APPLICATION OF SNOW DISTRIBUTION MODELS WITHIN THE LAGUNA NEGRA BASIN, CHILE

by

Brad J. Cadle
Roger C. Bales

August 1997

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Preface

This report is part of a series of publications issued by the University of Arizona, Department of Hydrology and Water Resources. The purpose of this series is to disseminate research findings related to natural resource systems to a broad audience of persons conducting research in natural resources. This particular report is based on the Master's thesis of the first author. Contact the author or the Department of Hydrology and Water Resources for further information.

Any opinions, findings, and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the agencies and individuals whose support we acknowledge.
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ABSTRACT

Spectral linear unmixing and binary regression trees were used to estimate the distribution of snow within the Laguna Negra basin in Chile. Spectral linear unmixing was performed for multi-band Landsat 5 images for the determination of sub-pixel snow fractions. We were interested in determining the number of bands needed for an adequate distribution of SCA. Results showed that for winter scenes (scenes with greater than 90% snow cover and portions of the basin covered by shadows) linear spectral unmixing can be used to model SCA using at least four bands with a rock, a snow and a shaded snow endmember, but that five bands, using two rock endmembers, a snow and a shaded rock endmember, are needed for the fall scenes (scenes with less than 10% snow cover and portions of the basin covered by shadows). The spring scenes (scenes with 50 percent and higher snow cover and no shadows) showed plausible results with three bands, but the need for a second rock endmember in the fall scenes suggest 4 bands may give a more accurate result.

A binary regression tree model was used to determine distributed SWE at peak accumulation in the Echaurren basin, a sub basin of Laguna Negra. Regression trees grown from field snow survey data did an excellent job at explaining the variation of SWE in two of the three surveys examined when resubstitution was used to evaluate the model, but did a poor job in all cases when cross validation was used. However, cross validation may over estimate the errors associated with the model. Basin-wide SWE maps resulting from the
application of the regression trees formed plausible structures. Normalized snow distribution was sufficiently different between years such that a “typical” SWE map could not be developed. Nonetheless, there were identifiable patterns that did occur in the SWE distributions from different years that gave insight into the factors affecting SWE in the basin. Such factors include a strong dependance on radiation in the lower portion of Echaurren for two of the years, and the presence of heavy SWE regions near cliffs. Insights such as these provided useful information on how the type of data and method of collection might be improved. The large SWE values near cliffs, for instance, suggest that use of an avalanche map might improve the modeled SWE distribution. The dependance of SWE on radiation in the lower basin suggest the SWE data should be obtained over the entire range of radiation values in the lower basin.
CHAPTER 1
INTRODUCTION

1.1 Previous Research

Water released from seasonally snow covered alpine areas is a major hydrologic concern for many areas around the world. Snow melt is relied upon for municipal water supplies, electric power generation, cooling systems and recreation [Akan, 1984; Yang et al., 1991]. In addition peak flows from snow melt are a concern in the planning of works to prevent flood related damages downstream from the snow pack [Martinec and Rango, 1992]. Clearly, the ability to predict the water release from these snowpacks is critical for water resource management.

Making predictions about the water released from melting snow requires estimates of the snow covered area SCA and snow water equivalent SWE. An estimation of the total water stored can be found by knowing the mean SWE of the snow covered portion of the basin and the total SCA. Environmental analysis of factors such as basin ecology [Buren et al. 1993], and soil and water chemistry [Walfud et al., 1996] requires knowledge of SWE for Snow melt. Predicting the timing and volume of runoff also requires a good understanding of the spatial variation of the snowpack properties [Elder, 1988]. The reason for this becomes clear when one understands, for example, that spatial variations in metamorphism (changes in the internal structure of the snow pack) will slow the release of
water until the temperature rises during the spring and summer seasons [Gray, 1981].

Obtaining accurate distributions of SCA and SWE in alpine regions is particularly important. There are several reasons why. First, evidence suggests that Snow melt from alpine regions produces more water per unit area than non-alpine regions [Alford, 1980]. Second parameters believed to control SCA and SWE, such as slope, radiation, and elevation, vary greatly in alpine regions resulting in a much more spatially (and temporally) heterogeneous snowpack [Elder 1988]. Failure to capture the spatial variations in SCA and SWE, can create errors in estimating runoff hydrographs.

Modern techniques of estimating SCA over basins generally rely upon either low oblique areal photography or multi-spectral satellite imagery. One of the advantages of using satellite photos is the ability to capture images in remote or inaccessible locations [Dozier and Marks, 1987]. The disadvantages of using satellite imagery vary depending on the platform. Landsat-5, for instance, covers a wide spectral range, but is only above a given location every 2 weeks. Hence, if there happens to be dense clouds on two sequential visits, no SCA data are attainable for that month.

Analyzing either satellite or areal photos for SCA typically has been performed by using binary classification schemes. In these schemes each pixel of an image is classified as either snow or not snow. The classifications are often performed by examining the relationships between the responses of two detectors and known land cover types. Threshold values are assigned based these responses. Holroyd III, et al [1989], for instance, looked at TM bands 3 and 5 and determined the relationships between the spectra of rock, snow and
vegetation.

Although a binary classification algorithm may be appropriate where snow covered area varies little over large distances, it may not be accurate for estimating snow cover in steep alpine regions [Dozier, 1989]. The reason for this is that a significant percentage of pixels in an image will likely contain both snow covered and snow free areas. Even the Landsat Thematic Mapper, with its 30 m resolution, will likely contain pixels with partial snow cover.

Rosenthal [1993] has looked at several variations on the technique known as linear spectral unmixing to predict the fraction of snow in each pixel of Landsat-5 images. His results suggested that linear spectral unmixing does an excellent job of determining snow fractions. A disadvantage with this type of analysis is that it can require the use of 5 or 6 bands of landsat data to classify the image. The more bands one must purchase the greater the cost of each scene. Nonetheless, linear spectral unmixing seems the most promising method of determining snow covered area accurately.

Obtaining the distribution of SWE in alpine regions is even more difficult than obtaining information on SCA. Although work has been done on developing remote sensing techniques for the estimation of SWE [Martinec et al, 1991; Shi et al, 1991; McManamon et al., 1993], no method of using solely air photos or satellite imagery has been developed that will give satisfactory results [Elder, 1995]. The U.S. National Weather Service (NWS) has an airborne snow survey program, whereby the attenuation of natural gamma radiation by the snow is used to give averaged SWE values over flight lines. Although fairly accurate, the
flight lines are not widely distributed in a single basin, and return only integrated values of SWE over the flight path. In addition, the method requires field measurements of soil and snow water content and reference measurements of gamma radiation over the flight line when snow is absent [Carroll et al, 1995].

Thus, the only currently feasible option for obtaining SWE values in many alpine regions is to conduct field surveys. Typical snow surveys involve obtaining snow depth measurements at numerous points throughout the basin of interest, and snow density values using either a Mount Rose sampler at those points or by digging snow pits and obtaining snow density profiles [Perla and Martinelli, 1978]. Once these values are obtained, some method for distributing the values throughout the basin required if you are interested in distributed SWE.

There are numerous methods that can be employed to distribute SWE throughout a snow covered basin. Various kriging techniques have been used by the U.S. NWS [Day, 1990; Carroll et al, 1995] and other researchers [Switzer, 1979, Dingman et al., 1988, Phillips et al., 1992] to predict SWE in locations where data is absent. Carroll and Cressie [1996] for instance, have compared two common methodologies for kriging. In some kriging models, covariance relationships are related only to the distance between survey points, but in others they are also related to geographic parameters. Elder [1988; 1991] used Bayesian maximum likelihood unsupervised classification to partition the Emerald basin in California into areas of similar SWE. Geographic parameters of elevation, slope, vegetation cover and land type, as well as solar radiation, were used to identify the structure of similar SWE groups in their
basin of study. The results were determined to be only partially satisfactory due an incomplete understanding of the factors controlling snow distribution, and inherent nonlinear relationships that were not captured by the model. Later work by Elder [1995] resulted in the application of binary regression trees to the modeling of SWE in the Emerald basin. This scheme proved quite effective at modeling the spatial distribution in all the survey dates examined since it explained 40-80 percent of the variance in the field data and appeared to capture the non-linearity present in some of the parameters. Since regression trees automatically determine relationships between model variables and SWE values, they are extremely useful in classifying the basin into area of similar SWE.

The potential usefulness of linear unmixing for determining SCA and regression tree analysis for distributing SWE prompted investigating their use in the Laguna Negra basin. First, we wanted to know if the conditions in Laguna Negra would allow research to proceed using only three bands of Landsat-5 data to unmix an image, thereby saving on purchasing costs. Second we also wanted to know if binary regression trees would adequately describe the distribution of SWE in the Echaurren basin, and if so, whether or not years of similar amounts of snow accumulation would result in similar patterns of snow.

1.2 The Study Site

The linear unmixing study was conducted in the Laguna Negra watershed in Chile
whose outlet is located at the UTM coordinates of 395350 meters easterly by 6275000 meters northerly (figure 1.1). The basin encompasses an area of approximately 53.7 km\(^2\) with 5.7 km\(^2\) of lake surface. The elevation ranges from 2540 m at the basin outlet to 4350 m giving a relief of 1810 m. The basin is generally south facing with lake Laguna Negra bounding the outlet of the basin to the south. There are several sub-basins within the Laguna Negra basin, including the Echaurren sub-basin, which was the study area for the regression tree study (figure 1.2). The Echaurren sub-basin, named for the Echaurren glacier, is rectangular in shape, possessing a poorly defined southeastern boundary. The elevation range of this sub-basin goes from approximately 2955 m to 4120 m. This gives a relief of 1165 m. There is a wide range of slopes in the Laguna Negra basin as shown on the histogram in figure 1.3. Indeed, as figure 1.4 reveals the Echaurren sub-basin possess a number of cliff faces (slopes > 55 degrees). There is little vegetation in the basin and total absence of trees. The land cover ranges from glacial ice that can be found in the Echaurren basin to wide spread talus and rock.

The snow accumulation period typically lasts for a period of 5 months from May to the end of September. During the accumulation period, snow often covers the entire basin except for the steep cliff sides. The snow melt season typically lasts from November to mid April. The runoff from the snow melt enters Laguna Negra. Overflow from the lake enters Rio Maipo, a major water supply for the city of Santiago. Indeed, accurate accounting of water available as runoff is a major concern for the metropolitan area.
Figure 1.1 Shaded relief map of the Laguna Negra Watershed and surrounding area generated from a 25 meter DEM. Black outline denotes approximate basin boundary.
Figure 1.2 Shaded Relief map of the Echaurren sub-basin and surrounding area generated from a 5 meter DEM.
Figure 1.3 Histogram of slopes for Laguna Negra
Figure 1.4 Shaded relief map of the Echaurren sub-basin illustrating the location of cliff faces. Black lines are cliff faces (slopes > 55 degrees).
CHAPTER 2.

Linear Unmixing for SCA in Laguna Negra

2.1 The Central Issue

The DGA (Direcccion General de Aguas) has a limited budget allocated for research in snow hydrology. One way to reduce the costs associated with determining water storage in alpine snowpacks is to make estimates of snow covered area on as few satellite bands as possible. Each band purchased adds to the total image cost. In addition, the processing costs for a 3 band scene is much lower than for a 5 or 6 band scene. The fewer the number of spectral bands of data that need to be purchased, the lower the total cost.

In order to get accurate values of SCA in Laguna Negra, estimates of sub-pixel snow fraction are desirable. This means that linear spectral unmixing using Landsat-5 TM images should be performed. Thus, I examined the question:

“Are three bands of Landsat-5 data sufficient to give an adequate description of snow covered area for Laguna Negra using linear spectral unmixing.”

A SCA map was deemed adequate if it met the following criteria:

1. The mean root mean square (RMS) error of the modeled image pixels is low.
2. The resulting snow covered area map is physically plausible.
The term plausible is a generic term implying realism. In this study, plausible means that the snow covered area maps must satisfy the following criteria. First, the total number of snow covered pixels should decrease from the time of peak accumulation to the end of the snow melt season. In other words, there should be a consistent time series for the snow covered area maps. Second, any given map during this period should have a snow distribution such that higher elevations generally have greater snow coverage than regions at much lower elevations. This is due to the fact that snow lines generally rise during melting. Although spatial variation in illumination can cause differences in snow coverage, a third criterium is that snow coverage in regions that receive shadows during part of the day should not be too different than the areas that are completely sunlit at the same elevation. There are exceptions to the third criterium, however, such as the presence of persistent snow on glaciers. Finally, small patches of snow on the modeled maps that are known, from field surveys, to be absent and cannot be explained were considered implausible.

Ideally, accuracy of the resulting snow cover maps should be evaluated by using an independent estimate of snow cover such as aerial photography. Unfortunately, no such data were available, so the two criteria of low RMS error and plausibility had to suffice.

2.2 The Process of Linear Spectral Unmixing

Linear spectral unmixing is a process whereby the electromagnetic spectrum measured by a detector is decomposed into the spectra of the individual components
(endmembers) believed to be responsible for the measured spectrum. The amplitudes of the decomposed spectra are then used to determine the fractional contribution from each endmember. An area consisting of just snow and rock, for example, ought to result in a spectrum that is a combination of the two if both land cover types are in the field of a single detector.

For a linear unmix to give a true representation of the fractional amount of a specific scene component several criteria must hold true. First, the scene components should consist of spectra that combine linearly. In other words, as the fraction of a scene component increases, the contribution to the resulting spectra must increase in proportion. This behavior holds true when the components in a pixel are spatially segregated and the incident photons interact with only one material. Second, the endmember spectra must be well chosen. If one is using image endmembers, i.e. spectral signatures chosen from the image to be modeled, the pixel chosen should consist only of the material desired. Finally, errors associated with modeling the image as a set of discrete components must be small.

The mathematical basis for performing an unmix on satellite imagery can be summarized in the following equation (The notation below is from Rosenthal, 1993),

\[
R_b = \epsilon_b + \sum_{n=1}^{N} r_{bn} f_n
\]

(2.1)

where \( R_b \) is the measured reflectance in band \( b \) of the satellite image, \( \epsilon_b \) is the error between the modeled and measured reflectance in band \( b \), \( r_{bn} \) is the reflectance of endmember \( n \) in
band \( b \), and \( f_n \) is the fraction of the endmember \( n \)'s spectrum that describes the spectrum of the modeled pixel.

The goal of the unmix is to solve a set of simultaneous equations of the form above for the values of each \( f_n \) for each pixel in the image. \( R_b \) is already given for each pixel and the values of \( r_{pn} \) are determined prior to the unmix. The number of simultaneous equations one must solve depends on the number of bands being used to perform the unmix. The optimal solution is found by least squares method. The specific means of extracting the values for all \( f_n \) is discussed in Rosenthal, 1993.

The quality of fit of the model to the data for a single image is described, as in Rosenthal, by the mean of the RMS errors of all the image pixels. Equation 2.2 summarizes this as,

\[
\overline{RMSE} = \frac{1}{P} \sum_{p=1}^{P} \left( \frac{1}{B} \sum_{b=1}^{B} \frac{1}{\epsilon_b^2} \right)_p
\]  

(2.2)

where \( P \) is the number of pixels in the image, \( B \) is the number of spectral bands in the image, and \( \epsilon_b \) is the error in band \( b \) as described in equation 2.1. Thus, equation 2.2 suffices for the second of my criteria for determining the answer to the question examined.

When using the mean RMS error as an indication of model fit, it is important to note that since \( \epsilon_b \) is an unknown parameter in the set of equations described by 2.1, the mean RMS error will generally decrease as the number of endmembers increases. By increasing the number of endmembers, there are more degrees of freedom and hence a better fit to the data.
can be attained. An increased fit may not necessarily mean an improved model, so the errors associated with an unmix should be minimized with respect to errors given by the same number of alternate endmembers. When the number of endmembers is equal to the number of bands, \( \varepsilon \) must be dropped as a parameter in equation 2.1 or the resulting model would have \( N+1 \) unknowns for \( N \) equations. The result is a unique value for each \( f_n \) with no measure of error. There is no set value for the optimal number of endmembers for images with a given number of bands. The number of endmembers depends on the dimensionality of the image. The goal is to find a small enough mean RMS error such that the number of endmembers being used is less than the number of bands of image data.

When the mean RMS error is small enough, the scaled endmember fraction should be calculated. Because of variations in the illumination of surface features, the values of \( f_n \) for any given pixel may not be equal to the fraction of endmember \( n \), even if the scene components mix linearly. Some slopes and aspects may contribute less radiation to the image detector than others. Hence, a fully snow covered pixel, for instance, at some given slope might result in a spectral signature with a smaller amplitude than a fully snow covered pixel directly facing the sun. If the spectra do mix linearly and the model represents the endmember spectral fractions well then equation 2.3 gives the fraction of endmember \( n \) in the pixel as,

\[
F_n = \frac{f_n}{\sum f_i}
\]

(2.3)
where \( F_n \) is the scaled snow fraction, \( f_n \) is the fraction of endmember n’s spectrum in the pixel, and the \( f'_n \)s represent the fraction of the other endmembers spectra in that pixel.

In other words, the decrease in illumination is taken account by noting that, if all the scene components are accounted for in a pixel, the fraction of the endmember n present should be the ratio of its amplitude to the total for that pixel.

Although this scaling works well in sunlit areas, deep shadows can cause problems by obscuring the relationship between the different endmembers. In such cases, alternate endmembers may have to be chosen. In addition to problems with deep shadows, band saturation issues can be important, especially for band 1 over snow. When this occurs the saturated bands must be disregarded or a special endmember of saturated snow must be chosen.

2.3 The Data Set

One 1992 and five 1991 Landsat 5 TM images of the Laguna Negra basin were available for this study. Of the six, three contained only three landsat bands, two contained five bands each, and one contains six bands (table 2.1) summarizes the six images in terms of the dates each image was taken and the bands present in each image. Each Landsat 5 image was taken at 10:00 local time.
In order to compare the results of the spectral unmixing on three bands versus using more than three bands, I chose the March 31st and August 8th images. These two scenes represent Fall and Winter, respectively. Figures 2.1 and 2.2 are false color composite images showing the degree of snow present in March and August respectively. Note that no cloud cover was present over the Laguna Negra basin at the time these images were taken.

In addition to the five and six band images, I performed unmixing on the three band images of November 10th and December 12th. These images were taken during spring near the beginning and middle of the melt season respectively. The sun is high in the sky at 10:00am in Chile at this time of year, thus there are no deep shadows present in Laguna

---

### Table 2.1 Database of Images

<table>
<thead>
<tr>
<th>Date of Image</th>
<th>LANDSAT 5 TM bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>7, 5, 4, 3, 2, 1</td>
</tr>
<tr>
<td>8/08/91</td>
<td>7, 5, 4, 2, 1</td>
</tr>
<tr>
<td>8/22/91</td>
<td>7, 5, 4, 2, 1</td>
</tr>
<tr>
<td>11/10/91</td>
<td>5, 4, 3</td>
</tr>
<tr>
<td>12/12/91</td>
<td>5, 4, 3</td>
</tr>
<tr>
<td>1/13/92</td>
<td>5, 4, 3</td>
</tr>
</tbody>
</table>
Negra. In addition, as was the case for the March and August images, there are no clouds present. Figures 2.3 and 2.4 are the corresponding false color composite images.

No unmix was performed on the January 13th image. The reason for this becomes clear by examining the color composite image of figure 2.5. The extensive cloud cover in the northern part of the basin, would force me to mask out a large portion of the basin. Since the purpose behind examining the images where only three bands are present is to make a physical determination of the adequacy of the image, it seemed pointless to use this in answering the question.

It is hoped that non linear spectral mixing contributes little to the resulting spectrum. Rosenthal [1993] outlined 6 potential sources of non-linear umixing in montane snow covered regions. Of the six, the two sources that seemed most relevant to the Chilean images are the potential for dust or soot contamination in the snow, and the possibility of a thin snow pack. The effect of a thin snowpack is particularly dominant on the VNIR band. Now dust is present in some alpine basins in the central Chilean Andes, however it is not known if significant amounts are present in the Laguna Negra basin. Thus the contribution of dust to unmixing the images is unknown. Rosenthal pointed out that the portion of a snow pack that is thin enough to contribute to non-linear mixing is generally small compared to the total area of a snow pack. This factor may be more important in the March scene, but it is still likely to be a small contribution.
2.4 Methods

The Landsat-5 images were imported into the Image Processing Workbench (IPW) [Frew and Dozier] by previous researchers. The radiance record in each band is quantized as an 8-bit digital radiance number (DN). The modeling and analysis of the Landsat images was performed entirely within IPW.

The basin mask for Laguna Negra was generated from a 25 meter digital elevation map (DEM). The DEM was generated using the GIS platform GRASS (version 4.1) and a topological map scanned in by the DGA. The elevation values between contours were determined by GRASS using an interpolation scheme. Sub-basins within Laguna Negra were identified by the GRASS algorithm r.watershed, and joined together manually using the GRASS scripting routine of r.mapcalc. Since the Landsat images were not geographically registered, I imported the March 31st image from IPW into GRASS4.1 and used i.rectify2 to coregister the image with a shaded relief map generated from the DEM. A second order polynomial function was used to map the satellite image to the relief map to account the fact that the detector track is not parallel to the curvature of the earth. The mapping function from the Landsat image to the shaded relief map was then inverted and applied to the Laguna Negra basin mask. A total of 13 points in and around the Laguna Negra basin were used for correlation. Figure 2.5 shows the registered basin mask, generated from r.watershed, used to separate the basin from the rest of the image for the analysis of the results.

Prior to unmixing the image, the effects due to the selective scattering of light by the
atmosphere were removed. In order to remove these effects, a histogram of pixel frequency versus radiance DN for each band was created and the values where the detector first starts responding significantly were located. The response was considered significant if the number of pixels at a specific DN in the Laguna Negra images was about 5 or greater. The DN values for which this occurs (path DN) are listed in table 2.2. The increasing value of path DN with respect to decreasing band number illustrates the fact that smaller wavelengths undergo greater scattering by the atmosphere. Once the path DN was determined for each band in an image, the detector response was shifted down by this amount. Any pixel values below the path DN were thrown into the bin containing the new zero response.

Table 2.2 Path DN values for each scene.

<table>
<thead>
<tr>
<th>Scene</th>
<th>Band 7</th>
<th>Band 5</th>
<th>Band 4</th>
<th>Band 3</th>
<th>Band 2</th>
<th>Band 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>8/08/91</td>
<td>0</td>
<td>0</td>
<td>9</td>
<td>NA</td>
<td>19</td>
<td>52</td>
</tr>
<tr>
<td>11/10/91</td>
<td>NA</td>
<td>0</td>
<td>12</td>
<td>27</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>12/12/91</td>
<td>NA</td>
<td>0</td>
<td>7</td>
<td>14</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

A short comment should be made regarding the path DN values for the March 31st scene. The estimated path DN’s listed seem a bit small. For instance, a path DN of about 20 is a more common value for band 2 in alpine regions during the fall [Rosenthal, personal communication]. The effect of the low path DN values is uncertain.
In order to perform the unmix on each image, the endmember spectra used to describe the image, the values of $r_{bn}$ for each $n$, were determined. Initially, the three band unmixes were performed restricting me to using only two endmembers. Since extensive vegetation was absent from the basin, I hoped that a snow and rock endmember would be sufficient for modeling the fractional contribution of snow in each pixel. In the unmixes using more than three bands, additional endmembers including shaded snow or rock were incorporated. The endmember spectra chosen to represent the various endmembers were determined from pixels in the image by using two methods. One technique, used specifically for the three band images, was to create a false color image using bands 5, 4, and either 3 or 2 depending on the image. Using an implementation of the software X view that supports IPW images, the pixel values of the snow covered and snow free areas were examined. Using figure 2.6 (Rosenthal, 1993) as guide to the relative amplitudes of snow and rock in each of the bands, pixels that appeared to consist of solely rock or snow could be picked out. Once the pixel coordinates were recorded, the pixel values (pixel spectra) were extracted using IPW. The second technique, which proved particularly useful in the unmixes using more than three bands, was to transform the multi spectral images into principal components space. The basic reason for using principal components analysis is that areas that are snow covered, or consist only of exposed rock, will tend to have points clustering along certain directions in the space of DN values where each axis is a band number. By determing a variance covariance matrix for this data, and calculating the associated eigenvectors, the dot product of the eigenvector and the image data is taken for each pixel to produce a new single band image where the extreme
points of the data cloud have the largest pixel values. These extreme points were used as a
guide for the selection of initial endmembers. One of the advantages of using principal
components analysis was that no prior interpretation of an adequate endmember is required.
Both methods for selecting image endmembers were only a starting point for determining the
optimal endmembers finally selected for each image. The final endmembers used were
determined after successive iterations by reducing the mean RMS error as much as possible.

The set of equations described by equation 2.1 were solved using the IPW program
sma (Spectral Mixture Analysis). A multi-band IPW image whose components are the TM
bands for the scene being modeled, and ASCII files containing the reflectance values of the
endmembers were the input data into sma. The output from sma consisted of fraction images
for each endmember spectra, and a single band error image containing the $\text{RMSE}$ for each
pixel. The method used to invert the matrix of reflectance values given by $r_{mn}$ was singular
value decomposition (SVD).

Sma offers two modes of operation with SVD to solve for the values of $f_n$ for each
pixel. The first is to set no constraints on the possible values of $f_n$. In a truly unconstrained
solution $f_n$ could take on any value, but sma outputs eight bit images so the range of pixel
values is restricted from -1.00 to 1.55 in increments of .01. Spectral fractions outside the
range of 0 to 1, can produce nonsensical results in equation 2.3 such as negative fractions
of snow. However, the presence of a large number of pixels outside this range can be an
indication of an inappropriate endmember. For this reason, all of the initial unmixes on each
image were performed using the unconstrained mode of sma. Histograms were generated to examine the range of fn for each n to help in evaluating the quality of the chosen endmembers. The second option for running sma in SVD mode is to constrain the values such that they fall between 0 and 1. This scheme will result in physically reasonable solutions to equation 2.3, but sacrifices some information about the quality of the chosen endmembers. This option was used once the image endmembers were found to be adequate.

The scaled snow fraction image described by equation 2.3 was calculated by using various IPW routines to add the fraction images together and divide their sum from the snow fraction image. The scaled snow fraction image was used as the resulting SCA map.

The specific procedures used in ipw to choose endmembers and determine SCA is outlined in Appendix A. Included in the appendix are all the steps needed to perform a manual spectral unmix starting with a Landsat-5 image that has been imported into IPW.

2.5 Results of the Three Band Unmixes

The endmember spectra that gave the best results for the three band unmixes are listed in table 2.3 below. The endmembers generally follow the trend indicated in figure 2-6.
Table 2.3 Endmember spectra

<table>
<thead>
<tr>
<th>Scene</th>
<th>Endmember</th>
<th>Band 2</th>
<th>Band 3</th>
<th>Band 4</th>
<th>Band 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>snow</td>
<td>184</td>
<td>NA</td>
<td>131</td>
<td>12</td>
</tr>
<tr>
<td>3/31/91</td>
<td>rock</td>
<td>93</td>
<td>NA</td>
<td>100</td>
<td>222</td>
</tr>
<tr>
<td>8/08/91</td>
<td>snow</td>
<td>236</td>
<td>NA</td>
<td>246</td>
<td>104</td>
</tr>
<tr>
<td>8/08/91</td>
<td>rock</td>
<td>91</td>
<td>NA</td>
<td>85</td>
<td>91</td>
</tr>
<tr>
<td>11/10/91</td>
<td>snow</td>
<td>NA</td>
<td>228</td>
<td>243</td>
<td>49</td>
</tr>
<tr>
<td>11/10/91</td>
<td>rock</td>
<td>NA</td>
<td>110</td>
<td>91</td>
<td>221</td>
</tr>
<tr>
<td>12/12/91</td>
<td>snow</td>
<td>NA</td>
<td>241</td>
<td>220</td>
<td>10</td>
</tr>
<tr>
<td>12/12/91</td>
<td>rock</td>
<td>NA</td>
<td>148</td>
<td>111</td>
<td>255</td>
</tr>
</tbody>
</table>

The SCA images produced from these endmembers, figures 2.7 through 2.10, show some interesting results. The March and August images suggest that the south facing ridges have greater snow coverage than other ridges. Indeed, the white areas on the August image correspond to snow coverage values slightly greater than 100 percent. The reason for the classification of pixels greater than 100 percent is that even using the constrained mode of sma, some pixels can get mapped as having values slightly less than zero. When the scaled snow fractions are calculated, this results in fractions greater than 1. The March image also shows a tendency to have large snow fraction values. The November and December images, on the other hand, do not have this bias. Comparing figures 2.7 and 2.8 to the color
composite images in figures 2.1 and 2.2, shows that slopes with the bias towards larger snow coverage are the ones that fall in deep shadows. Moreover, a comparison of the November and December SCA images to their color composite counter parts in figures 2.3 and 2.4 reveals that the absence of this bias in those two scenes is due to the absence of deep shadows. Clearly the bias in the March and August images are artifacts due to the presence of deep shadows in the image.

The mean RMS error associated for each unmix above is listed in table 2.4 below. Surprisingly, the March scene has the smallest mean RMS error, despite the fact that the regions in the shadows are being misclassified as snow.

Table 2.4 Mean RMS error for each scene.

<table>
<thead>
<tr>
<th>Scene</th>
<th>Mean RMS error for basin</th>
<th>Mean RMS error for image</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>2.00</td>
<td>2.83</td>
</tr>
<tr>
<td>8/08/91</td>
<td>7.93</td>
<td>8.61</td>
</tr>
<tr>
<td>11/10/91</td>
<td>12.84</td>
<td>10.44</td>
</tr>
<tr>
<td>12/12/91</td>
<td>7.30</td>
<td>6.26</td>
</tr>
</tbody>
</table>

The total snow coverage for the Laguna Negra basin in each scene is listed below in table 2.5. The November, December, and August images give reasonable values for the percentage snow cover, but the value for March is unlikely. A SCA of 27 % is not plausible
since the only source of snow for that time of year should be glacial ice. The high value of percentage snow cover is, of course, a result of the shadowed areas being misclassified as snow covered. This suggests that the August image, which also has an over classification of snow in the shadowed parts of the basin, probably has an estimated percentage snow cover that is a little too high. Since the basin is generally fully covered by snow this time of year, the error introduced by this over classification is probably small compared to the error in the March scene. The total SCA that is misclassified for the August scene is probably on the order of the areal coverage of the cliff faces in the shadows.

Table 2.5 Percentage of basin covered by snow (area covered by lake excluded).

<table>
<thead>
<tr>
<th>Scene</th>
<th>% SCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>27%</td>
</tr>
<tr>
<td>8/08/91</td>
<td>98%</td>
</tr>
<tr>
<td>11/10/91</td>
<td>84%</td>
</tr>
<tr>
<td>12/12/91</td>
<td>50%</td>
</tr>
</tbody>
</table>

2.5 Results using more than Three Bands

The endmembers that gave the best results using greater than three bands are shown in table 2.6. The March image required the addition of a two more endmembers. An endmember I call rock2 and an endmember I call shade. The rock2 endmember is substantially dimmer than the rock endmember, and the spectral signature is slightly
different. According to field surveyor Robert Harrington, the different soils include hard rock, talus and scree, and some meadow. It is more than likely that the endmembers rock and rock2 make up two of the three soil types listed above. The shaded rock endmember was chosen from a pixel believed to consist of primarily rock found in the deep shadows. The endmembers for the August image included the additional endmember of shaded snow. Band 1 was not used in the August unmix because it was found to result in an image where even the sunlit cliffs where modeled as snow. This may have been the result of band saturation problems. Since two endmembers were used for snow in this image the final SCA image is was the sum of the shaded snow and sunlit snow images.

<table>
<thead>
<tr>
<th>Scene</th>
<th>Endmember</th>
<th>Band 1</th>
<th>Band 2</th>
<th>Band 4</th>
<th>Band 5</th>
<th>Band 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>Snow</td>
<td>233</td>
<td>131</td>
<td>184</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>3/31/91</td>
<td>Rock</td>
<td>119</td>
<td>93</td>
<td>100</td>
<td>222</td>
<td>122</td>
</tr>
<tr>
<td>3/31/91</td>
<td>Rock2</td>
<td>20</td>
<td>17</td>
<td>69</td>
<td>63</td>
<td>15</td>
</tr>
<tr>
<td>3/31/91</td>
<td>Shade Rock</td>
<td>14</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>8/08/91</td>
<td>Snow</td>
<td>N/A</td>
<td>246</td>
<td>236</td>
<td>102</td>
<td>44</td>
</tr>
<tr>
<td>8/08/91</td>
<td>Rock</td>
<td>N/A</td>
<td>91</td>
<td>85</td>
<td>91</td>
<td>47</td>
</tr>
<tr>
<td>8/08/91</td>
<td>Shade Snow</td>
<td>N/A</td>
<td>36</td>
<td>27</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 2.11 and 2.12 show the SCA maps calculated for these unmixes. The March image shows that the shaded areas are no longer classified with extensive amounts of snow.
There are pixels throughout the image that seem inappropriately classified as snow but it is conceivable that this is due to noise in the data. Nonetheless, a substantial improvement in appearance was obtained by modeling the image with a greater number of endmembers.

The August image shows that the shadowed regions are no longer being classified as having snow fractions greater than 1. The shadowed regions still do not show any obvious cliff exposures however. Perhaps yet another additional endmember may help, but due to the problems with band one this hypothesis could not be tested. The image in figure 2.12 is the best that could be obtained. As was the case with the march image, this is an improvement over using only 3 endmembers.

The mean RMS error for the unmixes using more than three bands is shown in table 2.7. Comparing the errors of the March and August scenes for the three band unmixes (table 2.4) to those below shows that the mean RMS error decreased. Now the numbers of degrees of freedom for the images producing the results below is larger, but so is the number of bands of data.

<table>
<thead>
<tr>
<th>Scene</th>
<th>Mean RMS error for basin</th>
<th>Mean RMS error for image</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>1.43</td>
<td>1.67</td>
</tr>
<tr>
<td>8/08/91</td>
<td>4.02</td>
<td>5.03</td>
</tr>
</tbody>
</table>

Table 2.8 shows that the unmixes using more than 3 bands for the Laguna Negra
basin, produce a reasonable value for snow coverage. The March value of 4 percent is closer to what we would expect if the glacier was the only source of snow. The August value of 97 percent is slightly less than that for the image using three bands, but due to the absence of visible cliffs in the shadowed regions, the modeled SCA may still be slightly high.

Table 2.8 Percentage of basin covered by snow (area covered by lake excluded)

<table>
<thead>
<tr>
<th>Scene</th>
<th>% SCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/31/91</td>
<td>4</td>
</tr>
<tr>
<td>8/08/91</td>
<td>97</td>
</tr>
</tbody>
</table>

2.6 Discussion

Although the mean RMS error for the models of the 3 band images may be adequate the second criteria that the resulting snow covered area maps are physically plausible, was not met in all cases. The scenes in which extensive shadows were present, March and August, resulted in areas that were misclassified as containing high percentages of snow. The primary reason for this being that the application of equation 2.3 will result in snow fractions that are too high if all the components of a pixel are not taken into account. In order to get results that were physically plausible, I needed to use an additional two endmembers for the March image. Hence I had to resort to using a minimum of 5 bands for the August image and 4 bands for the March image. It is possible that by using a mask, one could run a mixing model on the shadowed and sunlit portions separately. In the case of the March
image however, the need for a second rock endmember still precludes the possibility of using three bands.

The adequacy of the method itself was difficult to gage given the lack of an independent estimate of SCA. Previous work by Rosenthal, has suggested that the method is quite robust. Nonetheless, since the models can produce erroneous results and still give a low a low values for $\overline{RMSE}$, an SCA map may look physically reasonable overall but individual pixels may not be precise. If the desired scale of an application is large enough and if the errors are randomly distributed, these errors may be unimportant.

In conclusion, the three band unmixes for November and December, although having mean RMS errors larger than the August and September images, seem to be adequate for the purposes of modeling snow melt in the Laguna Negra basin. Caution should be taken in using three bands even in scenes for this time of year, since the March results required the use of a second rock endmember. There may be incorrectly classified pixels that are not obvious in these scenes. The March and August images require the use of additional endmembers besides just snow and rock, therefore imply the need for a greater number of bands. Further analysis may show that a shade endmember can be removed if one analyzes the shaded and non shaded portions separately, but March still seems to require the use of a second rock endmember.
Figure 2.1 Histogram equalized false color composite image of Laguna Negra and surrounding area, March 31, 1991 (Landsat bands 5, 4, and 2). Blue areas are snow covered.
Figure 2.2 Histogram equalized false color composite image of Laguna Negra and surrounding area, August 8th 1991 (Landsat bands 5, 4, and 2). Blue areas are snow covered.
Figure 2.3 Histogram equalized false color composite image of Laguna Negra and surrounding area, November 10, 1991 (Landsat bands 5, 4, and 2). Blue areas are snow covered.
Figure 2.4 Histogram equalized false color composite image of Laguna Negra and surrounding area, December 12, 1991 (Landsat bands 5, 4, and 3). Blue areas are snow covered.
Figure 2.5 Histogram equalized false color composite image of Laguna Negra and surrounding area, January 13 1991 (Landsat bands 5, 4, and 3). Sharp blue areas are snow covered. Diffuse blue areas are cloud covered.
Figure 2.6: Upper figure shows endmember spectra for snow at various illumination angles. The lower figure shows endmember spectra for various rock types. The surface reflectance for rock is not very sensitive to illumination angle so this dependence was omitted.
Figure 2.7 SCA map for March 31st 1991 generated from 3 bands. Landsat 5 Bands 5, 4, and 2 were used. White represents 100% snow. Black represents no snow. White outline is the basin boundary. White area at the basin outlet is the lake.
Figure 2.8 SCA map for August 8th 1991 generated from 3 bands. Landsat bands 5, 4, and 2 were used. Off white represents 100% snow cover, Pure white represents more than 100% snow cover and black represents no snow. Black outline is the basin boundary.
Figure 2.9 SCA map for November 10th 1991 generated from 3 bands. Landsat 5 bands 5, 4 and 3 were used. White represents 100% snow and black represents no snow. Black outline is basin boundary.
Figure 2.10 SCA map for December 12th 1991 generated from 3 bands. Landsat bands 5, 4 and 3 were used. White represents 100% snow and black represents no snow. Black outline is the basin boundary.
Figure 2.11 SCA map for March 31st generated from 5 bands. Landsat bands 7, 5, 4, 2, and 1 were used. White represents 100% snow and black represents no snow. White outline is the basin boundary.
Figure 2.12 SCA map for August 8th 1991 generated from 4 bands. Landsat 5 bands 7, 5, 4, and 2 were used. White represents 100% snow and black represents no snow. Black outline is the basin boundary.
CHAPTER 3.

Regression Tree Based SWE Distribution

3.1 Introduction

Modeling snow melt requires a knowledge of both the area of a basin covered by snow, and the amount of water stored within the snow pack. Snow water equivalence (SWE) is the standard value for water storage. SWE is defined as the amount of water that results from instantaneously melting the entire depth of snow at a point. It is generally given in units of depth, but can be expressed as volume if extended over an area. SWE is calculated by multiplying the snow density, expressed as mass per volume, by the depth of the snow pack at the point of measurement, and dividing by the density of water. This results in the desired units of equivalent water depth.

Field surveys of the Echaurren sub-basin were done to develop snow depth and density data toward the end of the accumulation period. The data consist of point measurements of depth and either point measurements of density, or density profiles from snow pits. Since these are point measurements in a distributed parameter field, a method for distributing the SWE values had to be selected. Because of its ease of implementation, and apparent success at estimating distributed SWE at the Emerald lake basin in the Sierra Nevada [Elder, 1995] regression trees were grown on the survey data and used to estimate
distributed SWE in the basin for each year.

The questions addressed by the current research are whether or not the regression trees would give an adequate estimate of the spatial distribution of SWE in the Echaurren basin, and if so, are accumulation patterns similar from year to year.

3.2 Background on Binary Regression Trees

Binary regression trees consist of a set of binary decisions on independent variables (predictor variables) which result in a specific value predicted as a dependant variable (response variable). A simple binary regression tree, also known as a binary decision tree, is illustrated in figure 3.1. The basic procedure for traversing a tree is to start with a vector \( x \) in a measurement space \( X \) that consists of predictor variables. In the case illustrated in figure 3.1, the components (predictor values) of the vector would be an elevation and a radiation value. At each circle (node), starting with the top one, a binary decision is made based on one of the components of the vector. In figure 3.1 the first decision (split) would be on elevation. If the elevation component of \( x \) is less than or equal to 3873 meters than the vector is moved to the next decision on the left, otherwise it is moved to the right. The process continues down the tree until one of the square boxes (terminal nodes) is reached (see figure 3.1). At the terminal nodes a value is than assigned as the response value to \( x \), for example, 1.335 meters of SWE in figure 3.1.

To predict a response variable using this method, the relationship between the
response and the independent variable must be determined. The relationship falls naturally out of the process used to generate a regression tree. This process, referred to as growing a tree, involves recursively partitioning a set known responses on associated predictor values (the learning set). A partitioning algorithm will determine all possible ways to divide the response data into two sets based on a single predictor. If the predictor value takes on a continuous range of values, as is the case with elevation, the partition will occur exactly half way between two elevation values in the learning set. Thus, a learning set consisting of \( N \) data points, will have \( N - 1 \) possible partitions.

Once all possible partitions on a single predictor are determined, a measure of the change of impurity is calculated. Impurity is a general term that refers to the degree of heterogeneity in the response data. When all of the data points are in a single set the impurity is highest. Splitting the data into two subsets reduces the impurity by grouping together similar values of the response variable. When a tree is grown on numeric response data, a type of regression is used to classify the data in each subset (hence the term regression tree). In general, a least squares regression is used to classify the data and determine the degree of impurity. The impurity is estimated as the mean least squares error between a response value and its associated prediction. Once the change of impurity for each partition is calculated, the split that causes the greatest decrease in impurity is selected as the best split on the predictor being tested. The process than repeats for each predictor variable. In figure 3.1, the variables were elevation and radiation. Once the best split is found for each variable the variable with the split that reduces the impurity the most is chosen. This split becomes the
split for the node.

The procedures are recursively performed on each subsequent node (child node) until a stopping criterion is met. Stopping criteria generally consists of either one or both of the following rules. The first option is to stop splitting when the impurity of the child node is less than some fraction of the impurity of the root node (first node). The other option is to stop splitting when the number of observations in a subset is less than some minimum value. In practice the exact nature of the stopping rule is not crucial, as long as one prunes back an overgrown tree to some size decided to be the optimal size. The optimal number of terminal nodes can vary with application.

A detailed mathematical treatment of binary regression trees can be found in most texts covering decision trees. "Classification and Regression Trees" by Breiman et al [1984] includes a detailed mathematical description of regression trees as well nice discussions on their interpretation. Elder [1995], the original work on the use of regression trees in predicting SWE, includes a discussion on the aspects of the theory relevant to SWE prediction.

3.3 The Data Set

Once every year since 1992 a snow survey was conducted in the Echaurren basin. In 1993, however, only the lower part of the basin was surveyed since the heavy snow fall for that year prevented the surveyors from reaching the upper portion with the Echaurren
glacier. Since the data set for that year was relatively small, it was omitted from this study.

The date of each survey, listed in table 3.1, was somewhere within the last one to two months of the accumulation period, near the time of peak accumulation.

Table 3.1 Time period for each snow survey.

<table>
<thead>
<tr>
<th>Survey</th>
<th>Dates of survey</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>9/24 - 9/30</td>
</tr>
<tr>
<td>1994</td>
<td>8/17 - 8/20</td>
</tr>
<tr>
<td>1995</td>
<td>exact length of period not available</td>
</tr>
</tbody>
</table>

In each survey, snow depth measurements were taken from both the lower portion and the upper portions of the basin. The presence of cliffs made it impractical to get depth data from the central part of the basin. Figures 3.2 through 3.4 show the measurement locations for each survey. Table 3.2 lists the number of depth measurements from each one.

Table 3.2 Number of depth measurements performed on each survey.

<table>
<thead>
<tr>
<th>Survey</th>
<th>Number of survey points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>80</td>
</tr>
<tr>
<td>1994</td>
<td>81</td>
</tr>
<tr>
<td>1995</td>
<td>76</td>
</tr>
</tbody>
</table>

Two snow pits were dug during each survey to obtain profiles of snow density, one
pit for the upper half of the basin and one for the lower. Density values in the pits were determined by using stainless steel density samplers and taking two to three measurements at each 0.10 m depth interval. The locations of the snow pits can be seen in figure 3.5. The snow profiles for each survey are shown in figures 3.6 to 3.8.

The 1992 snow survey also included the measurement of mean density at each depth measurement by using a Mount Rose snow sampler. The Mount Rose data give mean values of snow density down to a depth of about 2 to 3 meters. A few of the depth measurements for 1992 did not have corresponding Mount Rose data, so there was only a total of 74 points.

No satellite imagery was obtained for the time of the snow surveys, so snow covered area maps could not be constructed. Even if Landsat 5 images were available it would have been difficult to scale down from the 30 meter resolution to the 5 meter resolution. Instead, one of the surveyors on the 1994 survey took photographs of several areas around the basin. These photographs were used as a qualitative indication of which portions of the basin were snow free during peak accumulation.

A 5 meter resolution contour map of the Echaurren basin with a contour interval of 10 meters was imported into the GRASS GIS by the DGA. Prior to the onset of the study, a DEM was created using the GRASS interpolation scheme referred to in chapter 2. A shaded relief map was generated using the grass program shade.rel.sh. The presence of deep canyons on the shaded relief map indicated that a few contours were incorrectly scanned. Since there were relatively few contours with this problem, the incorrect contours were removed from the map, and the DEM was recreated. The shaded relief map in figure 1.3 was
generated using the corrected DEM.

A hand drawn soils map was supplied by R. Harrington. The map was created using a combination of areas delineated from an areal photo, and his memory of the soil type in those areas. Harrington assigned the four soil classes of glacial ice, rock, talus and scree and meadow to the map.

The quality of the data set varied from year to year. Although the same kinds of instruments were used on each survey, issues relating to the positioning of the survey locations arose. Each survey used GPS to identify the starting locations of the transects for the depth measurements. The 1992 snow survey data, however, initially showed some problems related to reported coordinates of the transects. The upper measurements appeared to extend too far west going outside of the basin. The lower measurements appeared to extend too far north with measurable snow depths on cliff faces. The upper transects were corrected by a previous researcher, Remigio Gallagara, by shifting them 100 meters east to based on comments by the surveyors. I corrected the lower basin measurements by shifting them about 50 meters south based on comments from Dr. Gallagara. Thus, the 1992 transects possess the greatest uncertainty in terms of there positions relative to basin topology.
3.4 Methods

3.4.1 Predictor Variables

The predictor variables chosen for the model are the same used by Elder [1995] except that vegetation was not used, since the Echaurren basin has little vegetation. The small amount of vegetation present is merely shrubbery and its effect should be insignificant. The remaining variables used in both Elder’s work and this study are elevation, slope, radiation and soil type. The rationale for using each of these variables can be found in Elder [1988, 1995]. Although additional parameters influence the distribution of snow, most do not lend themselves to this type of analysis. Wind patterns in a specific basin, for instance, are not easily mapped. Other parameters that can be mapped may manifest themselves within the variables listed above. Relative air temperature differences may be taken into account by splits on elevation if the lapse rate is an important factor.

In order to determine the values of the predictor variables for each learning set and to generate a distribution of SWE from the resulting trees, geographically co-registered digital maps of each variable had to be generated. The elevation values were taken from the DEM discussed in section 3.3. The other three variables of radiation, slope and soil type were not supplied as independent digital maps, but were instead generated from the existing data.

Net radiation, as calculated from the IPW routine Topquad, was used as the
independent radiation variable. Topquad computes daily integrated radiation (solar or net) using a two stream atmospheric model with a 21 point Kron quadrature from sunrise to sunset. Among the input parameters Topquad requires are optical depth, single scattering albedo, scattering asymmetry parameter, reflectance of the substrate (surface albedo), date, longitude and latitude. The three atmospheric parameters of optical depth, single scattering albedo, and scattering asymmetry parameter were determined using LOWTRAN7 (Kneizy et al, 1988). As discussed in Elder [1995], only an index of radiation is needed to distinguish between the portions of the basin receiving high amount of radiation and those receiving low amounts. Hence, an assumption of simple meteorological conditions seemed justified as input for LOWTRAN7. The conditions include midlatitude winter, a rural extinction depth of 23 km, a solar zenith angle of zero (noon), background stratospheric profile and extinction, and the absence of clouds. Figure 3.9 shows the variation of the 3 atmospheric parameters with wavelength from .3 micrometers to 3 micrometers.

In addition to the input parameters described above, Topquad requires a 6 band image consisting of elevation, slope, azimuth, skyview factor, terrain configuration factor and surface albedo. The slope and azimuth images were generated from the elevation image using the IPW program gradient. The skyview and terrain configuration factor images were generated from the IPW routine viewf which also required the DEM as input. Since surface albedo varies with wavelength according to figure 3.10, several albedo images were generated covering the range of wavelengths from 0.3 to 3 micrometers. The albedo value assigned to each image was visually estimated from the figure by averaging vertically across
the full range of grain sizes and horizontally through each wavelength interval \textbf{Topquad} was run over. Appendix B summarizes the input values for the \textbf{Topquad} runs on each date.

Since the sun’s path in the sky changes as the accumulation period progresses, daily integrated radiation was computed for the 1st and 15th of each month starting from May 1st to the date of the snow survey for that year. The images were then integrated as a step function to give the total net radiation for the basin. Mean net radiation was calculated by dividing the total net radiation by the number of days in the accumulation period. This served as the radiation image. Figure 3.11 shows the net radiation image generated for the accumulation period of 1994. A low radiation region can be seen close to the north eastern boundary of the basin.

The slope image used in the regression tree models was computed within GRASS4.1 using the DEM. The routine \texttt{r.slope.aspect} uses a 3 by 3 neighborhood around each pixel in the DEM to determine slope along four principle directions. The algorithm then determines the aspect of the pixel and assigns the slope value for that aspect. Each slope was given in degrees of inclination above the horizontal.

The soil map provided by R. Harrington was digitized by \texttt{R.digit} in grass4.1. \texttt{R.digit} is a mouse driven routine which allows the user to generate boundaries for different regions by hand and assign categories to the pixel values within the boundary. Since Harrington’s map was based partly on memory, and the digital map was hand drawn from that map, the digital map presumably allows for only a rough indication of the relationship between SWE and soil type. Figure 3.12 shows the digital soils map. Although the detail and accuracy of
the map was poor, work by Elder suggested that at least in the Emerald basin, soil type is not an important predictor variable. The map was used in this study in case a strong relationship was found between soil type and SWE.

3.4.2 Generating the Regression Trees.

The statistical software Splus (version 3.3) generated the regression trees. The functional aspects of the tree-based algorithms in Splus are described in “Statistical Models in S” [Chambers and Hastie, 1988].

Grass4.1 was used to extract the predictor variables from each location a SWE value was calculated and saved as an S source file. First, the IPW total net radiation images were converted into Grass format. I used the total radiation images instead of the mean net radiation images because Grass uses only integer values and I wanted to maintain the same resolution in radiation values that IPW generated. The total net radiation values were converted back into the mean net radiation values when the site data was saved in the S format. Next, grass site files were created whose site names were the SWE values and whose site locations were registered to the DEM. Certain points along the survey transects were labeled as zero snow depth due to being a cliff or rock exposure. These were excluded because the relative scarcity of the data points would cause them to be grouped with nonzero SWE values and artificially lower the mean SWE of the snow covered regions. Moreover, the locations of snow free points are generally accounted for by the evaluation of snow covered area. The routine s.menu was used to generate the S source file, and a simple shell
Regression trees were grown for each survey and for several methods of calculating SWE. The methods used to calculate SWE are discussed in section 3.5.1. The impurity function used to grow the trees was the deviance function defined by equation 3.1.

\[ D(u, y) = (y - u)^2 \]  \hspace{1cm} (3.1)

where \(D(u, y)\) is the deviance, sometimes referred to as the square of the residual, between an observation \(y_i\) and its modeled prediction \(u_i\). Since the prediction is the mean of the predictors in a partition, the deviance is equivalent to the sample variance of the data without the weighting of \(n-1\). The default stopping rule for Splus was used, which ceases splitting child nodes when the number of observations drops below ten. This may be a liberal stopping rule, however the small size of the input data and the possibility of pruning full size trees to find an optimal tree allowed the use of this rule.

3.4.3 Analysis of the Regression Trees

Several schemes were used to analyze the regression trees for the ability to model the dependant variable. These schemes included qualitative estimates of the plausibility of the splits, estimates of the coefficient of determination for modeled data, and cross-validation testing. The trees were not examined to see if similar SWE distributions resulted from similar amounts of snow accumulation. This issue was best examined through SWE maps.
generated by the application of the trees.

Splits considered implausible included radiation splits where the snow exposed to
greater amounts of radiation resulted in larger SWE values, and splits on soil type where
glacial ice yielded in smaller values than rock. All elevation splits were considered plausible
since the basin topography was such that several factors related to elevation could result in
either an increase or a decrease in SWE. For example, although higher elevations generally
have greater amounts of snow deposited (greater snow depth means greater SWE), the lower
elevations of the basin were much warmer resulting in greater snow densities (increased
density can lead to increased SWE).

The amount of variance in the regressor data explained by a regressor model is often
described using the coefficient of determination. This quantity is generally calculated for
simple linear regression and multiple linear regression, however, Elder [1995] made use of
it for regression trees. Equation 3.2 shows the coefficient of determination as

\[
R^2 = \frac{\sum_{i=1}^{n} (u_i - \bar{y})^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \tag{3.2}
\]

where \(R^2\) is the coefficient of determination, \(u_i\) is the modelled SWE for observation \(i\) in the
learning set, \(\bar{y}\) is the mean SWE of all the SWE observations, \(y_i\) is the actual SWE
associated with observation \(i\), and \(n\) is the total number of observations. Equation 3.2 is
merely the ratio of the variance of the modelled values with respect to the mean of all
observations to the variance of the observed values.

An equivalent way to express the coefficient of variation can be found in appendix C as equation C.1. Included in the appendix is a proof that equation 3.2 and equation C.1 give identical results when applied to the values used to generate a single regression tree. The relationship between equation 3.2 and C.1 is true for general linear models.

The application of the coefficient of determination to the SWE regression trees was interpreted as how well the trees described the variation of SWE in the learning set. One pitfall in using the coefficient of determination with regression trees is that as the number of terminal nodes approaches the number of observations, the coefficient of determination will approach one. This results in an over estimate in the confidence of the model. The behavior of the coefficient as a function of tree size for each learning set was examined within Splus 3.3 for both adequacy of the tree and for the possibility of over determination. Another issue with using the coefficient of determination as a measure of model accuracy is that it is based on the same data used to generate the model. Thus it affords no opportunity to validate the model on other data sets.

Cross validation testing is a process that gives some indication of how trees perform on alternate data sets. Cross-validation is a process whereby the total learning set is split up into two subsets, one subset is used to grow a tree and the other is sent through the tree to give an estimate of the its regression error. Breiman et al [1984] details the process of cross-validation. In this study, the Splus routine cv.tree() was used to perform the cross-validation. The routine divides the data into several sets based on input from the user. It then
accumulates the deviance values for each set into a total deviance value. Since a sparse set of data points was used to generate the trees, the cross-validation was performed by withholding only one piece of data at a time. The total deviance calculated for a tree of a given size was then the total deviance of withholding each observation once. In addition to calculating cross validated deviance for the maximal trees in the learning subsets, Splus also calculates the deviances for tree sizes smaller than the maximal tree. The algorithm recursively prunes the maximal tree based on the reduction in size that increases the deviance the least. The results are the accumulated deviances for the optimal trees from two nodes to the maximal size. Plots of accumulated deviance versus tree size were generated for each survey and SWE method.

The three techniques were then applied to estimate an optimal tree size. The optimal tree size depends in part on the application for which the tree will be used. In snow melt modeling, the use of too many SWE classes may result in an excessive computation time. Hence, Elder used three criteria for deciding on tree size optimality. The first was to limit tree size to where an increase in the coefficient of determination was 0.005 with the addition of another node. The second was to remove splits that were not physically reasonable. Third, the choice of optimal trees was based on cross-validation, the change in the coefficient of determination and physical plausibility of the splits. The optimal tree based on cross validation results, is the tree size that generated the smallest accumulated deviance. In this study, the coefficient of determination and cross validated results were both examined for optimality and then, if needed, cut back further based on the plausibility of the splits.
3.4.4 Generating SWE Maps Based on the Regression Trees.

The SWE maps were generated using a shell script I wrote outside of Splus. Since the DEM, slope map, mean radiation map and soils map each consist of 183,122 pixels, it was not possible to read the values into a single vector within Splus due to size constraints. It would have been possible to divide the data into multiple groups and read them in separately, however programming a shell script was a viable alternative. The shell script makes use of IPW commands to extract the predictor values from the maps of the independent variables and rebuild the SWE map. It was therefore necessary to convert the slope and soils map from GRASS to IPW. Copying the output to a file was a necessary requirement for using this script. The prediction algorithm is contained within the script using a NAWK program which builds an array representation of the tree and reads in the predictor values for each pixel one by one.

Since no satellite imagery or air photos were available for identifying portions of the basin that were snow free during the time of the snow survey’s, a threshold value for slope was used. By running the algorithm for several threshold values and comparing the snow coverage with ground photos taken by Harrington in 1994, a threshold of 55 degrees was determined to be adequate. All pixels with slopes greater than or equal to 55 degrees were assigned zero SWE by the program. This technique was applied by Elder [1991] in the Emerald Lake basin.
3.5 Results

3.5.1 Point SWE Values

Three methods of calculating SWE at each depth measurement location were examined. The simplest scheme, applied to the 1992 data only, was to take the Mount Rose density data and multiply them by depth measurements at coincident locations. The problem with this method is that Mount Rose data often consist of only the first 2 to 3 meters of snowpack. Since the lower portions of the snow pack must support the weight of the snow above it, the density generally increases with depth. Occasionally, low density layers may be present, but generally ignoring the bottom portion of the snow pack may make the estimate of mean snow density at a location slightly low.

The other two methods combined snow density values with depth measurements using the density profiles from the snow pits. In one scheme the mean density of each profile from the surface to the bottom of the snow pack was determined. In the pits that were not deep enough to reach the bottom of the snow pack, the profile was extended using a fitting function of the form \( A - Be^{(c \cdot x)} \) where \( x \) represents the distance from the top of the snow pack. These profiles neglect the possibility of the presence of low density layers. This may not be justified, but if the low density layers are thin relative to the entire thickness of the snow pack, errors ignoring them should be small. In addition, all of the profiles, except for the lower pit in 1994, reached near the bottom of the snowpack. The extrapolation of density values for 1994 is shown in figure 3.13. Since this pit required the most extrapolation it is
also likely to be the least accurate. For each survey date, the mean density of the upper basin pit was applied to all of the depth measurements in the upper portion of the basin, while the mean density for the lower basin pit was applied to the depth measurements in the lower basin. The second scheme is similar to the first except that the snow density was averaged only along the length of the profile equal to the depth measurement. In cases where the depth measurement was deeper than the bottom of the snow pit, the average density of the bottom few points of the profile was used. This method neglects the possibility of low density layers. Averaging the snow density in this manner does allow for the fact that the deeper packs will generally have a higher percentage of dense snow than will the shallower packs.

The validity of using a single snow pit for each portion of the basin rests on two basic assumptions. The first is that the snow density in each portion of the basin does not indicate any obvious trends with respect to the predictor variables. Figure 3.14 shows the distribution of the 1992 Mount Rose data with respect to each of the four predictor variables. The first thing to note is that the snow density values show a marked decrease from the lower to upper portion of the basin. This is expected since the lower portion of the basin is warmer than the upper portion, so the snow pack in the lower portion will ripen earlier in the season. This elevational dependence is already taken into account by using the lower pit for the lower basin depth measurements and the upper pit for the upper basin measurements. There does not appear to be much dependence on slope, or surprisingly, on radiation. The apparent increase in density with talus versus rock or glacial ice is primarily because most of the lower
basin data were found on this soil type. Figure 3.15 shows the spatial distribution of the snow density values. Here too there is no indication of any major trends in the data other than the aforementioned division between the upper and lower portions of the basin, although there may be some slight dependance on radiation. In addition, the coarse sampling throughout Echaurren may be hiding smaller scale dependancies. The second assumption is that snow densities in each portion of the basin are distributed normally with a mean equal to the mean density along the profile. The histograms in figure 3.16 show that the Mount Rose data for 1992 are distributed approximately normally. There does appear to be some slight skew in the upper basin data, however. Ideally, a larger sample size should be used to prove the population is normal, but that was not available. In addition to being distributed normally, the mean snow density in each half basin, 500 kg / m³ for the lower and 370 kg / m³ for the upper, is nearly equal to the mean density for the corresponding profiles shown in figure 3.6. Hence, for 1992, the snow pits appear to capture the mean density of each half basin. Although the two criteria for using the snow pits is shown to hold for 1992, no data were available to show this was the case for the other surveys. Thus, there is no guarantee that the snow pits for the other two surveys capture the mean snow density for each portion of the basin.

3.5.2 Analysis of the Regression Trees

Physical plausibility of the regression tree splits

Figures 3.17 through 3.23 are the regression trees for all seven data sets. The first
obvious characteristic is the similar structure of each tree generated for a single survey date, with the exception of the 1995 trees. The 1992 tree generated using the Mount Rose density data (figure 3.17) even uses the same variables for the first three splits as the trees based on the snow pit density values. Figure 3.17 does differ from figures 3.18 and 3.19 in the lower portions of the tree, however. A second aspect is that the trees, except for figure 3.23, have an early split on elevation that divides the data into an upper and lower basin. The exact position of the division tends to vary however as sometime one or two of the lower data points gets grouped with the upper data. Unlike the results from Elder [1995], elevation appears to be the most crucial variable for splitting in this basin. Elevation may have ranked higher in this study due to the basins hypsometry and the data collection regime.

Although less important than elevation, the radiation variable appears quite frequently in the trees grown. In the 1992 trees, splits on radiation generally proceeded as expected with increases in radiation leading to decreases in SWE. There is a notable exception in figure 3.17, near the center of the page, and figures 3.18 and 3.19 earlier on. In each of these cases the split on radiation results in an increase in SWE which does not seem physically reasonable. In the 1994 trees the same behavior can be seen lower in the tree. In the 1995 trees, splits on radiation generally seem plausible except for one split near the bottom of each tree. These counterintuitive splits on radiation mean that either the learning data for that particular category is not statistically representative of the effects of radiation, that the radiation split is masking a relationship with some other variable not accounted for in growing the regression trees, or that the data registration is not correct.
Splits on slope occurred less frequently than either radiation or elevation. Moreover, except for figure 3.23, splits on slope appear much further down than splits on radiation. These facts suggest that splits on slope are less crucial than radiation. The splits that do occur sometimes have a negative relationship with SWE, and sometimes have a positive relationship with SWE. The splits on slope for the 1992 regression trees all predicted an increase in SWE with slope. The 1994 trees showed no relationship whatsoever with respect to slope, and the 1995 trees showed both relationships. After examining the positions of the survey points that generated the positive relationship with SWE, it was determined that most of the points were all located near areas just below cliff faces. Thus, a likely explanation is that the positive relationship of SWE with slope on those splits is due to the presence of snow piling up from sloughing from the cliff above. The sloughing would result in both an increased density and an increase snow depth. The splits resulting in the negative relationship with respect to snow may be due to avalanche events shifting snow from the steeper slopes to shallower slopes.

Splits on the soil predictor variable were sparse. The regression trees based on the 1992 survey show a relationship between soil type and SWE only for figures 3.18 and 3.19. They show an increase in SWE with respect to talus as opposed to the other soils types. On the other hand, the opposite relationship holds for the 1994 trees. The source of this behavior is uncertain, although the soil split for the 1992 trees occurs much further down. This suggests that the relationship between SWE and talus is more likely to be negative. A split on soil occurs in figure 3.23 as well, but this time it separates glacial ice from other soil
types. A factor that may be tied to the anomalous relationship between soil and SWE could be the poor quality of the soil map. As mentioned earlier, previous studies in other basins suggest that soil type is not as critical as other variables during the accumulation season. If the relationship between SWE and soil described by this tree are valid, soil does appear to be a less important variable. The importance of the meadow in Echaurren could not be concluded by this study, since the positions of the survey points usually resulted in only one or two depth measurements in the meadow. No splits would probably occur on meadow even if the meadow has a strong effect on SWE.

SWE variation explained by the model

Table 3.3 gives the coefficient of determination (R²) for each of the full size trees generated. The 1992 and 1995 regression trees explain 15-20 % more variation in the observed SWE values than the 1994 regression trees. This is particularly surprising since the positions of the 1992 survey points were known less precisely than the positions of the 1994 survey points. This suggests that, at least in this study, the accuracy of the survey locations is not the major contributing factor for R² differences in different surveys. The differences in R² values for the trees grown for a single survey was quite small, ranging from a 2 to 8 % change between the methods. There was no consistency in terms of which of the two snow pit techniques resulted in a better ability to describe the SWE variation. The 1992 snow survey data based on the Mount Rose measurements resulted in the largest R² value, but the lack of 1994 and 1995 Mount Rose data makes it impossible to conclude anything
about the relative ability of trees grown on Mount Rose to explain SWE variation.

Table 3.3 Coefficient of determination for the full size trees

<table>
<thead>
<tr>
<th>Regression tree</th>
<th>Coefficient of determination (R²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992 tree using Mount rose density data</td>
<td>0.63</td>
</tr>
<tr>
<td>1992 tree using first method with snow pits</td>
<td>0.60</td>
</tr>
<tr>
<td>1992 tree using second method with snow pits</td>
<td>0.56</td>
</tr>
<tr>
<td>1994 tree using first method with snow pits</td>
<td>0.44</td>
</tr>
<tr>
<td>1994 tree using second method with snow pits</td>
<td>0.42</td>
</tr>
<tr>
<td>1995 tree using first method with snow pits</td>
<td>0.56</td>
</tr>
<tr>
<td>1995 tree using second method with snow pits</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Figures 3.24 through 3.30 show the variation of $R^2$ with respect to tree size. In every case there is a large initial increase in $R^2$ and what looks like an approach towards an asymptote as tree size reached its largest value. This behavior is consistent with the plots of $R^2$ versus tree size in Elder's study. Since the change in $R^2$ with tree size is quite small near the full size trees and the value of the $R^2$ is still much less than one, the asymptotic behavior in these graphs is probably not an indication of an extreme over fit of the data. If the trees where grown to the point where every observation was a SWE class, a second rapid increase in $R^2$ would be observed as the number of SWE classes became too large. The asymptotic approach towards a value of one does not say however, that the later splits are valid. It is still likely that some of the splits are spurious. Nonetheless, since the change in $R^2$ becomes quite
small in this range, it does suggest that this is the most variation the model can explain.

**Cross validation testing**

Figures 3.31 through 3.37 show the accumulated deviance, as defined in section 3.4.3, versus tree size where each observation was removed from the learning set once. The results are not particularly encouraging. Although the variation in deviance with tree size shows a characteristic minimum, as is expected from this type of analysis, the magnitude of the total deviance remains quite large. Comparing the cross validated deviance values given in figures 3.31 through 3.37 to the total deviance of the unmodeled data (Table 3.4) suggests a poor ability of the trees to model new data. Since Elder [1995] did not present his cross validated results, it was not possible to determine how the quality of his regression trees compared to mine with respect to cross validation. Although on one sample plot included in his thesis, the minimum deviance achieved was only about 10% lower than the deviance of the unmodeled data. Large cross validation deviances may have important implications, but do not necessarily imply a poor ability to model basin wide SWE. This is discussed in more detail with respect to optimal trees.
Survey date and method of SWE calc. | Total unmodelled SWE deviance
--- | ---
1992, Mount Rose sampler | 19.60
1992, method 1 using snow pit data | 23.97
1992, method 2 using snow pit data | 27.49
1994, method 1 using snow pit data | 57.92
1994, method 2 using snow pit data | 61.69
1995, method 1 using snow pit data | 19.52
1995, method 2 using snow pit data | 27.24

**Optimal trees**

Despite the fact that three criteria were used to determine the optimal trees, only one tree was chosen for each method and for each survey. The 1992 cross-validation results (figures 3.31-3.37) suggested an optimal tree size of three of four nodes. Pruning the trees back to this size, however, over simplifies the distribution of SWE within the basin. Thus, cross validation was not used in choosing the optimal trees for the 1992 data. An optimal tree size of eleven nodes was chosen for the tree grown on the Mount Rose data. This choice was based on the fact that an additional node would increase the coefficient of determination by only 0.006 (figure 3.24) and it removed the implausible split on radiation found near the bottom of the tree (figure 3.17). The optimal trees based on the 1992 snow pit data were chosen at their full size since the increase in $R^2$ never quite makes it to under 0.005 and since
removing the implausible split on radiation would have reduced the tree size down the aforementioned 3 or 4 nodes. The problem with the radiation split had to be lived with. In the 1994 trees, cross validation results suggested a slightly larger tree size for optimality. The cross validation results from the first method of incorporating the density data into the depth measurements suggest an optimal tree size of 6 nodes (figure 3.20), while the cross validation results for the second method suggest an optimal size of eight nodes (figure 3.21). Although the diagrams of $R^2$ versus tree size suggest a larger optimal tree, that would mean including the implausible splits on radiation. Although the implausible split on radiation had to be lived with in the 1992 maps, I saw no reason to keep them in the 1994 maps. The optimal trees for 1994 did result in a substantial reduction in $R^2$. Both the cross validation results and the behavior of the coefficient of determination with tree size, suggested optimal trees around ten nodes for the 1995 data. These optimal trees did not have any obvious implausible splits.

The ability of the optimal trees in explaining the SWE variation of the learning data is listed in table 3.5. As was the case for the full size trees the statistic used was the coefficient of determination.
Table 3.5 Regression statistics of optimal trees.

<table>
<thead>
<tr>
<th>Survey date and method</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992, Mount Rose</td>
<td>0.62</td>
</tr>
<tr>
<td>1992, method 1</td>
<td>0.60</td>
</tr>
<tr>
<td>1992, method 2</td>
<td>0.58</td>
</tr>
<tr>
<td>1994, method 1</td>
<td>0.30</td>
</tr>
<tr>
<td>1994, method 2</td>
<td>0.38</td>
</tr>
<tr>
<td>1995, method 1</td>
<td>0.54</td>
</tr>
<tr>
<td>1995, method 2</td>
<td>0.61</td>
</tr>
</tbody>
</table>

The $R^2$ values suggest that the regression trees generate a fairly good fit to the data. The coefficient of determination for the optimal trees for 1992 and 1995 are just as good or better than the full size trees. Although the coefficient of determination for the optimal trees of 1994 is not bad, the values are substantially lower than the full size tree. This is not surprising since the optimal tree size has relatively few nodes.

As mentioned in a previous section, since the data used to generate the tree are also being used to evaluate the same tree, the coefficient of determination may result in over confidence in the model. This was the motivation for generating the cross validated estimates. Thus examination of some statistic based on the cross validation results would seem useful.

One option would be to generate statistics similar to the coefficient of determination
such as the expressions in equation 3.2 and C.1, but perform the sums over data left out of
the model. Although this can be easily done, care must be taken in the interpretation of the
results. First, the two ways of expressing $R^2$, although equivalent when modeled data are
resubstituted, are not necessarily equivalent when the sums are performed over observations
not used to grow the trees. This statement is immediately apparent by plugging in the values
of the modeled and unmodeled deviances of the 1994 cross validation results into the
numerator and denominator of the second term of (C.1). Since the modeled deviance is
larger than the unmodeled deviance the expression gives a negative value. Equation 3.2 on
the other hand can never be negative. This is a clear indication that the assumptions
discussed in appendix C do not hold. Thus the interpretation of the numerical values from
these expression must be different. The second concern is that the coefficient of
determination, as generally defined, is calculated over a single model (in our case a single
tree) while the sums performed for cross validation would be performed over an ensemble
of models (a separate tree for each time a data point is left out of the learning set). Now,
ideally, leaving a single point out of the learning set should not greatly alter the model;
However since the tree will be altered we must now interpret the numerical results with this
in mind. Due to the complexity of reinterpreting the precise meaning of the values, I
restricted myself to examining the ratio of the modeled deviance to the unmodeled deviance
described by equation 3.3 on the next page,
where $y_i$ is the SWE value of the observation left out of the learning set for cross validation or simply a given SWE value for a data point in the learning set for resubstitution, $u_i$ is the predicted value of the predictor associated with $y_i$, $\bar{y}$ is the mean SWE of all of the observations $y_i$, and $N$ is the total number of observations. In the case of cross validation it is clear that there would be $N$ different trees used to calculate the numerator of equation 3.3 since for each $y_i$ left out of the learning set a slightly different tree would result.

Calculation of the values for equation 3.3 is straightforward. For the resubstituted estimates, the values can simply be obtained by subtracting the coefficient of determination from one. For the cross validated estimates, the terms had to be calculated from those used to generate figures 3.31 to 3.37. Table 3.6 below shows the corresponding ratios for each survey date and method. The values are interpreted to be the percentage of the unmodeled deviance, or equivalently, the ratio of the mean square error using the model versus using a basin mean. The smaller the value, the smaller the average difference between the observed SWE and modeled SWE.
Table 3.6 Ratio of modeled to unmodeled deviance (percentage decrease in deviance).

<table>
<thead>
<tr>
<th>Survey date and method</th>
<th>cross-validated ratio</th>
<th>resubstituted ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992, Mount Rose</td>
<td>0.93</td>
<td>0.38</td>
</tr>
<tr>
<td>1992, Method 1</td>
<td>0.92</td>
<td>0.40</td>
</tr>
<tr>
<td>1992, method 2</td>
<td>0.81</td>
<td>0.42</td>
</tr>
<tr>
<td>1994, method 1</td>
<td>1.10</td>
<td>0.70</td>
</tr>
<tr>
<td>1994, method 2</td>
<td>1.07</td>
<td>0.62</td>
</tr>
<tr>
<td>1995, method 1</td>
<td>0.94</td>
<td>0.46</td>
</tr>
<tr>
<td>1995, method 2</td>
<td>0.81</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Since the coefficient of determination was shown to be fairly large for the modeled data, it's unsurprising that the resubstituted ratio's above are quite small. The 1992 deviances are close to 40% of what they were before the model is applied. This is a substantial reduction in model variance. The cross validated results, however, show little reduction in deviance. The 1994 data show the deviance of observations when they are not included in the model is actually larger than the unmodeled data. At first glance, this might suggest that the model does a poor job. After all, a ratio larger than one indicates that, on average, the data that were not included in the model would have resulted in a smaller error between the actual value and the modeled value if a simple mean was used instead of the regression tree. The fact that the cross validated deviances are so large does not necessarily mean that the model is a bad model to apply for the entire basin. A brief excursion into
examining the specific trees that result from leaving out a given data point uncovered sensitivity issues with the resulting trees. Some data points would result in nearly an identical tree with just a change in SWE at the terminal node which originally included the data point. The removal of other data points resulted in a drastically different tree structure for the lower portions of the tree. Although Elder examined sensitivity issues related to the error in the positions of the independent variables on a map, he did not look into the sensitivity of the trees to using a smaller learning set. Furthermore, I could not find cross validated deviance values for all of his trees, so it was not possible to determine how much better, if at all, his trees did on cross validation. There are two likely sources of the problem with large cross validation errors. One is that the decrease in deviance using splits on two or more different variables at a given node may be so close that the removal of a single data point can cause the split to be on another variable. The second is related to the fact that splits on a continuous variable, such as elevation, occur exactly halfway between two observations. The removal of a point that defines the specific value of a split could cause the division to occur at a different value on the same variable. Thus, when the removed observation is run back through the tree, it is thrown in a bin of data it was not in before. This could give a large deviance. The ultimate concern regarding SWE distributions is whether or not the large deviances resulting from leaving out observations from the learning set are an indication of significant changes in the SWE distribution. If the large deviances merely reflect minor shifts in the boundaries between SWE classes then the effect should be small. However, it is also conceivable that radically different distributions may result. Ultimately, the only way
to determine the magnitude of this effect would be to generate SWE maps leaving out each data point once. Since this would require the generation of 70 to 80 SWE maps, time constraints made it infeasible to examine this issue in detail. Nonetheless, I did look at a few of the maps resulting from regression trees that appeared to differ from trees using the full set of data. Preliminary examination of these maps indicate that many structures appeared consistently in the maps. This suggests that the extremely poor cross validated results are not a big concern. Further investigation of this issue should be performed in order to properly assess the validity of the statement above and to identify stable an unstable regions of the SWE map.

Figures 3.38 through 3.44 show the geographic distribution of errors between the modeled observations and modelled values. As can be seen in the figures, most of the models show no geographic bias in the errors. There is a tendency for the modelled values to underestimate the SWE near the cliffs along the northwestern basin boundary in all figures except for 3.41 and 3.42. Over an entire class, however, there is no bias, but the portion of the class near the cliffs does have one.

3.5.3 Analysis of the SWE Distribution Maps.

The SWE maps were analyzed for their consistency in capturing the mean SWE of the basin, the ability for the different SWE techniques to result in similar SWE distributions, and the similarities or differences in distribution from year to year.

The total SWE was examined for consistency to make sure the estimates from the
regression models were comparable to the mean of the observations. This allowed for an indication of whether or not the observation data were representative of the SWE classes in the basin. Table 3.7 shows the mean SWE values and total SWE volumes for the raw data, the maps generated from the full regression trees, and the maps generated from the optimal regression trees. The total SWE volume was calculated by taking the mean SWE and multiplying it by the total SCA of the basin. The total SCA for the basin was calculated by excluding the cliff faces estimated by the slope threshold. The SCA calculated in this manner for Echaurren is 4.186 X 10^6 square meters.

Table 3.7 Mean SWE (meters) and total SWE (meters^3).

<table>
<thead>
<tr>
<th>Survey Date</th>
<th>Full, optimal, or raw data</th>
<th>Mean SWE</th>
<th>Mean SWE</th>
<th>Mean SWE</th>
<th>Total SWE</th>
<th>Total SWE</th>
<th>Total SWE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mt. Rose</td>
<td>pit method 1</td>
<td>pit method 2</td>
<td>Mt. Rose</td>
<td>Pit method 1</td>
<td>Pit method 2</td>
<td></td>
</tr>
<tr>
<td>1992</td>
<td>Raw</td>
<td>1.82</td>
<td>1.87</td>
<td>1.89</td>
<td>7.62 x10^6</td>
<td>7.83 x10^6</td>
<td>7.91 x10^6</td>
</tr>
<tr>
<td>1992</td>
<td>Full</td>
<td>1.85</td>
<td>1.84</td>
<td>1.85</td>
<td>7.74 x10^6</td>
<td>7.70 x10^6</td>
<td>7.74 x10^6</td>
</tr>
<tr>
<td>1992</td>
<td>optimal</td>
<td>1.84</td>
<td></td>
<td>7.70 x10^6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>Raw</td>
<td>2.37</td>
<td>2.18</td>
<td></td>
<td>9.92 x10^6</td>
<td>9.13 x10^6</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>Full</td>
<td>2.25</td>
<td>2.49</td>
<td></td>
<td>9.42 x10^6</td>
<td>1.04 x10^7</td>
<td></td>
</tr>
<tr>
<td>1994</td>
<td>optimal</td>
<td>2.21</td>
<td>2.45</td>
<td></td>
<td>9.25 x10^6</td>
<td>1.03 x10^7</td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>Raw</td>
<td>1.63</td>
<td>1.74</td>
<td></td>
<td>6.82 x10^6</td>
<td>7.28 x10^6</td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>Full</td>
<td>1.58</td>
<td>1.75</td>
<td></td>
<td>6.61 x10^6</td>
<td>7.32 x10^6</td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>optimal</td>
<td>1.61</td>
<td>1.74</td>
<td></td>
<td>6.74 x10^6</td>
<td>7.28 x10^6</td>
<td></td>
</tr>
</tbody>
</table>

The 1992 values for mean and total SWE all come within 3% between the raw data,
the optimal tree and the full size tree for each method of incorporating snow density. The mean and total SWE values between the different methods of the 1992 data vary more greatly for the raw data, but not as great for the resulting SWE maps. The 1994 data on the other hand, show a much wider discrepancy between the raw data and the generated maps with differences between 5 to 12 percent. The maps generated from the optimal and full size trees are more closely related. There is also a fairly large difference between the two methods of incorporating the snow density data. The second method shows a much smaller mean SWE than the first based on the raw data. The primary reason for this is that most of the snow depth measurements were less than the length of the extrapolated snow profile, hence the denser snow contributes less. The modelled SWE however, show the opposite trend. The maps from the second method suggest a larger mean SWE. The reason for this will become clear by examining the SWE map. The 1995 values, like the 1992 data, all are within 3% of each other for a given method of determining SWE. Indeed the results from the map using the optimal tree indicate that the mean of the observation data for method 2 is identical at a precision level of the data. Furthermore, the mean and total SWE for method 2 is consistently higher than the mean and total SWE for method 1.

Thus, the results suggest that the regression tree classes assigned within the basin are representative of the observation data from 1992 and 1995. The mean and total SWE values from the 1994 observation data show a larger discrepancy with the basin wide results. This gives less confidence that the observation data used to generate the regression classes are representative of them over the basin. In all cases, however, the maps generated from the
optimal trees matched closer to the mean values generated from the observation data alone.

Since there are differences between the optimal trees and the full size trees, the maps were examined for the total amount of displaced SWE between them. As Elder states, “If 20% of the total SWE volume is moved from one portion of the basin to another the resulting hydrograph may be substantially different”. The displaced SWE volume, as calculated in the study by Elder, is given by

\[
SWE_{\text{disp}} = \sum_{i=0}^{M} \sum_{j=0}^{N} \left( (SWE_{Xij} - SWE_{Yij}) \right)^5
\]

where \(SWE_{Xij}\) is the SWE value from the optimal regression map at pixel coordinates \(i\) and \(j\), \(SWE_{Yij}\) is the SWE value from full size regression map \(Y\) at \(i\) and \(j\), \(M\) is the number of \(i\) coordinates, and \(N\) is the number of \(j\) coordinates. The displaced SWE volume represents the total SWE volume shifted between the maps. The displaced SWE volume thus quantifies the difference in SWE distributions. Table 3.8 shows the total and percentage displaced SWE volumes between the full size regression tree and optimal tree maps.
Table 3.8  Displaced SWE between optimal tree and full size tree.

<table>
<thead>
<tr>
<th>Regression Tree</th>
<th>Displaced SWE volume</th>
<th>% of optimal displaced</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992, Mount Rose data</td>
<td>$1.67 \times 10^5$</td>
<td>2</td>
</tr>
<tr>
<td>1992, method 1 snow pits</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>1992, method 2 snow pits</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>1994, method 1 snow pits</td>
<td>$7.12 \times 10^5$</td>
<td>10</td>
</tr>
<tr>
<td>1994, method 2 snow pits</td>
<td>$3.77 \times 10^5$</td>
<td>4</td>
</tr>
<tr>
<td>1995, method 1 snow pits</td>
<td>$2.51 \times 10^5$</td>
<td>4</td>
</tr>
<tr>
<td>1995, method 2 snow pits</td>
<td>$4.19 \times 10^4$</td>
<td>1</td>
</tr>
</tbody>
</table>

In general, the SWE maps generated from the full size and optimal trees show very little displaced volume between them. The optimal tree for 1994 using method one of incorporating the snow density shows the largest, with a displaced volume 10% of the total optimal SWE volume. All of the displaced volumes fall well below the 20% mark, but the ultimate test of the importance of the displaced SWE volume would come through a sensitivity analysis of the hydrographs from a snow melt model. Nonetheless the small amounts of displaced SWE suggest that the difference in the hydrographs resulting from the maps for the optimal and full size trees ought to be small.

Although the mean SWE gives an indication of the zeroth moment of the regression, the point of generating a regression tree is to determine an adequate distribution of SWE within the basin, and discover important topographic features controlling SWE distribution. A qualitative examination of the maps generated from the regression trees gave an indication
of important topographic features, as well as similarities and differences that result from using difference schemes of incorporating SWE.

Figures 3.45 through 3.51 are the SWE maps generated from the optimal trees. Yellows and oranges corresponded to the lowest SWE values, while browns, greys and blues corresponded to the largest SWE values. Several structures are immediately apparent in all of the 1992 maps. First, the largest SWE values occur in an elevation band between 3082 and 3174 meters. Upon examining the locations of the observation data at this elevation range, it was discovered that almost all of them lie within 75-100 meters of a cliff face. The proximity of the cliff faces suggest that avalanching may have been an important factor in depositing snow near those observation points. Another possibility is wind pushing snow towards the cliff faces. The next largest SWE values in each of the 1992 distributions, are located below the cliffs along the north western boundary of the basin. This suggests, as was the case for the cliffs in the lower portion of the basin, that avalanching played an important role here. In the lower portion of the basin, there is a clear division between an area consisting of the lowest SWE and an area consisting of higher SWE. This corresponds to differences in net radiation as can be seen in the regression trees of figures 3.45 to 3.47 and the radiation image of figure 3.11. Other features that the SWE maps have in common include the moderate SWE values in the southwestern portion of the basin.

Although there are many structural similarities in the 1992 SWE maps, there are also some differences. The cliff areas along the north eastern boundary of the basin suggest a large SWE compared to the south western boundary in figure 3.45. In figures 3.46 and 3.47
the SWE values along the north eastern boundary are slightly smaller than the south western boundary. The source of this problem is the counterintuitive split on radiation. In addition, figure 3.45 shows a portion in the middle of the basin where the SWE alternates between 1.97 meters and 1.66 meters. This result is due to a split on slope where the observation data consisted of some points near cliffs. The slope threshold is so low, however, that rolling hills are classified as having large SWE values in addition to those near cliffs. In figures 3.46 and 3.47, the split on slope is at a much higher threshold.

The 1994 SWE maps have a radical difference between them. In the method used to generate the first map, the central portion of the basin is classified with a relatively small SWE value for that year (figure 3.48). Figure 3.49 on the other hand classifies the same area with a large density relative to the other portions of the basin. This behavior is due to measurements from the upper basin being placed into the same class as the heavy measurements from the lower basin. The difference between the groupings extends from the fact that deeper snow in method 2 is emphasized more by the method used to generate the mean density. Since the snow pits in this year required the most extrapolation of snow density values beyond the bottom of the pit, it is conceivable that the densities were over estimated.

Despite the drastic difference, there are similarities between the two maps. Each map shows a band of lower SWE between the elevations of 2995 and 3047 meters. On either side of the band are the heaviest portions of the snow pack. Although, the heavy snow above the low SWE band is grouped with some observations from the upper portion of the basin, it is
conceivable that it is due to minor avalanching from the cliffs above as was the case in the 1992 maps. If the distribution can be believed, the heavy snow at the outlet of the basin may reflect a large avalanche event that shifted a large portion of the snow to the lowest points in the basin. Another similarity between the maps is the distinction in the upper portions between the talus and scree soil types and all other soil types. This may be a valid distinction due to the fact that most of the observations in the upper basin were above either talus or glacial ice. Glacial ice will tend to keep the snow above it cold, so that less water will be lost during warm days in the accumulation season. Due to the poor results from cross validation and from the coefficient of determination, the confidence of the resulting maps for the 1994 data is low.

The 1995 SWE maps are shown in figures 3.50 and 3.51. As was the case for the 1992 maps, there are divisions based on radiation in the lower basin. This time the figures suggest three categories of SWE based on the level of radiation. In the upper basin both maps suggest that the highest elevations have the largest amount of snow. Also both maps suggest the moderate elevations consist of light to moderate SWE in the high radiation portions of the basin, and heavier SWE in the shaded portions of the basin.

There are some obvious differences between the maps. First, the division between the lower basin and middle portions of the basin occur at a lower elevation in figure 3.50 than figure 3.51. This is due to the fact that all of the lower basin observations were classified within the three radiation classes in figure 3.51, while two of the observations were not classified as part or the lower basin data in figure 3.50. Another major difference between
the SWE classifications in the basin, is the extent of the coverage of the heaviest SWE class in the basin. This result extends from the fact that the heavy class was a result from a split on radiation in figure 3.50 and a split on slope in figure 3.51. Many of the observations that fell in the heavy category based on slope where near cliff faces. Some of the values that fell into the largest SWE category based on radiation where near cliff faces but not all. Thus, there is no consensus between the two methods as to the controlling factor for the heaviest SWE class.

Since there was no observation data taken from the central portion of the basin and the steep portions along the southwest and northeast boundaries, these sections of the SWE distribution maps are the most unreliable. First, there is little guarantee that the statistical relationships derived from the predictor variables in the two portions of the basin apply to these regions. Second, the lack of observation points in the central region causes the classification of that entire portion of the map to be sensitive to the values of the observations bordering that region. This can clearly be seen by the large differences in the classification between the 1994 maps using the two methods of incorporating snow density.

**Comparison of SWE distribution across different accumulation seasons**

In addition to examining the similarities and differences in the distributions generated from the different schemes of incorporating snow density, the similarities and differences between the different years were examined. A qualitative examination of snow distribution patterns was performed to determine if similar structures arose from year to year for a given
method of incorporating snow density. Any similarities that were found were subject to hypothesis testing to assess if these structures were indeed similar. In order to perform the hypothesis testing it was necessary to account for the difference in total snow accumulation between the different years. This was accounted for by dividing every SWE value by the mean SWE of its corresponding SWE map. I refer to the resulting map as a normalized SWE map. Any similar structures could than be compared by using Student’s t-Test for the hypothesis that two means of a population from the normalized map are identical. The normalized mean are referred to in this study as relative SWE values. A positive result from the hypothesis test indicates that the structures are similar. A negative result indicates they are not. By using Student’s t-test, I am assuming that a population from within a given SWE class from the map is distributed normally. Since the classified regions tend to be large, the assumption seems justified so long as the model is valid. Even if the data were distributed uniformly rather than normally, this would merely increase the likelihood of a positive result. Hence, a positive result from Student’s t-test indicates that the means of two overlapping classes are the same at some given significance level even if distributed normally. Since the true variance of the population within each class is unknown, Cochran’s approximation to Behrens-Fisher Student’s T-test was employed (State Water Resources Control Board, 1993). The test statistic was given as
(3.5) 

\[ t^* = \frac{|SWE_{Xnorm} - SWE_{Ynorm}|}{\sqrt{\frac{S_x^2}{N} + \frac{S_y^2}{M} \cdot S^2_{\text{year}}}} \]

Where \( SWE_{Xnorm} \) and \( SWE_{Ynorm} \) are the normalized SWE values from class \( X \) in the map from one year and class \( Y \) in the map from another year, \( S_x^2 \) and \( S_y^2 \) are the sample variances from the unnormalized SWE values in class \( X \) and class \( Y \), \( N \) and \( M \) are the number of observations that generated class \( X \) and \( Y \) respectively, and \( S^2_{\text{Xyear}} \) and \( S^2_{\text{Yyear}} \) are the mean SWE values of the basin from the map containing class \( X \) and \( Y \). The test statistic given above was compared to the weighted t statistic given as

\[ t_c = \frac{W_x * t_N + W_y * t_M}{W_x + W_y} \]  \hspace{1cm} (3.6)

where \( W_x \) and \( W_y \) are the sample variances of the observations in class \( X \) and \( Y \) from their respective years divided by the number of observation that generated each class, and \( t_N \) and \( t_M \) are the values from a two sided t distribution at some given significance level with \( N \) and \( M \) degrees of freedom respectively. All structures were tested for similarity at the .05 significance level. Table 3.9 shows the results of the hypothesis test on particular areas of overlap. In addition to performing hypothesis testing on the SWE maps transects of the normalized SWE maps were generated to compare the relative difference between structures...
on different maps.

It is clear by looking at the SWE maps from different years that none of the normalized SWE maps could be used to construct a specific SWE map for an alternate year. Even the 1992 and 1995 maps which were based on mean SWE values falling within 7 percent of each other, do not correlate particularly well. Nonetheless, it is informative to examine which areas of SWE are similar from year to year and which ones appear to be different. Table 3.9 summarizes the results of applying the Cochran’s approximation of the t-test to specific areas of overlap between years. Figures 3.52 and 3.53 show the transects generated for each normalized SWE map.

Table 3.9 Results of hypothesis testing.

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<th>$t_c$</th>
<th>sim / diff</th>
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</tr>
<tr>
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Table 3.9 (cont.)

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</table>

The 1992 and 1994 SWE maps using method 1 of incorporating the snow pit density data have some similarities. The avalanche region just below the cliffs in the lower basin, although occurring at slightly different elevation ranges, correspond extremely well according to the results of the hypothesis test. Indeed comparing the overlapping portion of the transects illustrates that the difference in the relative SWE is .05. In addition both maps suggest that the area of the basin with the lowest SWE can be found in lower portion of the basin. The hypothesis test on the overlapping region suggests that they are identical but with less confidence. As can be seen in the transects, the difference between the two values is
0.19. Now there are differences in the lower basin. The large SWE region at the outlet of the basin in the 1994 map is definitely absent from the 1992 map. Indeed, t test results show that the large SWE class from 1994 is different at the .05 level of significance. There is also a lack of a division between the shaded an unshaded portion of the lower basin. T test results suggest that the difference between the light region on the 1995 image and the moderate region in the shade on the 1994 image is not significant, however. The lack of a split on radiation may be partially due to the fact that the observation locations for that survey (figure 3.3) did not extend as far deep into the shadows compared to the other two surveys.

The central portion of the maps, the part the least likely to be accurate, does suggest a moderate SWE in both figures 3.46 and 3.48. The t test on the normalized maps passes suggesting that the unnormalized class of 1.63 meters in the 1992 map is similar to the unnormalized class of 1.82 meters. Indeed comparing the transects illustrates that the difference in relative SWE is only 0.06.

In the upper portion of the basin, there are large SWE values that appear just below the cliffs along the northwest border on both maps. Table 3.9 indicates, however, that at the 0.05 level of significance the relative SWE values are different. About 100 meters southeast of the cliffs there is an area of large SWE that is similar at that level of significance. The transects show that the relative SWE difference in this section is 0.08. In the southwestern basin boundary both maps show a moderate level of snow. The hypothesis test on the normalized values of the SWE classes of 1.60 meters in 1992 and 2.15 meters in 1994 pass, but not by as large a margin as other section of the maps.
The comparison between the SWE maps of the 1992 and 1994 data using the second method of incorporating the snow pit density data suggests less similarity between years. Although the low SWE sections in the lower basin were still determined to be similar, area in the upper basin tend to fail the hypothesis test in this case. In addition, the central portion of the basin now classified as heavy SWE is very different than suggested by the 1992 map. A hypothesis test on this region does suggest that the structures were similar however, the variance in this region for the 1994 data increased from .39 using method one to 1.33 using method two. With such a large variance it is no surprise that the two were identified as similar. The transects for these two maps do illustrate a significant difference in relative SWE between the two overlapping areas. It is difficult to make any definite conclusions regarding the similarities or differences between the two schemes. If the lower portion of the snow pack from 1994 does indeed possess denser snow then it is not surprising that differences would result between the lower and higher accumulation years. If the assumption of dense snow is too greatly exaggerated, then SWE values in the areas that appear similar ought to be closer as suggested by the first comparison. Since the 1994 survey produced the worst results both in terms of $R^2$ using resubstitution and for the deviance ratio for cross validation, any definite statements regarding the SWE distribution of the 1994 data are suspect.

A comparison between the SWE maps of 1992 and 1995 using method one of incorporating the snow pit density data fairs better than the 1994 maps. Both maps, as mentioned previously, show a dependance of SWE on radiation in the lower basin.
Hypothesis testing of the overlap between the lowest SWE class indicates the two structures are extremely similar at the .05 significance level. Indeed the transects show that the difference in the relative SWE is only .006. The SWE class of 1.63 meters in the 1992 map, the shaded portion of the lower basin, is similar to both the classes of 1.31 and 1.82 meters of the 1995 map. If these two categories were grouped together the similarity in the shaded region may be even stronger.

In the central portion of the basin, table 3.9 indicates that the SWE classes of 1.60 meters for 1992 and 1.37 meters for 1995 are quite similar. Indeed the difference in relative SWE, although difficult to discern from the transect, is merely 0.02. Both maps show an area of large SWE below the cliffs along the northwestern boundary. Hypothesis testing of the normalized classes suggest a strong similarity with a difference in relative SWE of .04.

Despite the similarities between the 1992 and 1995 SWE maps using method 1, there are differences. The most obvious is the lack of a heavy SWE band in the lower basin. Perhaps there was less avalanching. Other regions that show a distinct difference between the two maps are the regions along the southwestern and northeastern boundaries at higher elevations. T test results confirmed this.

The analysis of the SWE maps of 1992 and 1995 using method 2 of incorporating the snow density is quite similar to the maps using method 1. There are some differences, however, due to the differences associated with the 1995 map. First the region of overlap for the similar areas of the central portion of the basin is not as extensive. This result is of course due to the division on radiation occurring half way between the two regions. Second the
SWE class below the cliffs along the northwestern boundary in 1995 is much larger. Although it passed hypothesis testing, the values of the test statistic were quite large. The difference in relative SWE here is 0.19.

As was the case with the 1992 and 1994 SWE maps, the 1994 and 1995 SWE maps using method 1 of incorporating the snow pit density indicate that the area of overlap between the two smallest SWE classes are tested as similar. The extent of this area is small, however, as the high density region at the basin outlet in the 1994 map fails the test for similarity. The central portion of the basin with the unnormalized classes of 1.37 meters for 1995 and 1.82 meters for 1994 compare quite well. The difference in relative SWE for this region is only 0.06 meters as can be seen on the transects. The areas on the south western basin boundary do not pass the test, nor do the areas along the northeastern boundary, but there is little observation data for points among the cliffs on the northeastern boundary.

The hypothesis testing for the 1994 and 1995 SWE maps using method two of incorporating the snow pit data give similar results for the most part with that using method 1. Of course, the large SWE value in the central portion of the region would not be considered similar, even though it barely passes the t-test. Again, the passing of the t test for this region is due to the extremely large sample variance of the observation data in this class.

3.6 Discussion

The results from most of the analyses performed on the 1992 and 1995 regression
trees suggest a strong ability for the optimal trees to model the variation in the observation data. The coefficient of determination for these trees all fall within the range of values obtained by Elder in modeling SWE for the Emerald basin. The mean SWE of the observation data matches closely with the mean SWE of the modeled maps, suggesting that the observation data represent the variation of the modeled classes in the basin. Finally, with the exception of the cliffs, the error between the modeled and measures values shows no obvious geographic bias.

The results for the analyses of the 1994 trees suggest a poorer ability to describe the variation in the observation data. The $R^2$ values were generally below the range of values found by Elder even for the full size regression trees. In addition the mean SWE of the modeled maps tend not to correspond as well to the mean SWE of the observations. The distribution of errors, although greater in magnitude, did appear to be distributed uniformly however.

The results for cross validation on all of the trees suggested a poor ability for the trees to predict SWE, but certain unresolved issues related to the sensitivity of the splits to the removal of data from the learning set seems to be the source of the problem. There was no examination of this issue by Elder, so it was not possible to compare cross validated results to his trees. Preliminary results suggest that the fact that there is little decrease in cross validated deviance from the root node may not be a concern. Nonetheless, a comparison of cross validated results between the trees supports the notion that the 1994 regression trees model the data the worst. The different methods of incorporating snow density into the depth
measurements showed little difference in $R^2$ for the ability to model the SWE variation. There were some differences in the structure of the resulting SWE maps for 1994 and 1995. A further examination of the best method of incorporating snow density should be made.

The ability to distinguish SWE patterns from year to year was not as great as hoped. A quick examination of the SWE maps showed that the patterns were not consistent enough so that a normalized SWE map from one year could be substituted for another year if the total accumulation was known. Even the accumulation for 1994 and 1995, whose basin wide mean SWE fell within 10% of each other, did not prove to correlate closely. Since the location of the observation data did vary from year to year, it is possible that sampling different areas may have contributed to some of the differences between years.

Although differences in the predicted SWE distribution make it impossible to substitute maps, the patterns that did arise between the years provided useful information regarding the factors that control SWE in Echaurren. The 1992 and 1995 maps, for instance, both showed a strong dependance on radiation in the lower basin. Although this dependance was not visible in the 1994 maps, the poor ability of the model to explain variance in SWE and the fact that observation data did not extend as deeply into the shade, prevent the 1994 results from ruling out the relationship for higher accumulation years. All of the maps, except for figure 3.49, suggest that the central portion of the basin has a moderate amount of SWE. Of course the absence of observations in the very center of the basin suggest that this relationship may not completely valid. The large slopes present between the upper and lower basin may make the true SWE smaller than that predicted. With the exception of bands of
heavy SWE in the lower basin, the upper basin tends to consist of greater SWE than the lower basin. This result is entirely consistent with what one would expect. Finally many of the maps showed clusters of large SWE values below areas near cliffs. There was no consistent splitting mechanism for these large SWE values. The 1992, tree for instance, gave a split on elevation for the lower basin and a split on slope for the upper basin. The accumulation of snow beneath the cliffs suggest avalanching plays an important role in distributing SWE. These statements correspond well with field observations of avalanche debris.
Figure 3.1 Example regression tree
Figure 3.2 Location of depth measurements for 1992 snow survey. Background is the shaded relief map of the Echaurren basin and surrounding area.
Figure 3.3 Location of depth measurements for 1994 snow survey. Background is the shaded relief map of the Echaurren basin and surrounding area.
Figure 3.4 Location of depth measurements for 1995 snow survey. Background is the shaded relief map of the Echaurren basin and surrounding area.
Figure 3.5 Location of Snow pits. Snow pit locations for 1995 are missing. Background is the shaded relief map for the Echaurren basin and surrounding area.
Figure 3.6 Density Profiles for 1992 snow pits
Figure 3.7 Density Profiles for 1994 snow pits
Figure 3.8 Density Profiles for 1995 snow pits.
Figure 3.9 Atmospheric parameters assumed for Topquad
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Figure 3.11 Net radiation image for the accumulation period of 1994. Generated by IPW using Topquad.
Figure 3.12 Digital soils map
Figure 3.13 Extrapolated snow pack density for lower pit, 1994. Bar graph is the measured snow density profile. Solid line is the extrapolation using the function:

\[
density = 485.5 - 1.4 \times 10^{-15} \times e^{0.07 \times \text{depth}}.
\]
Figure 3.14 Mount Rose snow density data from the 1992 survey as a function of the independent variables
Figure 3.15 Distribution of Mount Rose Density data. Density values are in units of Kg/M$^3$. Background is a shaded map of the Echaurren basin surrounding area.
Figure 3.16 Histograms of Mount Rose density measurements for 1992 snow survey.
Figure 3.17 Full size regression tree grown from 1992 SWE data using Mount Rose samples for density.
Figure 3.18 Full size regression tree grown from 1992 SWE data using method 1 of incorporating the snow pit density data.
Figure 3.19 Full sixe regression tree grown from 1992 SWE data using method 2 of encoding the snow density data.
Figure 3.20 Full size regression tree grown from 1994 SWE data using method 1 of incorporating the snow pit density data.
Figure 3.21  Full size regression tree grown from 1994 SWE data using method 2 of incorporating the snow pit density data.
Figure 3.22  Full size regression tree grown from 1995 SWE data using method 1 of incorporating the snow pit density data.
Figure 3.23 Full size regression tree grown from 1995 SWE data using method 2 of incorporating the snow pit density data.
Tree size (number of terminal nodes)

\[ R^2 \]

Figure 3.24 Coefficient of Determination (resubstitution) versus tree size for 1992 SWE regression tree based on Mount Rose density data.
Figure 3.25 Coefficient of determination (resubstitution) versus tree size for 1992 SWE regression tree based on method 1 of incorporating snow pit density.
Figure 3.26 Coefficient of determination (resubstitution) versus tree size for 1992 SWE regression tree based on method 2 of incorporating snow pit density.
Figure 3.27 Coefficient of determination (resubstitution) versus tree size for 1994 SWE regression tree based on method 1 of incorporating snow pit density.
Figure 3.28 Coefficient of determination (resubstitution) versus tree size for 1994 SWE regression tree based on method 2 of incorporating snow pit density.
Figure 3.29 Coefficient of determination (resubstitution) versus tree size for 1995 regression tree based on method 1 of incorporating snow pit density.
Figure 3.30 Coefficient of determination (resubstitution) versus tree size for 1995 regression tree based on method 2 of incorporating snow pit density.
Figure 3.31 Accumulated deviance versus tree size for 74 fold cross validation of 1992 SWE data using Mount Rose density measurements.
Figure 3.32 Accumulated deviance versus tree size for 80 fold cross validation of 1992 SWE data using method 1 of incorporating snow pit density.
Figure 3.33 Accumulated deviance versus tree size for 80 fold cross validation of 1992 SWE data using method 2 of incorporating snow pit density.
Figure 3.34 Accumulated deviance versus tree size for 81 fold cross validation of 1994 SWE data using method 1 of incorporating snow pit density.
Figure 3.35 Accumulated deviance versus tree size for 81 fold cross validation of 1994 SWE data using method 2 of incorporating snow pit density.
Figure 3.36 Accumulated deviance versus tree size for 76 fold cross validation of 1995 SWE data using method 1 of incorporating snow pit density.
Figure 3.37 Accumulated deviance versus tree size for 76 fold cross validation of 1995 SWE data using method 2 of incorporating snow pit density.
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Figure 3.39 Distribution of errors between observed and modeled values for the regression tree from 1992 using method 1 of incorporating snow pit density. Background is the shaded relief map of the Echaurren basin.
Figure 3.40 Distribution of errors between observed and modeled values for the regression tree from 1992 using method 2 of incorporating snow pit density. Background is the shaded relief map of the Echaurren basin.
Figure 3.41 Distribution of errors between the observed and modeled values for the regression tree from 1994 using method 1 of incorporating snow pit density. Background is the shaded relief map of the Echaurren basin.
Figure 3.42 Distribution of errors between observed and modeled values for the regression tree from 1994 using method 2 of incorporating snow pit density. Background is the shaded relief map of the Echaurren basin.
Figure 3.43 Distribution of errors between observed and model values for the regression tree from 1995 using method 1 of incorporating snow pit density. Background is the shaded relief map for the Echaurren basin.
Figure 3.44 Distribution of errors between observed and modelled values for the regression tree from 1995 using method 2 of incorporating snow pit density. Background is the shaded relief map of the Echaurren basin.
Figure 3.45 SWE distribution from optimal tree, 1992 Mount Rose data. Black areas are snow free cliffs. Values are in meters of SWE.
Figure 3.46 SWE distribution from optimal tree, 1992 method 1. Black areas are snow free cliffs. Values are in meters of SWE.
Figure 3.47 SWE distribution from 1992 method 2. Black areas are snow free cliffs. Values are in meters of SWE.
Figure 3.48 SWE distribution from optimal tree, 1994 method 1. Black areas are snow free cliffs. Values are in meters of SWE.
Figure 3.49 SWE distribution from optimal tree, 1994 method 2. Black areas are snow free cliffs. Values are in meters of SWE.
Figure 3.50 SWE distribution from optimal tree, 1995 method 1. Black areas are snow free cliffs. Values are in meters of SWE.
Figure 3.51  SWE distribution from optimal tree, 1995 method 2. Black areas are snow free cliffs. Values are in meters of SWE.
Figure 3.52 Transects generated from normalized SWE distribution using method 1 of incorporating snow pit density. White line in the 1992 SWE map at upper left corner is the location of the transect.
Figure 3.53 Transects generated from normalized SWE distribution using method 2 of incorporating snow pit density. White line in the 1992 SWE map at upper left corner is the location of the transect.
CHAPTER 4

Conclusions and Future Directions

The examined methods for determining water stored in snow packs show some promise for use in the Chilean basins of Laguna Negra and Echaurren. The use of linear spectral unmixing for SCA in the Laguna Negra basin can result in plausible SCA maps if the proper number of endmembers is used. Statistical analysis of the regressed SWE data for Echaurren suggested that regression trees were effective at explaining the variance in the modeled data, however issues regarding the accuracy of the resulting SWE maps did arise.

The spectral unmixing study showed that three bands of Landsat data are adequate during the early melt season so long as extensive shadows are absent. Four bands are required for winter so that enough endmembers can be used to account for shadows. Finally, five bands were shown to be necessary in the late melt season since multiple soil types were exposed. Following these general rules when using linear spectral unmixing for Laguna Negra should ensure proper SCA maps.

The binary regression tree study showed that regression trees could be generated from Echaurren snow survey data such that 40 to 60 percent of the variation in the survey data could be explained by the tree models. Unfortunately, a comparison of distributed SWE maps that were generated from models using different survey data, showed that many regions of the maps were not statistically similar between years with similar degrees of accumulation.
This suggests that regression tree models generated from one year of survey data, cannot be used to generate a map for another year even if the total accumulation was known to be approximately the same. There were, however, large scale structures that did appear in multiple survey’s.

Despite the results suggested by the coefficient of determination, the large values for the ratio of modeled to unmodeled deviance using cross validated data suggests some problems that should be examined more closely. One way to examine it, is to generate all trees where each data point is left out once and compare the difference between the node an observation fell in for original tree and the node it would fall in for the trees where the observation was left out. This would indicate the reason for the large errors and may allow a qualitative judgment to their importance. Another scheme would be to generate SWE maps for each case in which data has been left out and determine the displaced SWE (equation 3.4) between the map using all the data and the maps with some of the learning set missing. This would give an indication of the total amount of snow that is sensitive to the changes in the learning set size. Since the purpose of generating distributed SWE maps for the Echaurren basin is to improve input for snow melt models, the best method of examining the significance of the changes in the SWE distribution would be to look at the sensitivity of the resulting hydrographs. Clearly an estimate of the total displaced SWE is not adequate since it does not indicate the location the SWE was shifted to. This requires a large number of SWE maps to be generated and a large number of runs of a snow melt model. Unfortunately, this is likely to be an extremely time consuming process.
In addition to the investigation of the large cross validated errors, there are additional issues which might be useful for future investigation. First, some determination of the proper method of incorporating snow density should be made. This was not critical for the distribution of SWE in 1992, but did appear to be for the 1994 and 1995 maps. In all cases, the method of incorporating SWE does have an effect of the mean SWE. Calculating point SWE values using the Mount Rose data produced the best value for the coefficient of determination. Since only one survey involved the use of the Mount Rose sampler, no conclusions can be made.

Analysis of the structures in the SWE maps showed that the presence of large SWE near cliffs sometimes resulted in splits which produced areas of large SWE nowhere near the cliff faces. The split on elevation in the lower basin of the 1992 trees is a good example of avalanche data resulting in a such a SWE structure. This might be improved by using a map of avalanche deposits in the model. This would require either a map generated by the field surveyors or a surrogate such as that used by Elder in modeling the SWE distribution of the Teton Glacier in Wyoming [Elder, 1995]. Allowing splits based on the tendency to accumulate avalanche debris might yield more accurate distributions.

Finally, there are possible improvements to be made in the data collection of the Chilean snow surveys. Efforts should be made to maximize the number of data points collected over the entire range of values for the independant variables used in the model. The 1994 survey, for example, neglected a large portion of the basin receiving the lowest values of radiation. This makes it less likely for a split to occur on radiation. Moreover,
investigating whether the meadow has a substantial influence on SWE, requires more measurements over the meadow. Digging two additional snow pits in each sub-basin could reveal changes in snow density with radiation. While this dependence did not appear great, it might improve the accuracy of the models slightly.
Appendix A: Manual unmixing for determination of SCA

Why Fractional SCA?

Satellite images of alpine snow covered regions often contain a high percentage of pixels whose spectra are the result of a combination of snow, rock, vegetation, and other materials. As a result, most pixels in these images cannot be accurately classified as pure snow or pure rock. The resulting spectra from such mixed pixel is usually a linear combination of the components of those pixels. Thus, given the spectra of pure snow, rock or whatever other components compose the image, it is possible to unmix the image and thereby obtain the fraction of a pixel that is covered with snow. In theory, such unmixing will produce a more accurate picture of snow covered area (SCA) in alpine regions.

Preparing the image for analysis

Before analyzing images for snow covered area, it is necessary to remove the effects of the scattering of light for different wavelengths. This can be done by simply performing a histogram analysis on each band and noting the DN value (Digital Number from 0-255) where the detector begins significantly responding to reflected light. This is known as the path DN. The path DN is then subtracted out of each pixel of each band in the images. In general, the band with the largest path DN will be the blue visible range and it will fall off to zero towards short wave infrared. The path DN is scene specific so histograms will have
to be done for each scene.

Suggested procedure for subtracting out the path DN using IPW:

1. Run Hist on a single band of the satellite image (ASCII output recommended)
2. View the output in vi and count the number of lines until a large number of pixels appears. (IE say, a jump say from 1 to 200).
3. Since the first line is a DN of zero subtract 1 from the number of lines. This is the path DN.
4. Run interp | mklut | lutx -I "image name" > "output image"
   0 0
   (path DN) 0
   255 (255 - path DN)

Choosing Endmembers

To perform spectral unmixing on any multi-band satellite image requires choosing spectral endmembers. A spectral endmember is the spectrum associated with a given material representative of the entire scene. Endmembers can be either image endmembers (an endmember that is characteristic of the scene) or reference endmembers (the spectra associated with a pure material). To determine SCA from only scene components it is necessary to find pure pixels. Pure pixels are pixels representing only one material such as a pixel fully covered with snow. If a pixel is pure then the image endmember is equivalent to the reference endmember. Alpine regions require at least two endmembers, a snow endmember and a rock endmember. Depending on the location, time of year, and scene composition, additional endmembers may be needed such as shaded snow, and vegetation.
The maximum number of endmembers possible depends on the number of linearly mixed components (dimensionality of the image), and the number of bands available. To prevent an over determined solution, there should always be at least 1 fewer endmembers than the number of bands in the image (see Chapter 3).

Determining proper endmembers is an iterative process more like art than science. It involves making some sort of initial educated guess on potential endmembers, performing the unmixing process, and looking at the results and errors associated with the unmix. These errors are then used as a guide to find better endmembers for the second attempt at unmixing. This process is continued until the image error, often judged by using the mean RMS error, is minimized.

Several rules of thumb can be used for the first guess at determining proper endmembers for alpine scenes. Begin with just two, snow and rock. If there are only three bands available for the satellite image that is all that is possible. In the range of visible blue to shortwave infrared (.45 µm to 2.35 µm) the spectral reflectance of snow and rock are complimentary; high amplitudes are measured for snow in the visible range while low amplitudes are measured for rock in the same range. The opposite holds true in the shortwave infrared range. If Thematic Mapper images from Landsat 5 are examined for snow, bands 1-3 and somewhat band 4 will have high DN values and bands 5-7 will have low DN values.

This behavior allows the use of false color composite images in choosing snow or rock endmembers. An RGB image of three of the bands can be a visual aid in locating areas of large snow cover, and large rock extent. For example, using Landsat TM images, RGB images of
bands 5, 4 and 2 respectively will show large snow cover areas as blue or blue green regions and snow free areas as red regions. Pixels can then be chosen from the image as potential endmembers.

Another method of finding potential endmembers is by performing a principal components analysis on the full multiband image. Principal components analysis is a method from basic linear algebra which allows the space defined by a matrix of values to be rotated so that the matrix is transformed into having only diagonal components. In terms of a multi-band image, picture the data (pixel values) plotted in a system where each axis represents a band. There will be a cloud of data distributed throughout the space defined by these axes. Ideally there will be clusters of data in certain directions based on the reflective properties of scene components and the response of the detector to these components. By using principal component analysis the axes describing this data space are rotated to coincide with skews in the data cloud, so long as the skews are nearly orthogonal. The basic procedure for performing a PCA is to start by running a statistic package on the image to determine the variance and covariance values along each the bands and between them (IE form a variance-covariance matrix). Next determine the eigenvalues of that matrix. The eigenvectors associated with these eigenvalues describe the axes in the directions of skew. To get useful information with these eigenvectors simply perform the dot product of the image pixels with an eigenvector. The resulting image has a single value for each pixel which represents the distance of the pixel along one of the new axis. One axis is likely to be related to the degree of snow cover in the image, so viewing the image and locating the brightest pixels should help identify a potential snow endmember. Similar trends may be true for
other endmembers. Bear in mind is that PCA axis are always perpendicular while the axis described by the endmembers may not necessarily be perpendicular. PCA should only be used as a starting point.

After potential endmembers are chosen an unmix is performed, as described in the next section of this paper, and the results are analyzed. New endmembers are chosen to minimize the RMS error. Once this is minimized, additional endmembers may be chosen using the methods described above to further minimize the error.

Suggested procedure for endmember determination using IPW:

I. By false color imaging
   1. use \texttt{mux} to combine short wave infrared (SWIR), near infrared (NIR) and visible (VIS) bands together in that order.
      (Eg, \texttt{mux band5 band4 band3 > false_color_image})
   2. View the image in XV, and use the mouse to get pixel values shown on screen.
   3. For the snow endmember, write down the coordinates of the pixel values that satisfy the trend: small DN, medium DN, large DN (The VIS DN should be as large as possible)
   4. For the rock endmember, write down the coordinates of the pixel values that satisfy the trend: large DN, medium DN, Small DN (The SWIR DN should be quite a bit larger than the VIS DN)
   5. Create a coordinates file with the coordinates reversed from XV (IPW coordinates are arranged with line number first followed by the sample number on that line. XV reports coordinates as the horizontal position first followed by the vertical position)
   6. Get the spectral signature of the pixels by using the IPW command \texttt{primg}.
      The three values returned are the intensity values for each of the three wavelengths. (Eg, \texttt{primg -c coordinate_file -I image_file > end_spect})
II. By principal component analysis.
   1. Use \texttt{mux} to combine all of the bands of the image into 1 image.
   2. Use the IPW command \texttt{mstats} on the entire image to create a file containing a variance-covariance matrix. (Eg, mstats full_image > full_image.stats)
   3. Run \texttt{eigen} on the mstats output to determine the eigenvalues and vectors of the variance-covariance matrix. The output from eigen places the eigen vectors from left to right in the same order as the corresponding eigenvalues. (Eg, eigen full_image.stats > full_image.eig)
   4. Use \texttt{lincom} to take the dot product of the each pixel in the full image with an eigenvector. This will perform a coordinate transformation of each pixel to a principle coordinate axis. Repeat the process for each eigenvector (IF there are N bands there are N eigenvectors). (Eg, lincom -c eigen_vect_component1, eigen_vect_component2, etc full_image > transformed_image_axis1)
   5. View the resulting images in XV and write down coordinates of the pixels with the highest value for images that appear to represent snow or rock.
   6. Create a coordinates file with the coordinates reversed from XV (IPW coordinates are arranged with line number first followed by the sample number on that line. XV reports coordinates as the horizontal position first followed by the vertical position)
   7. Get the spectral signature of the endmember pixels by using the IPW command \texttt{primg}. The values returned are the intensity values for each of the three wavelengths. (Eg, primg -c coordinate_file -i image_file > end_spect)

Unmixing the image

The unmixing of the image is the easy part. As mentioned previously it is based on the assumption that the spectra of the various scene components will mix linearly. This is generally the case for alpine snow covered regions. Equation 1 below, adapted from Rosenthal [1993], illustrates this relationship in vector notation.
\[ R = r f + \varepsilon \]  \hspace{1cm} (1)

In the equation above, \( R \) is a \( B \) dimensional vector representing the reflectance of a pixel for \( B \) bands, \( r \) is a \( B \) by \( N \) dimensional matrix representing the chosen endmember reflectance for \( B \) bands with \( N \) endmembers, \( f \) is a \( N \) dimensional vector representing the fractions of each endmember on a pixel, and \( \varepsilon \) is a \( B \) dimensional vector representing the unknown error in each band. Since \( r \) is a \( B \) by \( N \) dimensional matrix, each column in the matrix is the spectral signature of an endmember.

In light of equation 1 above, we see that linear unmixing is nothing more that solving for the vector of endmember fractions \( f \) for each pixel. In the case of a perfect model fit, where \( \varepsilon \) is identically zero, this can be done by inverting \( r \) and matrix multiplying it with the reflectance vector \( R \) of each pixel. Equation 2 below illustrates this in vector notation.

\[ f = r^{-1}R \]  \hspace{1cm} (2)

If the slope and aspect at every single location in an image were identical, and every component of the spectra was accounted for, the spectral fractions determined by solving equation 1 would all be less than and sum up to one. Naturally, this never occurs. Varying slopes and aspects change the intensity of the reflected light; since endmembers are generally the brightest of the chosen materials, slopes and aspects that result in less reflected light getting to the detector
will not add up to 1. Moreover since additional substances may be present in an image, some pixels would be expected to have components of \( f \) that are less than zero or greater than one. A bright metal roof, for instance, may be modeled as a fraction of the snow endmembers spectrum greater than one. To deal with these issues, constraints must be applied to the solution of equation 1. First, to deal with poorly modeled pixels all components of \( f \) are constrained to fall between zero and one. Any negative fractional values are remapped to zero and any fractional values larger than one are remapped to one. This does not solve the problem of these poorly modeled pixels but it does allow the further use of the resulting fraction images in applications where impossible fraction values may cause problems. In any case, a properly modeled image should not have too many pixels violating this condition. The second problem is that varying slopes and aspects will result in spectral fractions that do not sum up to one. Its solution is simple. Simply sum the individual endmember fractions for each pixel and use this as the denominator of a new fraction. The use of this technique is valid so long as the relative contributions of each material to the final spectrum does not change with slope or aspect and no other substances contribute much to the resulting spectrum.

In general, it is a good idea to perform the initial iterations of an unmix without any constraints. By using this unconstrained model, pixels that model the endmembers so poorly that they violate the condition that fractions be less than one and greater than zero can be easily identified. This can help in choosing an endmember for the next iteration. Once, the decision is made that the unmixed images have a minimum error, then a constrained run can be performed.
Suggested procedure for unmixing using IPW:

1. Using the endmember spectra previously determined, run the IPW routine \texttt{sma} in unconstrained mode using singular value decomposition to solve equation 1. Sma assumes that the endmember file names end in .em so make sure your files use this extension. The command line should resemble: \texttt{sma -e snow,rock -s image}.

2. Use the IPW routine \texttt{mux} to combine the endmember spectrum fraction images together and then use \texttt{lincom} to add the fractions for each pixel together. The command lines should resemble:
   \begin{verbatim}
   mux fract_image_1 fract_image2 > comb_image
   lincom -c 1,1 comb_image > sum_image
   \end{verbatim}

3. Use \texttt{mux} combine a fraction image with \texttt{sum_image} and use \texttt{mult} to divide the fraction image by the summed image. The command line should resemble: \begin{verbatim}
   mux fract_image_1 sum_image > comb_image
   mult -r 1 comb_image > new_fract_image
   \end{verbatim}

4. If you are satisfied with the results using the chosen endmembers repeat steps 1 through 3 where you have replaced the \texttt{-s} parameter for \texttt{sma} with the \texttt{-c} parameter. The \texttt{-c} parameter will solve for the fractional values using singular value decomposition with the constraint that all components of $f$ must be greater than zero and less than 1.

\textbf{Analyzing the unmixed images}

Several aspects of the resulting image must be examined in order to discover how well the chosen endmembers modeled the unmixing. The first and most obvious check of the unmix is to look at the output images and see if they reflect what you know about the area in question. For example, summer scenes will generally have less SCA than winter scenes. In addition, examining
the both the normalized and unnormalized output images can help identify areas were strong shading is occurring. To determine the total snow covered area or rock extent of an image, run a routine to evaluate the mean value of a pixel. Make sure any large bodies of water are masked out first.

Examining the RMS error for each pixel, often gives the most insight into areas of poor modeling. It is usually a good idea to generate a RMS error image with regions of high error indicated in white and low error in black. Viewing such an image can help identify potential endmembers for the next iteration. For example, if a snow endmember was chosen with low intensities for each band the higher intensity region will be poorly modeled. The next endmember will probably come from one of these areas of high error. To obtain an idea of the overall error of the image, use a routine to calculate the mean value of the RMS error.

It is important to keep in mind that the usefulness of examining the RMS error image is with respect to a previous iteration with the same number of endmembers. As the number of endmembers approach the number of bands, the calculated RMS error will always approach zero. Thus, if an additional endmember is added to an unmix, the RMS error will likely drop regardless of how good an endmember it is.

Finally, it is useful to examine the results of an unconstrained mix by creating a histogram of the pixel values in the fraction images. This allows the unmixer to see the degree at which pixels are classified with negative fractions or fractions greater than one. The better the unmix, the fewer pixels that will violate this condition. As mentioned previously, some pixels will always violate this condition, but rerunning the unmix with the constrained model will remove
these last few pixels.

Once, the RMS error is minimized, and the number of pixels in the image violating the fraction constraints is minimized, the image is properly unmixed and the SCA results can be used.

Suggestion for analyzing the results using IPW:

1. View the fraction images in XV and check if they “make sense”.
2. Run mstats on the fraction images for later comparisons. Run it for both the normalized and unnormalized images. Use the -m option with a mask for lakes.
2. View the RMS error image, and locate regions of high error.
3. Run mstats on the RMS error image and save to a file for later comparisons. This will give the mean RMS error for the image
4. On the command line use primg on the fraction images and pipe it to several UNIX utilities to create a histogram. The following command should be used:
   primg -a -I $1 sort -n | uniq -c
5. Exam the output in step 4 for the degree at which the fraction constraints are violated. The fraction image with the largest number of constraint violations is likely associated with a poor endmember.
6. On the basis of the RMS error image, in conjunction with the information obtained from either PCA or the false color image, choose new endmembers.
Appendix B - Script showing parameters used for TOPQUAD

The following pages consist of the shell script used to generate a radiation image for a single date. The script involves multiple calls TOPQUAD with varying values passed on each call. The switch -n tells TOPQUAD to calculate net radiation. The value after -z tells TOPQUAD the elevation of the optical depth measurement for the atmosphere while the value after -t is the optical depth for a specific wavelength interval. Both values are in meters. The value after -w specifies the single scattering albedo of the atmosphere for the wavelength interval, while -g is the scattering asymmetry parameter. The value after -r is the mean surface albedo for the given wavelength interval. The parameter -x specifies the wavelength interval of the incident radiation over which TOPQUAD is run. The parameters of -d, -b and -l are the date, latitude and longitude for the radiation image being calculated. The total incident radiation is determined on the last line as the sum of the individual contributions to the total power of each of the wavelength intervals.
Calculate radiation across watershed; for 5-m DEM

# total optical depth, SINGLE SCATTERING ALBEDO
# SCATTERING ASSYMMETRY PARAMETER and TRANSMITTANCE
# from LOWTRAN calculations
# average values used for each range of wavelength
# ECHAURREN GLACIER-CHILE, latitude, longitude data
# sent by Fernando Escobar on Sept 6, 1993
# total number of images for one day: 28
# range of wavelength: 3.0 - 0.3 micrometers
# interval of wavelength goes from 0.3 to 0.4
# 0.4 to 0.5
# 2.9 to 3.0
# 3.0 >

#VISIBLE range

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.95.ipw > esasta05

```
topquad -n -z 3480 -t 1.01 -w 0.787 -g .070 -r 0.95 -x 0.3,0.4 -d 1994,5,1 \
    -b -33,35,00 -l -70,08,00 esasta05 > rad03.out

topquad -n -z 3480 -t 0.330 -w 0.978 -g .148 -r 0.95 -x 0.4,0.5 -d 1994,5,1 \
    -b -33,35,00 -l -70,08,00 esasta05 > rad04.out

topquad -n -z 3480 -t 0.231 -w 0.727 -g .210 -r 0.95 -x 0.5,0.6 -d 1994,5,1 \
    -b -33,35,00 -l -70,08,00 esasta05 > rad05.out

topquad -n -z 3480 -t 0.142 -w 0.802 -g .337 -r 0.95 -x 0.6,0.7 -d 1994,5,1 \
    -b -33,35,00 -l -70,08,00 esasta05 > rad06.out
```

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.90.ipw > esasta05

topquad -n -z 3480 -t 0.142 -w 0.802 -g .337 -r 0.90 -x 0.7,0.8 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad07.out

topquad -n -z 3480 -t 0.101 -w 0.918 -g .387 -r 0.90 -x 0.8,0.9 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad08.out

# NIR rang

rn esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.70.ipw > esasta05
e
topquad -n -z 3480 -t 0.147 -w 0.554 -g .426 -r 0.70 -x 0.9,1.0 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad09.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.65.ipw > esasta05

topquad -n -z 3480 -t 0.065 -w 0.912 -g .420 -r 0.65 -x 1.0,1.1 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad10.out

# MIR rang

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.50.ipw > esasta05
e
topquad -n -z 3480 -t 0.166 -w 0.345 -g .453 -r .50 -x 1.1,1.2 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad11.out

rm esasta05
mux ~/Images/Chile/Radiation/echaurren_fix_elev
 ~/Images/Chile/Radiation/echaurren_fix_gradient
 ~/Images/Chile/Radiation/echaurren.viewf
 ~/Images/Chile/Radiation/albedo.0.40.ipw > esasta05

topquad -n -z 3480 -t 0.060 -w 0.772 -g .443 -r .40 -x 1.2,1.3 -d 1994,5,1 \ 
 -b -33,35,00 -l -70,08,00 esasta05 > rad12.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
 ~/Images/Chile/Radiation/echaurren_fix_gradient
 ~/Images/Chile/Radiation/echaurren.viewf
 ~/Images/Chile/Radiation/albedo.0.30.ipw > esasta05

topquad -n -z 3480 -t 0.64 -w 0.089 -g .457 -r .30 -x 1.3,1.4 -d 1994,5,1 \ 
 -b -33,35,00 -l -70,08,00 esasta05 > rad13.out

rm esasta05

topquad -n -z 3480 -t 0.361 -w 0.028 -g .465 -r .05 -x 1.4,1.5 -d 1994,5,1 \ 
 -b -33,35,00 -l -70,08,00 esasta05 > rad14.out

topquad -n -z 3480 -t 0.045 -w 0.639 -g .440 -r .05 -x 1.5,1.6 -d 1994,5,1 \ 
 -b -33,35,00 -l -70,08,00 esasta05 > rad15.out

topquad -n -z 3480 -t 0.039 -w 0.615 -g .432 -r .05 -x 1.6,1.7 -d 1994,5,1 \ 
 -b -33,35,00 -l -70,08,00 esasta05 > rad16.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
 ~/Images/Chile/Radiation/echaurren_fix_gradient
 ~/Images/Chile/Radiation/echaurren.viewf
 ~/Images/Chile/Radiation/albedo.0.07.ipw > esasta05
topquad -n -z 3480 -t 0.052 -w 0.481 -g 0.447 -r 0.07 -x 1.7,1.8 -d 1994,5,1 \
-b -33,35,00 -l -70,08,00 esasta05 > rad17.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.15.ipw > esasta05

topquad -n -z 3480 -t 0.918 -w 0.004 -g 0.435 -r 0.15 -x 1.8,1.9 -d 1994,5,1 \
-b -33,35,00 -l -70,08,00 esasta05 > rad18.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.05.ipw > esasta05

topquad -n -z 3480 -t 0.533 -w 0.004 -g 0.413 -r 0.05 -x 1.9,2.0 -d 1994,5,1 \
-b -33,35,00 -l -70,08,00 esasta05 > rad19.out

topquad -n -z 3480 -t 0.400 -w 0.003 -g 0.367 -r 0.05 -x 2.0,2.1 -d 1994,5,1 \
-b -33,35,00 -l -70,08,00 esasta05 > rad20.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.07.ipw > esasta05

topquad -n -z 3480 -t 0.042 -w 0.271 -g 0.398 -r 0.07 -x 2.1,2.2 -d 1994,5,1 \
-b -33,35,00 -l -70,08,00 esasta05 > rad21.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.10.ipw > esasta05

topquad -n -z 3480 -t 0.059 -w 0.0844 -g .373 -r .10 -x 2.2,2.3 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad22.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.07.ipw > esasta05

topquad -n -z 3480 -t 0.107 -w 0.025 -g .364 -r .07 -x 2.3,2.4 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad23.out

rm esasta05

mux ~/Images/Chile/Radiation/echaurren_fix_elev
~/Images/Chile/Radiation/echaurren_fix_gradient
~/Images/Chile/Radiation/echaurren.viewf
~/Images/Chile/Radiation/albedo.0.05.ipw > esasta05

topquad -n -z 3480 -t 0.104 -w 0.027 -g .408 -r .05 -x 2.4,2.5 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad24.out

topquad -n -z 3480 -t 2.45 -w 0.002 -g .357 -r .05 -x 2.5,2.6 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad25.out

topquad -n -z 3480 -t 4.33 -w 0.0003 -g .309 -r .05 -x 2.6,2.7 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad26.out

topquad -n -z 3480 -t 4.82 -w 0.0002 -g .267 -r .05 -x 2.7,2.8 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad27.out

topquad -n -z 3480 -t 2.818 -w 0.0002 -g .270 -r .05 -x 2.8,2.9 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad28.out

topquad -n -z 3480 -t 0.440 -w 0.001 -g .361 -r .05 -x 2.9,3.0 -d 1994,5,1 \ -b -33,35,00 -l -70,08,00 esasta05 > rad29.out

topquad -n -z 3480 -t 0.203 -w 0.004 -g .360 -r .05 -x 3.0,3.1 -d 1994,5,1 \
-b -33,35,00 -l -70,08,00 esasta05 > rad030.out

mux \nrad03.out rad04.out rad05.out rad06.out rad07.out rad08.out \nrad09.out rad10.out rad11.out rad12.out rad13.out rad14.out \nrad15.out rad16.out rad17.out rad18.out rad19.out rad20.out \nrad21.out rad22.out rad23.out rad24.out rad25.out rad26.out \nrad27.out rad28.out rad29.out rad30.out | lincom -c 1 > may01net_5
Appendix C - Equivalency of two forms for $R^2$

Equation C.1

\[ R^2 = 1 - \frac{\sum_{i=1}^{i=N} (y_i - \bar{u})^2}{\sum_{i=1}^{i=N} (y_i - \bar{y})^2} \]

Rewrite C.1 as a single term

\[ R^2 = \frac{\sum_{i=1}^{i=N} (y_i - \bar{y})^2 - \sum_{i=1}^{i=N} (y_i - \bar{u})^2}{\sum_{i=1}^{i=N} (y_i - \bar{y})^2} \]

Expand terms in parenthesis

\[ R^2 = \frac{\sum_{i=1}^{i=N} y_i^2 - 2\sum_{i=1}^{i=N} y_i \bar{y} + \sum_{i=1}^{i=N} y_i^2 - 2\sum_{i=1}^{i=N} y_i \bar{u} + \sum_{i=1}^{i=N} \bar{u}^2}{\sum_{i=1}^{i=N} y_i^2 - 2\sum_{i=1}^{i=N} y_i \bar{y} + \sum_{i=1}^{i=N} y_i^2} \]

Equation 3.2

\[ R^2 = \frac{\sum_{i=1}^{i=N} (u_i - \bar{y})^2}{\sum_{i=1}^{i=N} (y_i - \bar{y})^2} \]

Expand terms in parenthesis

\[ R^2 = \frac{\sum_{i=1}^{i=N} u_i^2 - 2\sum_{i=1}^{i=N} u_i \bar{y} + \sum_{i=1}^{i=N} \bar{y}^2}{\sum_{i=1}^{i=N} y_i^2 - 2\sum_{i=1}^{i=N} y_i \bar{y} + \sum_{i=1}^{i=N} y_i^2} \]
Combine like terms and simplify

\[ R^2 = \frac{-2 \sum_{i=1}^{i=N} y_i \bar{y} + N \bar{y}^2 + 2 \sum_{i=1}^{i=N} y_i \bar{u}_i - \sum_{i=1}^{i=N} u_i^2}{\sum_{i=1}^{i=N} y_i^2 - 2 \sum_{i=1}^{i=N} y_i \bar{y} + N \bar{y}^2} \]

Simplify expression

\[ R^2 = \frac{\sum_{i=1}^{i=N} u_i^2 - 2 \sum_{i=1}^{i=N} u_i \bar{y} + N \bar{y}^2}{\sum_{i=1}^{i=N} y_i^2 - 2 \sum_{i=1}^{i=N} y_i \bar{y} + N \bar{y}^2} \]

The two expressions above are equivalent when their numerators are equal. This is shown below as:

\[ -2 \sum_{i=1}^{i=N} y_i \bar{y} + N \bar{y}^2 + 2 \sum_{i=1}^{i=N} y_i \bar{u}_i - \sum_{i=1}^{i=N} u_i^2 = \sum_{i=1}^{i=N} u_i^2 - 2 \sum_{i=1}^{i=N} u_i \bar{y} + N \bar{y}^2 \]

\( N \bar{y} \) is on both sides of the above equation, hence canceling the terms gives the required condition for equation C.1 to be equivalent to equation 3.2. This is shown below as equation C.2.

**Equation C.2**

\[ -2 \sum_{i=1}^{i=N} y_i \bar{y} + 2 \sum_{i=1}^{i=N} y_i \bar{u}_i - \sum_{i=1}^{i=N} u_i^2 = \sum_{i=1}^{i=N} u_i^2 - 2 \sum_{i=1}^{i=N} u_i \bar{y} \]

In the case of evaluating regression trees using the data used to generate them, this condition holds due to the fact that the each prediction, \( u_i \), is the mean of a group
of observations, $y_i$, and that the same $y_i$'s are used in the expressions above.

Starting with the definition of a prediction for a binary regression tree we have,

\begin{equation}
\sum_{j=M_k}^{j=M_k} y_j
\end{equation}

Where $u_i$ is the prediction for a given $y_i$ which is given by terminal node $k$ of the regression tree, $j$ is a dummy index denoting an observation which generated node $k$, and $M_k$ is the total number of observations, $y_j$, which generated the node. Note that the observation $y_i$ is a specific instance of the observations $y_j$.

Any terms in equation C.2 that involve constants multiplied by the sum of all $u_i$ can be replaced by the equation below since the cumulative property allows one divide up the sum over $n$. This can be written as,

$$\sum_{i=1}^{i=N} u_i = \sum_{k=1}^{k=P} \left( \sum_{i=1}^{i=M_k} \left( \sum_{j=1}^{j=M_k} \frac{y_j}{M_k} \right) \right)$$

where $P$ is the total number of terminal nodes in the regression tree. Evaluating the summations inside the brackets gives,
\[
\sum_{i=1}^{i=N} \sum_{j=1}^{j=M} \left[ y_i \sum_{k=1}^{k=P} \frac{y_j}{M_k} \right] = \sum_{i=1}^{i=N} y_i
\]

But since \( M_k \) is constant over the inner summation \( j \) the inner \( M_k \) can be pulled out and canceled with the outer \( M_k \). Furthermore, for each unique combination of \( k \) and \( j \) there is a specific \( y_j \). Thus we have,

**Equation C.4**

\[
\sum_{i=1}^{i=N} u_i = \sum_{i=1}^{i=N} y_i
\]

A similar expression to the previous equation is shown below. This equation can be derived in a similar manner.

**Equation C.5**

\[
\sum_{i=1}^{i=N} y_i u_i = \sum_{i=1}^{i=N} u_i^2
\]

Using equations C.4 and C.5 to either replace all terms involving \( u_i \) or replace all terms involving \( y_i \) shows that both sides of the equality in equation C.2 are the same.

Equations C.4. and C.5 only hold, however, when the observations \( y_i \) are those that were used to generate the tree. If not, the \( y_j \)'s in equation C.3 are, in general, different than the \( y_i \)'s used in the summations for C.4 and C.5 and there is
no longer a relationship between the indices I and the indices j and k. Even for cross validation the correspondence in equation C.3 does not hold since for a given regression tree generated by leaving out an observation $y_i$, the sum of the predictions are done over the nodes for which the $y_i$ 's were left out. In the case of cross validation, the two expressions, C.1 and 3.2, must have different interpretations.
References


Myers, R. H., Classical and Modern Regression with Applications, PWS publishers, Boston, MA, 1986.


