/R_s$; y(t) is the signal plus noise at the input to the bank of filters (Fig. 1); and ω_i is the radian frequency for the i^{th} possible transmitted frequency. The probability of symbol error is given by

$$P_s = \frac{1}{M} \sum_{i=2}^{M} (-1)^i \binom{M}{i} \exp\left(-\frac{i-1}{i} \frac{E_s}{N_o}\right) \tag{3}$$

where

$$\frac{E_s}{N_o} = \left(\frac{S}{J}\right) \left(\frac{W}{R_s}\right) \tag{4}$$

and where W is the spread spectrum bandwidth and J is the total jamming power.

Because of the detector clipping at the signal level S, the best tone jamming strategy is to transmit tones at that level. Therefore, for N = 1, with tone jamming in m of the M filters, the probability of symbol error is

$$P_s = \begin{cases} 1 - \dfrac{1}{m+1} & \text{for no jamming tone in the signal filter} \quad (5a) \\ 1 - \dfrac{2}{3m} & \text{for a jamming tone in the signal filter and } m > 1 \quad (5b) \end{cases}$$

The first expression (5a) results from each jamming tone competing with the signal. The second expression (5b) is due to the jamming tones in the incorrect filters competing with the composite result of both a signal and the jammer being in the correct filter slot. This composite waveform has output power Y given by

$$Y^2 = 2S^2 (1 - \cos\theta) \tag{6}$$

where θ is the phase angle between the jamming tone and the signal. Because the clipping is at S, a symbol error will occur when $Y^2 < S^2$ i. e. when cos θ is less than 1/2, then all the m filter outputs will be equal due to clipping and the probability of symbol error is ((m-1)/m). Because cos θ is less than 1/2 two-thirds of the time, P_s = 1/3 + 2/3 ((m-1/m) = 1 - 2/3m.

For fast frequency hopping, that is N > 1, the exact symbol error probability for the binary (M = 2) case for full band noise jamming is well known[2]. For M > 2 a union bound is typically used:

$$P_s\left(M, N, \frac{E_s}{N_o}\right) < (M-1)\, P_s\left(2, N, \frac{E_s}{N_o}\right)$$
$$< (M-1)\,(1/2)^N \exp\left(-\frac{E_s}{N_o}\right)\sum_{i=0}^{N-1}\binom{N+i-1}{i}\left(\frac{1}{2}\right)^i F\left(-i, N, -\frac{E_s}{N_o}\right) \tag{7}$$

where F(- i, N, y) is the confluent hypergeometric function which in this case reduces to an i^{th} order polynomial

$$F(-i, N, y) = 1 + \frac{-iy}{N} + \frac{(-i)\,(-i+1)y^2}{(N)\,(N+1)2\,!} + \frac{(-i)(-i+1)(-i+2)y^3}{(N)(N+1))N+2)3\,!} + \ldots \tag{8}$$

To gain more insight into the relationship between P_s and N, an upper Chernoff bound on P_s can be used. This bound, which is asymptotically exact in exponent when the Chernoff bound parametric λ is optimized, has been shown to be[3]

$$P_s < \frac{M-1}{2}\sum_{k=1}^{N}\frac{\exp\left[-\frac{\lambda}{1+\lambda}\left(\frac{E_s}{N_o}\right)\frac{a_k^2}{N}\right]}{1-\lambda^2} \quad \text{for } 0 < \lambda < 1 \tag{9}$$

where a_k are the chip amplitudes.

For full band noise jamming, $a_k = 1$ and

$$P_s < \frac{M-1}{2}\left[\frac{\exp\left[-\frac{\lambda}{1+\lambda}\left(\frac{E_s}{N_o}\right)\frac{1}{N}\right]}{1-\lambda^2}\right]^N \quad \text{for} \quad 0 < \lambda < 1 \tag{10}$$

The tightest Chernoff bound can be obtained for

$$\lambda = \lambda_o = -(1/2)\ 1 + \left(\frac{1}{2N}\frac{E_s}{N_o}\right) + \sqrt{\left[(1/2)\left(1 + \frac{1}{2N}\frac{E_s}{N_o}\right)\right]^2 + \left(\frac{1}{2N}\right)\frac{E_s}{N_o}} \tag{11}$$

For tone jamming and fast frequency hopping, assume that the jammer hits m of the M demodulation filters for MFSK. Setting a clipping level L at the expected signal power S as before, the maximum value of any output envelope summed over N hops is NL =NS. Then, the output level for the signal filter is

$$S_t = L(N-t) + L\sum_{i=1}^{t} y_i \tag{12}$$

where t is the number of the N hops that are jammed in the signal filter and y_i is a random variable which depends on the phase between the jamming tone and the signal as shown in (6)

The output of the most jammed filter not containing the signal is

$$J_t = Lu \tag{13}$$

where u is the number of the N hops that are jammed in any filter other than the signal filter. The probability of error is

$$P_s(u,t) = \text{Prob}\left[(N-t) + \sum_{i=1}^{t} y_i \leq u\right] \tag{14}$$

where (14) is slightly pessimistic since ties are given to the jammer. Therefore, if $u = N$, the jammer always wins and a symbol error occurs. If $u > N-t$ or $u + t > N$, then the probability of error can be calculated, but this computation is extremely complicated unless N is very large and the Central Limit Theorem can be invoked. Hence, to obtain a simply calculable bound it will be assumed that a symbol error occurs whenever

$$u + t \geq N. \tag{15}$$

The two types of jammers discussed transmit noise or tones over the spread spectrum bandwidth. However, rather than spreading the jammer power over the total hop bandwidth, it may be a more effective strategy to increase the jammer power in some fraction of the band. For partial band jamming, the probability of symbol error is large when the signal is hopped into the jammed fraction of the band. The fraction of the band that is jammed must be larger than the required probability of error for the communication system to be jammed.

For a FH/MFSK signal with noise jamming in a fraction ρ of the hopped bandwidth, the total jamming power is J/ρ over the bandwidth ρW. Therefore, the signal is not jammed 1-ρ of the time and the probability of error is zero then. But ρ of the time the signal is jammed, degrading the signal-to-noise ratio (4) to $\rho E_s / N_o$. Thus the probability of symbol error for fast frequency hopping from (10) is

$$P_s < \left(\frac{M-1}{2}\right)\left[\frac{\rho \exp\left[-\frac{\lambda}{1+\lambda}\frac{E_s}{N_o}\frac{\rho}{N}\right]}{1-\lambda^2}\right]^N \tag{16}$$

where the worst case jammer occupies the fraction

$$\rho = \frac{3N}{E_s N_o} \leq 1 \tag{17}$$

of the spread bandwidth. When ρ < 1, then the tightest Chernoff bound occurs for $\lambda = \lambda_o = 1/2$. Therefore, the probability of symbol error for ρ < 1 is given by

$$P_s < \left(\frac{M-1}{2}\right) \left[\frac{4N}{e\, E_s / N_o}\right]^N \tag{18}$$

If ρ = 1, then the probability of symbol error is given by (10) and (11).

For frequency hopping with N = 1, the probability of symbol error from (3) is

$$P_s = \frac{\rho}{M} \sum_{i=2}^{M} (-1)^i \binom{M}{i} \exp\left(-\frac{i-1}{i}\,\rho\,\frac{E_s}{N_o}\right) \tag{19}$$

Tone jamming was discussed earlier and it was shown that the optimum jamming strategy was to set each jamming tone equal in power to the signal tone. For $E_s / N_o > 1$ m the jammer cannot jam all of the possible MFSK frequencies, and if $E_s / N_o > 1$, then the jammer cannot even jam each set of M frequencies. Therefore, typically the most effective tone jamming is partial band. There are $n = W/R_s$ possible orthogonal MFSK frequencies.

If m filters of the set of M filters are jammed, and if the fraction of band jammed is ρ, then the number of jamming tones is

$$q = \rho \frac{m}{M} n = \rho \left(\frac{m}{M}\right)\left(\frac{W}{R_s}\right) . \tag{20}$$

The power in each tone is

$$J_1 = J/q \tag{21}$$

Therefore, for optimum jamming

$$J_1 \geq S \tag{22}$$

Because it is a waste of jammer power for J_1 to be much greater than S, J_1 is taken to be infinitesimally larger than S (or equal to S and ties go to the jammer). Thus,

$$S = \frac{J}{q} = \frac{J}{\rho} \frac{M}{m} \frac{R_s}{W} , \quad \text{and} \tag{23}$$

$$\left(\frac{E_s}{N_o}\right)_{eff} = \frac{S}{J} \frac{W}{R_s} = \frac{M}{\rho m} \tag{24}$$

The probability of symbol error for tone jamming of slow frequency hopping from (5) is

$$P_s = \begin{cases} \rho\left(\frac{M-1}{2M}\right) = \dfrac{M-1}{2(E_s/N_o)_{eff}} & m = 1 \\ \rho\left[\left(\frac{M-m}{M}\right)\left(1 - \frac{1}{m+1}\right) + \left(\frac{m}{M}\right)\left(1 - \frac{2}{3m}\right)\right] \end{cases} \tag{25}$$

$$= \left[\frac{M-m}{m+1} + \frac{3m-2}{3m}\right] \Big/ \left(\frac{E_s}{N_o}\right)_{eff} \qquad 1 < m \leq M$$

For fast frequency hopping, and one of the M frequencies jammed, the probability of symbol error is

$$P_s = (M-1)\left[\left(\frac{1}{M}\right)\rho\right]^N \tag{26}$$

Alternately, if all M frequencies are jammed, then t = u and symbol error will occur when t + u ≥ N (15) or when t = u ≥ N/2. Therefore,

$$P_s = \sum_{t=\lceil N/2}^{N} \binom{N}{t} \rho^t (1-\rho)^{N-t} \tag{27}$$

where ⌈x is defined as the smallest integer greater than or equal to x.

The q jamming tones can also be spread across the hopping bandwidth randomly. In this case, the probability of hit, P_h , of any filter is

$$P_h = q/n \tag{28}$$

where n = W/R_s is the total number of possible orthogonal MFSK frequency slots. Since S = J/q, then

$$\left(\frac{E_s}{N_o}\right)_{eff} = \frac{S}{J}\frac{W}{R_s} = \frac{1}{P_h} \tag{29}$$

The probability of jamming any MFSK filter is an independent identically distributed event and hence

$$p(u,t) = p(u)\,p(t) = \left[\binom{N}{u} P_h^{\,u} (1-P_h)^{N-u}\right] \left[\binom{N}{t} P_h^{\,t} (1-P_h)^{N-t}\right] \tag{30}$$

Under the assumption that if t + u ≥ N, then a symbol error occurs, the probability of symbol error P_s is

$$P_s = 1 - \left\{ 1 - \left[p(N,0) + \sum_{u=1}^{N} \sum_{t=N+1-u}^{N} p(u,t) \right] \right\}^{(M-1)} \tag{31}$$

$$P_s = 1 - \left\{ 1 - P_h^{\,N}(1-P_h)^N + \sum_{u=1}^{N} \binom{N}{u} p_h^{\,u}(1-P_h)^{N-u} \sum_{t=N+1-u}^{N} \binom{N}{u} p_h^{\,t}(1-P_h)^{N-t} \right] \right\}^{(M-1)}$$

CODING TECHNIQUES AND PERFORMANCE

Error correction coding provides a powerful technique to decrease the effectiveness of a jammer against FH/MFSK. Convolutional dual-3 code using soft decisions with a maximum likelihood (Viterbi) decoder is discussed in this section. That is, the decoding algorithms for this technique uses the analog (quantized to at least three bits) outputs of each of the MFSK filters as inputs to the decoder for each hop time. Thus, the decoder performs the post detection combining of chips rather than the demodulator.

The dual-3 convolutional encoder is shown in Figure 2. After three data bits are shifted into the shift register, the 8-ary symbol out of the first modulo-2 adder set is transmitted m times and the 8-ary symbol out of the second modulo-2 adder set is transmitted n times. Therefore, N = m + n. The symbol transfer function V(D) describes the Hamming distance between codewords for any m and n is[4]

$$V(D) = \frac{7\ D^{2N}}{[1 - D^n - D^m - 5D^n]^2} \tag{32}$$

where D is a dummy variable whose exponent is the Hamming destance between code-words. If the division of the numerator by the denominator were carried out, the coefficients of the resulting power series in D would correspond to the total number of non-zero symbols associated with the total number of codewords which have the distance denoted by the exponent of D. Using Viterbi decoding, it has been shown[4] that the probability of symbol error is given by

$$P_s < V(D)$$

where D becomes a function of the channel statistics.

For partial band noise jamming, using the Chernoff bount as in (16), D is defined to be

$$D = \frac{\rho}{1-\lambda^2} \exp\left[- \frac{\lambda}{1+\lambda} \frac{E_s}{N_o} \rho \right] \tag{34}$$

where the worst case jammer occupies

$$\rho = \frac{3}{E_s/N_o} \leq 1$$

In the cases of interest the symbol rate is the hop rate. If $\rho < 1$, then the tightest Chernoff bound is when $\lambda = \lambda_o = 1/2$. Therefore, if $\rho < 1$, then

$$D = \frac{4\ \rho}{3\ e} \tag{36}$$

If $\rho = 1$ ($E_s/N_o \leq 3$), then

$$D = \frac{1}{1 - \lambda_o^2} \exp\left[- \frac{\lambda_o}{1+\lambda_o} \frac{E_s}{N_o}\right] \tag{37}$$

where

$$\lambda_o = -\frac{1}{2}\left(1 + \frac{1}{2} \frac{E_s}{N_o}\right) + \sqrt{\left[\left(\frac{1}{2}\right)\left(1 + \frac{1}{2} \frac{E_s}{N_o}\right)\right]^2 + \frac{1}{2} \frac{E_s}{N_o}} \tag{38}$$

Note that

$$N = m + n = \frac{1}{R} \tag{39}$$

where R is code rate, and the symbol rate is

$$R_s = \frac{R_b}{\ell R}$$

in terms of the information bit rate R_b. Thus,

$$\frac{E_b}{N_o} = \left(\frac{W}{R_b}\right)\left(\frac{S}{J}\right) = \frac{1}{\ell R} \left(\frac{W}{R_s}\right)\left(\frac{S}{J}\right) = \frac{1}{\ell R} \frac{E_s}{N_o} \tag{41}$$

For partial band tone jamming, if a jammer tone occurs in a filter that corresponds to a symbol in a codepath other than the correct codepath, then that incorrect codepath will have an increased metric. If a given incorrect codepath has enough symbols with jammer tones, then the Viterbi decoder will make a symbol error. The transfer function V(D) indicates the number of symbols in which each incorrect codepath differs from the transmitted coddword. If

$$D = \rho \tag{42}$$

then the probability of symbol error is bounded by (33). If all M filters are jammed or if the tones are randomly spaced with $\rho = P_h$, then from (24) or (Z9),

$$\frac{E_s}{N_o} = \frac{1}{\rho} \quad \text{for } \rho < 1 \tag{43}$$

MESSAGE LENGTHS

The performance of the communication systems for many applications is given in terms of probability of message error, P_M. A communication system is typically designed for $P_M = 10^{-3}$. When $P_M = 10^{-1}$ the system performance is considered marginal. The communication system is considered completely jammed at $P_M = 0.99$.

In order to determine P_M in terms of P_W or P_b as given in the previous sections, the message lengths must be defined. For purposes of this paper, let the message length be 40 ASCII characters which are stripped of the ASCII parity bits leaving 6 bits per character. Therefore, the message is 240 bits. However, the messages will be assumed to have error detection coding which will add 24 parity bits. Thus, the total message length to the communication system is 264 bits.

COMMUNICATION SYSTEM PERFORMANCE CALCULATIONS

In this section the communication system performance is calculated for $P_M = 10^{-3}$, 10^{-1} and 0.99 for a variety of modulation/coding alternatives under the jamming environments described previously. The three configurations considered are Dual-3 R = 1/3 coded slow-hopped 8-ary FSK, Dual-3 R = 1/6 coded slow-hopped 8-ary FSK, and Dual-3 R = 1/8 coded fast-hopped 8-ary FSK.

In this paper, the Dual-3 convolutional codes are used with slow wideband hopping at 5 hps and fast wideband hopping at 200 hps. When the Dual-3 code is used with slow wideband hopping the 8-ary symbols are transmitted at 100 symbols/sec using the m = 2, n = 1, R = 1/3 Dual-3 code or the 8-ary symbols are transmitted at 200 symbols/sec by the Dual-3 code.

In the case of the slow wideband hopping, the bit rate is 75 bps but by using the R = 1/3 Dual-3 code at 100 symbols/sec or the R = 1/6 Dual-3 code at 200 symbols/sec, there can be non data symbols per wideband hop for time synchronization uncertainty and frequency hop setting time. Figure 3 illustrates the format for the wideband hop. Note there are 15 data symbols per wideband hop with the R = 1/3 Dual-3 code and 30 data symbols with the R = 1/6 Dual-3 code. At the beginning and end of each wideband hop, 20 msec is allowed for lack of time synchronization between the transmitter and receiver. At the beginning of the hop, 10 msec is allowed for wideband hop settling time. The overhead non data time in the wideband hop format is 3/4 corresponding to 1.25 dB loss in signal energy from the data.

FULL BAND NOISE JAMMING (ADDITIVE WHITE GAUSSIAN NOISE)

The performance of the Dual-3 convolutional coded systems in full band noise jamming (AWGN) is given by Eqs. (32) and (33) for probability of symbol error where D and λ_o are given by (37) and (38) with the proper normalization for code rate. The probability of message error can be calculated from the probability of symbol error P_s by

$$P_M = 1 - (1 - P_s)^{L_c} \tag{44}$$

where

$$L_c = \frac{L+3}{3}$$

and L is the message length in bits, the 3 bits added to L corresponds to the message tail needed to flush the Dual 3 encoder and the 3 dividing the message length corresponds to ℓ = 3 bits per 8-ary symbol.

Table I presents the performance of the Dual-3 codes in AWGN not including the rate loss associated with the tail bits and with any overhead in the hop format. Note that the lower the code rate R, the larger the required E_b/N_o to achieve a given probability of message error. This increase in E_b/N_o can be attributed to the increase in noncoherent combining losses as the rate decreases and the number of 8-ary symbols per bit increases. It may be observed in Table I that the performance of the R = 1/6 code is about 0. 7 to 0. 8 dB worse than the R = 1/3 code and that the R - 1/8 code is about 0. 3 to 0. 4 dB worse than the R = 1/6 code in AWGN or full band noise jamming. Also presented in Table I is the performance of uncoded 8-ary FSK symbols frequency hopped at the same hop rate as the equivalent Dual-3 code, At $P_M = 10^{-3}$, the uncoded data performance is about 2.1 - 2.2 dB worse than the equivalent Dual-3 coded data performance. At $P_M = 10^{-1}$ the Dual-3 coded data performance is about 1.8 - 1.9 dB better than the uncoded data performance. At P_M = 0. 99, the Dual-3 coded data performance is better than the uncoded data performance by 1.1 dB.

FIXED PARTIAL BAND NOISE JAMMING

The performance of the Dual-3 code in fixed partial band noise jamming is given by (44), (45), (32), (33), (35), and (36). Table II presents the performance of the Dual-3 codes and the equivalent uncoded performance. The Dual-3 coded performance is better than the uncoded performance at all values of P_M. At $P_M = 10^{-3}$, the performance of the Dual-3 codes are better than the equivalent uncoded performance by 8.6 dB, 4.3 dB, and 3.2 dB for the R = 1/3, 1/6, and 1/8 Dual-3 codes, respectively. Note that for $P_M = 10^{-3}$, the best choice of N is 3 or 6 and the best choice of R is 1/3. The performance of the Dual-3 codes at P_M = 0. 99 is 1.6 dB, 1. I dB, and 1.1 dB better than the uncoded performance at R = 1/3, 1/6, and 1/8, respectively.

FIXED PARTIAL BAND TONE JAMMING

The performance of the Dual-3 codes for fixed partial band tone jamming is given by (44), (45), (32), (33), (42), (43), and (41).

Table III presents the performance of the Dual-3 codes and the equivalent uncoded performance in fixed partial band tone jamming. The Dual-3 codes provide significant improvement in performance over the uncoded data. At $P_M = 10^{-3}$, the Dual-3 code with the best performance is the R = 1/6 or R = 1/8 Dual-3 code which is 1.9 dB better than the R = 1/3 code. The coding gains at $P_M = 10^{-3}$ for the Dual-3 codes over the equivalent uncoded data is 17.3 dB for the R = 1/3 code, 10.4 dB for the R = 1/6 code, and 9.7 dB for the R = 1/8 code. At $P_M = 10^{-1}$, the Dual-3 code with the best performance is the R = 1/6 code which is 0.3 dB better than the R = 1/3 code and 0.4 dB better than the R = 1/8 code. The coding gains at $P_M = 10^{-1}$ for the Dual-3 codes are 10.4 dB, 6.8 dB, and 6.7 dB for the R = 1/3, R = 1/6, and R = 1/8, respectively.

SUMMARY OF PERFORMANCE IN JAMMING

Comparing the Dual-3 code performance against fixed partial band tone jamming in Table III with the Dual-3 code performance against fixed partial band noise jamming in Table II, it may be observed that the worst case jammer is fixed partial band noise jamming. The modulation/coding technique with the best performance against the worst case jammer is slow hopping with the Dual-3 R = 1/6 code for low P_M For high P_M (= 0.99), the best performance is obtained with the slow hopping with the Dual-3 R = 1/3 code:

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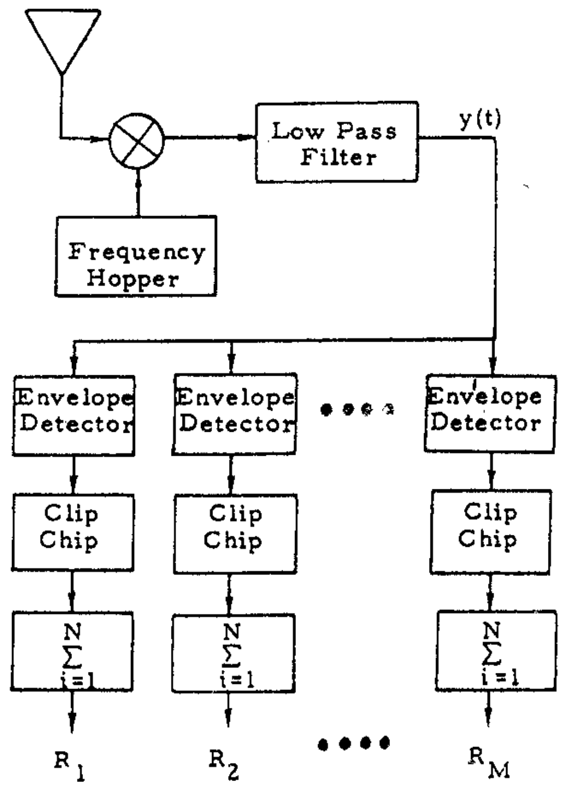

Figure 1. FH/MFSK Demodulator

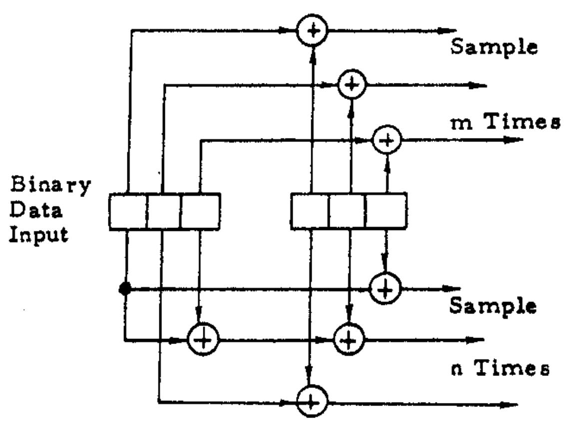

Figure 2. Dual-3 Encoder

Table I. Full Band Noise Jamming (AWGN) with Slow and Fast Wideband Frequency Hopping

Probability of Message Error P_M	Coding	E_b/N_o
10^{-3}	Uncoded, N = 3	10.7
	Dual 3, R = 1/3	8.5
	Uncoded, N = 6	11.3
	Dual 3, R = 1/6	9.2
	Uncoded, N = 8	11.6
	Dual 3, R = 1/8	9.5
10^{-1}	Uncoded, N = 3	9.1
	Dual 3, R = 1/3	7.2
	Uncoded, N = 6	9.8
	Dual 3, R = 1/6	7.9
	Uncoded. N = 8	10.1
	Dual 3, R = 1/8	8.3
0.99	Uncoded, N = 3	7.0
	Dual 3, R = 1/3	5.9
	Uncoded, N = 6	7.8
	Dual 3, R = 1/6	6.7
	Uncoded 3, R = 8	8.2
	Dual 3, R = 1/8	7.1

Table III. Fixed Partial Band Noise Jamming with Slow and Fast Wideband Frequency Hopping

Probability of Message Error P_M	Coding	Partial Band Jamming ρ	E_b/N_o (dB
10^{-3}	Uncoded, N = 3	0.030	20.0
	Dual 3, R = 1/3	0.216	11.4
	Uncoded, N = 6	0.248	13.8
	Dual 3, R = 1/6	0.666	9.5
	Uncoded, N = 8	0.420	12.8
	Dual 3, R = 1/8	0.381	9.6
10^{-1}	Uncoded, N = 3	0.143	13.2
	Dual 3, R - 1/3	0.449	8.2
	Uncoded, N = 6	0.539	10.5
	Dual 3, R = 1/6	0.963	7.9
	Uncoded, N = 8	0.752	10.3
	Dual 3, R = 1/8	1.0	8.3
0.99	Uncoded, N = 3	0.498	7.8
	Dual 3, R = 1/3	0.728	6.2
	Uncoded, N = 6	1.0	7.8
	Dual 3, R = 1/6	1.0	6.7
	Uncoded, N = 8	1.0	8.2
	Dual 3, R = 1/8	1.0	7.1

Table III. Fixed Partial Band Tone Jamming with Slow and Fast Wideband Frequency Hopping

Probability of Message Error P_M	Coding	Partial Band Jamming ρ	E_b/N_o (dB)
10^{-3}	Uncoded, N = 3	0.002	27.1
	Dual 3, R = 1/3	0.106	9.8
	Uncoded, N = 6	0.030	18.3
	Dual 3, R = 1/6	0.327	7.9
	Uncoded, N = 8	0.046	17.6
	Dual 3, R = 1/8	0.432	7.9
10^{-1}	Uncoded, N = 3	0.020	17.0
	Dual 3, R = 1/3	0.220	6.6
	Uncoded, N = 6	0.098	13.1
	Dual 3, R = 1/6	0.463	6.3
	Uncoded, N = 8	0.122	13.4
	Dual 3, R = 1/8	0.570	6.7
0.99	Uncoded, N = 3	0.135	8.7
	Dual 3, R = 1/3	0.357	4.5
	Uncoded, N = 6	1.270	8.7
	Dual 3, R = 1/6	1.603	5.2
	Uncoded, N = 8	1.286	9.7
	Dual 3, R = 1/8	1.684	5.9