WRITE NOISE IN MAGNETIC RECORDING

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ABSTRACT

A novel source of noise is identified and described in this study. If a continuous recording medium is less than perfectly uniform, a given quantity may be recorded differently at different locations in the medium. Inadvertent "encoding" occurs, embedding noise in the signal. Symmetrical sideband noise power results from amplitude and phase modulation of the signal stream by the nonuniform recording medium. "Write noise", so-called because writing is required, is correlated in amplitude with signal amplitude, and its mean frequency is the signal frequency. It is the dominant noise source for the current generation of recorders and tapes; its power spectrum is almost the same as the power spectrum of the signal. The ratio of standard deviation to mean value of the signal envelope when recording cw signals is an absolute measure of tape quality independent of record level, tape speed, and track width, and establishes an available signal-to-noise ratio which cannot be exceeded. It is assumed that the recorder output has a normal amplitude distribution about its mean value. Theory is confirmed by experiment, within experimental error, for cw and digital recording. (Key words: Write Noise, Magnetic Recording, Recording Theory.)

INTRODUCTION

Prior researchers have stated that tape noise is due to deviation from the mean magnetization, and have assumed that this is caused by additive white gaussian magnetization uniformly distributed throughout the recorded region of the medium. This approach was initially used by Stein (1) and Daniels (2), and further refined by Mallinson (3,4,5). More recently, Thurlings (6,7) investigated the effects of particle "clumping" in the emulsion. When he forced agreement between theory and experiment at long wavelength on tape he observed that the ratio of theoretical to experimental noise power

* A particulate medium cannot be isotropic, but if all particles are chemically and physically identical, oriented in one direction and spaced equally, the result of recording is predictable, and, though wavelength dependent, free of tape noise.

density at large wave numbers was 20 dB, and "too large to be accounted for by the present theory", i.e., it could not be explained by clumping.

When recording cw it is observed that the envelope of the recorder output is variable. (See Figure 5a.) The ratio of the standard deviation to the mean value of the envelope is found to be essentially independent of record level within the linear range of the recorder. (This observation is described in detail under "Experiment".) It is concluded that tape noise is not additive, for, if it were, this ratio would decrease as record level is increased.

In any continuous recording process the medium is imperfect – a given quantity may be recorded differently at different locations in the medium, the result of many independent random variables due to particle metrics, chemistry, and metallurgy. The signal is amplitude and phase modulated by the medium, producing symmetrical sideband noise power. Since this occurs in the act of writing it is called "write noise". It is the subject of this study.

THEORY

We assume that the write/read process is nominally linear, and that the read head output has a normal amplitude distribution about a mean value, η , with standard deviation σ . For the moment we neglect additive noise. The read head output in a narrowband filter may then be described as:

$$q(t) = c(t) e \cos \left| k v t + \emptyset(t) \right| \tag{1}$$

where t is time on tape, ecos (kvt) is the write head input, k is wave number $= 2\pi/\lambda = 2\pi$/wavelength, v is tape speed, t = x/v, and x is location on tape. c(t) and Ø(t) are amplitude and phase functions, respectively, which describe the recording medium and its variability. These quantities exhibit negligible change during a signal period. Equation (1) describes both signal and noise output of the process; E {Ø(t)} = 0 by definition, and the mean noise frequency must be the signal frequency. Figure 1 shows the vector relationships of expected value, η, output, q(t), and noise, ω(t), at the read head terminals. If ω(t) has a Rayleigh amplitude distribution, and is uniformly distributed over 2π radians in phase, the distribution requirements on q(t) and Ø(t) are met. Then:

$$f_{\omega(t)}(\omega) = (\omega/\sigma^2)\, \epsilon^{-\omega^2/2\sigma^2}\, U(\omega) \tag{2}$$

$$f_{\theta(t)}(\theta) = (2\pi)^{-1}$$

Write noise is deceptive – phase modulation entails no change in the envelope. If amplitude modulation is present, and if: im (σ / η) → constant as η increases, write noise exists. If we construe "variance" to be the "expected value of noise power", classical analysis is possible. With Figure 1 and the law of cosines, instantaneous read head power output is:

$$q^2(t) = \eta^2 + 2\eta w(t)\cos\theta(t) + w^2(t) \quad ** \tag{3}$$

The variance is:

$$\sigma^2_{q(t)} = E\left\{ \left| q^2(t) - \eta^2 \right| \right\} \tag{4}$$

$$= E\left\{ \left| 2\eta w(t)\cos\Theta(t) + w^2(t) \right| \right\} \tag{5}$$

or, the expected absolute value of the sum of phase and amplitude sideband noise power. When noise sidebands are summed over the signal spectrum, wideband noise is the result.

If η > > σ:

$$\sigma^2_{q(t)} \simeq E\left\{ \left| 2\eta w(t)\cos\Theta(t) \right| \right\} \tag{6}$$

Since ω(t) and ⊖(t) are independent random variables, with Equations (2) we have:

$$\sigma^2_{q(t)} = \sigma\,\eta(8/\pi)^{1/2} \tag{7}$$

or

$$\sigma^2_{q(t)}/\eta^2 = (\sigma/\eta)\,(8/\pi)^{1/2} \tag{8}$$

The reciprocal of Equation (8) is the available signal-to-write-noise power ratio. As noted above σ / η is independent of the record level, more generally independent of η, the mean signal output level, and therefore independent of track width and tape speed.

* However, in the present problem some amplitude modulation is required in order to provoke phase modulation.

** This equation describes amplitude and phase correlation characteristics of these problems.

From Equation (3) and η > > σ:

$$\left| q(t) \right| \simeq \eta + w(t) cos\theta(t) \tag{9}$$

$$\sigma^2_{\left| q(t) \right|} = E\left\{\left[\left| q(t) \right| - \eta\right]^2\right\} \tag{10}$$

$$= E\left\{w^2(t)cos^2\theta(t)\right\} \tag{11}$$

With Equations (2):

$$\sigma^2_{\left| q(t) \right|} \doteq \sigma^2 \tag{12}$$

σ is the standard deviation of the envelope of q(t) when recording cw signals and η > > σ . This corresponds to the “large signal” condition treated by Rice (8) and Papoulis (9), for which they obtained this result. A convenient method for the measurement of σ is provided below.

Rice and Papoulis estimated the distribution of the phase of signal plus gaussian noise. Sideband noise power due to phase modulation is, from Middleton (10):

$$\frac{\sigma^2_{q(x)}}{\eta^2} \simeq 2E\left\{1 - J_o^2(\varnothing)\right\} \quad \frac{1}{2\gamma}$$
$$\simeq E\left\{\varnothing^2\right\}, \eta > > \sigma \tag{13}$$

J_o is the Bessel function of the first kind and zero order. The distribution of ∅ is given by Papoulis, Equation (14 63).

$$f_{\varnothing(x)}\varnothing = \frac{\epsilon^{-2\gamma}}{2\pi} + \frac{\sqrt{\gamma} cos\varnothing\, \epsilon^{-2\gamma sin^2\varnothing}}{\sqrt{2\pi}} (1 + 2\, erf 2\sqrt{\gamma} cos\varnothing)$$

$$erf \beta = \frac{2}{\sqrt{\pi}} \int_0^\beta \epsilon^{-r^2} dr \tag{14}$$

Phase modulation noise calculated by this method is in agreement with Equation (8).

Phase modulation noise is much greater than amplitude modulation noise; their power ratio may be estimated with Equations (7) and (12):

$$(PM/AM) \simeq (\eta/\sigma)(8/\pi)^{\frac{1}{2}} \tag{15}$$

This ratio is large for the current generation of magnetic tape recordings ($\simeq$ 17.6 dB for Ampex 795 tape).

Signal recovery from a magnetic recording was described by Wallace in his classical treatment of the problem (11). His model is shown in Figure 2, which also defines the parameters. He assumes that recording occurs in the plane normal to the tape velocity vector at the center of the write head gap. The expected output of a read head at the tape surface is:

$$E\left\{q(t)\right\} = \bar{c}e\epsilon^{-ka}(1 - \epsilon^{-kd}) \tag{16}$$

The variance of the output of the read head is the sum of the variances of the elemental layers transferred to the surface of the recording by a separation function, g(z), which differs from the signal separation function within the recording medium. We reason that $g(z_1 + z_2) = g(z_1) \cdot g(z_2)$ provided $(z_1 + z_2) \leq d$, and g(z) must be a negative exponential function.

Because of the Faraday effect, (S/N) for an elemental layer is very small, and the variance of an elemental layer is the variance of the Rayleigh distribution. With Equations (2):

$$\begin{aligned} \sigma_\omega^2 &= (2 - \pi/2)\sigma^2 \\ &= \upsilon\sigma^2 \\ \upsilon &= (2 - \pi/2) \end{aligned} \tag{17}$$

Let:

$$g(z) = \epsilon^{-\upsilon kz} \tag{18}$$

Then:

$$\sigma^2_{|q(t)|} = \sigma^2 \epsilon^{-2ka} \int_o^d \nu \epsilon^{-\nu kz} kdz^{*} \tag{20}$$

Equation (20) satisfies Equations (17) and (12). With Equations (8), (16), and (20):

$$\frac{\sigma^2_{q(t)}}{\eta^2} = \frac{\sigma \epsilon^{-ka}(1 - \epsilon^{-\nu kd})^{\frac{1}{2}}}{\bar{c} e \epsilon^{-ka}(1 - \epsilon^{-kd})} (8/\pi)^{\frac{1}{2}} \tag{21}$$

Let:

$$\xi = \sigma/(\bar{c}e) \tag{22}$$

and:

$$\frac{\sigma^2_{q(t)}}{\eta^2} = \frac{\xi(1 - \epsilon^{-\nu kd})^{\frac{1}{2}}}{(1 - \epsilon^{-kd})} (8/\pi)^{\frac{1}{2}} \tag{23}$$

A parametric plot of Equation (23) is shown in Figure 3. The ratio of write noise to signal is within 0.5 dB of a constant value when kd $\geq$ 0.5, or at least 96% of the passband of the IRIG standard recorder (12). It is concluded that:

$$\sigma^2_{q(t)} / \eta^2 \simeq \xi (8/\pi)^{\frac{1}{2}} \tag{24}$$

If ξ is independent of wave number, the power spectrum of write noise is almost the same as the power spectrum of the signal.

If read-head first-circuit noise is negligible, degaussed tape noise is the power output of a narrowband filter, in the limit, as the signal level is reduced to nil. It derives from the entire emulsion, not only the recorded portion. Noise power density is a linear function of track width, tape speed, and particle population density, i.e., of $\bar{c}$. It is also proportional to ξ_2.

[*] We may sum noise power due to amplitude modulation sidebands, and, consequently, infer phase modulation noise.

The power density per Hz is:

$$\sigma^2_{dtn} = A\,\bar{c}\,\xi^2 \epsilon^{-2k\alpha} \int_o^{\delta} \upsilon \epsilon^{-\upsilon k z} k dz \tag{25}$$

The notation is explained by Figure 2. Integrating:

$$\sigma^2_{dtn} = A\,\bar{c}\,\xi^2 \epsilon^{-2k\alpha}\left(1 - \epsilon^{-\upsilon k\delta}\right) \tag{26}$$

A is a constant which justifies Equation (26). A small amount of tape noise from the regions $\alpha \geq z \geq a$ and $d \geq z \geq \delta$ is neglected in Equation (24), making that result slightly optimistic. Competent management of the record level will assure that degaussed tape noise is negligible in comparison to other noise sources. The power spectrum of Equation (26) is useful in determining α and δ, the emulsion separation and depth, respectively. The method is described by Hedeman and Law (13).

The attenuation of write noise due to its separation function is $20\pi\upsilon\log\epsilon \simeq 11.7$ dB/λ, very modest in comparison to the signal separation effect of 54.6 dB/λ found by Wallace. In effect, write noise is derived from a larger volume of the emulsion than signal.

Phase distortion in the recording process attenuates signal recovery (14), but leaves write noise unchanged because of its random phase. This causes an apparent increase in ξ in the top octave of the recorder baseband. The signal decrease is predictable and may be corrected; a measured standard deviation may be corrected to obtain ξ as a function of wave number.**

We know that the variance of $|q(t)|$ for large signals is σ^2, and for small signals is $\upsilon\sigma^2$. The measured value of standard deviation must be corrected at long wavelengths on tape to obtain ξ. We have not derived this correction since it is required over a negligible fraction of the recorder basebandwidth, 4% maximum. The correction may be made with the help of Rice (8) or Papoulis (9). If it is necessary to consider additive noise power, it may be summed with write-noise power. Additive noise is the power out of a filter with tape moving, signal “off”, and bias “on”.

[*] Filar and Wright (15) report that degaussed tape noise in a narrowband filter has a Rayleigh amplitude idstribution.

[**] Azimuth misalignment between record and reproduce heads also produces this effect, which may be corrected by head alignment.

EXPERIMENT

The experiments described here were performed by Mr. E. L. Law of the Pacific Missile Test Center, Pt. Mugu, CA. They are but a small part of his work. We hope that he will report in detail on his findings. The experimental configuration is shown in Figure 4. The normalized standard deviation ξ , is the quantity measured as a function of record level and wave number. The envelope function, q(t), is the output of the low pass filter following the linear half-wave rectifier. This output was sampled 2048 times over a 12-inch length of tape while recording cw signals. The data were quantized and processed by minicomputer to determine the amplitude distribution function, the mean value, and the standard deviation. Quantization noise was negligible.

Figures 5a and 5b show typical results for an amplitude function and its distribution function, which tends to be normal, but exhibits a maximum value of the independent random variable. This indicates that particle orientation which has this property, possibly dominates the distribution. Table I summarizes the results of a number of experiments. The normalized standard deviation is independent of record level at 600 µin and 67 µin wavelength on tape, but increases with larger wave numbers and decreases with smaller wave numbers; the possible reasons for this were discussed under "Theory". With estimated corrections ξ is constant within 0.5 dB over the recorder basebandwidth. Tape noise cannot be additive, for, if it were, ξ would be inversely proportional to record level.

For the last two lines in Table I two cw signals were multiplexed in order to record on the same section of tape. Figures 6a and 6b show the amplitude functions for 3000 µin and 600 µin. Figures 7a and 7b show the results for 600 µin and 67 µin for a different section of tape than that used for Figures 6a and 6b. The envelope function, except for a constant multiplier, is independent of wave number. This function is the "signature" of a magnetic recording medium, evidence of quality and identity.

Table I – Normalized Standard Deviation (ξ) at Various Record Levels

Record Level,dBN*	Wavelength, µin 3000	600	300	150	80	67
0		.028	.035	.039		.060
-6		.028				
-12		.025				.062
-6(MUX)		.030			.055	
-6(MUX)	.020	.028				

*with respect to IRIG normal record level.

From this experimental work $\xi \simeq .028$ at a wavelength of 600 μin. The available signal-to-write-noise ratio $\simeq$ 13.5 dB, from Equation (24), for the reel of Ampex 795 oriented particle tape used.

The available signal-to-noise power ratio may also be estimated with a noise-added method similar to that used to measure receiver noise figure. It has been shown by Equation (23) that the power spectrum of write noise is approximately the power spectrum of the signal. Then write noise may be simulated by a white gaussian noise generator followed by a filter whose passband is shaped to the signal spectrum, and may then be added to the output of an equalized recorder. If the signal is a pseudorandom number (PRN) sequence in the non-return to zero (NRZ) code, the signal power spectrum and the filter shape are $(\sin\mu/\mu)^2$, where μ is a linear function of wave number. The experimental method finds Δ_1, the noise added to a noisy signal which results in bit error probability B, excluding bit synchronizer slippage, and Δ_2, the noise added to a noise-free signal which results in bit error probability B. Figure 8 shows the experiment configuration. S is the mean signal power at the input to the error detector, and W the mean write noise power when recording the PRN sequence at normal record level, and:

$$P = S + W \tag{27}$$

If the mean power at the input to the error detector due to the noise-free signal is also P:

$$S / (W + \Delta_1) = P/\Delta_2 \tag{28}$$

It can be shown that:

$$S / W = (P + \Delta_1) / (\Delta_2 - \Delta_1) \tag{29}$$

The method is viable if Δ_2 is much greater than additive noise in the recording process at the input to the bit error detector.

This method was used by Mr. Law with a PRN sequence in the Manchester code (11), and the determination made that S/W $\simeq$ 13.4 dB for the Ampex 795 tape. This was the average of two experiments at data rates within the passband characteristics of the recording system and related by a factor of two.

A body of experimental work has been devoted to the study of catastrophic signal dropouts (16). One such encountered by Mr. Law is shown in Figure 9. It amounts, almost, to signal extinction on a 50 mil track over a 1/16-inch length of tape. Microscopic inspection revealed a physical aberration. It is the only catastrophic dropout on the track over the full length, 9600 ft., of the tape. This suggests that there might be errors due to populations other than write noise. Theory and experiment which account for these observations are wanting.

CONCLUSIONS

Write noise is caused by the variable magnetic characteristics of the recording medium which phase- and amplitude-modulate a signal being recorded. It appears as symmetrical signal sidebands. It is a gaussian process with a normal amplitude distribution. The ratio of signal-to-write-noise is independent of tape speed, track width, and record level. The write noise power spectrum is almost the same as the signal power spectrum. The fraction of signal energy converted to write noise is an emulsion characteristic which establishes an available signal-to-noise ratio, the best possible result. Additive tape noise due to imperfect degaussing is negligible compared to write noise when the record level is competently managed.

ACKNOWLEDGEMENTS

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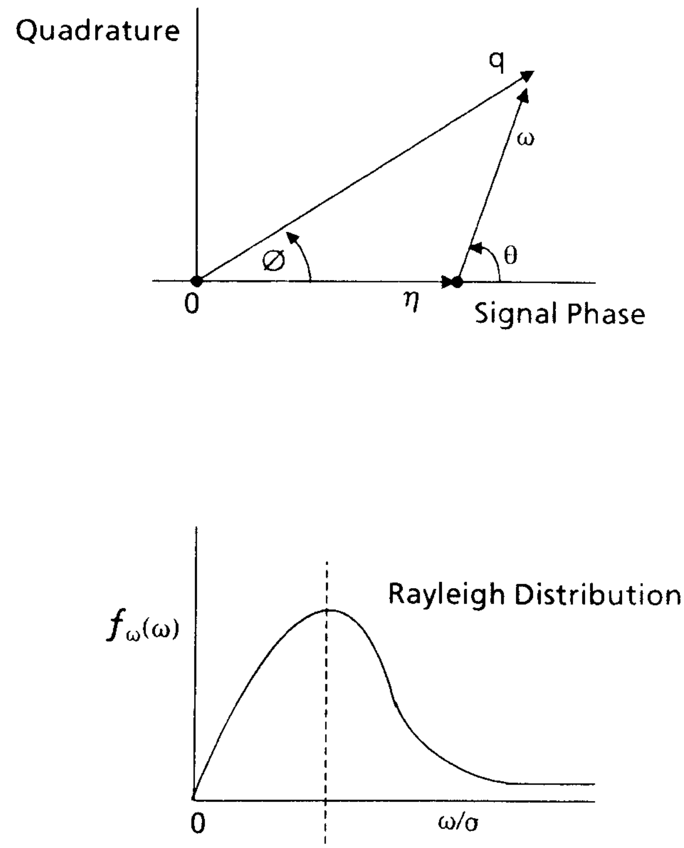

Figure 1 –Vector Relationships

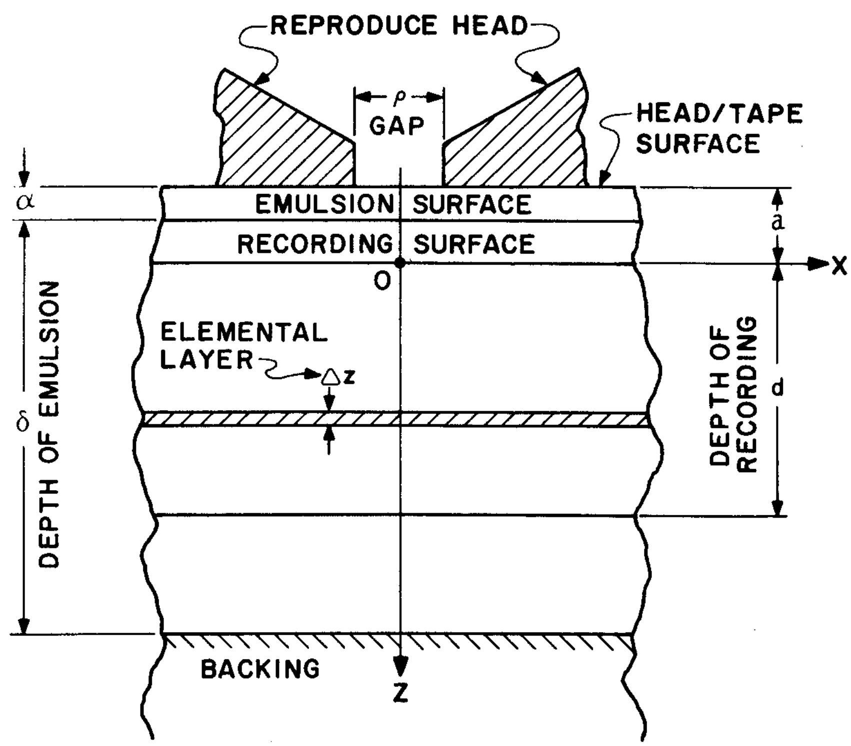

Figure 2 – Recorder Model

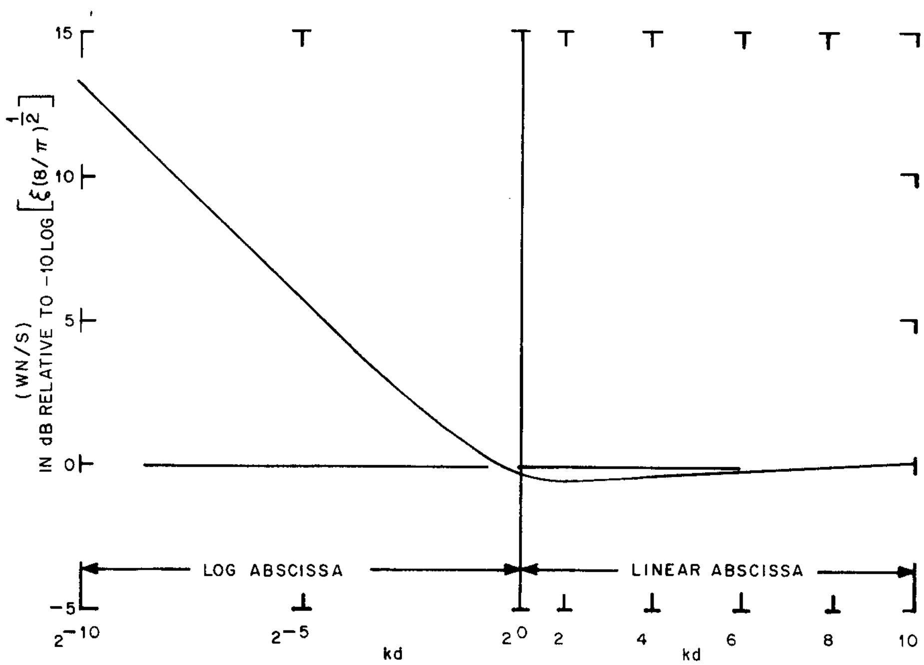

Figure 3 – Write-Noise-to-Signal Ratio (WN/S)

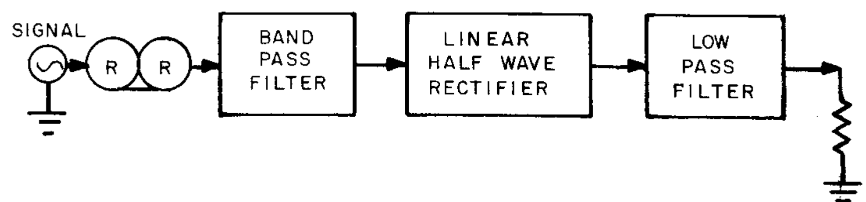

Figure 4 – CW Recording Experiment

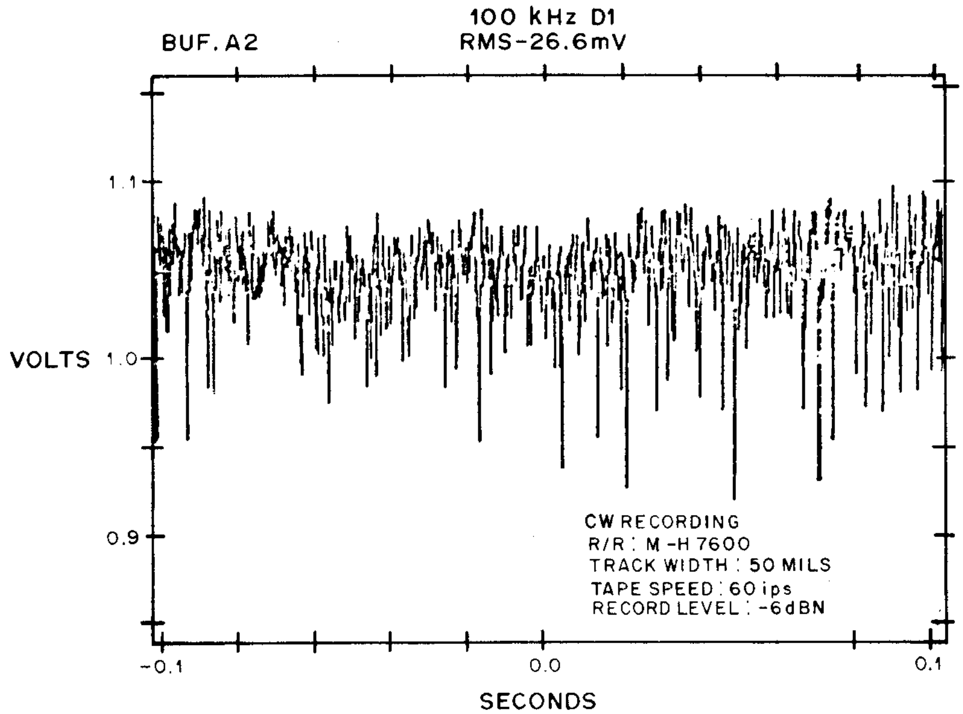

Figure 5a – Amplitude Function

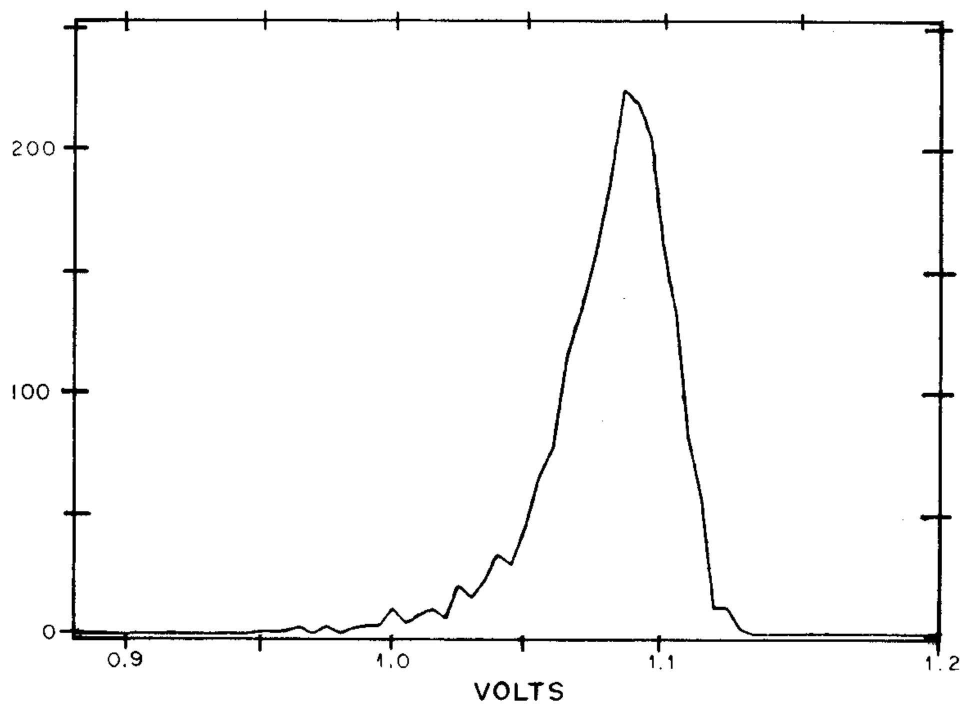

Figure 5b – Distribution Function

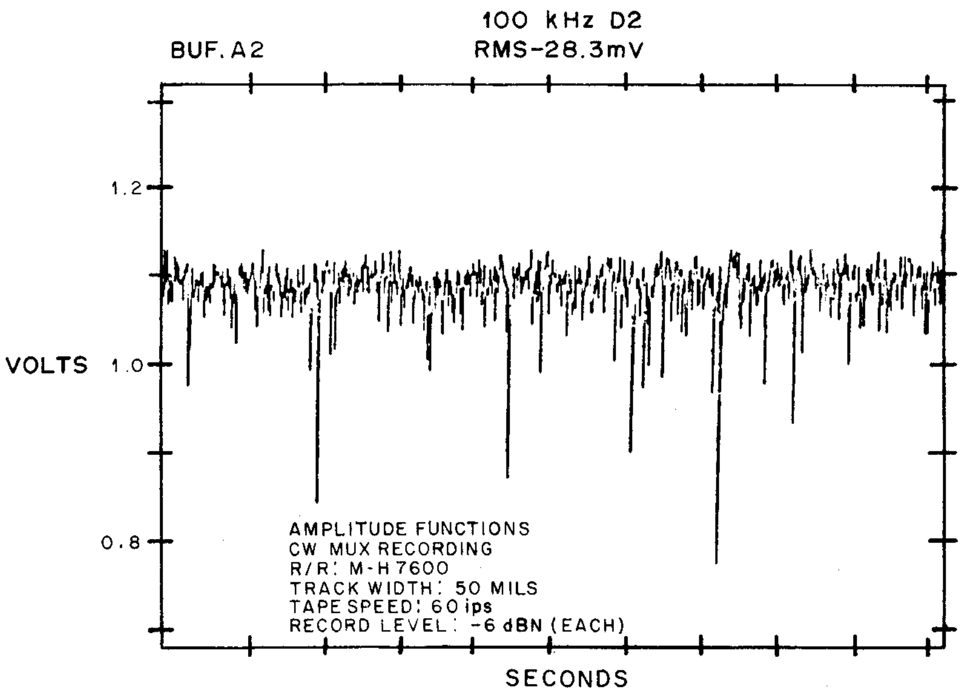

Figure 6a – Amplitude Function (λ = 600 µin)

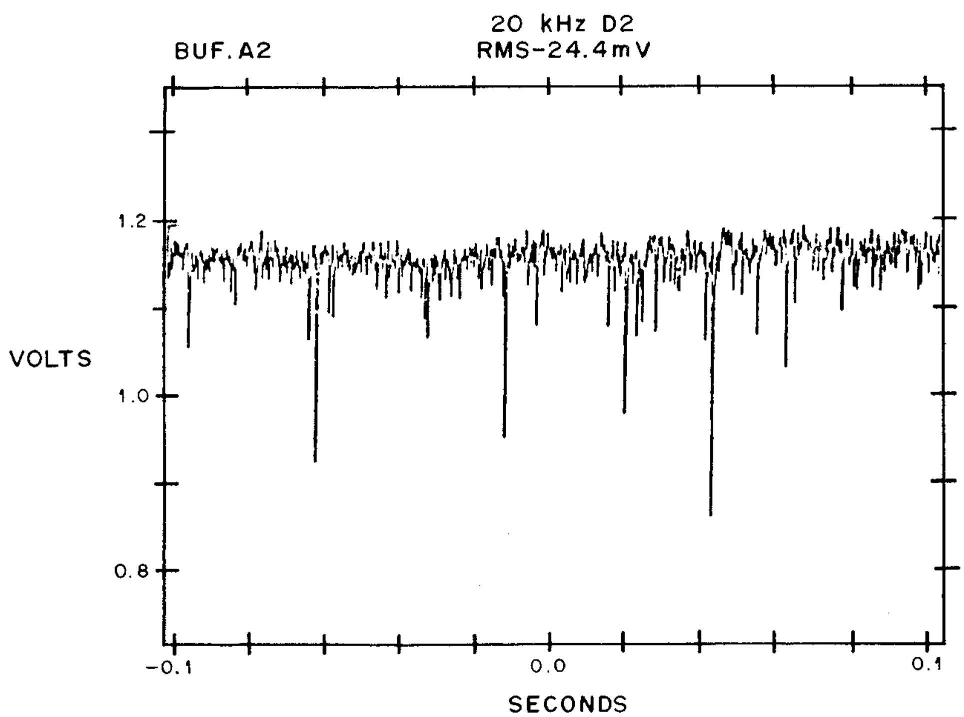

Figure 6b – Amplitude Function (λ = 3000 µin)

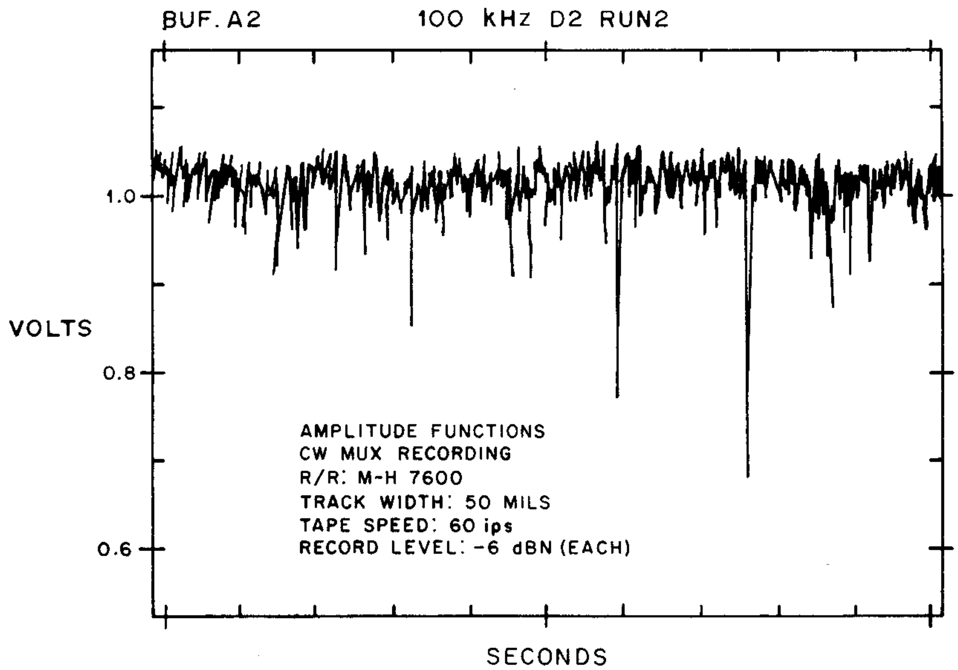

Figure 7a – Amplitude Function (λ = 600 µin)

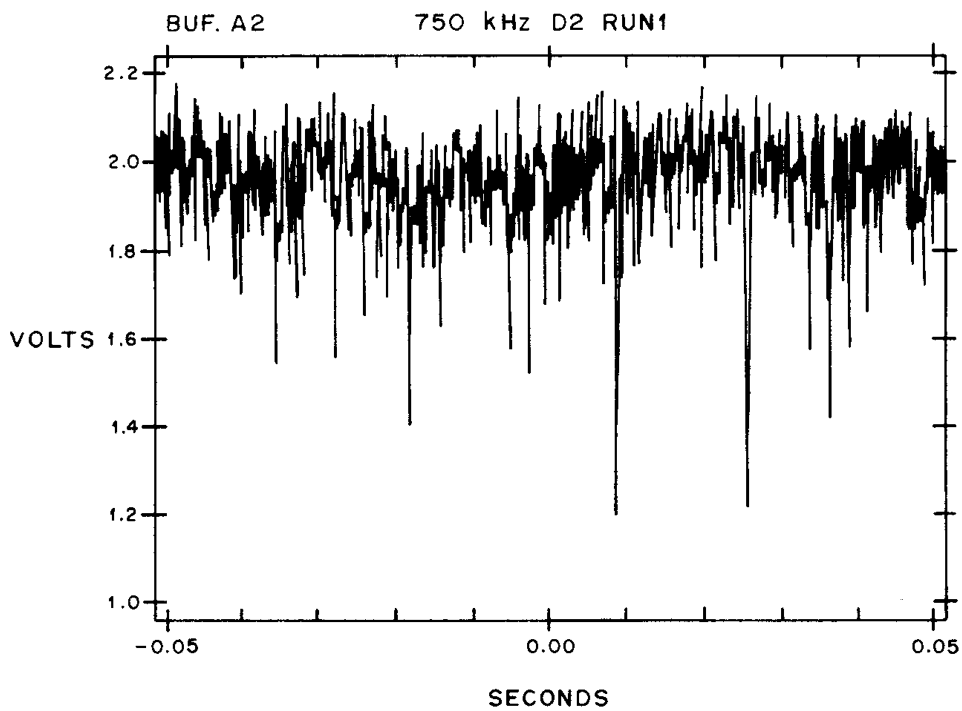

Figure 7b – Amplitude Function (λ = 80 µin)

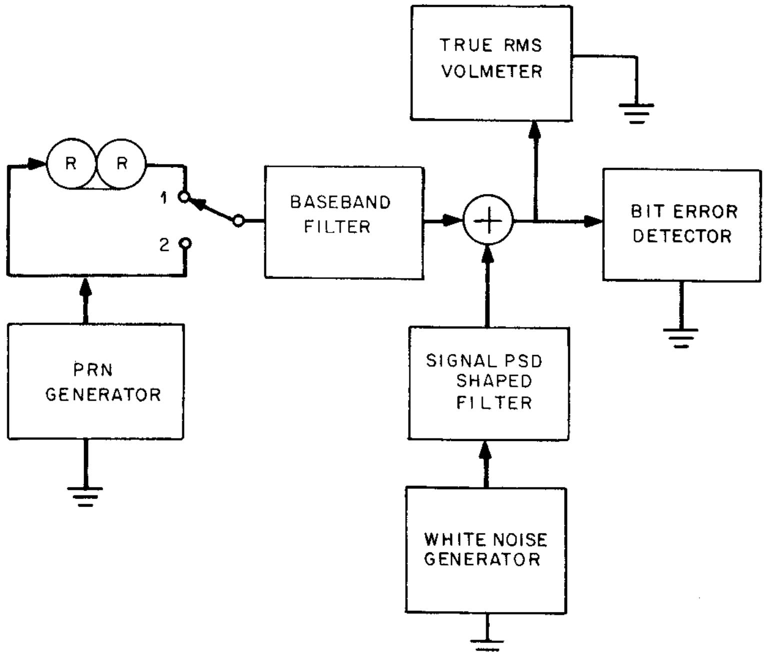

Figure 8 – Write Noise Added Experiment

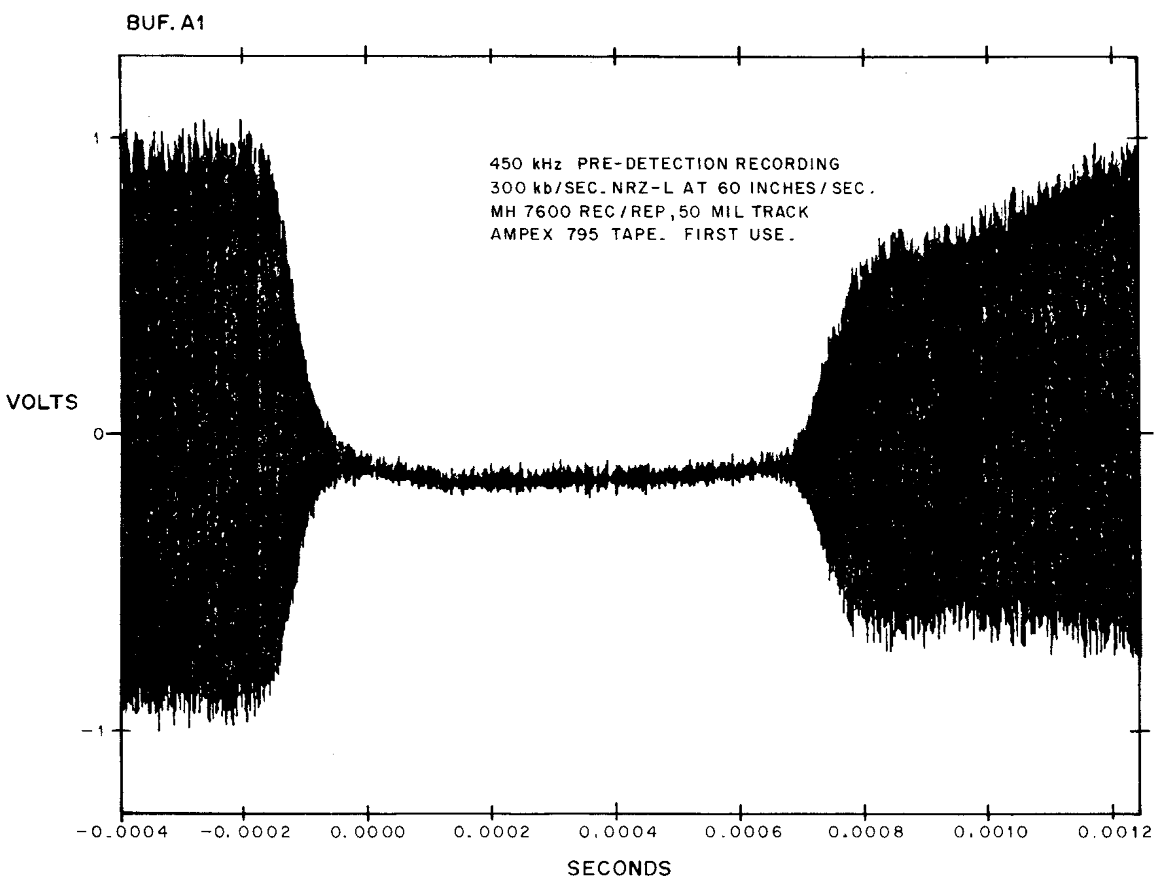

Figure 9 – Catastrophic Signal Dropout