THE SIMULATION OF HAAR TELEMETRY SYSTEM WITH MICROCOMPUTER

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ABSTRACT

The mathematical basis which can form a telemetry system is orthogonal functions. The Haar function set forms a complete set of orthogonal functions. Assording to the princeples of orthogonal multiplexing, a new telemetry system can be formed with Haar functions.

Haar functions assume the values +1, 0 and -1, multiplied by powers of $\sqrt{2}$. In this paper we first make a modification to the Haar functions. The functions are so modified that it is suitable for computer to simulate and it also can be easily realized by hardware. The orthogonality of the orthogonality of the modified Haar functions is unchanged. Then,the simulation of Haar telemetry system with microcomputer is given by software. Finally, we have proved that the design of Haar telemetry system is workable.

INTRODUCTION

The basic problem of a multiplexing system is how to transmit many information signals over a single wire or radio link and recover them without interference between channels.

Non-interference between channels can be guaranteed in a multiples system if the selected set of subcarriers are mutually orthogonal, meaning that the product of any two subcarricr waveforms integrates to zero over some characteristic interval T. Any two subcarriers P(n,t) and P(m,t) are orthogonal if they satisfy the relationship:

$$\frac{1}{T}\int_0^T P(n,t)\ P(m,t)\ dt = 0$$

An optimum multiplex system is shown in figure 1. In communication systems, frequency division multiplex, time division multiplex and code division multiplex are usually used nowadays. Their common property is the utilization of orthogonal functions. The difference is that the orthogonal function sets used are different. For example, since and cosine functions are used as subcarriers in frequency division multiplex, block pulses are used as subcarriers in time division multiplex, Walsh functions are used as subcarriers in code division multiplex. It is proved that all orthogonal function sets may in principle be used for multiplexing. In practical applications, there are some factors which should be considered in choosing waveforms, such as ease of generation, detectability, and flexibility in engineering.

THE HAAR FUNCTION

The Haar function set forms a complete set of orthogonal rectangular functions similar in several respects to the Walsh function. Haar functions were introduced by the Hungarian mathematician Alfred Haar in 1910 [1]. The amplitude values of the Haar functions assume a limited set of values, 0, ±1, $\pm\sqrt{2}$, ±2, $\pm 2\sqrt{2}$, ±4 etc. If we consider the time base to be defined as $0 \le t \le 1$, then the Haar function {har(n,k,t)} may be written as follows:

$$\mathrm{har}(0,0,t) = 1, \qquad \text{for } 0 \le t \le 1$$

$$\mathrm{har}(0,1,t) = \begin{cases} +1 & \text{for } 0 \le t \le 1/2 \\ -1 & \text{for } 1/2 \le t \le 1 \end{cases}$$

$$\cdots\cdots \qquad \cdots\cdots$$

$$\mathrm{har}(n,k,t) = \begin{cases} 2^n & \text{for } \frac{2k-2}{2^{n+1}} \le t < \frac{2k-1}{2^{n+1}} \\ -2^n & \text{for } \frac{2k-1}{2^{n+1}} \le t < \frac{2k}{2^{n+1}} \\ & n=0,1,2,\ldots\ldots \quad k=1,2,\ldots,2^n \\ 0 & \text{elsewhere} \end{cases}$$

Then the Haar functions can be referred to by order, k, and degree, n, as well as time , t. The degree, n, then denotes a subset having the same mumber of zero crossings in a given width, $1/2^n$, thus providing a form of comparison with frequency and sequency terminology. The order, k, gives the position of the function within this subset. All members of the subset with the same degree are obtained by shifting the first member along the time axis by an amount proportional to its order. The first eight Haar functions are shown in Figure 2.

THE MODIFIED HAAR FUNCTION

From Figure 2 we know that the values of Haar function are multiplied by powers of $\sqrt{2}$. since $\sqrt{2}$ is not easy to simulate precisely by computer, a proper modification is needed.

The orthogonality of the modified Haar function is unchanged. The modified function is defined as follows:

$$\mathrm{har}(0,t) = 1 \qquad \text{for} \quad 0 \leqslant t < T$$

$$\mathrm{har}(1,t) = \begin{cases} +1 & \text{for} \quad 0 \leqslant t < T/2 \\ -1 & \text{for} \quad T/2 \leqslant t < T \end{cases}$$

$$\mathrm{har}(2^i+j,t) = \begin{cases} +1 & \text{for} \quad 0 \leqslant t < T/4 \\ -1 & \text{for} \quad T/4 \leqslant t < T/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\cdots\cdots \qquad\qquad \cdots\cdots$$

$$\mathrm{har}(2^i+j,t) = \begin{cases} +1 & \text{for} \quad \frac{jT}{2^i} \leqslant t \leqslant \frac{(j+1/2)T}{2^i} \\ -1 & \text{for} \quad \frac{(j+1/2)T}{2^i} \leqslant t \leqslant \frac{(j+1)T}{2^i} \\ 0 & \text{elsewhere} \end{cases}$$

$$i=1,2,\ldots;\quad j=0,1,\ldots,2^i-1$$

Now, let us prove the orthogonality of Haar functions.

(1) $m=n$

$$\int_0^T \mathrm{har}(m,t)\,\mathrm{har}(n,t)\,dt = \int_{\frac{jT}{2^i}}^{\frac{(j+1/2)T}{2^i}} 1\cdot 1\,dt + \int_{\frac{(j+1/2)T}{2^i}}^{\frac{(j+1)T}{2^i}} (-1)\cdot(-1)\,dt$$

$$= \int_{\frac{jT}{2^i}}^{\frac{(j+1)T}{2^i}} dt = (j+1)T/2^i - jT/2^i = T/2^i$$

(2) $m \neq n$

The first case, $m=2^i+j_1$, $n=2^i+j_2$, $j_1 \neq j_2$ that is: the same i and the different j. $\because$ f or any J_1, $j_2 \in (0,1,\ldots,2^{i-1})$, $j_1 \neq j_2$ there is $\mathrm{har}(2^i+j_1,t)\mathrm{har}(2^i+j_2,t) = 0$ $\therefore$ $\mathrm{har}(2^i+j,t)\mathrm{har}(2^i+j,t)dt=0$

The second case, $m=2^i+j_1$, $n=2^i+j_2$, $i_1 \neq i_2$ that is : for the different i, for any i_1, $i_2 \in(1,2,\ldots)$, $i_2 > i_1$, (if $i_1 > i_2$ will make no difference) and $j, \in (0,1,\ldots,2^i-1)$, $j_2 \in (0,1,\ldots,2^i-1)$, by the definition of modified Haar function we can find that:

$$har(2^{i_1}+j_1,t)\ har(2^{i_2}+j_2,t)=\begin{cases} har(2^{i_2}+j_2,t), & \text{for } \frac{j_1 T}{2^{i_1}} \leqslant \Omega \leqslant \frac{(j_1+1/2)T}{2^{i_1}} \\ -har(2^{i_2}+j_2,t), & \frac{(j_1+1/2)T}{2^{i_1}} \leqslant \Omega \leqslant \frac{(j_1+1)T}{2^{i_1}} \\ 0, & \text{elsewhere} \end{cases}$$

Ω is the area where $|har(2^{i_2}+j_2,t)| \neq 0$

In this case we have:

$$\int_0^T har(2^{i_1}+j_1,t)\ har(2^{i_2}+j_2,t)\ dt = 0$$

By the combination of the above results:

$$\int_0^T har(m,t)\ har(n,t)\ dt = \begin{cases} T/2^i, & \text{for } m=n \\ 0, & \text{for } m \neq n \end{cases}$$

Where i is degree of Haar function, i = 0,1,2,...
The last equation shows that the orthogonality of the modified Haar functions is unchanged.

SYSTEM DESIGN

Microcomputer is widely used. In our system, the microcomputer is used to simulate the Haar waveform generator, the mulitiplier and the adder and so on. The diagram of the system is shown in Figure 3. The working principle is roughly the same as the system described in Figure 1, but this system belongs to digital system instead of analogue one.

The Simulation of Haar Waveform

For the first 15 channels of Haar waveform. We define [H] as 15x16 degree of Haar discrete matrix, and H_i as the I^{th} line element in [H], i=1,2,...,15. We divide the orthogonal period [O,T) in to 16 sub-intervals, and each of the sub-intervals is equal to T/16. We suppose T=16. Then we can get the simulated Haar waveform. For example, har(3,t) is equal to H_3. That is:

$$Har(3,t) = H_3 = [1\ 1\ 1\ 1\ -1\ -1\ -1\ -1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]_{1x16}$$

Har(5,t) is equal to H_5, that is:

$$Har(5,t) = H_5 = [1\ 1\ -1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]_{1x16}$$

And so on, and so forth.

So, the matrix [H] is the simulated Haar waveform, and the microcomputer can easily process it.

The Program in the Transmitting Terminal

The feafure of Haar waveform is that: at the any interval in its orthogonal period, the waveforms with non-zero value only take a small part in all channels. Take the 16 channels system for example, the sum of signals at any time is only the sum of 4 channels. To take advantage of this feature in program making, we first modulate those waveforms whose non-zero waveforms are on the left side of time axis, and then middle, right side. For each sub-interval, the narrow one is modulate first, and then the wide, the wider, and the widest one. The order of modulation is shown in Figure 4 on the left side. By so doing, first, we have solved the problem of the shortage of reg isters in microcomputer; second, we can make full use of CPU’S working time, and shorten its waiting time which is caused by the limited speed of A/D converter.

After four channels of signal are transfered and put into the µp(µp=microcomputer) by A/D, the µp starts to process them very quickly and soon after that the sum of four processed signals is sent out. Immediately, the µp begin to process the following input signals. The program chart is shown in Figure 5. Sixteen times of the above procedure are needed for processing a frame the sum signals.

The Program in The Receiving Terminal

We take the advantage of Haar waveform’s feature in programing in the receiving terminal. Since the sum of signals received by the receiver is modulated in such way as described, we are going to demodulate it in the same way. We first use those waveforms whose non-zero waveforms are on the left side of time axis to multiply the sum of signals and the rest of the procedure of demodulating; and then we use middle ones, and so on. The order of demodulation is also shown in Figure 4 on the right side. The advantage of making program in this way is that it is very easy to synchronize the softwares in both terminals. What is more, time is saved and the system workes more efficiently. But the time delayed for each signal is not the same. We can find by examining the numbers on the both side of Figure 4. The different time delay will make no difference. The program chart is shown in Figure 6.

Data Transmission and Synchronization

There are three kinds of way to transmit data in µp: synchronization (or: uncondition) , polling (or: condition or asychronization) and interruption. The last two ways are more reliable for data transmission, but they take more CPU time than the first one. The first means of deta transmission is adopted here. The advantage of unconditional transmission is that it can save time and simplify the interfacing circuits; but the softwares in transmitting and receiving terminals are required to be strick syschronized, otherwise the deta will be lost or mixed up. For example, if before the last deta been taken a new data comes, the last one will be lost. If the µp wants to take a new data before it comes, the µp will take the old one or last one twice. Both cases are shown in Figure 7.

To avoid the above cases, the program must be made very carefully. Every instruction's excuting time has to be take into consideration. The time when the receiving µp takes a data should be a little delay to the time when the transmitting µp sends the same data, and all SEND and TAKE must be one by one, as shown in Figure 8. The program made accorking to Figure 8 is in principle workable. But, sometimes the crystal clock frequency in µps is not exactly the same. So even the program made accorking to Figure 8, the mistakes shown in Figure 7 can probably be occured. During the data transmission. For example, if the clock frequeny in the receiver is a little bit higher than that of transmiter's, Δt in Figure 8 will become less and less, and finally the data-repeat-taken happens. On contrary, if the clock frequency in the transmiter is higher than that in receiver, t in Figure 8 will become less and elss, and the data will be lost. To solve this problem, a changing-steps means of receiving and transmitting is adopted in the system, as shown in Figure 9.

Addorking to Figure 9, if the clock frequency in the receiver is higher than that in transmiter, we still have $\Delta t_k \cong$ constant, (k=1,2,...) because when j>i, there is $\Delta t_j > t_i$. Therefore the mistake of data-repeat-taken is avoided. If the clock frequency in the transmiter is higher, we still can have t_μ=constant, (k=1,2,...) because when j>i, there is $t_j > t_i$. So the data will never be lost. Besides the changing-steps, the frame synch ronization is done for every frames of data. After these measures are taken, the data transmission is perfectly reliable.

CONCLUSION

We have made a system as shown in Figure 3 in our laboratory. After carefully making the microcomputer's interfacing circuits and the system software. The system works well and the results come out quite good: eight channels of signal can simultaneously be transmited; the highest frequency of signals to be transmited is $200H_z$. The experimental results demonstrate that the design of Haar telemetry system with microcomputer is workable.

ACKNOWLEDGEMENT

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REFERENCES

1. K.G.Beauchamp, "Walsh Function and Their Applications" Academic press, New York, 1975, pp. 84-87

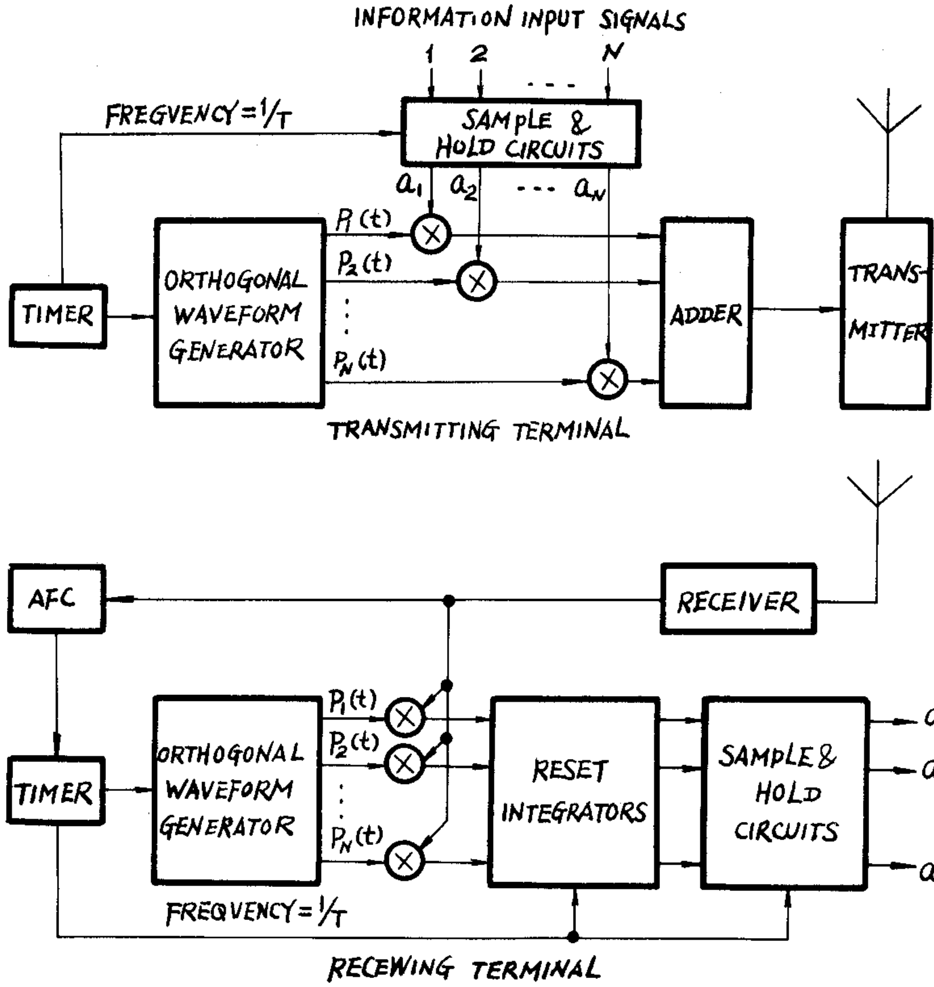

Figure 1. Multiplex System.

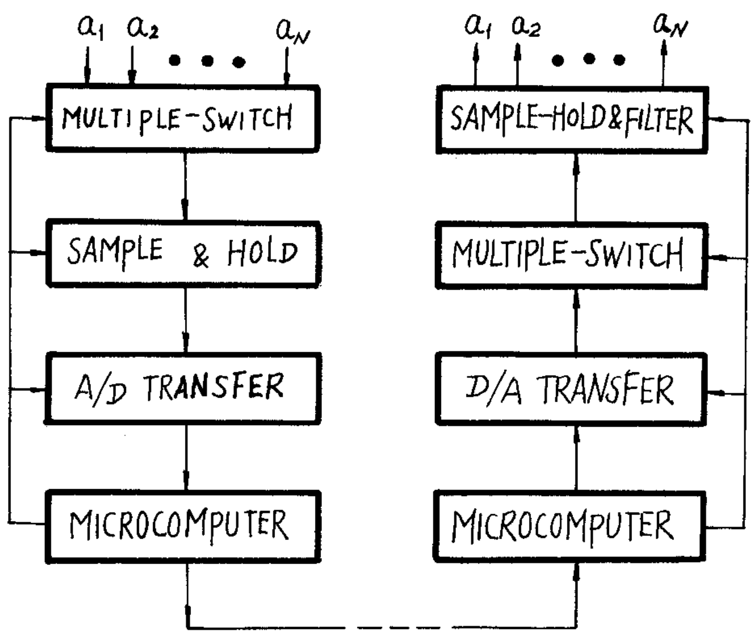

Figure 3. Simulated System

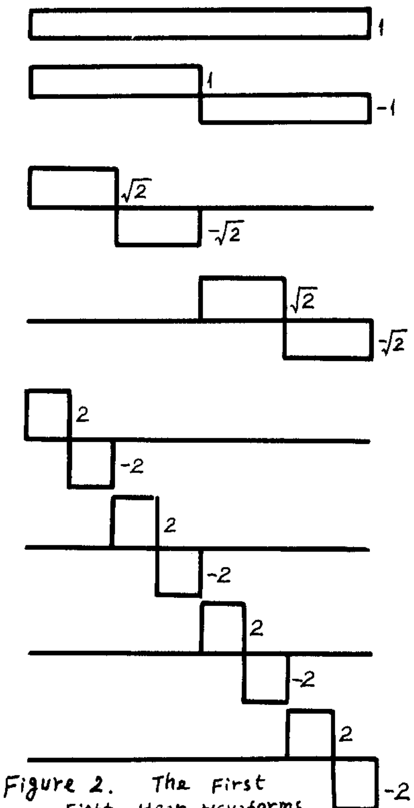

Figure 2. The First Eight Haar waveforms

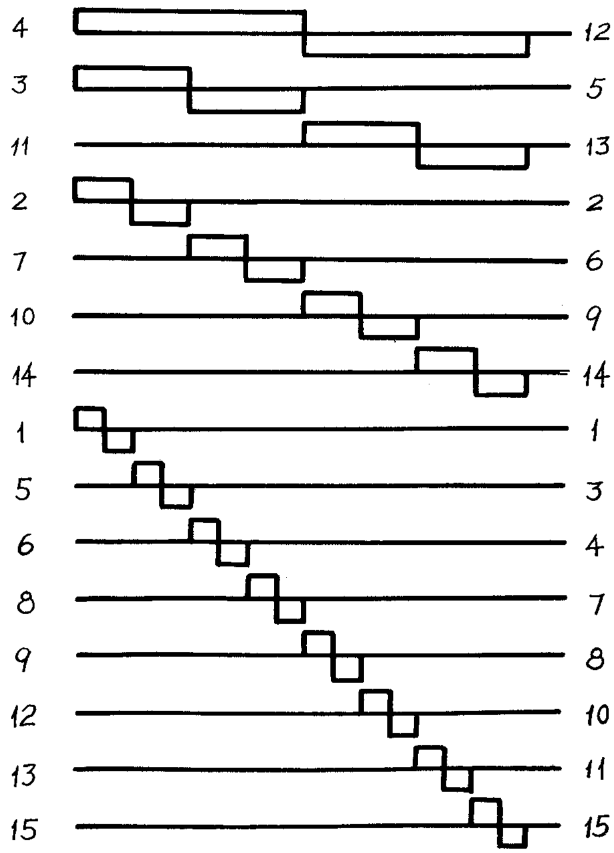

Figure 4. The order of modulation and Demodulation

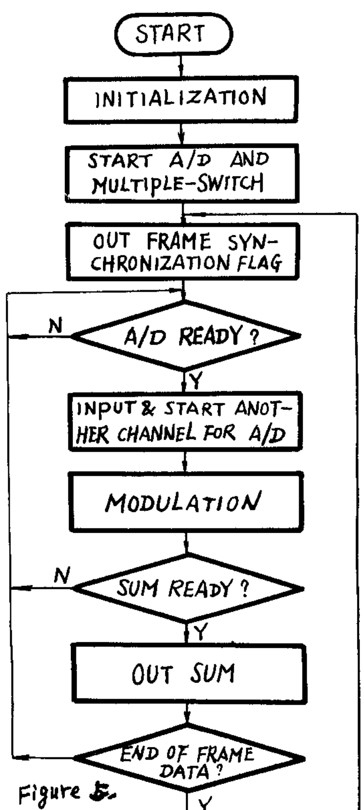

Figure 5. Transmitting Terminal Program Chart

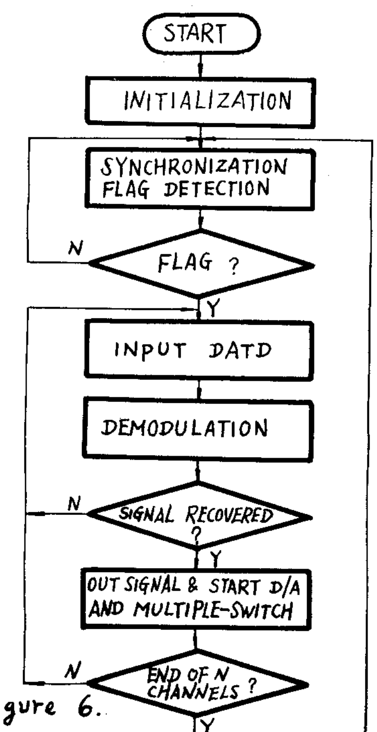

Figure 6. Receiving Terminal Program Chart

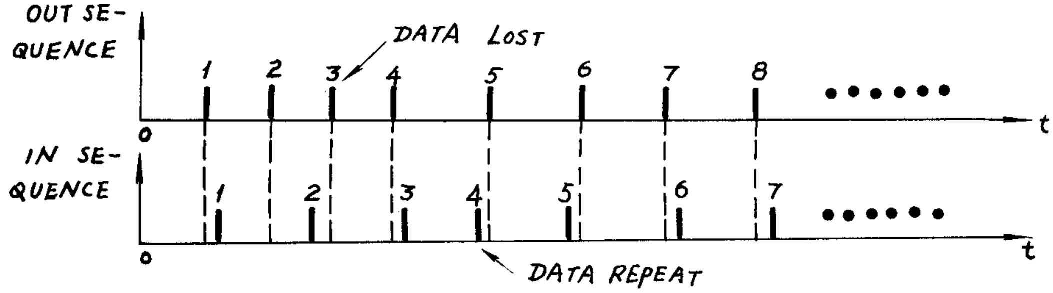

Figure 7. Data Lost and Data Repeat

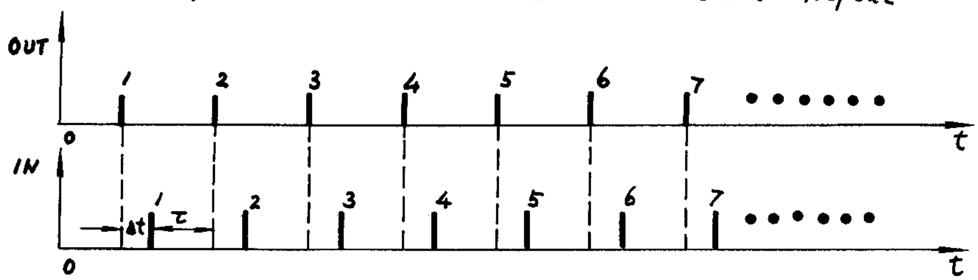

Figure 8. Synchronization Sequence

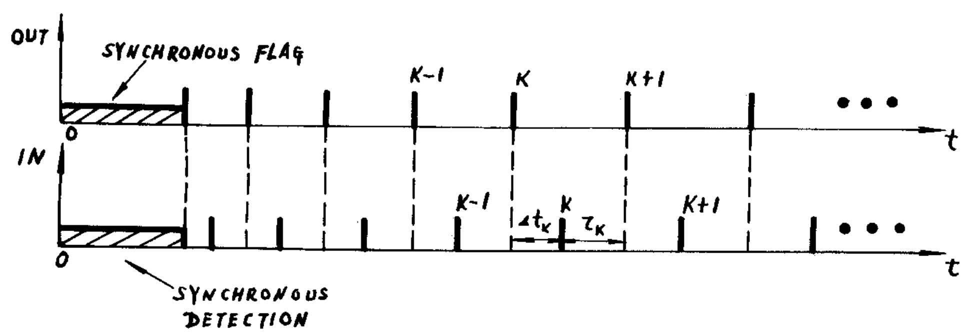

Figure 9 The Changing Steps Synchronization Sequence $\begin{cases} \Delta t_j > \Delta t_i \\ \tau_j > \tau_i \end{cases}$, for $j > i$