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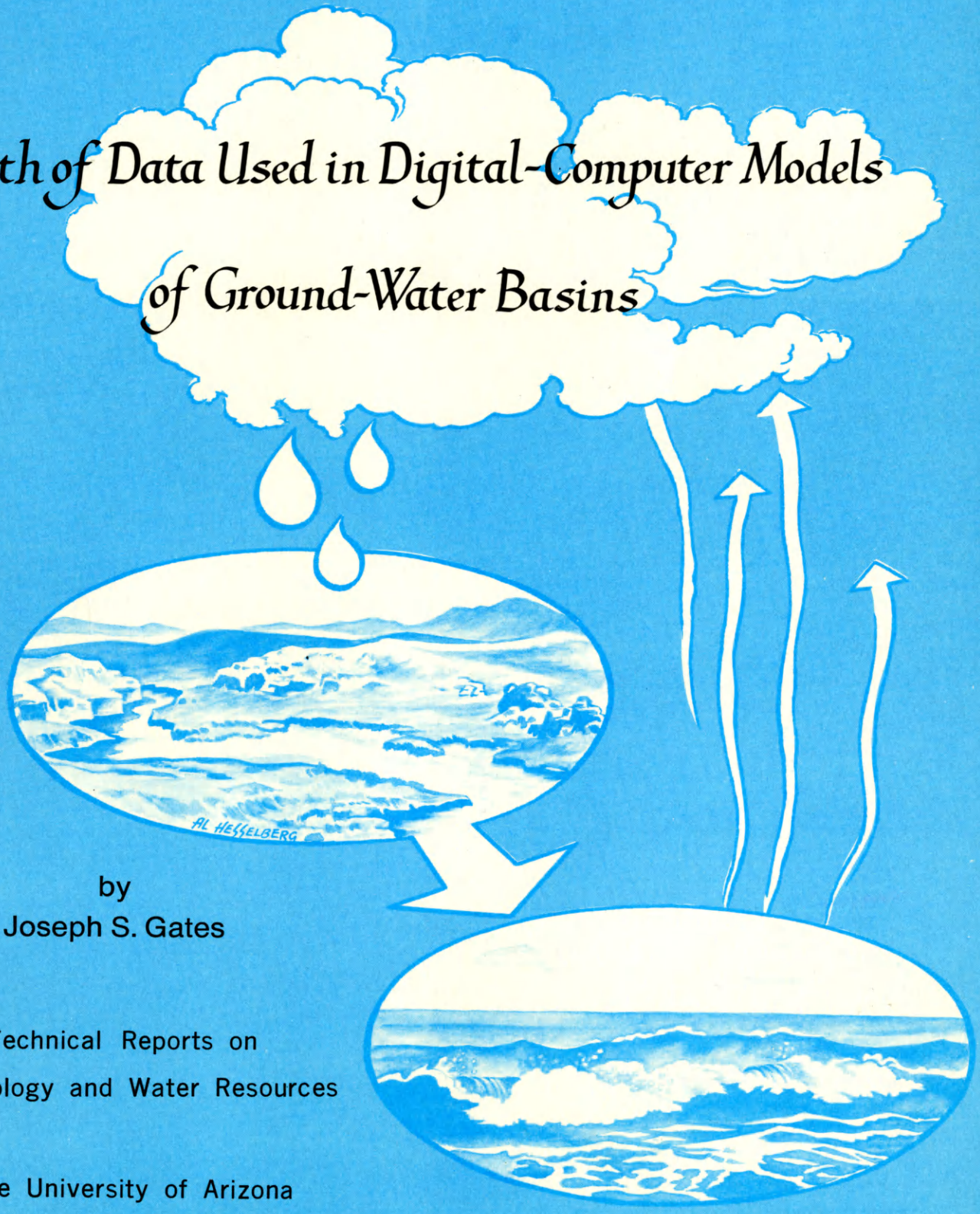
Worth of Data Used in Digital-Computer Models of Ground-Water Basins



by
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Hydrology and Water Resources

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WORTH OF DATA USED IN DIGITAL-COMPUTER MODELS OF GROUND-WATER BASINS

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Joseph Spencer Gates

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PREFACE

This report constitutes the doctoral dissertation of the same title completed by the author in April, 1972 and accepted by the Department of Hydrology and Water Resources.

This report series constitutes an effort to communicate to practitioners and researchers the complete research results, including economic foundations and detailed theoretical development that cannot be reproduced in professional journals. These reports are not intended to serve as a substitute for the review and referee process exerted by the scientific and professional community in these journals. The author, of course, is solely responsible for the validity of the statements contained herein. A complete list of currently-available reports may be found in the back of this report.

ACKNOWLEDGMENTS

I am indebted to several persons who gave advice and information during the course of this study. Dr. C. C. Kisiel provided overall guidance and frequently furnished more ideas than I had time or ability to utilize. Other members of my committee, especially Drs. E. S. Simpson, J. R. Sturgul, and J. W. Harshbarger, also gave assistance. I am especially grateful for the help I received from fellow graduate students. It was a privilege to work with Allen F. Moench and William M. Little of the Department of Hydrology and Water Resources, Robert L. Knickerbocker of Computer Science, and Donald R. Davis, Robert E. Lovell, and Norval Baran of Systems Engineering, among others. I also would like to thank E. S. Davidson and T. W. Anderson of the U. S. Geological Survey and E. C. Rebuck, W. G. Matlock, and P. R. Davis of the Department of Agricultural Engineering at The University of Arizona for furnishing data and advice. I also benefited from attending a seminar presented by Thomas Maddock III of the U. S. Geological Survey, and from discussions with J. F. Rauscher, Chief Engineer of the Tucson Department of Water and Sewers. Mr. Rauscher also furnished a copy of a letter from Mark Wilmer, attorney for the firm of Snell and Wilmer, Phoenix, Arizona, which contained data on water use in the southern part of the Tucson basin. I also am grateful to Donald W. Peaceman, of the Esso Production Research Company, who furnished a set of lecture notes for a NATO Symposium on "Hydrocarbon Reservoir Simulation by Computers" to the Department of Hydrology and

Water Resources. My wife, Connee, and daughters cannot, of course, be thanked enough for their patience and encouragement. My wife typed and assisted in editing the first draft of the dissertation and Miss Bonnie Jo Barthold typed the manuscript.

This study was supported in part by funds from a matching grant (B-007-ARIZ) on "Efficiency of Data Collection Systems in Hydrology and Water Resources for Prediction and Control"; and to a lesser extent an allotment grant (A-010-ARIZ) on "Operations Research Study of Water Resources Development and Management of the Tucson Basin." These funds were provided to the Water Resources Research Center, University of Arizona, by the Office of Water Resources Research, U. S. Department of the Interior under the authorization of the Water Resources Research Act of 1964. I was also supported during the period September 1970 - June 1971 under the graduate school training program for employees of the U. S. Geological Survey, and during August and September 1971 by a contract between The University of Arizona and the city of Tucson on digital modeling of the Tucson basin.

Computations for this study were done on the CDC-6400 installed at The University of Arizona Computer Center.

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ABSTRACT

Two digital-computer models of the ground-water reservoir of the Tucson basin, in south-central Arizona, were constructed to study errors in digital models and to evaluate the worth of additional basic data to models. The two models differ primarily in degree of detail -- the large-scale model consists of 1,890 nodes, at a 1/2-mile spacing; and the small-scale model consists of 509 nodes, at a 1-mile spacing.

Potential errors in the Tucson basin models were classified as errors associated with computation, errors associated with mathematical assumptions, and errors in basic data: the model parameters of coefficient of storage and transmissivity, initial water levels, and discharge and recharge. The study focused on evaluating the worth of additional basic data to the small-scale model.

A basic form of statistical decision theory was used to compute expected error in predicted water levels and expected worth of sample data (expected reduction in error) over the whole model associated with uncertainty in a model variable at one given node. Discrete frequency distributions with largely subjectively-determined parameters were used to characterize tested variables. Ninety-one variables at sixty-one different locations in the model were tested, using six separate error criteria. Of the tested variables, 67 were chosen because their expected errors were likely to be large and, for the purpose of comparison, 24 were chosen because their expected errors were not likely to be particularly large.

Of the uncertain variables, discharge/recharge and transmissivity have the largest expected errors (averaging 155 and 115 feet, respectively, per 509 nodes for the criterion of absolute value of error) and expected sample worths (averaging 29 and 14 feet, respectively, per 509 nodes). In contrast, initial water level and storage coefficient have lesser values. Of the more certain variables, transmissivity and initial water level generally have the largest expected errors (a maximum of 73 per feet per 509 nodes) and expected sample worths (a maximum of 12 feet per 509 nodes); whereas storage coefficient and discharge/recharge have smaller values. These results likely are not typical of those from many ground-water basins, and may apply only to the Tucson basin.

The largest expected errors are associated with nodes at which values of discharge/recharge are large or at which prior estimates of transmissivity are very uncertain. Large expected sample worths are associated with variables which have large expected errors or which could be sampled with relatively little uncertainty. Results are similar for all six of the error criteria used.

Tests were made of the sensitivity of the method to such simplifications and assumptions as the type of distribution function assumed for a variable, the values of the estimated standard deviations of the distributions, and the number and spacing of the elements of each distribution. The results are sensitive to all of the assumptions and therefore likely are correct only in order of magnitude. However, the ranking of the types of variables in terms of magnitude of expected error and expected sample worth is not sensitive to the assumptions, and

thus the general conclusions on relative effects of errors in different variables likely are valid.

Limited studies of error propagation indicated that errors in predicted water levels associated with extreme erroneous values of a variable commonly are less than 4 feet per node at a distance of 1 mile from the tested node. This suggests that in many cases, prediction errors associated with errors in basic data are not a major problem in digital modeling.

CHAPTER 1

INTRODUCTION

This study is an attempt to evaluate the worth of additional hydrologic data on a ground-water system. The work focused on potential errors associated with digital-computer models of ground-water basins and on the worth of data on aquifer parameters, initial conditions (water levels), and output/input (discharge and recharge) to a model of the Tucson basin, Arizona. As Bibby and Sunada (1971, p. 2) pointed out, "this [type of] deterministic model is frequently used in situations in which nothing is known of the accuracy of the input data to the model or how errors in the input data are related to the accuracy of the results."

Meteorologists also are interested in these problems as related to models which predict weather conditions. Hammond (1971, p. 394) noted that "numerical experiments are being done to answer questions about the amount and type of data that are most useful, about the effect on predictability of observational errors in the data, and about the methods by which data are to be incorporated into models."

The study consisted of two major parts: construction of the model of the Tucson basin and evaluation of worth of additional data to the model. A complex model of an actual basin was used for the study instead of a small, idealized model, such as has been used in other related studies (Bibby 1971, McMillan 1966), in order to gain

additional insight into actual modeling problems. To a considerable extent, this goal was realized. During construction and calibration of the model many actual and potential errors were discovered and studied. There are, however, marked disadvantages to using a large complex model. In general, sophisticated mathematical techniques cannot be applied because they would use excessive amounts of computer time. However, the aim here was to develop a technique that could be utilized by practicing field hydrologists (a category which includes the writer), so relatively practical methods which could be used in actual modeling efforts were developed, rather than methods for experimentation.

Worth of data was studied using basic concepts of statistical decision theory. Statistical decision theory has been developed over the past two decades to aid in making decisions with uncertain information. An important basis of the theory, Bayes Theorem, is by no means recent, however, as it was proposed by Thomas Bayes in the 1700's. Use of statistical decision theory has been primarily confined to business and industrial decisions, and it is just beginning to be used in scientific problems. Folayan (1969), for example, used decision theory to evaluate the reliability of predicted soil settlement. The full power of this body of theory could not be applied in the study reported here because of the complexity of the basin model; so a relatively simple application of Bayes Theorem using subjectively-determined, discrete frequency distributions was used.

The question addressed by this study is one commonly posed by a field hydrologist -- "What kinds of data on a ground-water basin

should I collect and where should I collect them in order to most improve my ability to predict the future behavior of the system?" This question is usually answered, if answered at all, by applying experience and intuition rather than by using any quantitative or more formal techniques. It is doubtful whether such approaches can be tolerated in the future, as the demands made on the limited funds for hydrologic studies likely will intensify. This study is one of a few beginning attempts to provide objective methods for planning programs for collecting hydrologic data.

Perhaps studies such as this and the work by Meyer (1971) also will stimulate the use of preliminary models of ground-water basins to guide basin investigations by pointing out which aspects of the ground-water system are significant in predicting effects of development.

The Tucson Basin

The Tucson basin (fig. 1) includes an area of about 1,000 square miles in south-central Arizona (see index map, fig. 9), and is traversed by the Santa Cruz River and its principal tributaries, Rillito Creek, Pantano Wash, and Canada del Oro. The most recent and comprehensive evaluation of the water resources of the basin is the several chapters of U. S. Geological Survey Water-Supply Paper 1939. Most of the material in this summary section on the geography, geology, and water resources of the basin was taken from a report which will be published as one of these chapters (Davidson 1970).

Geography and Geology

The Tucson basin is an alluvial valley bounded primarily by the Tortolita and Santa Catalina Mountains on the north and northeast, the Rincon Mountains (which include Tanque Verde Ridge) on the east, the Santa Rita Mountains on the southeast, and the Sierrita Mountains and Tucson Mountains on the west. Parts of the boundary are low passes between the Tucson basin and adjacent alluvial basins, such as San Pedro Valley to the east and Avra-Altar Valley to the west. As defined for this study, the basin extends about 50 miles from the town of Rillito on the north, where the Santa Cruz River leaves the basin, to the Pima County-Santa Cruz County line on the south, where the river enters the basin. Along the Santa Cruz River, the altitude ranges from about 2,000 feet at Rillito to about 3,000 feet at the county line. Tucson, the only large city in the basin, is in its north-central part.

The climate of the basin is semiarid and warm. Precipitation over the basin ranges from 11 to 12 inches per year in the vicinity of Tucson to more than 25 inches in the adjacent Santa Catalina Mountains. About 65 percent of the precipitation falls between May and October and about 50 percent in thunderstorms in July and August (Davidson 1970, p. 38). The annual potential evapotranspiration is several times the annual precipitation.

Geologically, the basin is an elongated structural valley filled with unconsolidated alluvial deposits and older semi-consolidated and consolidated alluvial deposits. These deposits, which are more than 2,000 feet thick in parts of the basin, include the Pantano Formation

and Tinaja beds of Tertiary age and the Fort Lowell Formation and surficial deposits of Quaternary age. The mountains are composed primarily of metamorphic, intrusive igneous, and volcanic rock and to a lesser extent consolidated sedimentary rock. Structurally, the basin has been downfaulted with respect to the mountain blocks, which was a necessary condition for the accumulation of the basin fill. Faulting continued during the deposition of the fill, as beds of Tertiary and Quaternary age are offset.

Water Resources

The primary source of water in the Tucson basin is obtained from its ground-water reservoir. Tucson is one of the largest cities in the United States that is totally dependent on ground water, and thus knowledge of the ground-water reservoir is extremely important to the city and all residents of the basin. Realization of this fact has led to virtually continuous study of the water resources of the basin by the city of Tucson, The University of Arizona, the U. S. Geological Survey, the U. S. Bureau of Reclamation, and the U. S. Army Corps of Engineers, among others. The investigation reported here is a small part of this continuing effort to understand and manage the basin's ground-water reservoir.

The ground-water reservoir has been defined as a single, unconfined aquifer which includes all of the unconsolidated and semi-consolidated sediments which make up the basin fill. A vast amount of water is stored in this reservoir -- estimated by Davidson (1970, p. 13)

to be about 52 million acre-feet to a depth of 1,000 feet below the water table.

In 1965 about 160,000 acre-feet of water was pumped from the basin. More than 50 percent was used for irrigation, about 35 percent for public supply, and about 15 percent for industrial purposes (Davidson 1970, p. 14). Ground water in the aquifer is partly replenished by infiltration of streamflow to the channel of the Santa Cruz River and its tributaries and by subsurface inflow. Of the estimated 110,000 acre-feet per year of recharge to the basin during the 1960's, about 51,000 acre-feet was supplied by streamflow infiltration (Davidson 1970, p. 213).

Streamflow is not used directly for water supply in the basin because it is too erratic in time, duration, and volume of flow. The Santa Cruz River and Rillito Creek, for example, are dry on the average of 320-335 days per year (Davidson 1970, p. 163). Flow in the streams is mainly in response to summer thunderstorms or winter frontal storms, and individual flow events rarely last more than a few days. The mean annual streamflow past gaging stations on the major streams of the basin is about 10,000 to 20,000 acre-feet; the mean annual streamflow out of the basin is about 17,000 acre-feet (Davidson 1970, p. 10).

History of Modeling Fluid Reservoirs in Porous Media

Pinder and Bredehoeft (1968) presented a good summary of the development of reservoir modeling. Electrical-analog computers made up of resistor-capacitor networks were used originally in the early

1940's to model the flow of heat but soon were adopted by the petroleum industry to solve problems involving oil and gas reservoirs. In the early 1960's analog computers were employed to study ground-water flow, and since then the U. S. Geological Survey and the Illinois State Water Survey, among others, have used these computers extensively.

Digital computers first were used to attack problems of oil and gas reservoirs in the early 1950's. Digital models are very similar to analog models in that they both solve the partial differential equations of fluid flow by applying finite-difference approximations. When high-speed computers with large memories became available they soon were utilized to study reservoirs of fluids in porous media. Stallman (1956) first discussed the application of numerical analysis to ground-water problems, but the first large-scale use of digital computers to study the dynamics of ground-water basins was by the California Department of Water Resources (Tyson and Weber 1964). Tyson and Weber employed a relaxation technique, essentially the Gauss-Seidel iteration method, to solve the set of equations that represents a ground-water basin. Since then, digital computers have been used increasingly to solve fluid-reservoir problems. Many new solution techniques have been developed, primarily by the petroleum industry, although the finite-element (as opposed to finite-difference) method was taken from structural engineering.

A method that is currently popular in the petroleum industry (Peaceman and Rachford 1955) and in ground-water studies (Pinder and Bredehoeft 1968) is the alternating-direction-implicit technique. This

method is faster computationally than the Gauss-Seidel method and often requires less computer memory.

Little has been published, however, on studies of errors in digital and analog models and on the best methods of reducing errors, although Landau (1963) studied the accuracy of analog models used in heat-flow studies. Most investigators have been content to apply their numerical or analog technique to a relatively uncomplicated problem for which an analytical solution can be derived, and if the results match reasonably well, they assume the technique also will give good solutions to complex problems. This procedure shows that the particular computer model is a valid way to approximate an analytical solution which is itself an idealized model, but it tells less about the errors in modeling a complex hydrogeologic system. A few studies have touched on this problem but the writer knows of none that have dealt with it in a comprehensive way. Another common check is to compare the results from an analog and digital model, although this really only validates the procedures used because the two methods are theoretically similar and will have equivalent errors. Limited analyses of the sensitivity of the results to variations in parameters also commonly are done in operational studies to estimate the possible variation from the "true" results.

Another aspect of modeling on which little has been published is model calibration. Calibration is the process in which initially assumed model parameters, initial conditions, and input/output functions are modified so that the model reproduces the known response of the physical system being modeled over some historical time period.

Calibration commonly is done by trial and error methods (Allison 1967, p. 12) although the writer knows of no published accounts of specific techniques used in trial and error calibration.

Some workers have attempted to devise automatic, "objective" calibration procedures using mathematical techniques and computers; such as, for example, Coats, Dempsey and Henderson (1968); Haimes, Perrine and Wismer (1968); Pliska (1968); and Y. Emsellem and G. de Marsily (1971). Lovell (1971, pp. 13-16) evaluated these methods and concluded that each of them depended on some mathematical assumptions or simplifying assumptions about the physical system that made them of little use for the large, complex model of the Tucson basin.

Previous Studies

The only known previous work that is directly concerned with the subject matter of this investigation was done by Bibby (1971) who studied prediction errors in digital models of ground-water basins, and by Meyer (1971) who investigated the use of digital models to guide collection of ground-water data. In addition, McMillan (1966) studied the effects of random variations in transmissivity in a digital model on predicted water levels. None of these studies focused on quantification of the worth of additional data to such models.

Bibby (1971) assumed that the values of the variables of a digital model of a ground-water system -- in his study the variables were hydraulic conductivity, aquifer thickness, initial water level, discharge, and storage coefficient -- were random and, using statistical techniques, related the accuracy of the variables to the accuracy

of predicted water levels at a point in time. His method consisted of a Monte Carlo technique to generate a random sample of the final water level, computation of a tolerance-limit width and a coefficient of variation on the final water level which were used as indicators of water-level accuracy, and a regression analysis to determine a relation between the accuracy of the variables and accuracy of the final water level. Bibby concluded (1971, pp. 71-72) that when only one variable at a time over the whole model is considered erroneous (for a confined aquifer), the error in the final water level is of the same order of magnitude as the error in initial water level; but he found that for all other variables, the errors in final water levels are one to two orders of magnitude less. When all variables are considered erroneous simultaneously (for both confined and unconfined aquifers), the error in initial water level is the only significant cause of error in final water levels at any one node.

There are a few similarities and several differences between the approach of Bibby and that used in this study. Both studies assumed errors at different nodes were independent, although as is pointed out below in Chapter 4, "Use of Statistical Decision Theory to Evaluate Worth of Ground-Water Data," this is commonly a poor assumption. Bibby used an idealized, 20-node rectangular model with a nodal spacing of either 1,000 or 10,000 feet for his studies, possibly because use of an actual basin model would have been too costly in terms of computer time. He used only normal distributions for his variables because he assumed that the only errors in the data were those associated with

measurement; which, as is discussed in Chapter 3 in the section on "Errors Associated with Basic Data," may not always be valid. Bibby used relatively short periods of time for his studies, commonly less than 120 days, although some simulations were as long as 440 days; whereas this study used a 20-year simulation period.

Bibby assumed that errors in a variable occurred at all nodes in the model, which is certainly a more realistic assumption than introducing errors one node at a time, as was done for this study. However, it is then difficult to study how errors in various parts of the model affect model results, or difficult to study error propagation. Bibby did not describe any extensive effort to determine what typical values of error would be, or in other words what typical values of the standard deviation of variable distributions would be, although he stated (p. 65) that data used were typical of aquifers in Colorado.

Bibby made no attempt to evaluate the worth of additional data, although he pointed out (1971, pp. 67-69) how his methods might be used to attack this problem.

Meyer (1971) observed that preliminary digital models could be used to guide the collection of ground-water data for a more definitive model, and developed a practical, qualitative approach to evaluating worth of data. Essentially, Meyer generated errors in model variables over an entire model of an actual basin using Monte Carlo techniques and triangular or log-triangular probability distributions for the variables. He made little attempt, however, to determine how the parameters of the probability distributions would vary over space and for

different variables, other than presenting a table of average error ranges for hydrologic variables in California (Meyer 1971, table 1). He did not develop a quantitative measure of the effect of errors, but used hydrographs comparing "true" and erroneous predicted water levels and maps of errors over the model to evaluate uncertainty in data.

McMillan (1966) studied the effects of random variations in transmissivity on resulting potential, i.e., water-level, distributions, using two- and three-dimensional digital models with rectangular boundaries and up to 576 nodes in two dimensions. He showed that a random variation in transmissivity produced potential distributions that did not vary significantly from those computed using constant transmissivity. McMillan assumed, in his primary numerical experiments, that transmissivity was log-normally distributed over the basin area and varied the log of the mean of transmissivity from 0 to 3 and the log of the standard deviation from 0.1 to 0.9. He also assumed that errors are statistically independent at adjacent nodes. McMillan used a steady-state system for his studies; his results, therefore, may not be applicable to the transient-state system studied in this investigation. He studied only transmissivity and not any other types of basic data, although he investigated the effects of variations in hydraulic gradient and model-grid design on predicted potentials.

McMillan concluded (p. 103a) that "for a wide range of groundwater basin conditions, extensive areas may be considered to be homogeneous without serious error in predicted potential values." He stated (p. 102), however, that serious errors can arise if the potential gradient and the nodal spacing are large and transmissivity is

highly uncertain. As an example, using a potential gradient of 1 foot per 100 feet, a nodal spacing of 10,000 feet, and a log-normal distribution of transmissivity with a mean of 0 and a variance of 1, McMillan computed the standard deviation of the differences between water levels obtained under homogeneous and heterogeneous conditions of transmissivity to be 1,410 feet. This degree of uncertainty in water levels is unacceptable for an operational study of a ground-water basin.

CHAPTER 2

THE DIGITAL MODELS OF THE TUCSON BASIN

The digital-computer models of the Tucson basin were developed, primarily by the writer and A. F. Moench, to use in studies of the application of operations-research techniques to management of ground-water resources, in studies of modeling errors, and in studies of the worth of ground-water data and efficiency of data-collection systems. Two digital models were constructed: the original large-scale model with 1,890 nodes of 1/4-square-mile area each and a small-scale model with 509 nodes of 1 square mile each. The less-detailed model was developed to reduce computation times during worth-of-data studies. The large-scale model covers about 470 square miles over a length of about 50 miles of the basin north of the Pima County-Santa Cruz County line. Figure 1 shows the area included in the large-scale model as well as the area of the electrical-analog model of the basin constructed by the U. S. Geological Survey (Anderson 1968), from which much of the data for the digital models were obtained. The small-scale model covers a slightly larger area of 509 square miles.

Essentially another "model" was modeled, in that the starting point for the digital models was the two-dimensional, quasi-linear, parabolic, time-invariant differential equation of incompressible flow through saturated porous media:

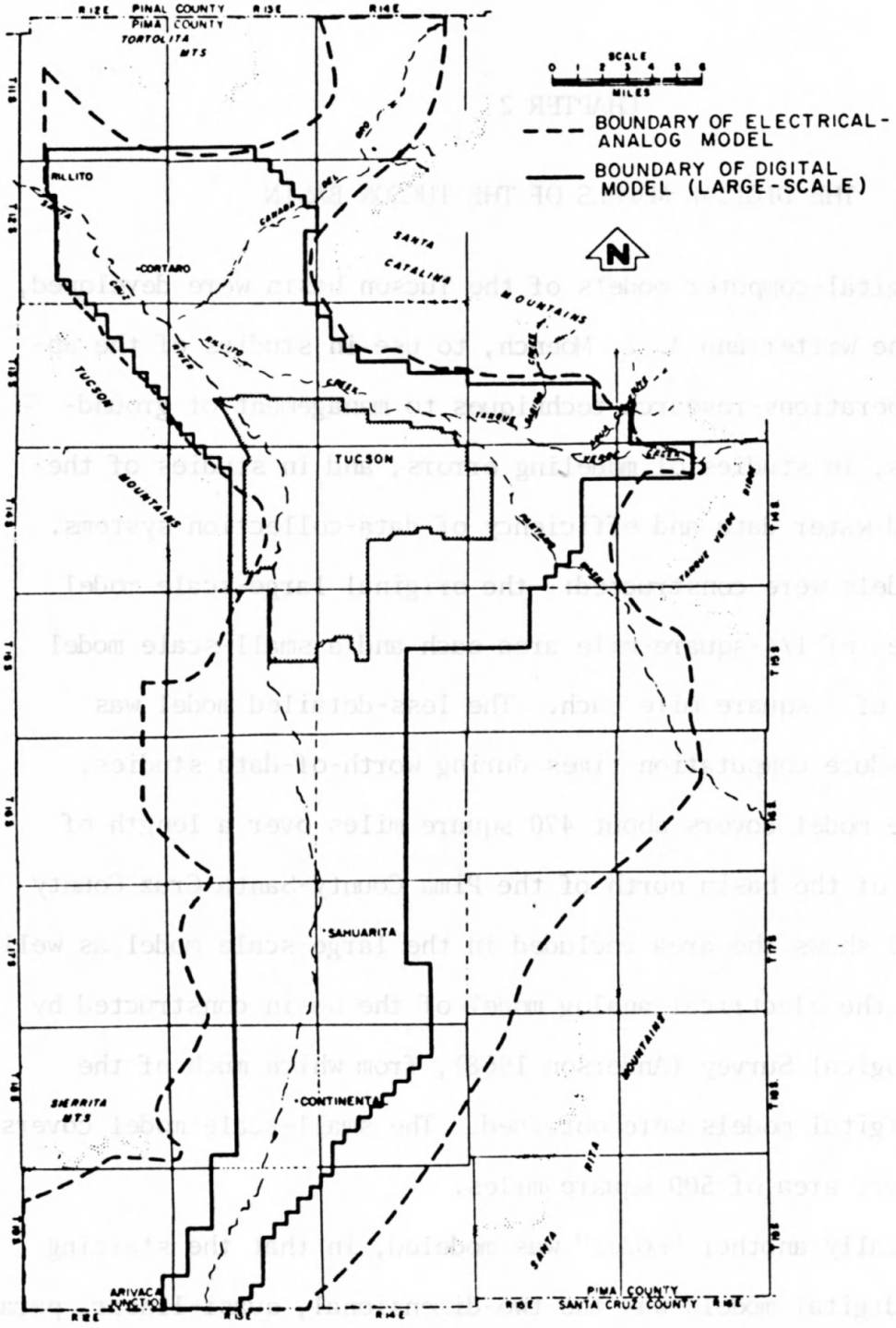


Figure 1. Map of the Tucson basin, Arizona, showing the areas included in the electrical-analog computer model and the digital-computer model.

$$\frac{\partial}{\partial x} T\frac{\partial h}{\partial x} + \frac{\partial}{\partial y} T\frac{\partial h}{\partial y} = S(x,y) \frac{\partial h}{\partial t} + QR(x,y) , \qquad (1)$$

in which h = head or water level (PY + z), in units of length (L),

S = coefficient of storage (dimensionless),

T = transmissivity (L^2t^{-1}), and

QR = inflow or outflow (Lt^{-1}).

The digital model "models" this equation using a finite-difference approximation (after some rearrangement of terms):

$$\frac{T_{i-\frac{1}{2},j}(h^{n+1}_{i-1,j} - h^{n+1}_{i,j}) + T_{i+\frac{1}{2},j}(h^{n+1}_{i+1,j} - h^{n+1}_{i,j}) + T_{i,j-\frac{1}{2}}(h^{n+1}_{i,j-1} - h^{n+1}_{i,j}) + T_{i,j+\frac{1}{2}}(h^{n+1}_{i,j+1} - h^{n+1}_{i,j})}{(\Delta x)^2}$$

$$= S(h^{n+1}_{i,j} - h^{n}_{i,j})/\Delta t + QR(i,j) , \qquad (2)$$

where the i,j notation is a standard matrix or grid reference system, n refers to the time step, $\Delta x(=\Delta y)$ = nodal spacing, and t = time-step size. The model consists of a set of these implicit equations, one per node. The set of simultaneous equations was solved using two separate methods: the Gauss-Seidel iterative algorithm and the alternating-direction-implicit algorithm.

Gauss-Seidel Algorithm

W. M. Little, at The University of Arizona, developed the first version of the digital-computer model that was later applied to the Tucson basin (written communication, 1968). He used a finite-difference

approximation of equation (1) similar to that given by Tyson and Weber (1964, p. 72), and verified that the Gauss-Seidel iterative technique would converge to a solution of such a set of simultaneous equations.

The Gauss-Seidel method solves a system of equations that can be represented by a pentadiagonal matrix, or a matrix with five unknowns per equation. These five unknowns are the water level at a given node and the four surrounding nodes, as can be seen in equations (2) or (3). Solution of a system of equations represented by a pentadiagonal matrix using a direct method, such as Gaussian elimination, can involve a large amount of computer time. Therefore an iterative technique, such as Gauss-Seidel is commonly used, even though Gauss-Seidel commonly requires many iterations and uses a considerable amount of computer time (Carnahan, Luther and Wilkes 1969, p. 452).

Little's finite-difference equation (including some minor notational changes by the writer) was:

$$\sum_{i=1}^{m} \left(h_i^{n+1} - h_B^{n+1}\right) Y_{i,B} - A_B (QR)_B^{n+1} = \frac{A_B S_B}{\Delta t} \left(h_B^{n+1} - h_B^{n}\right) \quad (3)$$

where B = the node for which water-level change is being computed,

i = number of an adjacent node,

A_B = area of node B,

$Y_{i,B} = T_{i,B} W_{i,B} / L_{i,B}$ is internodal conductance,

$W_{i,B}$ = width of flow path (width of boundary between nodes),

$L_{i,B}$ = length of flow path (distance between nodal centers),

$T_{i,B}$ = transmissivity between nodes B and i,

S_B = coefficient of storage of nodal area,

m = number of adjacent nodes,

n = number of time intervals,

$(QR)_B$ = net withdrawal or recharge,

t = time interval over which state changes are being calculated,

h^n = water-level elevation at the previous time interval, and

h^{n+1} = the water-level elevation being computed (present time interval).

Equation (3) is actually an alternate way to express equation (2), as the $Y_{i,B}$ terms in (3) are equivalent to the T terms in (2), and A_B in (3) is equivalent to $(\Delta x)^2$ in (2).

For purposes of computation the basic equation can be rearranged to give:

$$\sum_{i=1}^{m} h_i^{n+1} Y_{i,B} - h_B^{n+1}\left[\sum_{i=1}^{m} Y_{i,B} + \frac{A_B S_B}{\Delta t}\right] = -\frac{A_B S_B h_B^{n}}{\Delta t} - A_B (QR)_B^{n+1} . \qquad (4)$$

Little and N. E. Baran (written communication, 1968) collaborated in developing a computer program utilizing this equation. In addition, they prepared an alternate program that treated transmissivity as a variable. After each time-step is solved, the change in water-table elevation is used to recalculate transmissivity (assuming transmissivity is linearly related to saturated thickness of aquifer) and the new value is used in the subsequent time-step. Little and Baran also prepared modifications that could account for boundary nodes with constant potential instead of the impermeable boundaries of the

basic program. The programs finally developed for the Tucson basin, however, assumed transmissivity constant with time (see below pp. 66-67); and treated all physical boundaries as impermeable. The models simulated recharge and discharge at boundaries as simple input or output, identical to a simulated pumping or recharging well, at each boundary node where subsurface flow occurs. This procedure involves less programming than holding a potential gradient constant across a boundary, although it is then more difficult to simulate variable subsurface flow in response to changes in water-table gradient. Because such changes in flow at boundaries are conjectural, especially along mountain-front boundaries, the simpler approach was used.

The program, like that of the California Department of Water Resources, is readily adaptable to irregularly-shaped areas, and individual nodal areas can be polygons of various sizes with a variable number of sides.

R. L. Knickerbocker (written communication, 1969) modified the Gauss-Seidel method by adding an overrelaxation coefficient. The Gauss-Seidel method for solving the set of m simultaneous linear equations generated at the m nodes of the model can be characterized by the equation:

$$Ah = B \ ,$$

where A = matrix of coefficients $(A_{i,j})$, i = 1 to m, j = 1 to m,

h = vector of unknown head values $(h_i)^T p$, i = 1 to m (T_p indicates the transpose of the matrix), and

B = vector of constants $(b_i)^T p$, i = 1 to m.

The system is solved iteratively by solving the i^{th} equation for the i^{th} unknown as follows:

$$h_i^{(k+1)} = \frac{1}{a_{i,i}} \left[b_i - \sum_{j=1}^{i-1} a_{i,j} h_j^{(k)} + \sum_{j=i+1}^{m} a_{i,j} h_j^{(k)} \right], \qquad (5)$$

where i indicates the unknown sought and k indicates the number of iterations. The equation can be rewritten as:

$$h_i^{(k+1)} = h_i^{(k)} + \frac{1}{a_{i,i}} \left[b_i - \sum_{j=1}^{m} a_{i,j} h_j^{(k)} \right], \qquad (6)$$

by adding and subtracting $h_i^{(k)}$ to the right-hand side of the equation.

The Gauss-Seidel method can be modified by the use of an over-relaxation coefficient (the method is then commonly referred to as the successive-overrelaxation method or SOR). The i^{th} equation is solved for the i^{th} unknown using:

$$h_i^{(k+1)} = h_i^{(k)} + \frac{\omega}{a_{i,i}} \left[b_i - \sum_{j=1}^{m} a_{i,j} h_j^{(k)} \right] . \qquad (7)$$

For overrelaxation ω ranges from 1 to 2. An optimum value of ω (ω_{opt}) can be calculated for the Tucson basin model but the method took more computer time than was justified. An estimate of ω_{opt} of 1.8 gave a decrease in the number of iterations necessary for convergence that was deemed sufficient.

Alternating-Direction-Implicit Algorithm

Knickerbocker (written communication, 1970) determined that for a grid of square nodal areas, such as the Tucson basin digital model, the alternating-direction-implicit algorithm would be computationally

more efficient than the Gauss-Seidel algorithm. The alternating-direction-implicit method was devised to solve a system of equations represented by a pentadiagonal matrix by converting the system into two systems of equations, each with a tridiagonal matrix. A tridiagonal system can be solved directly, using a method such as Gaussian elimination, without resorting to time-consuming iterative procedures. The method employs two finite-difference equations which are used in turn over successive time-steps, each of duration $\Delta t/2$ (Carnahan, Luther and Wilkes 1969, p. 452). The first equation includes 3 unknowns, the water level at the given node and at the two adjacent nodes in the same row (or column), the second also includes 3 unknowns, the water level at a given node and at the nodes in the same column (or row). The solution of the first equation furnishes values used in the second equation, the solution of which yields water levels at the end of the entire time-step, Δt.

Knickerbocker wrote a computer program, following the discussion of the algorithm by Pinder and Bredehoeft (1968), that would solve a ground-water flow problem in a rectangular basin composed of 400 nodes, 20 nodes by 20 nodes in size.

Comparison of the Algorithms

Moench (written communication, 1969) devised a ground-water flow problem in a rectangular, homogeneous aquifer with three impermeable boundaries and one recharge boundary, and obtained an analytical solution using heat-flow theory. Moench, Knickerbocker, and the writer

utilized both the Gauss-Seidel (as modified by successive overrelaxation) and alternating-direction methods to solve this problem (using one time-step) and compared the results with the analytical solution (table 1). The area modeled in the problem is 10 square miles and was approximated by 400 nodes, each of which represents 1/4 square mile. Transmissivity was assumed constant over the model at 500,000 gpd/ft (gallons per day per foot) and the storage coefficient was assumed to be 0.15. The recharge was defined as 0.15 feet of water per day for a simulation period of one year in a row of nodes along one boundary. This totals to 0.15 ft/day x 43,560 sq ft/acre x 160 acres/quarter-square mile x 20 quarter-square miles or 20,900,000 cu ft/day (2.06 gpm/ft (gallons per minute foot) of boundary).

The alternating-direction method agreed best with the analytical solution overall, although the Gauss-Seidel method gave better results in the center of the model. Both methods gave poor results, in terms of percent error, at the boundary opposite the recharge boundary, although the alternating-direction method was much better there, yielding an absolute error of only 0.28 feet. In addition, the alternating-direction method used about 1/6 as much central processor time on the computer, although some of the difference was due to a simpler form of data input and output in the alternating-direction program.

The writer and R. L. Knickerbocker then experimented with the Gauss-Seidel and alternating-direction techniques to see how to approximate more closely the analytical solution. Several approaches were

Table 1. Comparison between the Analytical Solution of a Ground-Water Flow Problem and its Solution by Finite Differences Using the Gauss-Seidel[a] and Alternating-Direction Algorithms.

Distance from Recharge Boundary, in miles	Water-level Rise, Analytical Solution, in feet	Water-level Rise, Gauss-Seidel Method,[b] in feet	Percent of Analytical	Water-level Rise, Alternating-Direction Method, in feet	Percent of Analytical
9.5	0.35	2.42	690	.63	180
9.0	0.39	2.53	650	.69	177
8.5	0.52	2.74	527	.80	154
8.0	0.74	3.07	415	.98	132
7.5	1.09	3.54	325	1.25	115
7.0	1.62	4.16	257	1.62	100[c]
6.5	2.37	4.96	210	2.13	90
6.0	3.43	5.97	174	2.83	82.5
5.5	4.89	7.23	148	3.76	77
5.0	6.84	8.80	129	5.02	73.4
4.5	9.43	10.76	114	6.71	71.1
4.0	12.78	13.17	103[c]	8.97	70.2
3.5	17.05	16.14	94.7	12.01	70.5
3.0	22.40	19.82	88.5	16.07	71.7
2.5	23.99	24.33	84	21.50	74.2
2.0	36.97	29.89	81	28.78	77.8
1.5	46.48	36.73	79	38.52	83
1.0	57.64	45.13	78.3	51.56	89.5
0.5	70.55	55.48	78.5	69.02	97.9
0.0	85.25	68.20	80	92.39	108
Average Deviation		2.24	134.0	2.07	26.9

[a]Modified by successive overrelaxation.
[b]Error tolerance is an average of 0.001 foot per node.
[c]Value closest to analytical.

tried using both the Gauss-Seidel and alternating-direction methods. For Gauss-Seidel, experiments included dividing the one-year time period into time-steps -- starting with a time interval of one minute and doubling each subsequent interval for a total of 20 time-steps -- and requiring a minimum of 10 iterations per time-step. In order to avoid large computation times, the error tolerance was reduced from an average of 0.001 to 0.01 foot per node. This reduced accuracy and thus partly offset the increase in accuracy obtained by using time-steps and other modifications. Therefore, the original solution of the problem by Gauss-Seidel cannot be directly compared to the solution using modifications.

For the alternating-direction method, experiments included dividing the time period into nine time-steps (an initial time interval of one day) and 20 time-steps (an initial interval of one minute) and reducing the nodal spacing to 1/4 and 1/8 mile.

Comparison of results for these changes with analytical results are shown in table 2 for Gauss-Seidel and table 3 for alternating-direction. These very limited experiments on a simple, idealized problem suggest that: (1) the alternating-direction method is still superior to the Gauss-Seidel with the time period divided into 20 time-steps, although reducing the error tolerance had an unknown effect; (2) checking the error at each node instead of the sum of the nodal errors, or requiring a certain number of iterations for the early time-steps (when the absolute value of water-level change is small) in the Gauss-Seidel method makes little difference in the result; (3) dividing the time period into steps gives better results for both the Gauss-Seidel

Table 2. Comparison between the Analytical Solution and Various Modifications of the Gauss-Seidel Algorithm.

Distance from Recharge Boundary, in miles	Water-level Rise, Analytical Solution, in feet	Water-level Rise, Gauss-Seidel[b,c] in feet	Percent of Analytical	Water-level Rise, Gauss-Seidel[d,e] in feet	Percent of Analytical	Water-level Rise, Gauss-Seidel[d,f] in feet	Percent of Analytical
9.5	0.35	1.05	300	1.02	290	1.02	290
9.0	0.39	1.06	272	1.10	282	1.10	282
8.5	0.52	1.20	230	1.27	244	1.27	244
8.0	0.74	1.48	200	1.54	208	1.54	208
7.5	1.09	1.88	172	1.93	177	1.93	177
7.0	1.62	2.45	151	2.46	152	2.46	152
6.5	2.37	3.19	135	3.18	134	3.18	134
6.0	3.43	4.16	121	4.14	121	4.14	121
5.5	4.89	5.42	111	5.39	110	5.39	110
5.0	6.84	7.04	103[g]	7.01	102.5[g]	7.01	102.5[g]
4.5	9.43	9.14	96.9	9.10	96.5	9.10	96.5
4.0	12.78	11.82	92.6	11.78	92.1	11.78	92.1
3.5	17.05	15.22	89.4	15.18	89	15.18	89
3.0	22.40	19.51	87.2	19.47	81.9	19.47	81.9
2.5	28.99	24.86	85.8	24.83	85.7	24.83	85.7
2.0	36.97	31.48	85.2	31.46	85.1	31.46	85.1
1.5	46.48	39.59	85.2	39.56	85.1	39.56	85.1
1.0	57.64	49.36	85.6	49.33	85.5	49.33	85.5
0.5	70.55	60.97	86.4	60.95	86.4	60.95	86.4
0.0	85.25	74.59	87.5	74.53	87.5	74.53	87.5
Average Deviation		2.22	45.7	2.23	47.3	2.23	47.3

[a]Modified by successive overrelaxation.

[b]Error tolerance is an average of 0.01 foot per node.

[c]20 time-steps.

[d]Error tolerance is 0.01 foot per node.

[e]20 time-steps, error tolerance checked at each node.

[f]20 time-steps, error tolerance checked at each node, at least 10 iterations per time-step required.

[g]Value closest to analytical.

Table 3. Comparison between the Analytical Solution and Various Modifications of the Alternating-Direction Algorithm.

Distance from Recharge Boundary, in miles	Water-level Rise, Analytical Solution, in feet	Water-level Rise, Alt. Direction[a], in feet	Percent of Analytical	Water-level Rise, Alt. Direction[b], in feet	Percent of Analytical	Water-level Rise, Alt. Direction[c], in feet	Percent of Analytical	Water-level Rise, Alt. Direction[d], in feet	Percent of Analytical
9.5	0.35	.28	80	.32	91.4	.31	88.5	.31	88.5
9.0	0.39	.33	84.5	.37	95[e]	.37	95	.38	97.4
8.5	0.52	.44	84.6	.47	90.4	.50	96.2[e]	.51	98
8.0	0.74	.63	85.2	.65	88	.69	93.2	.72	97.3
7.5	1.09	.91	83.5	.93	85.2	.99	90.8	1.03	94.5
7.0	1.62	1.33	82	1.33	82	1.43	88.2	1.49	92
6.5	2.37	1.94	81.8	1.93	81.5	2.07	87.4	2.16	91.2
6.0	3.43	2.81	82	2.77	80.7	2.99	87.1	3.11	90.6
5.5	4.89	4.02	82	3.95	80.7	4.26	87.1	4.44	90.8
5.0	6.84	5.69	83.1	5.58	81.5	6.02	88	6.27	91.7
4.5	9.43	7.93	84	7.79	82.5	8.41	89.1	8.74	92.5
4.0	12.78	10.89	85.2	10.73	84.1	11.56	90.5	12.01	94.1
3.5	17.05	14.69	86	14.56	85.4	15.64	91.8	16.21	95.1
3.0	22.40	19.51	87.1	19.42	86.8	20.79	92.7	21.50	96
2.5	28.99	25.49	88	25.46	87.9	27.15	93.5	28.04	96.8
2.0	36.97	32.81	88.8	32.84	89	34.89	94.4	35.96	97.2
1.5	46.48	41.60	89.5	41.69	89.7	44.14	95	45.40	97.6
1.0	57.64	51.98	90	52.13	90.5	54.99	95.4	56.46	97.8
0.5	70.55	64.05	90.8	64.25	91	67.54	95.8	69.24	98
0.0	85.25	77.90	91.4[e]	78.11	91.6	81.85	96	83.76	98.2[e]
Average Deviation		2.23	14.5	2.23	13.3	1.18	8.2	0.60	5.7

[a] 9 time-steps.

[b] 20 time-steps.

[c] 20 time-steps and 1/4-mile nodal spacing.

[d] 20 time-steps and 1/8-mile nodal spacing.

[e] Value closest to analytical.

and the alternating-direction methods; (4) the greatest improvement (shown experimentally only for alternating-direction) is made by decreasing the nodal spacing; and (5) alternating-direction takes less central processor time than Gauss-Seidel.

At present the Gauss-Seidel method, as used for the Tucson basin model, takes about 75 percent of the computer memory storage (exclusive of storage used by the computer for control and other uses) required for the alternating-direction method. This is largely because of the irregular boundaries of the Tucson basin model. Gauss-Seidel needs storage only for the interior nodes of the model whereas the alternating-direction technique needs storage for nodes outside of the model, so that the whole model has a rectangular shape. It may, however, be possible to modify this requirement so as to reduce required storage. For a model for which all interior nodes form a rectangular shape, such as the 20 by 20 grid used for the recharge problem, the alternating-direction method requires only about half as much storage (exclusive of computer needs) as Gauss-Seidel.

The writer adapted the basic alternating-direction program for the Tucson basin model by modifying the program so that it could solve models with non-rectangular outlines, such as the irregularly-shaped Tucson basin. In addition the transmissivity data were modified so that values could be read in directly for each node instead of reading in values between each node and all its adjacent nodes, as required for the Gauss-Seidel method. The data-input format also had to be modified so that data for the interior of the model -- within the

irregular boundaries of the model proper -- could be positioned "inside" the larger data arrays which include nodes outside the model proper. Thus, it was not necessary to punch large numbers of zeros on cards to represent nodes outside the model proper.

The computer program using the alternating-direction algorithm was converted to a subroutine in the computer program prepared for the worth-of-data studies (Appendix A). The essentials of the alternating-direction algorithm and data output are included in subroutine ALDIRS, while the essentials of data input are included in the main program WODATA.

Data for the Models

In September 1968, A. F. Moench began collecting data, from various sources, for the specific model of the Tucson basin and compiling them on computer punch cards. He divided the basin into 1,890 nodal areas of 1/4 square miles each -- a grid of square nodes spaced 1/2 mile apart. Moench decided not to use polygonal nodes because (written communication, 1969):

(1) compilation was simplified, thereby making it possible for persons with little experience in hydrology to assemble data easily;

(2) the nodal areas correspond with the nodal areas of the electrical-analog model constructed by the U. S. Geological Survey, although the University model covers only about two-thirds of the area covered by the Survey model;

(3) the internodal conductance of equation (3) is equal to transmissivity since the distance between nodes (L) equals the length of the

side of each node (W), or $Y_{i,B} = \frac{T_{i,B}W_{i,B}}{L_{i,B}} = T_{i,B}$;

(4) nodal area is constant, saving computer storage and eliminating the need to measure individual nodal areas; and

(5) computer print-out of data is simplified in that results can be printed out directly in map form and computer storage is not needed to record nodal locations.

Moench pointed out that using a grid of equal-sized nodes has disadvantages, namely that the whole grid has to be fine enough to give good results in areas where potential gradients are steepest and/or much data are available. Thus many "unnecessary" nodes are included in areas where gradients are flat or data are few, leading to extra compilation and an impression of accuracy in these areas that really does not exist.

The area of the large-scale digital model corresponds fairly well to the area of the analog model of the U. S. Geological Survey (figure 1) except that the digital model includes less of the Canada del Oro valley, less of the area between the Tucson and Sierrita Mountains, a narrower part of the Santa Cruz valley south of Continental, and omits a large area on the eastern side of the basin between Pantano Wash and the Santa Rita Mountains. The model boundaries are not smooth curves but are irregular approximations using the sides of the 1/4 square-mile nodal areas, as can be seen on figure 1.

Data on Coefficient of Storage and Transmissivity

The coefficient of storage of a ground-water basin model is commonly assumed to be constant over the whole basin because few data are available to assess its variability. Transmissivity can be obtained from a test on a pumping well alone, but to obtain good values of storage coefficient, one or more suitable observation wells should be available. Consequently, there commonly are fewer values of S from aquifer tests than values of T.

The storage coefficient for the Tucson basin, which is virtually equivalent to specific yield because the aquifer in the basin is unconfined, is commonly assumed to be between 0.05 and 0.20 by various workers, depending upon their experience or predilections. The storage coefficient likely varies over the basin, depending on the lithology of that part of the aquifer where the water table is declining. Initially the coefficient of storage was assumed to be 0.15 for all nodes of the digital model, as was initially assumed for the electrical-analog model.

Aquifer tests in the basin commonly indicate that storage coefficients are less than 0.01, probably because the tests have short pumping periods, in the order of hours, and delayed drainage causes water-level declines to be too great and thus calculated storage coefficients to be too small (Clyma, Rebuck and Shaw 1968). Even when they used methods of analysis which attempt to account for delayed drainage, Clyma et al. (1968, table 2) computed values of storage coefficient which ranged only from 0.01 to 0.07. Apparently no long-term tests have

been run in the Tucson basin that have yielded realistic values of the coefficient of storage.

Transmissivity data developed for the analog model were obtained from the U. S. Geological Survey. The Survey had compiled these data in the form of a map showing areas of equal transmissivity, subdivided into areas where transmissivity is (1) less than 10,000 gpd/ft, (2) 10,000 - 50,000, (3) 50,000 - 100,000, (4) 100,000 - 180,000, and (5) more than 180,000. From this map a value was read for the internodal transmissivity between each node and every one of its adjacent nodes for the Gauss-Seidel algorithm, and read at each node for the alternating-direction algorithm. Values of 7,500 gpd/ft, 30,000, 75,000, 140,000, and 250,000 were assigned to the map intervals for the purpose of specifying nodal or internodal transmissivities.

The Department of Agricultural Engineering of the University of Arizona compiled the results of aquifer tests made during the period 1961-68. Figure 2 shows the distribution of tests over the basin. This was the main source of data used by the U. S. Geological Survey for their map.

Water-Level Data

Water-level and water-level-change data were obtained from the Agricultural Engineering Department. They have prepared contour maps of the water-table surface for almost every year since 1947 and water-level-change maps for selected periods, both commonly using 10-foot contour intervals, although some change maps use 5-foot intervals. Initial

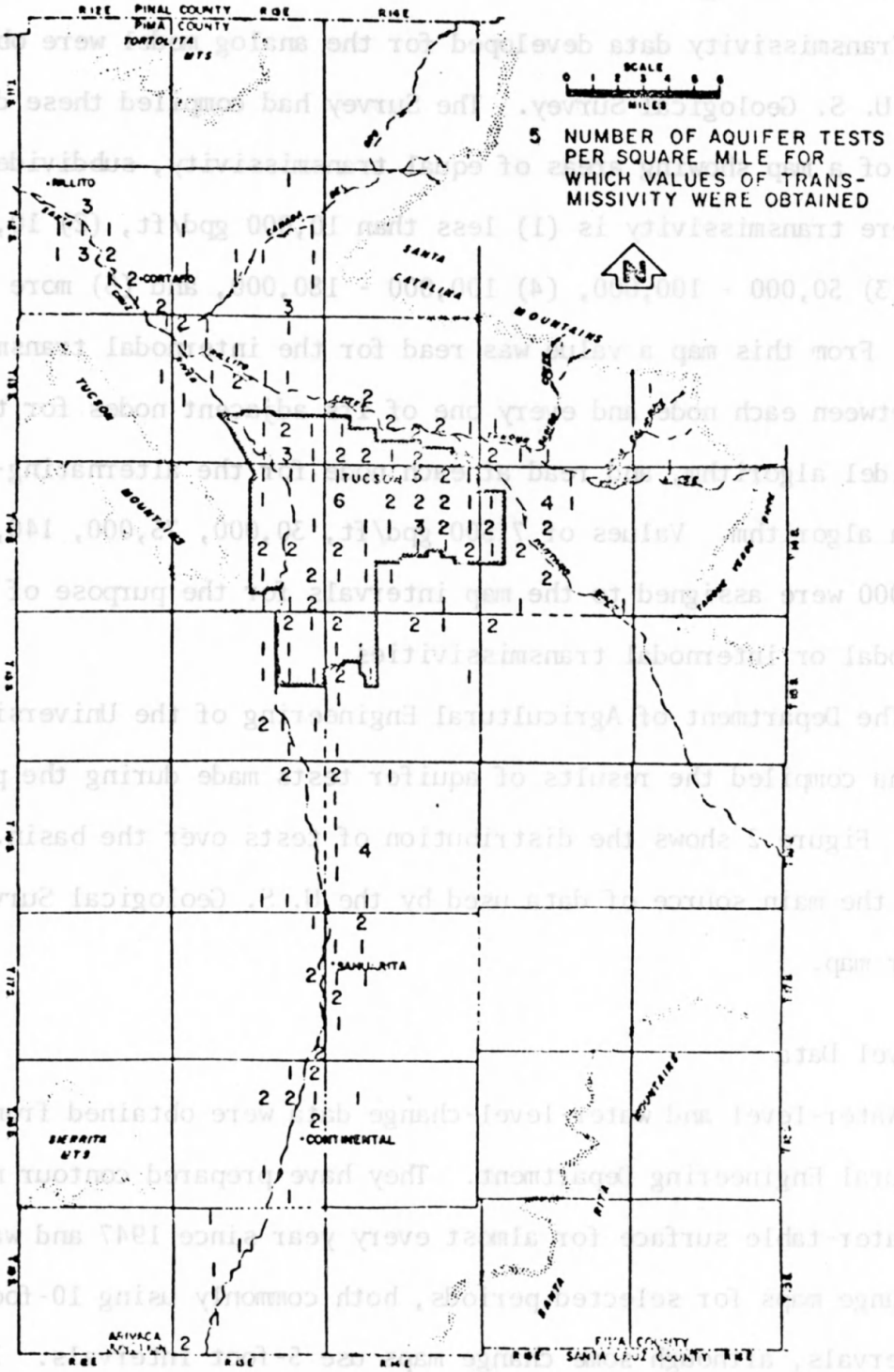


Figure 2. Map of the Tucson basin, Arizona showing the distribution of aquifer tests.

conditions, in terms of water levels (h), for any period can be interpolated for each node using the water-table elevation maps, and values of change at each node for selected periods can be interpolated from the change maps for use in model calibration. In this dissertation, H(i,j) refers to the initial water level at any point, whereas h(i,j) refers to the predicted water level at a point at some future time.

Water levels in about 1,500 wells in the drainage basin of the Santa Cruz River are measured annually by the Department of Agricultural Engineering (Schwalen and Shaw 1961). Levels commonly are measured in the winter or spring when pumping in the basin is at a minimum, and thus are the best approximation of the basin's annual static (nonpumping) water levels. Many of the observation wells, however, are outside that part of the Tucson basin included in the digital model. In addition, data from all of these wells are not available for specific periods of water-level change, since every well was not measured both at the beginning and end of every period. As an example of the distribution of data, figure 3 shows the locations of the approximately 500 observation wells over the basin for which 1947-66 water-level-change data are available. Data are concentrated along the streams and in the city of Tucson.

Values of historical water-level change rather than historical water-table elevations were used for model calibration for several reasons. The computer could print out values of change on one sheet of paper because such values have a maximum of two digits while elevations have four. Also contour maps of change were believed to be much

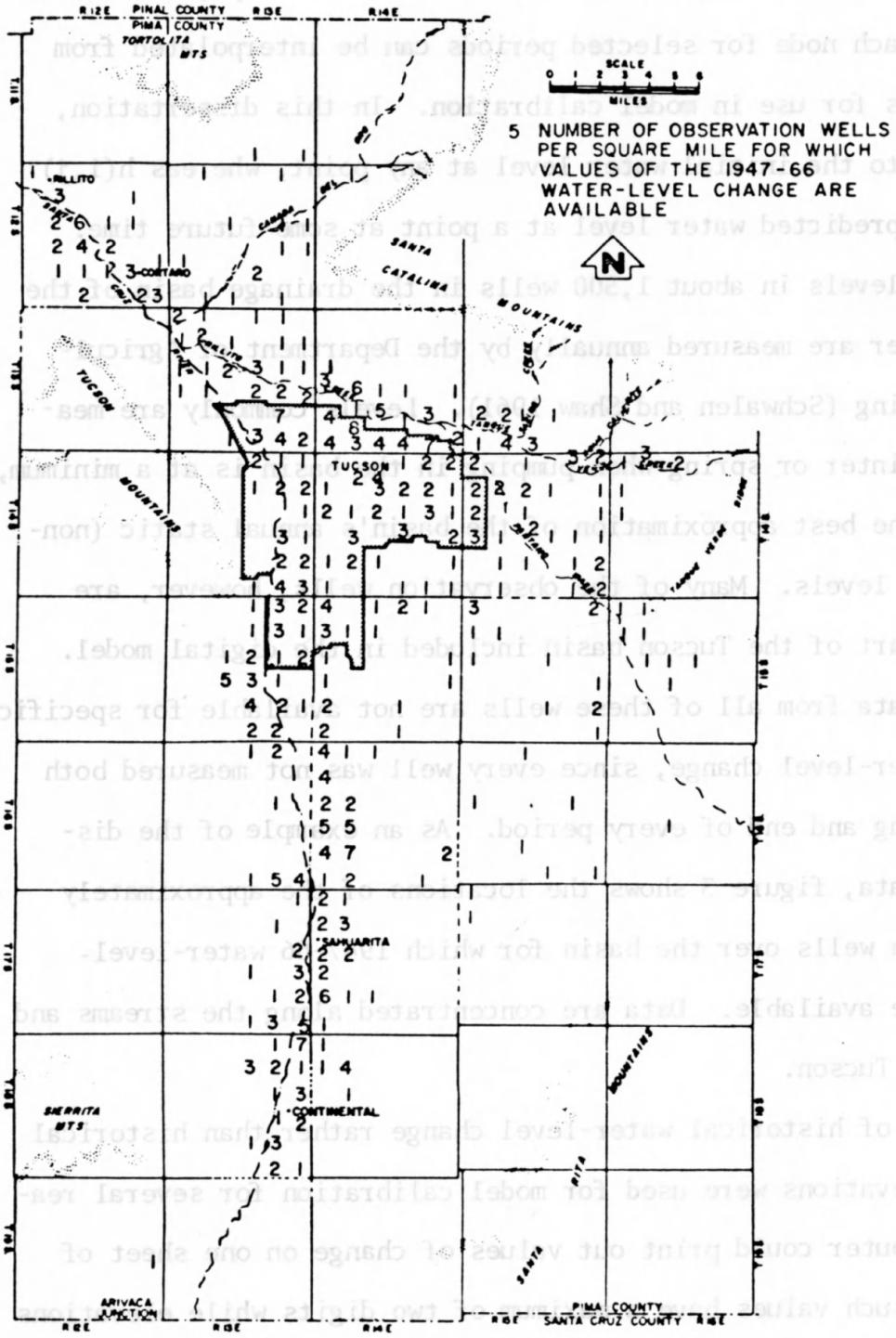


Figure 3. Map of the Tucson basin, Arizona showing the distribution of observation wells for which values of the 1947-66 water-level change are available.

more reliable than elevation maps. Values of change are commonly related to factors such as pumping and recharge that can be evaluated readily, and thus subjective interpretation of contour spacing and position is simplified. Values of change also commonly decrease toward boundaries of a basin, away from centers of pumping, and can be estimated reasonably well from sparse data.

Discharge (Pumpage) Data

The pumpage (Q) data used in the electrical-analog model were obtained from the U. S. Geological Survey and were used in the digital model. The data were compiled for each 1/4 square-mile nodal area for nine time periods: 1940, 1941, 1942-46, 1947-49, 1950, 1951-52, 1953-57, 1958-61, and 1962-65. The pumpage was considered constant within each of these time periods. The Survey made these estimates using field measurements of pumpage from some of the wells in the basin, pumpage records of the city of Tucson, and estimates of irrigated acreage. Anderson (1968, p. 22) revised pumpage in a few areas during analog-model calibration so that the model results corresponded better in a visual sense with actual changes.

Recharge Data

Data on infiltration into stream channels were obtained from a report by Burkham (1970, table 5). He studied channel losses for the period 1936-63 and estimated the average annual infiltration per mile of channel for various streams in the Tucson basin. These data were used

in the digital model, assuming initially that infiltration equalled recharge (R) and that recharge was constant for any given period.

"Boundary" or "mountain-front" recharge is defined here as that water moving into the basin from bounding mountain ranges or from tributary basins. In the digital model, this is the water moving into the model from areas outside the model, including subsurface flow from the Tortolita, Santa Catalina, Rincon, Empire (located northeast of the Santa Rita Mountains), Santa Rita, Sierrita and Tucson Mountains; subsurface flow from the Canada del Oro and upper Santa Cruz valleys; and any flow from the areas between the Rincon and Empire Mountains (San Pedro Valley), between the Sierrita Mountains and the southern edge of the model, and between the Sierrita and Tucson Mountains (Avra-Altar Valley).

Data on boundary recharge were obtained from Anderson (1968, pp. 20-22), who used the electrical-analog model of the basin to estimate subsurface inflow and outflow. He assumed that inflow to and outflow from the basin were in balance in 1940 -- that is, no ground water was being withdrawn from storage, or that the flow system was in a steady state. He then adjusted values of transmissivity and subsurface inflow and outflow until the model duplicated the 1940 water-level elevation map. The resulting inflow and outflow values initially were assumed to be the correct average quantities for the digital model.

Anderson (1968, fig. 4) estimated that there was no subsurface inflow from the Tucson Mountains or from the area between the Tucson and Sierrita Mountains, probably because annual precipitation over these

areas is low. Preliminary results from the digital model indicate that there may be some flow from these areas, although it is likely small. The subsurface outflow at Rillito in the northwest corner of the Tucson basin was also obtained from the analog model calibration. This quantity is actually a form of discharge, but data on subsurface outflow were compiled with recharge data, and commonly will be discussed in conjunction with recharge data in this dissertation, because they were derived from the same source.

Calibration of the Models

After initial estimates were made for parameters, initial conditions, and input/output at each of the 1,890 nodes, the model was calibrated by adjusting these estimates until the model reproduced historical water-level changes fairly well for the period from the spring of 1947 to the spring of 1966. This period was selected because it was the longest period for which all types of data were available in significant amounts. Prior to 1947, water-level data are sparse; and the U. S. Geological Survey had not compiled pumpage data after 1965 for each node of the analog model. A fairly long period is needed for calibration so that historical water-level changes are at least 10's of feet. If changes at each node over the time period used are only a few feet, the model cannot be calibrated well because errors in the interpolated values of historical change will be of the same order of magnitude as the changes themselves. In this case, an analysis of differences between computed and historical change to indicate what model adjustments to make is not meaningful.

Allison (1967, pp. 100-101) attempted to obtain, for a given total budget for a digital model of the southern San Joaquin Valley, California, the optimal combination of the number of time periods used in calibration, the number of nodes, and the number of calibration runs. He estimated that the optimal combination was 3-4 time periods, 440-480 nodes, and 260-280 calibration runs.

It would have been preferable to calibrate the Tucson basin model over several separate time periods to obtain independent estimates of parameters, initial conditions, and input/output. These independent values could then be averaged to provide estimates that would be representative of more than one set of basin conditions. However, for calibrating over separate time periods, pumpage data specific to each period should be available. If pumpage is lumped over a period, separate calibration on parts of the period is not meaningful. For the Tucson basin, the 1947-66 period could have been divided into subperiods because pumpage data were available for 1947-49, 1950, 1951-52, 1953-57, 1958-61, and 1962-65. However, water-level changes for these subperiods are small, and errors in the data and in contour maps made from the data likely would be a significant proportion of the change. For this reason, calibration over subperiods of 1947-66 was not done. In future work with the model it might be instructive to divide the 1947-66 period into two subperiods and compare calibrations over them.

Some modelers prefer to calibrate using only measured values of historical change or water-level elevation (Allison 1967, p. 12). In this procedure, only the changes or elevations at nodes which include

observation wells are matched. This method enables a modeler to calibrate over short time periods because there are no errors that are associated with interpolating data to other nodes and that would complicate the analysis. However, if the observation wells are poorly distributed over the basin being modeled, adjustments made in areas of few or no wells are likely unreliable. Another disadvantage to calibrating solely with measured changes is that information can be lost, specifically the knowledge and experience of the hydrologist. It is not clear how much information such knowledge represents in comparison to measured data, but it likely is significant. Interpolating water levels to all nodes in a model for use in calibration necessarily incorporates some of this knowledge because water levels are not interpolated mechanically, but by exercising judgment.

In this study, calibration was done using water-level-change data interpolated to all nodes in the model because observation wells are not evenly distributed over the Tucson basin (figure 3), and because it was judged that a significant amount of added information is obtained by using interpolated water-level data.

The calibration procedures used for the two Tucson basin models were subjective and to a large extent trial and error. Subjective calibration is defined here as adjusting model variables largely using individual judgment; whereas objective calibration would involve setting rigid criteria to control the adjustment process, which process probably would be done automatically by computer.

At present subjective trial and error methods most commonly are used in calibration of ground-water models (Allison 1967, p. 12). Lovell (1971), using the southern end of the Tucson model, developed a semi-objective method of calibration which uses a computer to aid the hydrologist in selecting the nodes at which variables should be adjusted and in determining the size of the adjustment. Lovell did his work during and after the calibration discussed here, and thus his techniques were not used.

When the data used to construct the electrical-analog model were obtained from the U. S. Geological Survey, the Survey's calibration process almost had been completed. This process was discussed by Anderson (1968). He calibrated the analog model in two stages, a steady-state analysis for the year 1940, and a transient-state analysis for the period 1940-65. In the steady-state analysis, Anderson assumed that the ground-water flow system was in equilibrium, in the sense that water levels were constant over time and inflow equalled outflow from the basin. In making initial estimates for recharge, he used the entire amount of streamflow losses by infiltration as an estimate of recharge from streams. Anderson then varied the analog-model recharge and subsurface outflow on a trial and error basis until he obtained the best match between the model potential field and the 1940 water-table contour map (Anderson 1968, figure 1). He later used the derived values of recharge and subsurface outflow as initial estimates in the transient-state analysis. Anderson also adjusted some values of transmissivity in the steady-state analysis.

In the transient-state analysis, Anderson used the analog model to simulate changes in water levels during 1940-65. He varied values of pumpage, recharge, and storage coefficient until the best match was obtained between changes in the analog-model potential field and measured changes in water levels during four subperiods of the period 1940-1965.

Although many of the data used in the digital model were derived from the calibrated analog model, the digital model did not compute water-level changes for the 1947-66 period that matched, in some sense, actual changes, and therefore it also had to be calibrated. Because much of the data on recharge, subsurface outflow, and pumpage were derived from analog calibration, no steady-state analysis was made using the digital model.

The reasons for the lack of correspondence between the calibrated analog model and the digital model are not entirely clear but are likely related to the following factors: (1) the topographic areas encompassed by the two models do not correspond exactly; (2) initial water-level data and water-level-change data used in the digital model were obtained from maps drawn by the Department of Agricultural Engineering, while the Survey used their own data as well as University data and prepared their own maps; (3) changes in the initially-assumed values of stream-channel recharge and constant storage coefficient in the analog model were not incorporated in the digital model; and (4) the two models likely would not produce identical changes even if all other factors were equal because the digital-model results are affected by round-off error (see

Chapter 3 below, "Errors in Digital Modeling," p. 62), while analog model results are affected by errors in electrical components. In addition, the digital model uses a finite-difference approximation for the $\partial h/\partial t$ term in equation 1, while the analog model does not. These four factors likely account for most of the lack of correspondence between results of the analog and digital models.

Large-Scale Model

The writer calibrated the large-scale model of the Tucson basin during the spring and summer of 1970. In initial test runs of the model, the average of the absolute values of the nodal error -- calculated as the difference between computed values and historical values of 1947-66 water-level change at each node -- was 24.4 feet and the maximum error was 190.6 feet, located in the northwestern corner of the model. In several large areas, all nodes had errors of more than 50 feet, and along Pantano Wash errors were up to 110 feet. In comparison, maximum historical changes in water level for the 1947-66 calibration period were 80 feet. Only 26 percent of the 1,890 nodes had errors less than 10 feet, 48 percent had errors less than 20 feet, 65 percent had errors less than 30 feet, and 82 percent had errors less than 40 feet. A summary of these errors for initial and final runs of both the large-scale and small-scale models of the basin is given in table 4.

For each calibration run the computer printed maps of the value of error at each node and of nodes where errors were more than 20 feet, computed the average absolute error and average squared error over the whole model, and counted the number of nodes with errors greater than

Table 4. Comparison of Initial and Final Errors in the Large-Scale and Small-Scale Models.

	Average Nodal Error, in feet	% Nodes w/ Errors < 10 Feet	% Nodes w/ Errors < 20 Feet	% Nodes w/ Errors < 30 Feet	% Nodes w/ Errors < 40 Feet	Maximum Error, in feet
Large-Scale Model (1,890 nodes)						
Initial Run	24.4	26	48	65	82	190.6
Final Run	5.3	87	99.5	100	--	28.6
Small-Scale Model (509 nodes)						
Initial Run	6.2	80	89	99.5	100	34.4
Final Run	5.7	82	99.5	100	--	24.7

10 feet, 20 feet, 30 feet, and 50 feet. These data were used to guide the calibration process, and indicated whether adjustments made to model parameters, initial water levels, and discharge/recharge were successful in reducing errors. The computer also punched values of error for each run on cards. These data were used as input to the following run so that a map of differences in errors for successive runs could be printed. These maps also were used to evaluate the effects of adjustments.

A total of 38 separate computer runs of the model were made, reducing the average error from 24.4 to 5.3 feet, and the maximum error from 190.6 to 28.6 feet. At the final run, 87 percent of the nodes were in error by less than 10 feet, and 99.6 percent had less than 20 feet of error. Figure 4 shows the average error and the principal type of model adjustment for each run. Between two and three man-months of time were spent on model calibration and the cost of computer time was on the order of $100 - $150.

In more detail, the calibration process was started by correcting errors in initial data. These were primarily card-punching errors in recharge, pumpage, and transmissivity. During calibration more such errors were found periodically and doubtless a few errors of this type remain in the model.

After this, the calibration process attempted to eliminate large (greater than 20 feet) errors in computed water-level change. These errors seemed to be more "deterministic" than "random" in that errors were concentrated in specific areas rather than being scattered over the model, and appeared to be related to specific causes, such as errors

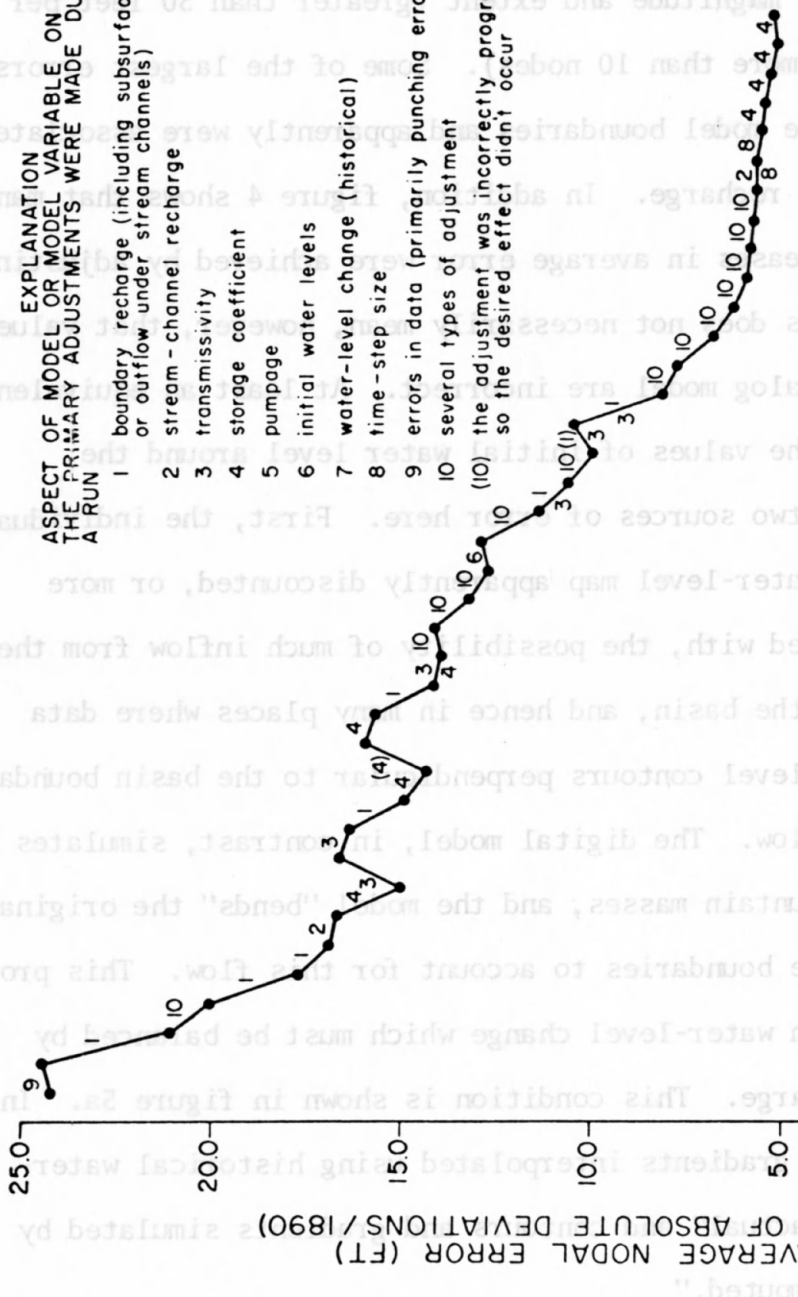
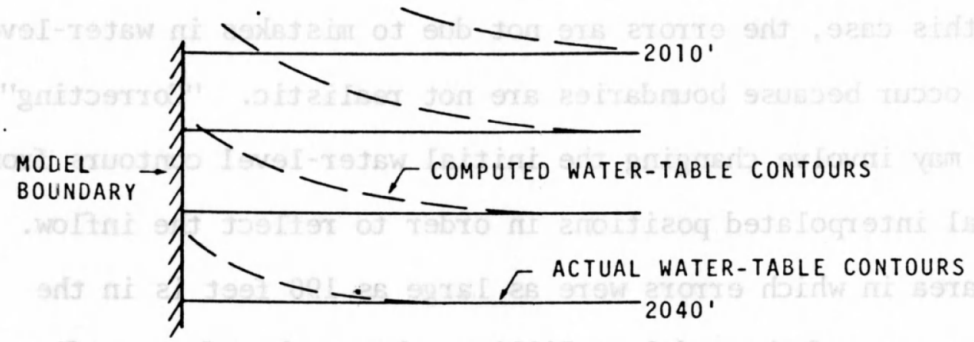


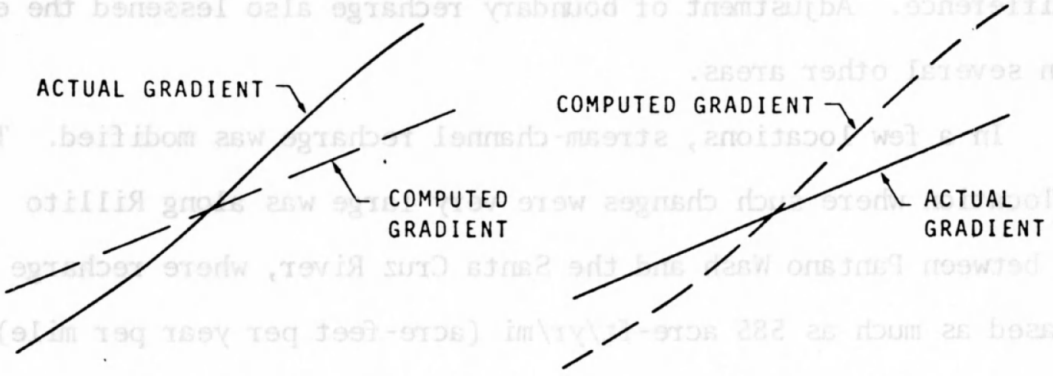
Figure 4. Graph showing average nodal error of successive computer runs during calibration of the Tucson basin model.

in input data in local areas. When model variables were adjusted to reduce errors, care was taken so that adjustments did not put variable values outside a range judged to be reasonable.

The first few calibration runs were aimed at eliminating the several errors of large magnitude and extent (greater than 50 feet per node and extended over more than 10 nodes). Some of the largest errors were concentrated on the model boundaries and apparently were associated with errors in boundary recharge. In addition, figure 4 shows that many of the significant decreases in average error were achieved by adjusting boundary recharge. This does not necessarily mean, however, that values of recharge from the analog model are incorrect. At least an equivalent source of error is in the values of initial water level around the boundaries. There are two sources of error here. First, the individuals who prepared the 1947 water-level map apparently discounted, or more likely were not concerned with, the possibility of much inflow from the mountain masses around the basin, and hence in many places where data were scarce drew water-level contours perpendicular to the basin boundaries, indicating no inflow. The digital model, in contrast, simulates subsurface flow from mountain masses, and the model "bends" the original contours parallel to the boundaries to account for this flow. This produces apparent errors in water-level change which must be balanced by modifying boundary recharge. This condition is shown in figure 5a. In figure 5a, contours and gradients interpolated using historical water-level data are termed "actual" and contours and gradients simulated by computer are termed "computed."



A. PLAN VIEW OF DIFFERENCES IN WATER-TABLE CONTOURS CAUSED BY SIMULATING BOUNDARY RECHARGE WHERE NONE WAS ASSUMED IN CONTOURING.



B. CROSS-SECTIONAL VIEW OF DIFFERENCES IN WATER-TABLE GRADIENT CAUSED BY MODEL TRANSMISSIVITY BEING MORE THAN ACTUAL.

C. CROSS-SECTIONAL VIEW OF DIFFERENCES IN WATER-TABLE GRADIENT CAUSED BY MODEL TRANSMISSIVITY BEING LESS THAN ACTUAL.

Figure 5. Diagrams of differences between actual (interpolated from historical data) and computed water-table contours or water-table gradients caused by errors in model data.

A related problem occurs when the model boundaries do not coincide with the physical boundaries of the basin. In such cases, when contours are perpendicular to these assumed boundaries, and inflow is simulated, errors occur for the same reasons illustrated in figure 5a. However, in this case, the errors are not due to mistakes in water-level contours but occur because boundaries are not realistic. "Correcting" these errors may involve changing the initial water-level contours from their original interpolated positions in order to reflect the inflow.

The area in which errors were as large as 190 feet is in the northwestern corner of the model at Rillito, where subsurface outflow leaves the basin. The initial estimate of outflow of 17,500 acre-ft/yr (acre-feet per year) was reduced to 12,700, which eliminated most of the difference. Adjustment of boundary recharge also lessened the error in several other areas.

In a few locations, stream-channel recharge was modified. The only location where such changes were very large was along Rillito Creek between Pantano Wash and the Santa Cruz River, where recharge was increased as much as 585 acre-ft/yr/mi (acre-feet per year per mile) to raise water levels. This amounts to an increase of about 70 percent over Burkham's estimate (1970, table 5) of 820 acre-ft/yr/mi of stream-channel infiltration in this reach of Rillito Creek or a total of 1,405 acre-ft/yr/mi. Moench and Kisiel (1970, table 1), however, estimated recharge along Rillito Creek from one 10-day flow event, beginning in December 1959, to be from 1,770 to 2,840 acre-ft/mi, so the new figures may not be unreasonable.

The other major type of adjustment that was made in the model was in values of transmissivity. The most effective type of change was in lowering values of T to increase the slope of the water table or in raising values of T to decrease the slope. If computed water levels down-gradient from a node were too high and levels above were too low, lowering T at the node would help correct both problems. Conversely, raising T at a node would raise water levels below and lower levels up-gradient. These relations are shown in figures 5b and 5c.

In three areas, transmissivities were modified significantly to help calibrate the model. In a large area south of Rillito Creek, west of Pantano Wash, east of the Santa Cruz River and north of Davis-Monthan Air Force Base (the Base is in the southeastern part of T.14S., R.14E.), initially-computed water levels were as much as 110 feet above historical levels. The Geological Survey (Anderson 1968, p. 22) had a similar discrepancy in this area, as they lowered its storage coefficient from 0.15 to 0.045 to lower computed water levels. Hydrologists have long noted (Schwalen and Shaw 1957, p. 85-87) a steep water-table gradient along Pantano Wash and in an arc, although at a lesser gradient, to the south and then southeast, through the southern part of the Air Force Base. The model could not simulate this gradient using the data initially given; so the gradient was flattened during a run, resulting in large water-level rises, and thus errors, west of Pantano Wash. In order to maintain the gradients, minimum T values along the wash were lowered from 7,500 to 3,000 gpd/ft. These changes along with changes in recharge to the north, east, and south, changes in pumpage, and

changes in the storage coefficient, eliminated much of the error. This probably was the most difficult area in the basin to model. The maximum nodal error at the final calibration run, 28.6 feet, was in this area, indicating that the problem has not been resolved completely yet. The geologic factors causing this steep gradient are not fully known. The area east of the wash may be the upthrown side of a fault which lifted less permeable material close to the land surface. Flow over such a fault may cause the steep gradient, in effect, creating a ground-water "cascade." It is also possible that flow through material of low permeability or a small storage coefficient in the area of low water-level causes the steep gradient.

Transmissivity was also significantly modified around and to the north of the Tucson International Airport (which is in the west-central part of T.15S., R.14E.). In this area, computed water levels were as much as 60 feet below historical levels. Increases in transmissivity south of the airport and decreases in minimum transmissivities to the north (from 7,500 to 4,000 gpd/ft), along with changes in the storage coefficient, eliminated much of the error. In a third area, just southeast of the confluence of the Santa Cruz River and Rillito Creek, transmissivities were increased to the south and decreased to the north to raise water levels in the area.

In addition, transmissivities were modified in an area around and north of Sahuarita, and along Rillito Creek in T.13S., R.14E. In most of the areas in which T was modified, cross-sections of the water table were drawn between calibration runs by the writer to indicate

where changes in T would be most effective. Transmissivities were also changed at other nodes in the model to eliminate minor errors.

Coefficients of storage were changed in two ways. First, S was modified to lessen errors in two specific localities. In the area west of Pantano Wash discussed previously under changes in T, computed water levels were too high. Values of S had to be reduced, from 0.15 to 0.075, to lower computed water levels because pumpage predominates over recharge there. The Geological Survey made this same type of adjustment in this location, lowering values of S to 0.045. There are no geologic or hydrologic data that suggest such changes -- they were made solely to calibrate the model.

In the area around the Tucson Airport discussed under changes in T, values of S were raised to as much as 0.30 to raise computed water levels. Again, no data indicated that such a change was justified. However, this is also an area of low T, and may be underlain by much fine-grained material. Possibly over long periods of time, slow drainage from these deposits may yield relatively large amounts of water, even though T values are low. If this is the case, higher values of S may be realistic.

Values of S over the rest of the model were adjusted slightly (from 0.150 to 0.156) during the last few calibration runs to balance the volume of water removed from the aquifer corresponding to computed water-level declines (equal to the computed dewatered volume of aquifer), with the volume removed according to historical declines (equal to the historical dewatered volume). The model always balances the net of

discharge from and recharge to the basin with the computed dewatered volume, as this is essentially the way the set of simultaneous finite-difference equations (one for each node) is solved. However, the historical dewatered volume does not necessarily check with this quantity. By raising or lowering S, the volume of water removed according to the fixed historical decline can be increased or decreased to match the net of discharge and recharge. Such a raising or lowering of S decreases or increases, respectively, computed water-level declines. These changes in computed declines generally reduce the average error slightly and improve the model. This second type of change in S is equal over the whole model, and is not varied according to area.

Values of initial water level were modified where analysis of errors suggested that initial levels were incorrect. These areas mostly were around the boundaries of the model, and commonly resulted from contouring that had not accounted for the possibility of boundary recharge, as was previously illustrated (figure 5a). At one location in the Tucson Mountains, however, an initially large water-table gradient resulted in the simulation of a large quantity of recharge in an area that likely furnishes little recharge. A check of the water-level data indicated an alternate interpretation which lessened the gradient, resulting in a more realistic quantity of recharge. In addition, initial water levels at a few locations in the interior of the model were adjusted where errors in computed change coincided with places where interpretation of contours of the 1947 water table seemed questionable in relation to observation-well data.

Values of historical water-level change for 1947-66 also were adjusted at a few locations in the model. Again, wherever significant errors in computed change for 1947-66 coincided with places where the contour map was questionable, historical contours were modified.

In the Tucson basin, much of the pumpage is not measured and must be estimated. Therefore, amounts and assumed locations of pumpage are subject to error and were adjusted during model calibration; although such changes were commonly minor. Changes in pumpage were made when analysis indicated that an error in pumpage was the most likely cause of an error in computed water level. Three kinds of adjustment were made: one involving only the amount at a given node; another involving location of pumpage, and commonly involving amount as well; and a third involving a change in amount of pumpage over a fairly large area.

The first type of change was made when there was pumpage at a node which had a significant error in computed water-level change. Adjustments of as much as 90 percent in pumpage and as much as 1,050 acre-ft/yr (650 gpm) were made to lessen errors. The second type of change was made when centers of significant water-level decline did not correspond with concentrations of pumpage or vice versa, and when such discrepancies were coincident with significant errors in computed water-level change. Because pumpage is the main cause of long-term water-level decline in the basin, centers of significant decline should correspond with concentrations of pumpage; and conversely, concentrations of pumpage should produce some water-level decline. The locations of a few pumpage concentrations were shifted slightly, on the order of a mile, and the

amounts were adjusted where such changes were warranted. The third type of change was made in the area west of Pantano Wash, where pumpage over the whole area was increased by 25 percent to increase computed water-level declines. Davidson (1970, pp. 119-120A) discussed the difficulties in simulating historical water-level declines in this area with the electrical-analog model. He stated that the likely sources of error were in pumpage data and in estimated values of the coefficient of storage. The U. S. Geological Survey decided to adjust S only in this area, but for the study reported here a combination of adjustments in pumpage and S was made.

The number and size of the time-steps used in simulating the 1947-66 period were varied to see how model results would be affected. In the early stages of calibration, use of 30 time-steps (initial step of one minute and successive steps doubled) and three time-steps (either an initial step of 2.72 years and successive steps doubled, or three equal steps of 6.33 years) were compared. The 30-step runs used about 100 seconds of computer time each while the three-step runs used about 20 seconds each. The runs using different time-steps gave significantly different results at some nodes but the overall model results were very similar. During the rest of the calibration runs, three steps with an initial size of 2.72 years were used in order to minimize costs. At the end of calibration, a run using 30 equal steps of 231.3 days each was made. The model results were not significantly different from those of a three-step run, so the model was not calibrated further using 30-step runs.

Few published accounts document calibration of digital groundwater models, and the writer knows of no detailed accounts of calibrating a complex model such as that of the Tucson basin. It therefore is difficult to evaluate the techniques used on this model. The writer had no previous experience in calibrating models, so doubtless this slowed progress. On the other hand, using data from the calibrated electrical-analog model probably shortened the calibration effort significantly. Allison (1967, figure 3.11) showed the relation between mean water-level error and number of calibration runs for the Chino-Riverside basin model constructed by the California Department of Water Resources. The initial mean error was about 200 feet, and the mean error did not reach the Tucson initial mean error of about 25 feet until about 30 runs had been made. Subsequently it took about 25 additional runs to reduce the mean error of the Chino-Riverside model to about 5 feet, the point at which calibration of both models stopped. These data suggest that the availability of the electrical-analog data cut the number of calibration runs of the Tucson model approximately in half.

Figure 4 suggests that the average error in the Tucson basin model cannot be reduced much under 5 feet, since the curve of average error versus number of runs approaches 5 feet asymptotically. This is somewhat misleading, however, in that calibration involved attempts to eliminate errors greater than 20 feet. If the emphasis was on eliminating errors greater than 10 or 5 feet, the mean error could doubtless be reduced to less than 5 feet.

The parameters, initial conditions, and input/output of the model likely could be manipulated until the difference between computed and

historical water-level change was virtually zero at each node. Because the model cannot exactly reproduce the physical ground-water system of the Tucson basin, a model so calibrated would give a false impression of accuracy. A model that matches a 50- to 100-foot historical change within 10 feet at most nodes, as the Tucson model does, probably is adequately calibrated.

It is difficult to assess how closely the calibrated model approximates the true parameters, initial conditions, and input/output of the physical system. There is no guarantee that values of model variables adjusted during calibration are close to true values, or even that adjusted values are improved relative to initial estimates. Many combinations of various values of parameters, input and output functions, and initial conditions can produce an identical water-level or water-level change configuration, so in effect the true values are indeterminate. In other words, a set of values obtained during calibration are non-unique. Adjusted values in the interior of the model, and especially along the major streams and in the city of Tucson, probably are best because it is in these areas that most of the hydrogeologic data have been collected. Estimated values around the boundaries of the model are less reliable because there are few observation wells and few aquifer tests have been made there. Simulated values of boundary recharge, for example, could be greatly in error if the water-table gradients around the boundaries are incorrect.

It probably should be stressed that the emphasis during construction of the Tucson basin model was on developing an experimental or

research tool quickly, rather than a model which was the best possible representation of the basin. In order to make the model more representative, every modification that has been made should be checked against all available hydrologic and geologic data to insure that the changes are valid. This was done in a general way during model calibration, but should be done more thoroughly.

Small-Scale Model

During planning for studies of the worth of ground-water data for the Tucson basin, it became apparent that the 1,890-node model was too detailed because its use would consume too much computer time. It was decided, therefore, to develop a model with a 1-mile nodal spacing instead of the 1/2-mile spacing of the original model. The computer time needed for a run of the less-detailed model is 1/4 to 1/3 that of the original, and made the worth-of-data studies less expensive.

Fortunately, R. E. Lovell (1971) had written a computer program to reduce a model to a coarser nodal spacing so his program was used to made the reduction automatic. The small-scale model was constructed by combining 4-node groups of the large-scale model into single nodes of one-square-mile area. Wherever there were 1, 2, or 3 nodes remaining on a boundary these were made into one node. Thus, instead of a 472.5 node model (1/4 of 1,890) the reduced model has 509 nodes. Construction was begun by taking the northwestern-most two nodes of the original large-scale model as the first node of the reduced model. In this way, much of the western boundary of the reduced model coincides with that of the original model, while muchof the eastern boundary is extended

1/2 mile to the east. This was done because the boundaries of the original model are fairly close to the physical boundaries of the basin on its western side, but to the east, the model boundaries fall short of reaching the physical boundaries.

The input data were transferred to the 509-node model by combining values of each type of data for all the 1/4-mile-square nodes included in a given one-square-mile node. For recharge and discharge, the value for the new node was the sum of values for the included original nodes. For other data (transmissivity, storage coefficient, initial water level, and historical water-level change) the new value was obtained by summing values from all included nodes and dividing by the number of nodes included (essentially calculating the average and applying the average to any extra area included).

The initial run of the small-scale model had errors larger than the final run of the large-scale model -- 6.2 feet average error as compared to 5.3 feet, and a maximum error of 34.4 feet as compared to 28.6 feet (table 4). About 80 percent of the nodes had less than 10 feet error, 89 percent had less than 20 feet error, and two nodes had more than 30 feet error. One problem was that in combining groups of four original nodes into one node, some of the original boundary nodes were combined into nodes which were not on the boundary of the small-scale model. This resulted in transferring some of the original boundary recharge into the interior of the new model, and was the major source of the 34.4 foot error. All original boundary recharge then was moved into adjacent boundary nodes of the reduced model. If more than one adjacent

node was on the boundary, recharge was moved to the one with the largest error in computed water-level change.

These changes, however, did not lessen the model error much, and further adjustments in boundary recharge, transmissivity, and overall coefficient of storage were made in the course of six calibration runs. The model then had an average error of 5.7 feet, a maximum nodal error of 24.7 feet, 82 percent of the nodes with less than 10 feet error, and 99.5 percent with less than 20 feet error. Changes in transmissivity were necessary in two locations where averaging T over 4 nodes had effectively eliminated low values that were necessary to maintain steep water-table gradients. The overall coefficient of storage was lowered from 0.156 to 0.153 (or lowered 0.003 if the original S was larger than 0.156) in order to achieve water-volume balance.

The small-scale model can simulate the 19-year period, 1947-66, using three time-steps, in a run of about six seconds at a cost of less than $1.00.

CHAPTER 3

ERRORS IN DIGITAL MODELING

For this study, an error in digital modeling is defined as the absolute difference, at a given time, between the water level computed at a given model node and the true water level at the corresponding point in the physical system being modeled. This definition is shown in equation 8:

$$e_{t,i,j} = h_{t,i,j} - \hat{h}_{t,i,j} , \qquad (8)$$

where $e_{t,i,j}$ = the modeling error at node (i,j) at time t,

$h_{t,i,j}$ = water level computed by the digital model at node (i,j) at time t,

and $\hat{h}_{t,i,j}$ = true water level at the corresponding point in the physical system at time t.

Modeling errors can be classified as (1) errors associated with computation, (2) errors associated with mathematical assumptions, and (3) errors caused by errors in basic data. Errors in basic data are defined as the difference between the estimated or measured value of a model variable and the corresponding true value of the physical system being modeled. The classification of modeling errors, as specifically applied to the Tucson basin, is shown in more detail in table 5. Although all of these errors will be discussed in a general way, this

Table 5. Errors Associated with the Digital-Computer Model of the Tucson Basin.

- I. Errors associated with computation and related effects
  - A. Roundoff
  - B. Truncation (discretization)
  - C. Algorithm
- II. Errors associated with major assumptions of the mathematical model
  - A. Two-dimensional representation
  - B. Constant transmissivity and coefficient of storage with time
  - C. Confined aquifer
  - D. Miscellaneous
- III. Errors associated with basic data
  - A. Parameters
    1. Coefficient of storage
    2. Transmissivity
  - B. Initial and final conditions (water levels)
  - C. Input and output functions
    1. Discharge
       a. Value
       b. Location
       c. Variation with time
    2. Recharge
       a. Value
       b. Location
       c. Variation with time
  - D. Boundary configuration and idealization

study focused on errors in basic data, and specifically on the worth of additional data in reducing the errors in computed water levels caused by errors in existing basic data.

Errors Associated with Computation

Errors of computation result when a problem is solved by a digital computer, and include roundoff errors, truncation or discretization errors, and errors peculiar to the algorithm used. Roundoff errors occur when the computer rounds numbers, due to finite word-size, during arithmetic operations. The simplest method of evaluating roundoff would be to compute water levels in both single and double precision and to compare the results. Computing in double precision, instead of the normal single precision, would add about twice the number of digits to each computed number. Roundoff would affect mainly the added digits in each number, so the original digits would be relatively accurate. It was not possible to evaluate roundoff error for the Tucson basin model because sufficient computer storage was not available. Roundoff, however, is not likely a major source of error. Carnahan, Luther, and Wilkes (1969, p. 442) discussed roundoff error for the algorithm used to solve the set of simultaneous flow equations for the Tucson basin model, and concluded that for most choices of nodal spacing and time-step size, roundoff error is small in comparison to truncation error.

Truncation, or discretization, error results from the approximation of a differential equation by a finite-difference equation, and essentially results from approximation of derivatives by assuming linear

changes in head between nodes and between time-steps. A mathematical expression for an approximation of truncation error for the Gauss-Seidel algorithm, as applied to the Tucson basin model, can be derived using a Taylor series. If equation 1 is simplified by assuming T constant over space to yield:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t} + \frac{QR}{T}, \tag{9}$$

then the local (at a given node) truncation error (E_{LT}) is:

$$E_{LT} \simeq \frac{(\Delta x)^2}{12}\left[\frac{\partial^4 h}{\partial x^4} + \frac{\partial^4 h}{\partial y^4}\right] - \frac{S}{T}\frac{\Delta t}{2}\frac{\partial^2 h}{\partial t^2}, \tag{10}$$

providing $\Delta x = \Delta y$ is the nodal spacing. This can be derived easily using methods given by D. W. Peaceman (written communication, 1969, p. 34) in a set of notes for lectures at a NATO School on "Hydrocarbon Reservoir Simulation by Computers." This equation shows that truncation error is proportional to the algebraic sum of the square of the nodal spacing ($(\Delta x)^2$) and the time-step size (Δt). An approximation for truncation error for the alternating-direction-implicit algorithm is more complicated to derive, as the algorithm involves solution in two steps. However, Carnahan et al. (1969, p. 453) stated that the discretization error for this method is proportional to the algebraic sum of the square of the nodal spacing and the square of the time-step size.

It might be possible to obtain a rough estimate of local truncation error at given nodes utilizing finite-difference expressions for

$\frac{\partial^4 h}{\partial x^4} + \frac{\partial^4 h}{\partial y^4}$ and $\frac{\partial^2 h}{\partial t^2}$. However, as Peaceman points out (written communication, 1969, p. 35),

> As a practical matter, for problems involving any complexity at all, estimates of the [truncation] error are best obtained by solving the difference equations with different mesh sizes [nodal spacings], varying both Δx (and Δy and Δz) as well as Δt to determine their effects on the solution. In many cases, practical values of Δx and Δt (wherein the computational work is not excessive) may be so large that the error does not appear to decrease as rapidly as predicted by the formulas for local truncation error. The reason for this is that expressions for the order of the error describe the asymptotic behavior as Δx and Δt approach zero and really say nothing about the behavior of the error for large mesh sizes. Consequently, we must content ourselves frequently with empirical estimates for the errors obtained by running the same problem with several different mesh sizes. We would then run the remainder of the cases with that mesh size which balances the risks associated with the apparent error against the cost of running with a smaller grid size.

Tests of this type were not done for the Tucson basin model itself, but were done, to a limited extent, on a 400-node model, as discussed in Chapter 2 above in the section, "Comparison of the Algorithms." Tables 1, 2 and 3 show that the error can be more than 10 feet out of a total water-level change of 50-90 feet, near an input or output source. Since in these experiments the mathematical model was assumed correct and the basic data were assumed correct, the error was associated only with computation, and likely was chiefly truncation error.

The third component of computational error is defined here as algorithm error. This category includes errors that do not seem to fit into the classification of roundoff or truncation error. For example, in the Gauss-Seidel method, a solution is considered to be adequate when the sum of the nodal errors is less than a set quantity

or tolerance level. However, this tolerance level can be varied, and the difference in solutions for various tolerances is here classified as algorithm error.

In addition, there will be differences in solutions depending on the method of computing the average or internodal transmissivities. Bouwer (1969, pp. 394-396) estimated average values of K, hydraulic conductivity (for a definition of K see p. 107 below), using the arithmetic, harmonic, or geometric means. He showed that the arithmetic mean is applicable when flow through porous media is parallel to regions of different K, and the harmonic mean is applicable when flow is across regions of different K, or series flow. Bouwer concluded that the intermediate value given by the geometric mean might be the best estimate of average K (or T) when flow is a combination of parallel and series flow. Lovell (1971) decided that the harmonic mean was most applicable to the Tucson basin model, but for this study the arithmetic mean was used, as was used by Pinder and Bredehoeft (1968, p. 1075). These differences in computing average K or T might also be classified as algorithm errors, for lack of a better classification system.

During this investigation, the only study of algorithm errors was in comparing results using different ways of computing tolerance levels, as discussed in Chapter 2 above in the section on "Comparison of the Algorithms." Defining tolerance at each node instead of over all nodes made little difference in results, but algorithm errors merit more study.

Errors Associated with the Assumptions of the Mathematical Model

Using a relatively simple mathematical model, equation 1, to represent the complex ground-water flow system of the Tucson basin involves many assumptions, some obvious and others more subtle. Equation 1 is a quasi-linear, time-invariant, 2-dimensional equation, in that T and S are constant over time and vertical components of flow are ignored. Some implicit assumptions of the equation are that wells (or points of input/output via the term QR) fully penetrate the aquifer, that water is released from storage instantaneously with decline in head (this also refers to S being time-invariant), that the laws of Darcy and Hooke hold, and that temperature is constant. In addition, the aquifer system in the Tucson basin is unconfined, but strictly speaking, equation 1 applies only to confined aquifers.

Some of the errors associated with these assumptions have been studied by those who have worked with the Theis equation (equation 24, below), a solution of a special case of the general flow equation. For example, Jacob (1950, p. 384) concluded that if the total head change is small relative to the total saturated thickness, equation 1 can be used to describe an unconfined system. The total effective saturated thickness over the Tucson basin is not well known, so this assumption is difficult to evaluate.

Two major assumptions that probably are violated for the Tucson basin, at least at some locations for some time periods, are (1) that components of vertical flow are not significant and (2) that transmissivity is constant with time, based on the assumption that the saturated

thickness of the unconfined aquifer is constant. At locations where recharge or discharge is large, vertical components of flow may be significant, and a three-dimensional model would better describe the system. However, in a large-scale model, vertical components of flow in local areas might not be a significant source of error. Sufficient computer storage was not available to construct a three-dimensional model of the basin; and in addition, it is not likely that there are enough data on variations in hydraulic conductivity with depth to make such a model meaningful.

It would be possible to include an approximation to a time-varying transmissivity in the Tucson basin model, at the cost of a relatively small increase in computation time. The model could recalculate transmissivity after each time-step, based on the change in saturated thickness during that time-step. The relation involved is:

$$T = Kb \ ,$$

where K = hydraulic conductivity (defined more completely below, p. 107), in ft/day or gpd/sq ft; and b = saturated thickness of aquifer, in feet. However, K at each node might have to be considered constant over b because, as mentioned above, there likely are few data on the actual variations in K with depth. Therefore a comparison of results using a constant T and time-varying T might not give a good approximation of the actual error.

The coefficient of storage also may undergo an apparent change with time because of the slow release of water from storage in relatively fine-grained sediments. These changes have been observed in the Tucson

basin (Clyma et al. 1968). However, after long periods of time (months or years) such as are simulated with the model, the apparent S should approach the true S and little error should result from assuming it constant with time.

Although there has been little attempt in this investigation to study rigorously the errors associated with all the assumptions of the mathematical model, it is an area that needs further study.

Errors Associated with Basic Data

Errors in the basic data used in models probably are one of the major sources of error, and have been the focus of much of the work in this study. Remson, Hornberger, and Molz (1971, p. 65) are of the opinion that "errors in approximation [truncation errors] are generally outweighed by the inaccuracies due to the uncertainties of the specification of subsurface hydrologic parameters." Most errors in basic data are well-recognized by modelers, although some, such as variations in discharge and recharge over relatively short time periods and errors in boundary configurations, commonly receive little attention.

In general, errors in data can be of several types, such as instrumental or measurement error, interpolation error, and errors due to data not being representative of the aquifer. Instrumental or measurement error probably is present always, although it likely is a minor problem. Interpolation errors arise when field data are contoured to yield estimates at all nodes, as is commonly done for the coefficient of storage, transmissivity, and initial water levels. Some field data may not be representative of or even may not be from the aquifer being

modeled. Measurements of water levels in wells that are being affected by local pumping, or in wells tapping perched water bodies, for example, will not be representative of aquifer conditions. Errors due to interpolation and non-representative data are likely significant problems.

Even if measurement errors and errors due to interpolation and non-representative data were not present, estimates of the parameters and initial water levels at a model node still would be in error because of imperfect sampling in space. Values of parameters and initial water levels of the physical system being modeled will vary naturally over a nodal area, and any sampling procedure can only approximate the true value. The problems discussed above suggest a need for study of optimal design of networks for collecting ground-water data.

Parameters

Errors in estimating the coefficient of storage over the Tucson basin model are due mainly to a lack of data, as discussed in "Data for the Models" (in Chapter 2 above). Even if data were available from properly-designed aquifer tests, errors would be associated with measurement of well-discharge and water levels during tests, interpretation of test results, and interpolation of data from tested to untested nodes.

For transmissivity, errors arise for the same reasons: from erroneous aquifer-test data, errors in interpretation, and faulty interpolation of these data to all nodes of the model. In preparing the map of transmissivity used to estimate T at each node, the U. S. Geological Survey likely used sources of data other than aquifer tests, such as

geologic data from test wells and general knowledge about patterns of sedimentation in the basin. However, these sources of data and interpretations made from them likewise are not free of error.

For the Tucson basin, transmissivities estimated using data from short-term aquifer tests and using methods of analysis which do not account for delayed drainage likely will be larger than actual values. Clyma et al. (1968, pp. 13-14) demonstrated that delayed drainage during the first few hours of pumpage lessens the rate of water-level decline, and that using these water-level data leads to unrealistically large values for T.

Initial and Final Conditions

In order to compute water-level changes for any period, initial water levels for each node must be estimated from a contour map. At this point in the model construction, interpolation of data is always required, unless water-level data are available for every node. The maps are contoured using measurements of water levels in observation wells. The estimated values at each node may be in error because of errors in measurement or because data are not representative of the aquifer. Contouring water-level data over the Tucson basin was a subjective process, and errors certainly were introduced during contouring.

Digital models commonly are calibrated by adjusting model parameters and other data so that computed water levels match historically-measured levels, which could be termed "final conditions," at one or more points in time. Errors also enter the model because historical "final" water levels include measurement errors or are non-representative.

In addition, if historical water-level data were interpolated to all nodes in a model, such as was done for the Tucson basin model, interpolation or contouring errors also are present.

Input and Output Functions

Errors in assumed values of discharge and recharge lead to errors in the model. Such errors can be classified as errors in the quantity of or in the assumed location of discharge or recharge and errors related to time-variations of discharge and recharge not accounted for by the model.

In many basins pumpage is measured, so the major error is in measurement with no error in location. In the Tucson basin, however, pumpage, except for that by the city of Tucson, largely is estimated and there are errors both in the quantity and assumed location of pumpage. In addition, the model assumes pumpage constant over long periods of time. The smallest time periods over which the U. S. Geological Survey estimated pumpage is one year; thus, actual variations in pumpage within a season, week or day cannot be included in the model and can lead to errors in predicted water levels.

Another form of discharge, evapotranspiration from the water table, is not included specifically in the model. Although such discharge is small over most of the basin, it may be significant along some stream channels. However, this discharge likely was at least partly accounted for in the model by adjusting values of recharge along streams during model calibration.

The various categories of recharge are also in error with respect to quantity, location, and variation with time. Recharge from streams was estimated primarily using data from infiltration studies by the U. S. Geological Survey, and any errors in the assumptions or in the measurements made during those studies will lead to errors in the model. In addition, the amounts of infiltration into stream channels are not equivalent to recharge to the water table because some water is lost by evapotranspiration and some is used to satisfy soil-moisture requirements. Any method of estimating recharge from infiltration data necessarily will include errors.

The model also assumes recharge constant for long reaches of streams, when in fact recharge probably varies along a reach because of variations in the hydraulic conductivity of sediments beneath the stream-channels. Stream-channel recharge also varies with time, contrary to the model assumptions. For ephemeral streams such as those in the Tucson basin, significant recharge occurs during only a few months of the year and commonly during only a few days of those months. Total stream-recharge also varies from year to year and the proportion of recharge contributed by a given channel reach also varies from year to year.

There is also some lag between the time of infiltration to the stream-channel bed and the time when water actually reaches the water table, for which the model does not account. This lag apparently is of the order of a few days for the reach of Rillito Creek studied by Moench and Kisiel (1970, figure 3). The lag, of course, would be greater if the water table were deeper than the less than 50-foot depth at the time

of the flow event studied by Moench and Kisiel. In predicting regional water levels at the end of a long time period with a digital model, however, local space and time variations in recharge assumed constant may not cause significant errors.

Boundary recharge, including recharge directly from mountain masses or from stream-channel infiltration in the foothills, and subsurface inflow under stream-channels from tributary basins, can be estimated roughly at best, as these quantities cannot be measured directly. Boundary recharge and subsurface inflow chiefly were estimated as the quantities necessary to achieve model calibration, but as was previously discussed in Chapter 2 (in "Calibration of the Models"), lack of data around the boundaries makes such estimates unreliable with respect to quantity and location. Subsurface outflow, actually a form of discharge, also was estimated in this way and includes similar errors.

Boundary recharge, and especially recharge from mountain masses, may not vary with time significantly because variations in precipitation on the mountains may be largely damped as the water moves into the alluvial basin. This is also true, although perhaps to a lesser extent, of subsurface flow from tributary basins. The assumed nature of the boundary probably also leads to error. The boundary is assumed to be impermeable in the model and rates of recharge thus are not affected by changes in hydraulic gradient at the boundary. In reality, an increase in hydraulic gradient, such as is caused by water-level declines in the basin, likely will increase recharge, mostly due to withdrawal of water from storage in the bedrock of the mountains or in tributary basins.

The Tucson basin model does not now simulate any recharge from infiltration of excess irrigation water. The electrical-analog model calibration process suggested that prior to 1958, about 25 percent of pumpage was infiltrated to the water table in the southern part of the basin, but that this percentage lessened after 1958 (Anderson 1968, p. 24). This same recharge, however, may have been at least partly accounted for by adjusting pumpage during calibration of the digital model.

Boundary Configuration and Idealization

Model boundaries commonly are delineated along contacts between permeable alluvium and rock of low hydraulic conductivity in the mountains around a basin, or at water-table divides between adjacent hydraulically connected alluvial basins. However, geologic data on effective contacts between permeable and less permeable material, or on estimated water-table divides, may be in error. Perhaps a larger source of error is that model boundaries often cannot be placed exactly at geologic contacts or water-table divides, either because of limitations on the total size of the model or because smooth lines cannot be closely approximated by the model grid.

In summary, all types of data used in a digital model of a ground-water basin contain some error of which modelers should be well aware. Additional errors are introduced by the process of computation and because of the simplifying assumptions of the mathematical model. During model calibration, when model parameters, initial conditions, and input/output are adjusted so that the computed water-level change matches

historical change, compensation is made for these various errors. In other words, errors are "eliminated" by altering values of storage coefficient, transmissivity, initial water levels, pumpage, and recharge.

The calibration process initially may move estimated values of basic data closer to true values, but eventually, if calibration proceeds until computed changes approach the exact historical changes, a point will be reached where calibration yields adjusted values of basic data that will move away from true values. This will happen, however, only when computational errors and errors due to mathematical assumptions are significant. In addition, as stated previously, many combinations of various values of parameters, initial conditions, and input/output can produce identical water-level-change values, especially for a single historical time period; so that the true basic-data values are indeterminate, and a set of values derived from calibration is thus nonunique. If many historical matching periods are available, the calibrated basic-data values may approach some mean values which adequately predict water levels in the future, but these will not be identical to the true values because they are in part still compensating for other model errors and assumptions. If future conditions in the basin, such as water levels, pumping and recharge patterns, etc., vary greatly from those in the periods used for model calibration, the calibrated values for model parameters may not predict future water levels accurately. As Lovell (1971, p. 11) pointed out, "continued withdrawal of water from the aquifer below the level where data have previously been available would produce behavior not encountered at the time of calibration and, therefore, not incorporated in the adjustment program."

After calibrating the Tucson basin model, it was observed that calibrated values of transmissivity commonly were far from sample values obtained from aquifer tests in corresponding nodal areas. The means of sample values of T at the 57 nodes in which more than one aquifer test had been made were compared to the calibrated values of T. The mean ratio of the absolute value of the difference between the sample mean and the calibrated value to the sample mean was 0.47. For example, if the sample mean was 100,000 gpd/ft, the calibrated value tended to be about 47,000 larger or smaller. At 8 of the 57 nodes compared, the difference between the sample mean and the calibrated value was about the same magnitude as the sample mean.

These relatively large differences, of course, may not be caused primarily by compensation, during calibration, for errors in computation, errors due to mathematical assumptions, and errors related to the algorithm. As is pointed out in the present discussion (see pages 69 and 107), sample values of T may be in error for several reasons. Delayed drainage during the aquifer-test period, for example, may result in sample values of T being too large. This particular problem may be the cause of a large part of the observed difference between sample means and calibrated values, because 40 of the 57 sample means were larger than the corresponding calibrated values.

CHAPTER 4

USE OF STATISTICAL DECISION THEORY TO EVALUATE WORTH OF GROUND-WATER DATA

This study focused on a problem often faced by field hydrologists -- given that error exists in estimates of parameters, initial conditions, and input/output for a ground-water basin, what additional data collected at what locations in the field would add the most knowledge about the basin? For this study the question was rephrased to ask -- what new data collected in the Tucson basin would yield the most improvement in the digital model? It probably would be necessary in any case to evaluate improvement, or worth of new data, in terms of a digital or other type of model because these tools presently offer the best method of estimating the response of a complex ground-water flow system to development of water. Davis and Dvoranchik (1971) and Davis (1971) evaluated the worth of additional surface-water data using statistical decision theory, and their approach has been modified here to study the worth of additional ground-water data to a digital model.

For the present worth-of-data studies values of parameters, initial water levels, and discharge/recharge were assumed to be still far enough from true values so that additional sampling of the actual physical system would tend to improve model data. Although this assumption likely would be good during the early stages of studies of a basin, later it would be difficult to be sure that it was valid. If this assumption

were invalid, of course, worth-of-data studies such as these would be of little value, because additional sampling might yield variable values that would result in a poorer model, in the sense that predicted water levels would be less accurate, even though the variable values would be more representative of the physical system being modeled.

Statistical decision theory was used in this study because some more or less objective method was needed to compare the effects of errors in different kinds of variables. Sensitivity analyses, such as those proposed by Meyer (1971), can be used to evaluate the sensitivity of the model to an error introduced in a variable at a given node, but this sensitivity cannot be compared directly with sensitivities of different variables because there is no way to choose exactly equivalent errors, representing the same degree of uncertainty, in two variables at the same or different nodes. Statistical decision theory provides a relatively objective method of choosing equivalent errors, in that errors located at the same number of standard deviations from the mean can be considered equivalent.

In this study, errors in one variable at one node at a time were evaluated, and data at all other nodes were assumed correct. Thus errors in a given variable at different nodes were considered independent of one another. Where variable values at each node are measured separately, such as is commonly done for pumpage, sometimes done for initial water levels, and which theoretically could be done for all other variables, the assumption that errors are independent may be reasonable. However, if data on a variable are not available at each

node, such as is common for storage coefficient, transmissivity, initial water levels, and recharge; estimates commonly are made at nodes without data. These estimates can be interpolated from a map showing contoured values of a variable, such as is common for transmissivity and water levels, or by using point measurements to estimate values over wide areas or along zones, such as is common for coefficient of storage and recharge. In these cases, errors at one node are not independent of one another. For this study, data on variables were judged to be insufficient to estimate joint probabilities of dependent errors, and all errors were considered independent even though this was somewhat unrealistic. In addition, use of dependent errors and use of the technique described in this report would consume a prohibitive amount of computer time (see p. 147). However, study of the dependence of errors at adjacent nodes resulting from the contouring process merits more work.

Errors in different variables at the same or different nodes were also assumed independent. This assumption is reasonable because variables commonly are measured independently. Even though transmissivity and storage coefficient can be obtained from a single aquifer test, values of S from aquifer tests in the Tucson basin are unreliable and were not used (see p. 30). Errors in recharge are the only errors that might be dependent on errors in other variables. Recharge for the Tucson digital model was derived largely from calibration of the analog and digital models, and thus values of, and errors in, R depend on values of, and errors in, initial water levels, transmissivity, and storage coefficient along model boundaries and stream channels.

Loss Functions

In order to use statistical decision theory, a loss or objective function must be specified. As used in this study, this function is an attempt to quantify the cost of an error in predicted water levels. The basic loss function (L) was defined:

$$L(V^{n}_{k,p,q},m) = \sum_{t=1}^{TT} \sum_{i=1}^{I} \sum_{j=1}^{J} C(e_{i,j})_{i,j} \cdot e_{t,i,j,k,p,q}; \qquad (11)$$

where $V^{n}_{k,p,q}$ = the nth (n=1,2,...N) possible value of the kth (k=1,2,...K) variable V (in the Tucson model K was assumed to be 4 and the K variables are storage coefficient, transmissivity, initial water level, and discharge/recharge) at a given single node (p,q) in a digital model;

m = mth value of V_k (m can be any of the N values of V_k), assumed to be its true value;

t = time-step;

TT = total number of time-steps in the simulation period;

i = row location in grid (north-south coordinate);

I = total number of rows in model grid;

j = column location;

J = total number of columns;

and $C(e_{i,j})_{i,j}$ = cost per foot of water-level error at node (i,j) as a function of the magnitude of that error.

The magnitude of water-level error at node (i,j) for time t caused by an

error in the kth variable (V_k) at a given node (p,q) was further defined as:

$$e_{t,i,j,k,p,q} = |h^n_{t,i,j,k,p,q} - \hat{h}^m_{t,i,j,k,p,q}| \; ; \qquad (12)$$

where $h^n_{t,i,j,k,p,q}$ = predicted water level (head or potential) at node (i,j) for time t computed by a digital model using the nth possible value of the kth variable at node (p,q), ($V^n_{k,p,q}$);

and $\hat{h}^m_{i,j,k,p,q}$ = water level at node (i,j) for time t computed assuming that the mth value of $V_{k,p,q}$ is the true value.

Thus L is the loss over all nodes (i, i = 1,2,..., I; j, j = 1,2,..., J) associated with using the nth value of V_k at node (p,q) instead of the "true" mth value. This equation implies that the errors at each node (i,j) at each time t have independent effects and that they can be summed to yield a total effect. If the ground-water basin is operated as a single unit, these assumptions are reasonable. If this is not the case, the cost coefficient $C_{i,j}$ can be set equal to zero for any node at which an error does not affect a given water user.

Although the basic loss function probably gives the most information about loss, other functions were judged to be necessary to give a more complete evaluation. For example, the basic function, equation 11, yields the same results if (1) all nodes have a moderate error or (2) most nodes have a small error while a few nodes have a very large error.

Therefore in addition to the basic loss function, five alternate loss functions were derived and used in the worth-of-data studies. The first is a quadratic loss function:

$$L = \sum_{t=1}^{TT} \sum_{i=1}^{I} \sum_{j=1}^{J} C(e_{i,j})_{i,j} \, (e_{t,i,j,k,p,q})^2 \; ; \qquad (13)$$

the second is the loss associated with the maximum nodal error in the model:

$$L = \max_{t,i,j} C(e_{i,j})_{i,j} \cdot e_{t,i,j,k,p,q} ; \qquad (14)$$

and the last three are losses associated with numbers of nodes in error by specified quantities:

$$L(u) = \sum_{t=1}^{TT} \sum_{i=1}^{I} \sum_{j=1}^{J} C(e_{i,j})_{i,j} \cdot NN(u)_{t,i,j} \; ; \qquad (15)$$

$$\text{where } NN(u)_{t,i,j} = \begin{cases} 1 & \text{if } e_{t,i,j,k,p,q} \geq u \; ; \\ 0 & \text{otherwise} \end{cases} \qquad (16)$$

$$u = 5, 10, 25.$$

Equation 13 defines the loss associated with an error in $V_{k,p,q}$ as the sum of the squares of the errors in water levels over all nodes and time-steps; equation 14 defines the loss as the maximum water-level error over all nodes and time-steps; and equation 15 defines the loss (for u=25, for example) as the number of nodes over the model, for all time-steps, at which the error in water levels was equal to or greater than 25 feet.

If the variable value $V^n_{k,p,q}$ that would minimize the loss as expressed by equation 14 were chosen, it could be viewed as an application of the minimax decision criterion. This procedure consists of minimizing the maximum possible error (in this case it would be the maximum expected error). Minimax is commonly a more conservative decision criterion because, for example, a higher overall level of error over a digital model might be accepted in return for a lower maximum error.

These loss functions are all symmetrical, in that positive and negative errors of equal size are considered equivalent. For specific management problems this may not always be true. For example, suppose the problem was to forecast when the water level would fall below the bottom of a well, necessitating its deepening or replacement. The cost of predicting the water level too low, so that the well was replaced prematurely, would be different than the cost of predicting the level too high, so that the well went out of production before it could be replaced. However, specific management problems of this type were not considered in this study, so asymmetric loss functions were not derived.

In the basic worth-of-data studies, losses were computed or evaluated only at the end of the simulation period, and were summed over time only for a sensitivity test, primarily because summing or evaluating loss over all time-steps used too much computer time (see Appendix A). In addition, the cost coefficient $C(e_{i,j})_{i,j}$ was set equal to 1.0 at all nodes and was not made a function of $e_{i,j}$. In order to define a meaningful cost-coefficient function, a specific management problem would

have to be considered. Except for an idealized management problem, for which a simple cost coefficient was assumed, management problems were not defined for the worth-of-data studies, and the cost coefficients were set to unity for simplicity.

Normally loss functions are defined as the economic loss pertaining to a given decision in light of the unknown true state of nature. However, the determination of true economic loss was judged to be beyond the scope of this study, and loss was defined in terms of feet of error in predicted water levels, $e_{t,i,j}$. A loss or objective function in true economic terms might be expressed as the difference between all benefits derived from the use of a bit of additional data and all costs expended in obtaining it. Costs could be determined relatively easily, but determining all the future primary economic benefits, let alone secondary benefits, from an added bit of data would be very difficult. This subject, however, deserves a detailed formal study.

If the ground-water resources of a basin were controlled by one organization or manager, and this manager could assign an economic cost per foot of prediction error at each node of a digital model, then the loss functions defined previously would yield true economic loss. However, to the writer's knowledge there has been little research in determining costs of prediction errors. In fact, it is not entirely clear what level of accuracy is necessary in model studies of ground-water basins. A modeler may require that the model reproduce historical water-level elevations or change within 10 feet, but such results may be

more (or less) accurate than those needed by water-resource administrators. The cost of pumping ground water for irrigation in central Arizona is about $0.03 per acre-foot per foot of lift (Nelson and Busch 1967, p. 36). If predicted water levels were in error by 10 feet, the resulting error in estimated pumping costs would be $0.30 per acre-foot. This error is only three percent of the value of $10.00 per acre-foot for water used to irrigate low-value crops, and even a lesser percentage if the water were applied to high-value uses. This suggests that models do not need to be particularly accurate, especially if constructing and operating accurate models is costly.

However, such a conclusion may ignore other aspects of ground-water basin operation. Fairly accurate knowledge of water levels may be necessary for scheduling well-deepening or replacement, for planning artificial recharge operations, for prediction of the migration of poor-quality water or of land subsidence, and other activities.

The simple cost coefficient, $C(e_{i,j})_{i,j}$, as defined in the loss functions, can be used in a general way to approximate economic loss. If water-level errors in one part of the model are judged to cause more harm than in other parts, the cost coefficient can be used to weight the losses accordingly.

In larger perspective, it is possible that errors in knowledge of non-hydrologic aspects of ground-water basin development and operation, such as economic, legal, political, or institutional factors, may be more significant than errors in hydrologic data. Generally similar conclusions were drawn by James, Bower and Matalas (1969) in

relation to use of the water resources of the Potomac River, and tentatively drawn by Thomas Maddock III in relation to a problem involving irrigation with pumped ground water (oral communication, 1970).

Risk

Loss cannot be computed directly, however, because the true value of the variable is not known. Risk (RK) is the expected value of loss given any choice of a value of a variable, and is a more useful concept:

$$RK(V^{n}_{k,p,q}, P_{pr}) = \underset{m}{E(L)}$$

$$= \sum_{t=1}^{TT} \sum_{m=1}^{N} \sum_{i=1}^{I} \sum_{j=1}^{J} C(e_{i,j})_{i,j} \cdot e_{t,i,j,k,p,q} \cdot P_{pr}$$

$$\{V^{m}\}\ ; \qquad (17)$$

where E = the expectation operator;

N = the total number of possible values of V; and

$P_{pr}\{V^{m}\}$ = the probability of occurrence of the m^{th} value of V which is distributed $N(\mu_{pr}, \sigma^{2}_{pr})$ (normally with mean μ_{pr} and variance σ^{2}_{pr}) or $LN(\mu_{pr}, \sigma^{2}_{pr})$ (log-normally), a prior probability in Bayesian terms where pr signifies prior.

The risk given any choice of a value of a variable is computed by summing the losses over all possible true values of the variable

weighted by the prior probabilities of the true values. This definition requires the variable to be a random variable that can be described by a probability distribution, in this example a discrete distribution. The above definition means that risk is evaluated using an expected-value criterion. Benjamin and Cornell (1970, p. 531-541) concluded that expected value is a logical basis for choosing among alternatives in engineering decisions.

The use of continuous distributions for the variables was considered, but the expected costs of computation were judged to be too great. For each point on a distribution of a variable, a complete set of water levels must be computed by the digital model. Although continuous distributions are more representative, many more points are needed to define them adequately, and computing sets of water levels for these extra points would be costly. In addition, data on the variables likely are insufficient to define adequately their distributions.

Admittedly, the use of discretized and truncated distributions requires careful evaluation in the context of Bayesian statistical decision theory. Little work was done on these problems in this study, although some of the results of the sensitivity tests (see p. 151) indicated that discretization did not well approximate the frequency distributions of the model variables.

Expected Opportunity Loss

Opportunity loss was defined by Benjamin and Cornell (1970, p. 528) as the loss associated with not making the best possible choice of action in light of the true state of nature. Opportunity loss is then the difference in benefit (or cost) of the choice actually made and the benefit (or cost) of the choice that would have been made if the true state of nature had been known.

Because the true state of nature is not known, expected opportunity loss (EOL) is a more useful concept. EOL was defined for this study as:

$$EOL = \underset{n}{\text{Min}} \, (RK), \tag{18}$$

where $\underset{n}{\text{Min}}$ = the minimum value of risk over the N values of $V_{k,p,q}$. EOL is thus the expected loss associated with the value of $V_{k,p,q}$ that yields the minimum risk, or $V^{*}_{k,p,q}$ (under the assumption that there is no loss if knowledge of the variable is perfect). This, under normal conditions, is the value of $V_{k,p,q}$ with the highest probability of occurrence and would be the logical choice for the variable value if no further sampling were possible. EOL also can be characterized as the expected error over the model associated with the uncertainty in a given variable at a given node $V_{k,p,q}$.

Expected Worth of Sample Data

The goal of this analysis was to estimate the improvement that could be made in a model by sampling for more data. This improvement was defined as the difference between EOL (or expected error) before

sampling and EOL after sampling. However, the so-called expected value of the expected opportunity loss after sampling (EEOL) can be estimated, without doing any actual sampling, by computing EOL for every possible sample result. First, for every possible result of sampling an unknown $V_{k,p,q}$, a new probability distribution, called a posterior probability distribution, can be computed for the variable by means of Bayes Theorem (Benjamin and Cornell 1970, p. 556; Schmitt 1969, p. 62-65). This theorem, put in the context of our example is:

$$P_{ps}\{V^m | V^x_{k,p,q}\} = \frac{P_{pr}\{V^m\}\, P_\ell\{V^x | V^m\}}{P\{V^x\}}, \qquad (19)$$

where $V^x_{k,p,q}$ = the x^{th} possible result of sampling $V_{k,p,q}$ ($x = 1, 2, \ldots N$);

$P_\ell\{V^x | V^m\}$ = the probability of sampling V^x given that V^m is the true value of $V_{k,p,q}$, distributed N or $LN(V^m, \sigma^2)$ where σ^2 is the variance of a sample (P_ℓ is a likelihood function in Bayesian terms where ℓ signifies likelihood); and

$$P\{V^x\} = \sum_{m=1}^{N} P_{pr}\{V^m\}\, P_\ell\{V^x | V^m\}, \qquad (20)$$

the total probability of observing a sample V^x.

$P\{V^x\}$ acts as a normalizing factor in equation 19. Therefore P_{ps} is the probability of a value V^m being the true value given that a sample yields a result V^x, a posterior probability in Bayesian terms, where ps signifies posterior. Equation 19 expresses the idea that posterior probability is proportional

to the product of prior probability and likelihood. In general, if V^m is really the true value of V, successive sampling increases the posterior probability that V^m is the true value given the available data V^x on $V_{k,p,q}$.

Using these distributions, EEOL can be computed as defined in equation (21):

$$EEOL = \underset{x}{E}(EOL) = \sum_{x=1}^{N} \left[\min_{n} \sum_{t=1}^{TT} \sum_{m=1}^{N} \sum_{i=1}^{I} \sum_{j=1}^{J} C(e_{i,j})_{i,j} \cdot e_{t,i,j,k,p,q} \cdot P_{ps}\{V^m|V^x\} \right] P\{V^x\} . \quad (21)$$

EEOL is determined using equation 21 by (a) computing the risk for each choice of a variable value V^n assuming a given sample result, (b) determining the value with the minimum risk for each possible sample result (V^*), and (c) weighting the sum of these minimum risks by the probability of observing each sample result.

The expected worth of sample data (EWSD) was defined as:

$$EWSD = EOL - EEOL. \quad (22)$$

This is the difference between expected opportunity loss before and after sampling. The optimum bit of data to collect for the model is the bit with the largest EWSD ($EWSD^*$), defined as:

$$EWSD^* = \max_{k,p,q} EWSD , \quad (23)$$

where $\max_{k,p,q}$ = the maximum EWSD over all k variables (k = 1,2,....K) at each node (p,q) (p = 1, 2, ...I), (q = 1, 2, ...J). Alternately,

the variables at various locations could be ranked in order of the worth of additional samples of data on V for improving model results.

The equations given in this chapter were incorporated in a computer program (Appendix A) and used to estimate worth of additional data to the Tucson basin model. This technique includes basin dynamics in estimating worth of additional data, by means of using the digital model to compute all values of predicted and "true" water levels included in the loss function.

In actual practice, evaluating worth of data might proceed in stages. A preliminary or initially-calibrated digital model could be used to choose the data that would most improve the model. This data would be collected, if possible, and used to modify the model, which then would have to be recalibrated to some extent. The process could be repeated until model improvement was judged, by some objective criterion, to be of less value than the cost of collecting additional data. However, the techniques developed in this study likely are adequate only to indicate, in the initial stages of model building, which data are most critical to the model.

CHAPTER 5

WORTH OF DATA FOR THE TUCSON BASIN MODEL

As an example in using statistical decision theory to approximate the worth of collecting additional hydrogeologic data to improve a digital model of a ground-water basin, variables of the small-scale Tucson basin model were tested to determine their associated expected error and expected worth of sample data.

Major Assumptions of the Worth-of-Data Studies

The major assumptions made in the worth-of-data studies include the assumptions inherent in the digital model and the assumptions of the method used to compute worth of data. The main assumptions in the digital model are discussed briefly in Chapter 3 in the section on "Errors Associated with the Assumptions of the Mathematical Model." This section summarizes the assumptions of the method, although specific assumptions also have been discussed in the text where they are made.

First, only the worth of additional data to a digital model is evaluated, and not the worth of data to any other kind of evaluative tool. Secondly, worth of added data is evaluated only in terms of feet of reduction in error in predicted water levels over the model, and not in terms of economic benefits resulting from reduced error. In addition, only the worth of added hydrologic data is considered; the worth of added data on legal, political, or institutional factors was not studied.

Thirdly, the statistical criterion used to evaluate error and reduction in error is expected value, and not the maximum likelihood, the minimization of the maximum error, or some other criterion (see pp. 83 and 87). Fourthly, this study assumes that a digital model is in a relatively early state of calibration, so that collection of added data will tend to improve the model, rather than yield values which will result in a poorer model.

Some additional detailed assumptions of the method are listed below. (1) Errors at individual nodes are assumed to be statistically independent, or not related to each other. (2) Only one variable at one node at a time is considered in error; all other model variables are assumed to be correct. (3) Functions of loss due to error are assumed to be symmetrical -- in other words, positive and negative errors are given equal weight. (4) Errors are assumed to be additive in that the model error over all nodes is an algebraic sum of errors at individual nodes. The computer program, however, has the capability of weighting errors at individual nodes if such weighting is justified. (5) Functions of loss due to error are computed only at one point in time, although the program has the capability of approximating the integration of loss functions over time. (6) The frequency distributions of the model variables are assumed to be either normal or log-normal, and to be adequately represented by discrete and truncated distributions. Parameters of the distributions had to be subjectively estimated because few sample data are available at individual nodes.

Time Period Used

In order to make the worth-of-data studies as realistic as possible, a time period in the future was used. A hydrologist interested in determining which types of additional data at what locations would most improve the predictive capability of his model would have to select a time period to use for his worth-of-data studies. He probably would select the future time period over which he wished to predict for his studies. The period selected for this study was the 20 years from the spring of 1970, assumed to be the "present," to the spring of 1990. In order to use this period, additional basic data had to be compiled for the Tucson basin model. The coefficients of storage and transmissivity over the model were assumed to be the same as for the 1947-66 calibration period. Recharge to the model was likewise assumed equal to that previously determined, and constant over the 1970-89 period. Initial water levels and discharge, however, had to be recompiled.

A map of the contours of water-table elevations for the spring of 1970 was obtained from the Department of Agricultural Engineering and used to estimate representative water levels at each node of the 509-node model. These initial levels were used to predict water-level change for an arbitrary 19-year period in order to check their compatibility with the computer model. A 19-year period was assumed because the already compiled 1947-66 pumpage and recharge data were used in the test simulation. Predicted changes were unrealistic at several places around the model boundary, due to the same problems discussed in Chapter 2 in the section on "Calibration of the Models." All measurement data

used in preparing the contour map were then plotted on the map of 1970 water levels and used to make reasonable adjustments of the contours around the model boundaries.

Estimating pumpage for the 1970-89 period was not as straightforward, primarily because nobody has made comprehensive estimates of future pumpage from the Tucson basin. The U. S. Geological Survey made an estimate of 1962-65 pumpage over the basin and assigned values to each node of their electrical-analog model. In addition, they made lumped estimates of pumpage from the basin as a whole for the years 1965 through 1969. J. F. Rauscher, Chief Engineer of the city of Tucson Department of Water and Sewers, has made the only basin-wide predictions of water use in a chart entitled "Table of Water Requirements in Acre-Feet, period 1970-2030," dated May 16, 1968. In addition, the Department of Agricultural Engineering supplied their available, although incomplete, data on locations of the wells owned by the mining companies in the southern part of the basin, and the city of Tucson furnished data on well locations and current and projected pumpage for the mines.

All these data were used in making a rough estimate of pumpage for 1970-89. A better estimate could have been made but would have taken considerable time and effort. Since the purpose of this study was primarily to test the method using realistic data rather than to obtain the best possible worth-of-data values for the Tucson basin, the rough estimates of pumpage were deemed sufficient.

The estimates were made using the following procedure. First, the 1962-65 values at each node in the 1,890-node model (distributed values) were summed and compared with the lumped 1965 estimate. The 1965 total was about 10 percent greater, so 1962-65 distributed values were increased to match by first adding any pumpage for the mines that did not appear to have been included in the Geological Survey analog model, and then increasing each node by the remaining seven percent difference.

The 1965 distributed values were then adjusted to match the 1969 lumped value. The Geological Survey subdivided their lumped values into irrigation, municipal, and industrial uses. From 1965 through 1969 irrigation use declined slightly, municipal use increased by about 30 percent, and industrial use almost doubled, primarily due to increased pumpage for the mines. The 1965 values were adjusted to 1969 by (1) adding the pumpage by the mining companies, (2) assuming that all the irrigation decrease was accounted for by wells taken out of production on irrigated farm land retired by the mining companies, and (3) increasing pumpage over the area in and around Tucson in which the city has production wells. In addition, some pumpage was added to account for new wells drilled by the city in their Santa Cruz well-field between Tucson International Airport and Sahuarita. The 1970 values of pumpage were assumed to be equal to 1969 values, except for wells of the mining companies, for which actual 1970 estimates were available.

The 1970 distributed values then were adjusted to give an estimate of 1989 values of pumpage. This was probably the poorest of the

estimates, as the only projections available node by node were those for wells of the mining companies. Even the estimates by the mining companies are likely too high because the companies apparently assumed they would do no recycling of water. The adjustment to 1989 was made by increasing the total pumpage for the mines from 35,000 to 43,000 acre-ft/yr -- this corresponds with the city of Tucson's estimate of pumpage for the mines rather than the mining companies' own estimate of 53,000 acre-ft/yr. In addition, some wells drilled in the city's Santa Cruz field and presently held in reserve were assigned some pumpage. Pumpage over the rest of the basin was assumed equal to 1970 values. This may seem a poor assumption in view of the general belief that population in the Tucson basin will continue to grow over the next 20 years. However, J. F. Rauscher of the city's Department of Water and Sewers (oral communication, 1971) claimed that basin pumpage will likely be declining by 1980 because of availability of alternate supplies outside the basin, such as water from the Central Arizona Project and Avra Valley. However, because some degree of doubt exists about when and whether either of these supplies will be available, the basin discharge for 1989 was not decreased from the 1970 values. The assumption that it will be about the same is perhaps as reasonable an assumption as could be made currently.

The average pumpage for 1970-89 used for the worth-of-data studies with the 509-node model was derived by averaging the annual values for 1970 and 1989, multiplying by 20 years and converting the data from the 1,890-node grid to the 509-node grid.

Basic Worth-of-Data Studies

Estimation of Parameters for the Prior Distributions and the Likelihood Functions

In order to use statistical decision theory to estimate worth of data to the Tucson basin model, parameters (μ, σ) of the assumed normal frequency distributions -- or log-normal for transmissivity -- for the variables studied had to be determined or estimated for both prior distributions and likelihood functions. Unfortunately, few sample data commonly are available within given nodal areas for the four variable types studied, so the parameters had to be estimated, and to a large extent, subjectively estimated. This was true even though the Tucson basin is typical, or even well-endowed, in the amount of hydrogeologic data available. For problems of this type, therefore, such subjective estimates invariably will be necessary.

Subjective estimates of parameters are those made primarily on the basis of the experience, judgment, and intuition of the estimator; whereas objective estimates are based only on sample data. Subjective estimates of probability are especially useful where repetitive sampling to obtain objective estimates of relative frequency is not possible. Benjamin and Cornell (1970, pp. 40-41) contrasted estimating the probabilities associated with tossing coins, which can be done by experiment, with estimating the probability that the material at a depth of 30 feet beneath a bridge footing is clay. This probability must be estimated subjectively prior to drilling, which will settle the question once and

for all. In this case the probability is not a relative frequency but expresses an individual measure of the relative likelihood of an outcome.

The process of subjective estimation can be made more "objective" by employing standard techniques, such as those discussed briefly by Benjamin and Cornell (1970, pp. 538-539; pp. 541-544). Folayan (1969, pp. 26-33) obtained subjective estimates of parameters for the distribution of the in-situ compressibility of a soil by questioning engineers familiar with the soil. Such techniques were not used in this study because the writer made all necessary subjective estimates. Subjective estimates were needed for parameters of distributions of variables at each node of the digital model. Making the approximately 2,000 necessary estimates was considered to be too formidable a request to make of local practicing hydrologists, and unnecessary for a study which was primarily to develop a general approach for evaluating worth of ground-water data.

A prior distribution, P_{pr}, for a given variable at a given node represents the best estimate on the distribution of possible true or representative values of the variable, based on available sample data and/or the experience and intuition of the hydrologist making the estimate. A likelihood, P_{ℓ}, is the probability of observing a given sample assuming the mean of its distribution has a given value. For the discrete distributions used in the worth-of-data studies, a likelihood is the probability of observing, or sampling, one of the discrete values of the variable being tested, assuming that one of these possible

values is the true value, or $P_{\ell}\{V^{x}|V^{m}\}$. Each likelihood was estimated by using the assumed true value as the mean of a normal, or log-normal, distribution. However, estimating the standard deviation of this distribution, or likelihood function is more complicated.

Initially a "sampling" standard deviation ($\sigma_{\ell ms}$, see p. 125) was estimated for each variable to be tested. This is the standard deviation of the likelihood function associated with collecting one more sample, assuming that the model value of the variable, or the value assumed true in the model, is the true mean. The first set of likelihoods, then, is the probability of observing each of the possible values of the variable, including the model value, if the model value is the true mean. Subsequently, each of the other possible values is assumed to be the true mean and a likelihood function is derived. The method used here to estimate the standard deviations of these likelihood functions was different for each variable type.

As an example, assume that the value of T at a node in the model is 50,000 gpd/ft, and six alternate, or erroneous, values are assumed to be 5,000, 15,000, 30,000, 75,000, 125,000, and 200,000 gpd/ft. The first likelihood function consists of the probability of obtaining each of the seven values of T from an aquifer test if the mean, or expected or true, value of T for the node is 50,000. These probabilities are determined from a standard normal probability table, using μ_T = 50,000 and the assumed value of $\sigma_{T_{lms}}$. In this study, however, the logarithms of the values of T were used as T was assumed to be log-normally distributed. The next set of likelihoods is determined by assuming that the

mean or expected nodal value is 5,000 gpd/ft and again computing the probability of obtaining each of the seven values of T from an aquifer test. This process is repeated for each of the other five values of T to obtain the complete set of likelihood functions.

One difficulty in discretization is associated with the discretization of likelihood functions. If the extreme lower (or upper) value of $V_{k,p,q}$ is assumed to be the mean, all the alternate values like above (or below) the mean. In fact, for only the model value as mean are there equal numbers of alternate values above and below the mean. Therefore, almost 50 percent of the area under the probability curve is not assigned initially to any value of $V_{k,p,q}$. Normalization distributes this "unused" probability to each variable value, but the resulting probabilities are not equivalent to probabilities computed when the mean is centrally located. Because of the asymmetry, the assumed mean and its two closest values have a higher probability, and the four values farthest from the mean have a lower probability, than if values were symmetrically distributed around the mean. Additional alternate values could be selected so that each assumed mean were centrally located, but then the total number of variable values would be 43 -- six extra alternate values for each of the six original alternate values plus the original seven values. This would require computing 43 sets of water levels over the model instead of 7, and would be much more costly.

Coefficient of Storage

The "true" value of storage coefficient that is uncertain at a given node is not a physical entity that could be measured if means were available, but it is the representative storage coefficient for the node. The coefficient of storage is defined as the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head (Lohman and others 1970, p. 38). The value is already somewhat of an average, representing an integration of the specific storage -- defined by Lohman and others (1970, p. 37) as the volume of water released from or taken into storage per unit volume of the aquifer per unit change in head -- over the saturated thickness of the aquifer. The representative S for a node might be defined as that value which when used as the nodal value, results in correct water-level changes corresponding to given water-volume changes in the node. Providing that the digital model of the aquifer is a good approximation of the physical system, this representative S should be close to an average of S values measured over the node. If an aquifer test were made in a node so that the cone of depression extended over the entire node, the derived value of S also might be a good approximation of the representative S.

Unfortunately, as previously discussed in Chapter 2 in the section on "Data on Coefficient of Storage and Transmissivity," there are few reliable samples of S in the Tucson basin. Therefore the prior mean and standard deviation and the standard deviations of the likelihood functions had to be estimated for each node in the model. The

estimation of means was discussed under "Data on Coefficient of Storage and Transmissivity," and the standard deviations were estimated subjectively.

Frequency Distribution. There has been little research on what type of frequency distribution samples of S might follow. Transmissivity is commonly assumed to be distributed log-normally, as will be discussed subsequently, and it might be argued that S also is distributed log-normally over nodal and basin areas because many of the features of sediments that affect transmissivity affect S in the same way. For example, well-rounded, well-sorted, coarse, uncemented sediments tend to have both high S (Johnson 1967, table 17) and high transmissivity. However, the relation between the two, if any direct relation exists, is complex because the highest S values are commonly observed in medium or coarse sand and slightly lower values are observed in gravel (Johnson 1967, table 29), which presumably have higher values of transmissivity. In addition, fine-grained sediments which are being compacted may, over long periods of time, yield significant quantities of water, and thus have a relatively high value of S along with a low transmissivity.

The writer knows of no study in which measurements were made of S at random over an aquifer and then plotted to obtain a frequency distribution. Johnson (1967, table 11) reproduced a table of specific yields (for the unconfined aquifer of the Tucson basin, specific yield is virtually equivalent to storage coefficient) of core samples from California. The distribution around the mean specific yield of each

textural classification -- sand, silt, etc. -- apparently is symmetrical because the mean and median of each class are virtually the same. These data suggest that S is not log-normally distributed; however, the statistics are not computed for data from random samples so no conclusion is warranted.

The distribution of S over nodal areas was assumed to be normal for lack of data suggesting any other distribution. Because normal distributions extend from values of minus infinity to plus infinity, truncated normal distributions commonly are used to avoid negative values when they are physically impossible, unreasonably low values, or unreasonably high values. In this study discrete distributions were used for all model variables, and the computer program automatically eliminated any alternate variable values that were infeasible. For this reason, truncated normal distributions were not needed.

Estimation of Parameters. For the purpose of this study, standard deviations of both prior distributions and likelihood functions of S were estimated to be constant over given intervals of S, as shown in table 6. Estimates were made for two classes, nodes in the interior of the basin and nodes near the boundaries. Uncertainty about given values of S was assumed to be somewhat less in the interior of the basin, where hydrogeologic data are more plentiful -- because of more wells, and thus more geologic data, aquifer tests, water-level measurements, and pumpage data -- than near the basin boundaries, where data are sparse. Presumably judgment and experience would be more effective in estimating S in areas of much hydrogeologic data. Therefore, standard

Table 6. Estimated Standard Deviations around Mean Values of the Coefficient of Storage.

Interval of Storage Coefficient in which Mean Occurs	Standard Deviation, for Nodes in the Interior of the Basin	Standard Deviation, for Nodes near the Boundaries of the Basin
0.0-0.0375	0.05	0.07
0.0375-0.1125	0.06	0.08
0.1125-0.1875	0.07	0.09
0.1875-0.2625	0.08	0.10
> 0.2625	0.09	0.11

deviations in the interior of the model were assumed to be uniformly 0.02 less than those on the boundaries.

Standard deviations also were assumed to be proportional to the magnitude of the mean S, although there were no data to use in obtaining an objective estimate of such a relationship. If S was in the artesian range, for example, the standard deviation around a value of 0.0001 would certainly be less than that around a value in the water-table range of 0.01.

The standard deviations of the prior distribution and the likelihood function were assumed equal for a given value of S. This implies that the amount of information, or number of samples or aquifer tests, used to estimate the prior probability distribution is equal to the amount collected in an additional sample. For example, this could be interpreted as assuming that the equivalent of one sample of S in professional judgment and experience was used to estimate its value in a nodal area. Sampling to modify this prior estimate to obtain a new, or posterior, probability then is assumed to involve collecting the equivalent of one additional sample.

The assumptions made in estimating parameters for frequency distributions of S, or for that matter, any of the variables studied, are inadequately justified and represent very subjective judgment. They are, however, the kinds of assumptions that will have to be made in using this type of technique to estimate worth of additional data to a digital model of a ground-water basin; and they were believed adequate to illustrate the technique. If the goal of this study had been to

obtain the best worth-of-data estimates possible, the parameters could have been defined better by utilizing such methods as interviewing hydrologists familiar with the basin and eliciting their opinion on the distributions of S, or other variables, at various points in the basin. Folayan (1969) used questionnaires to obtain similar opinions from engineers about soil properties. Research to determine the type of frequency distribution followed by the coefficient of storage also would be useful.

Transmissivity

The "true" value of transmissivity desired at a node of a model is the value that best represents T over the whole nodal area. Similar to storage coefficient, T is already somewhat of an averaged quantity in that it is defined as the rate at which water of the prevailing kinematic viscosity is transmitted through a unit width of aquifer under a unit hydraulic gradient (Lohman and others 1970, p. 41). Transmissivity thus refers to the entire thickness of aquifer whereas hydraulic conductivity, K, refers to a specific volume of aquifer. A medium has a hydraulic conductivity of unit length per unit time if it will transmit in unit time a unit volume of water, at the prevailing kinematic viscosity, through a cross-section of unit area, measured at right angles to the direction of flow, under a hydraulic gradient of unit change in head over unit length of flow path (Lohman and others 1970, p. 9). Therefore, T might be considered as representing an integration of K over the aquifer thickness, although T also depends on the degree of interconnection of beds with high (or low) hydraulic conductivity.

If point measurements of T could be made over a nodal area, the representative value of T probably could be estimated by averaging these measurements; providing, of course, that the digital model of the aquifer was a reasonably good representation of the physical aquifer system. Estimates of T commonly are obtained by aquifer tests, and an equivalent to sampling T over the nodal area might be an aquifer test for which the cone of depression extended over the whole nodal area. The length of test required can be roughly estimated using a variant of the Theis equation (Ferris and others 1962, pp. 92-94):

$$s = \frac{Q}{4\pi T} (W(u)) \ , \qquad (24)$$

where $u = r^2S/4Tt$,

r = radius, in feet,

s = drawdown, in feet, at a radius r,

S = coefficient of storage,

T = transmissivity, in cubic feet per day per foot (cu ft/day/ft),

t = time, in days,

Q = pumpage, in cu ft/day,

and $W(u) = \int_u^\infty (e^{-u}/u)\, du$.

If a well is pumped at the center of a node, it can be computed that the cone of depression will take on the order of nine days to reach the boundaries of the node -- assuming values typical of the Tucson basin such as T = 10,000 cu ft/day/ft or 75,000 gpd/ft,

S = 0.15,

and Q = 192,000 cu ft/day or 1,000 gpm,

and that r = 3,190 ft or 0.6 miles and a significant s = 0.005 ft.

Choosing 0.005 as a significant value for drawdown is somewhat arbitrary, but 0.005 is about the limit of precision for the wetted-tape method of measurement.

However, few aquifer tests of this length have been made in the Tucson basin. Most of the available tests were several hours in length, and the cone of depression for a 6-hour test, using the assumed values in the previous computation, would have a radius of about 525 feet -- which includes only about three percent of a nodal area. Therefore, even with several aquifer tests per node, considerable uncertainty remains about the representative value.

There are only 168 nodes in the digital model in which aquifer tests have been made, and 341 without tests. Of the sampled nodes, 45 have two aquifer tests, and only 12 have more than two tests, the maximum being five. For only a handful of nodes, then, are data adequate for even a rough estimate of the natural variability of T. The individual test results are, of course, subject to error, primarily due to errors in measurement of discharge and in the subjective interpretation of test results by the hydrologist, mainly in the curve-fitting procedures.

Figure 2 shows the distribution of aquifer tests over the basin. The numbers on the map indicate the tests per land section or square mile, which is not equivalent to the tests per nodal area because the nodal areas are not exactly equivalent to sections. For this figure, only the aquifer tests analyzed by the U. S. Geological Survey were used, which include 94 percent of the total number of tests. The map

illustrates that most of the aquifer tests have been made in the city of Tucson and in the irrigated areas along the Santa Cruz River.

Frequency Distribution. McMillan (1966, pp. 8-17) summarized available research on the frequency distribution of permeability (hydraulic conductivity, K) and transmissivity. He pointed out that the variation in K depends on the volume of material considered. On a microscopic scale K could vary from zero to infinity depending on whether the volume considered was impermeable rock or a pore space. As sample volume increases, the possible limits of the variation in the average K for the volume lessen, which is a result predicted by the central limit theorem of statistics. However, when the sampling procedure, such as an aquifer test, obtains information from more than one geologic unit, the variation may well increase. In the Tucson basin, where the aquifer material consists of basin-fill deposits which are made up of small (measured in tens of feet) individual units largely of alluvial origin, the cone of depression of even a short-term aquifer test inevitably will extend across several units.

McMillan (1966, pp. 10-15) discussed research by several workers in petroleum reservoir engineering which indicated a log-normal distribution for K. He also plotted data (pp. 15-17 and figures 2.3-2, 2.3-3, and 2.3-5) on transmissivity from ground-water basins in California and concluded that T was approximately log-normally distributed. McMillan did not speculate on why K and T are log-normally distributed, although he mentioned that explanations for the natural occurrence of the distribution have been based on the assumption that

the effects of an underlying random variate are multiplicative (p. 11). Benjamin and Cornell (1970, pp. 262-263) further discussed how the log-normal distribution represents aspects of a breakage process, such as the transport of sediment in streams. The final size of a particle depends on collisions with particles of many sizes traveling at various velocities. This multiplicative process may produce a particle-size distribution that is log-normal. Since hydraulic conductivity is related to particle size, (Todd 1959, p. 51), the log-normal distribution of K may result from a log-normal distribution of particle sizes. Although this relation seems logical for K for small volumes of aquifer, it does not necessarily explain a log-normal distribution of K or T over a large ground-water basin. A log-normal distribution for values of T representing large subareas of a basin implies that there is more coarse than fine sediment in an aquifer than there would be if sediment size was normally distributed around some mean.

Data on T from aquifer tests over the entire Tucson basin were compiled (table 7) and the cumulative percentages of T values were plotted against T on log-normal probability paper (figure 6). The data fall on a straight line over most of their range, indicating the values are log-normally distributed. The line is curved, however, at its extremes. At its upper end, the curve indicates fewer very high values of T than if T were distributed purely log-normally. For example, the curve shows that 99.5 percent of the T values are less than 700,000 gpd/ft; whereas if the straight-line portion of the curve were extended, 97 percent would be less than 700,000. At the lower end of the line,

Table 7. Probability Data on Values of Transmissivity Derived from Aquifer Tests in the Tucson Basin

Value of Transmissivity, in gallons per day per foot	Cumulative Numbers of Values of Transmissivity from Aquifer Tests that are less than values in the first column[a]				
	Values from the Entire Basin	Values from Areas in which Transmissivities are Estimated to be less than 50,000[b]	Values from Areas in which Transmissivities are Estimated to be from 50,000 to 100,000[b]	Values from Areas in which Transmissivities are Estimated to be from 100,000 to 180,000[b]	Values from Areas in which Transmissivities are Estimated to be more than 180,000[b]
1,000	0	0	-	-	-
2,000	1	1	-	-	-
4,000	1	1	-	-	-
7,000	4	4	0	-	-
10,000	8	7	1	0	-
20,000	25	20	4	1	-
40,000	62	43	16	3	-
70,000	100	56	37	7	0
100,000	127	60	52	14	1
150,000	161	67	63	28	3
200,000	177	68	66	39	4
300,000	195	71	67	40	17
400,000	211	71	67	44	29
500,000	220	72	67	44	37
700,000	225	-	69	45	39
1,000,000	226	-	-	-	40

Table 7--continued

Value of Transmissivity, in gallons per day per foot	Cumulative Percentages of Values of Transmissivity from Aquifer Tests that are less than Values in the first column				
	Values from the Entire Basin (%)	Values from Areas in which Transmissivities are Estimated to be less than 50,000 (%)	Values from Areas in which Transmissivities are Estimated to be from 50,000 to 100,000 (%)	Values from Areas in which Transmissivities are Estimated to be from 100,000 to 180,000 (%)	Values from Areas in which Transmissivities are Estimated to be more than 180,000 (%)
1,000	0	0	-	-	-
2,000	0.4	1.4	-	-	-
4,000	0.4	1.4	-	-	-
7,000	1.7	5.6	0	-	-
10,000	3.5	9.7	1.4	0	-
20,000	11.0	27.8	5.8	2.2	-
40,000	27.4	59.7	23.2	6.7	-
70,000	44.2	77.8	53.6	15.6	0
100,000	56.1	83.3	75.4	31.1	2.5
150,000	71.2	93.1	91.3	62.2	7.5
200,000	78.3	94.4	95.7	88.9	10.0
300,000	86.3	98.6	97.1	91.1	42.5
400,000	93.4	98.6	97.1	97.8	72.5
500,000	97.3	100.0	97.1	97.8	92.5
700,000	99.5	-	100.0	100.0	97.5
1,000,000	100.0	-	-	-	100.0

[a]Aquifer-test data from a compilation by the Department of Agricultural Engineering, University of Arizona; only those tests evaluated by the U. S. Geological Survey were used in this table (this includes 94 percent of the tests).

[b]From a map prepared by the U. S. Geological Survey showing areas in which the transmissivities are estimated to be between given values.

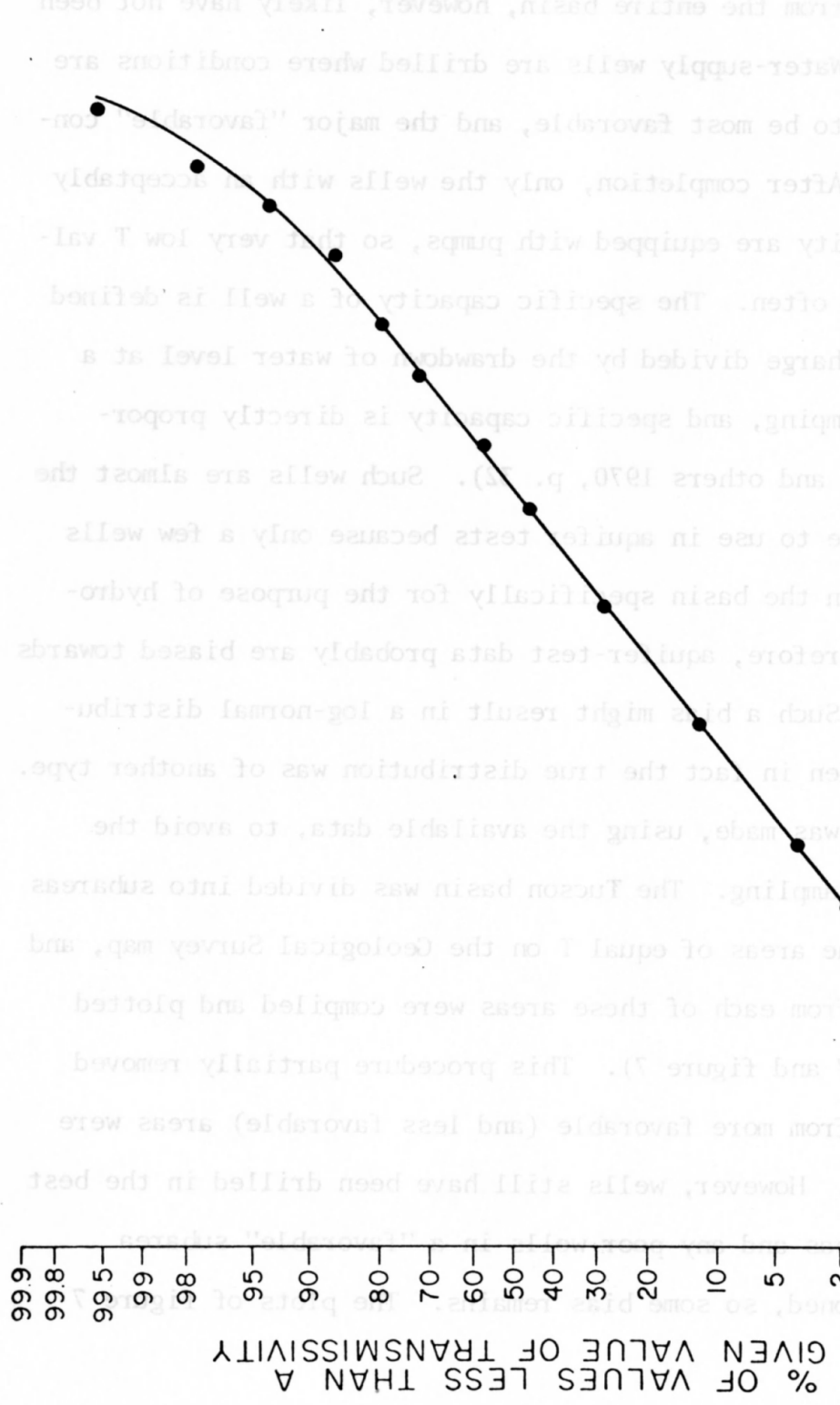


Figure 6. Graph of the probability that a given transmissivity value will result from an aquifer test in the Tucson basin.

there is a slight indication of more low values of T than if T were distributed log-normally.

These data from the entire basin, however, likely have not been sampled randomly. Water-supply wells are drilled where conditions are known or predicted to be most favorable, and the major "favorable" condition is high T. After completion, only the wells with an acceptably high specific capacity are equipped with pumps, so that very low T values are not sampled often. The specific capacity of a well is defined as its rate of discharge divided by the drawdown of water level at a given time after pumping, and specific capacity is directly proportional to T (Lohman and others 1970, p. 32). Such wells are almost the only wells available to use in aquifer tests because only a few wells have been drilled in the basin specifically for the purpose of hydrologic testing. Therefore, aquifer-test data probably are biased towards high values of T. Such a bias might result in a log-normal distribution of T values when in fact the true distribution was of another type.

An attempt was made, using the available data, to avoid the potential bias in sampling. The Tucson basin was divided into subareas corresponding to the areas of equal T on the Geological Survey map, and the aquifer tests from each of these areas were compiled and plotted separately (table 7 and figure 7). This procedure partially removed bias because data from more favorable (and less favorable) areas were studied separately. However, wells still have been drilled in the best parts of the subareas and any poor wells in a "favorable" subarea probably are abandoned, so some bias remains. The plots of figure 7

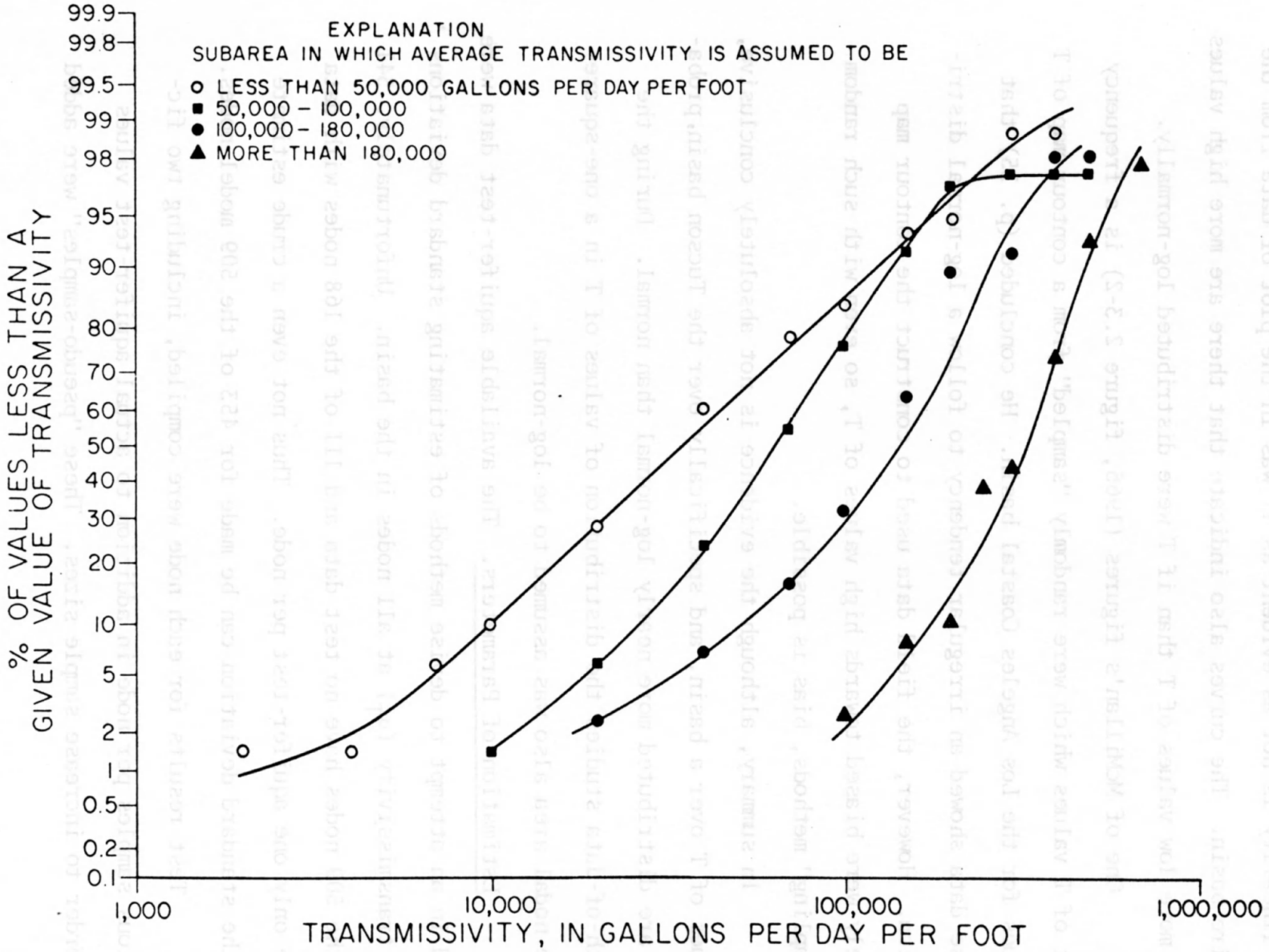


Figure 7. Graph of the probability that a given transmissivity value will result from an aquifer test in a subarea of the Tucson basin.

show generally linear trends, suggesting log-normal distributions, but the linearity is not as evident as it was in the plot of data from the entire basin. The curves also indicate that there are more high values and more low values of T than if T were distributed log-normally.

One of McMillan's figures (1966, figure 2.3-2) is a frequency plot of T values which were randomly "sampled" from a contour map of T values for the Los Angeles Coastal basin. He concluded (p. 15) that these data showed an irregular tendency to follow a log-normal distribution. However, the field data used to construct the contour map likely were biased towards high values of T, so even with such random "sampling" methods, bias is possible.

In summary, although the evidence is not absolutely conclusive, values of T over a basin, and specifically over the Tucson basin, probably are distributed more nearly log-normal than normal. During the worth-of-data studies the distribution of values of T in a one-square-mile nodal area also was assumed to be log-normal.

Estimation of Parameters. The available aquifer-test data were used in an attempt to devise methods of estimating standard deviations for transmissivity (σ_T) at all nodes in the basin. Unfortunately, 341 of the 509 nodes have no test data and 111 of the 168 nodes with data have only one aquifer-test per node. Thus not even a crude estimate of the standard deviation can be made for 453 of the 509 model nodes.

Test results for each node were compiled, including two fictitious samples per node in addition to actual aquifer-test values in order to increase sample sizes. These "pseudo-samples" were added

only to nodes with some actual data, and were added primarily so that σ_T at the 111 nodes with only one test could be estimated crudely. The pseudo-samples were (1) the nodal value shown on the map of T compiled by the U. S. Geological Survey and (2) the nodal value after calibration of the digital model -- at 71 nodes these two values were equal. Addition of these pseudo-samples was believed to be justified somewhat because (1) considerable geologic knowledge and professional judgment were used in preparing the T map and (2) the value after calibration was one that improved the model's ability to match historical water-level changes and thus may represent additional information.

An attempt was made to estimate σ_T at unsampled nodes by relating σ_T to some factor which could be measured at all nodes. It was hypothesized (1) that σ_T was proportional to the magnitude of T, and (2) that σ_T was proportional to the local variability in T. It was believed reasonable, for example, that if σ_{T_1} around a mean value of 10,000 gpd/ft (μ_{T_1}) were 5,000, then σ_{T_2} around a mean value of 100,000 (μ_{T_2}) would be nearer to 50,000, which assumes $\sigma_{T_2} = \sigma_{T_1} \times \mu_{T_2}/\mu_{T_1}$, than it would be to 5,000, which assumes σ_T = constant. It was also believed reasonable that σ_T would be greater in an area where values of T varied greatly over short distances, because results from short-term pumping tests likewise would be variable. Such a marked variability in T might be encountered in areas where deposition was controlled by several different processes, or deposits came from several source areas, such as along a stream channel or near a mountain front.

Assuming the above hypotheses were true, an attempt was made to relate σ_T to (1) the magnitude of the nodal T from the U. S. Geological

Survey map, T_M, as this was the only available prior estimate of T at all nodes; (2) the maximum difference between T_M and values at the four adjacent nodes -- essentially, the local maximum T "gradient," and an estimate of local variability; and (3) both of these factors combined. In addition, the uncertainty in T, and therefore σ_T, was assumed proportional to the distance to the nearest sample of T, as the farther a node was from a sampled node, the more uncertain its value likely would be. Although this attempt to estimate σ_T at unsampled nodes was unsuccessful, as discussed below, the estimated values of σ_T at sampled nodes were used as a general guide to standard deviations and therefore the procedure will be discussed here.

A computer program was written to compute the information required to analyze the T data. The program first computed the two statistics, the "prior" sample mean $\bar{x}_{T_{pr}}$, based on both actual and pseudo-samples; and the prior sample standard deviation, $s_{T_{pr}}$, of T at each node, in arithmetic units. The program also computed the standard deviation of the likelihood function for the model value as mean, the "sampling" standard deviation or s_{T_ℓ}. This represents the standard deviation associated with collecting one additional sample, and was computed using the equation:

$$s_{T_\ell} = s_{T_{pr}} \sqrt{n} \quad , \tag{25}$$

where n = the number of samples per node. Equation 25 is an adaptation of the standard statistical formula:

$$\sigma_x = \sigma_{\bar{x}} \sqrt{n} \quad , \tag{26}$$

where σ_x = the population standard deviation, and

$\sigma_{\bar{x}}$ = standard deviation of a group of n samples.

Here $s_{T_{pr}}$ is assumed equivalent to $\sigma_{\bar{x}}$, as it represents a standard deviation from prior sampling or equivalent information; and s_{T_ℓ} is equivalent to σ_x in that s_{T_ℓ} is related to the uncertainty in collecting one more sample. The computer program also calculated the distance between each unsampled node and the nearest sampled node. Equation 26 is applicable only if the samples of T obtained by aquifer tests are statistically independent. If the hydraulic properties of the aquifer vary significantly over distances as small as tens of feet, as is likely in the Tucson basin, then values of T from short-term aquifer tests spaced more than a few hundred feet apart likely are independent.

After these initial computations, the program computed the mean and standard deviation of the 168 values of $s_{T_{pr}}$ ($\bar{x}_{s_{T_{pr}}}$ and $s_{s_{T_{pr}}}$). The maximum $s_{T_{pr}}$ at an individual node was 706,000 gpd/ft, $\bar{x}_{s_{T_{pr}}}$ was 56,000 and $s_{s_{T_{pr}}}$ was 75,000. These results showed that about two-thirds of the values of $s_{T_{pr}}$ varied between 0 and 130,000 gpd/ft, and indicated significant variability in the results.

In order to determine whether σ_T was related to the magnitude of T and/or the local variability in T, all T data at a node were transformed by dividing them by (1) T_M, (2) the local maximum T gradient, and (3) both of these factors. Then the program recomputed values of $\bar{x}_{T_{pr}}$ and $s_{T_{pr}}$ for each node, and $\bar{x}_{s_{T_{pr}}}$ and $s_{s_{T_{pr}}}$ over all nodes. If the magnitude and variability of T were hypothesized to be related to standard deviation, then the value of $s_{s_{T_{pr}}}$ would be proportionally less for the transformed T data than for the original data. For

example, if a perfect relation were found between σ_T and T_M, and all sample values of T were divided by T_M, then σ_T would be constant over all sampled nodes and σ_{σ_T} would equal zero. The standard statistical relation:

$$\sigma_{ax}^2 = a^2\sigma_x^2 \quad , \text{ or alternately } \sigma_{ax} = a\sigma_x \tag{27}$$

shows that transforming individual sample data, where $a = 1/T_M$, will modify the standard deviation of the samples by the same ratio. If this statistic were less for transformed than for original data, σ_T might be estimated at unsampled nodes using derived relations between σ_T and the magnitude and local variability of T.

Unfortunately, the values of $s_{s_{T_{pr}}}$ obtained from the transformed data on T were all proportionally larger than $s_{s_{T_{pr}}}$ computed using the original data, using both arithmetic and logarithmic units for T. This may have been due to factors such as (1) the small sample sizes; (2) inadequate justification for using the two "pseudo-samples" per node; (3) the values of T_M were significantly different from the mean nodal values of the samples -- this was checked at several nodes and found to be the case; and (4) the maximum T gradients were not well approximated, because eithe the values of T_M were not representative of the physical system or the real maximum gradients were over smaller distances than the nodal spacing. Although this attempt was unsuccessful, the results (primarily $s_{T_{pr}}$) were used as a general guide in estimating σ_T at both sampled and unsampled nodes. The results also suggested that the values of T from the short-term aquifer tests were not

representative samples of the values of T_M, or that the values of T_M did not represent well the physical system.

The prior mean, or model, values of T used in the worth-of-data studies were the final values at each node after model calibration. The prior standard deviation of T ($\sigma_{T_{pr}}$) at each sampled node was estimated subjectively after taking into account (1) the computed statistic $s_{T_{pr}}$ at each node, (2) the local variability in T, and (3) the interval around the model value in which about two-thirds of the samples were included. Of the three components used in estimation, (3) was given the most weight.

The standard deviation of the likelihood function for the model value as mean (σ_{T_ℓ}) was estimated using (1) the computed s_{T_ℓ} at each node, (2) the local variability in T, and (3) an interval about $\sqrt{n}$ times the interval around the model value in which about two-thirds of the samples were included. For nodes in which no aquifer tests had been made, the values of $\sigma_{T_{pr}}$ and σ_{T_ℓ} were set equal, which implied that the amount of data involved in estimating the prior was equivalent to the data obtained from one more sample (see above, p. 106). In estimating $\sigma_{T_{pr}}$ and σ_{T_ℓ} at unsampled nodes the following factors were considered: (1) estimates of $\sigma_{T_{pr}}$ and σ_{T_ℓ} for sampled nodes, (2) the distance to the nearest sampled node, and (3) the local variability in T.

The technique used to estimate the standard deviation of a likelihood function for an alternate, or erroneous, value of T as the mean for a node was more complex. It was recognized that the value of σ_{T_ℓ} for the model value as the mean might not be appropriate for

extreme erroneous values. For example, if the model value of T is 200,000 gpd/ft, and σ_{T_ℓ} for the model value is also 200,000, σ_{T_ℓ} for the likelihood for which an erroneous value of 50,000 was the mean likely is less than 200,000. For this reason, a relatively objective method of estimating alternate values of σ_{T_ℓ} was devised.

The writer estimated standard deviations subjectively for a series of 16 values of T ranging from 2,000 to 500,000 gpd/ft. These were judged to be values typical of the aquifer in the Tucson basin, assuming that the sampling procedure was an aquifer test of several hours duration, similar to the actual tests in the basin. A second set of 16 estimates was made under the assumption that the test was of several days duration, or close to the time required to obtain a reasonably good test of T for a nodal area. These estimates were "educated guesses" by the writer. The values, in logarithmic units, of the first set of standard deviations are about double the values of the second set. The two sets of estimates then were plotted against the logarithm of T (figure 8) and equations derived for the assumed linear relationships. These relations are equations for straight lines on semi-logarithmic paper, for which the first term is the intercept of the line on the vertical axis, the coefficient of the second term is the slope, in logarithmic units, per the three log cycles, and the part in brackets is the fraction of the three log cycles over which the interpolation of σ_T is made. For any given value of T, then, these equations can be used to estimate a "typical" standard deviation

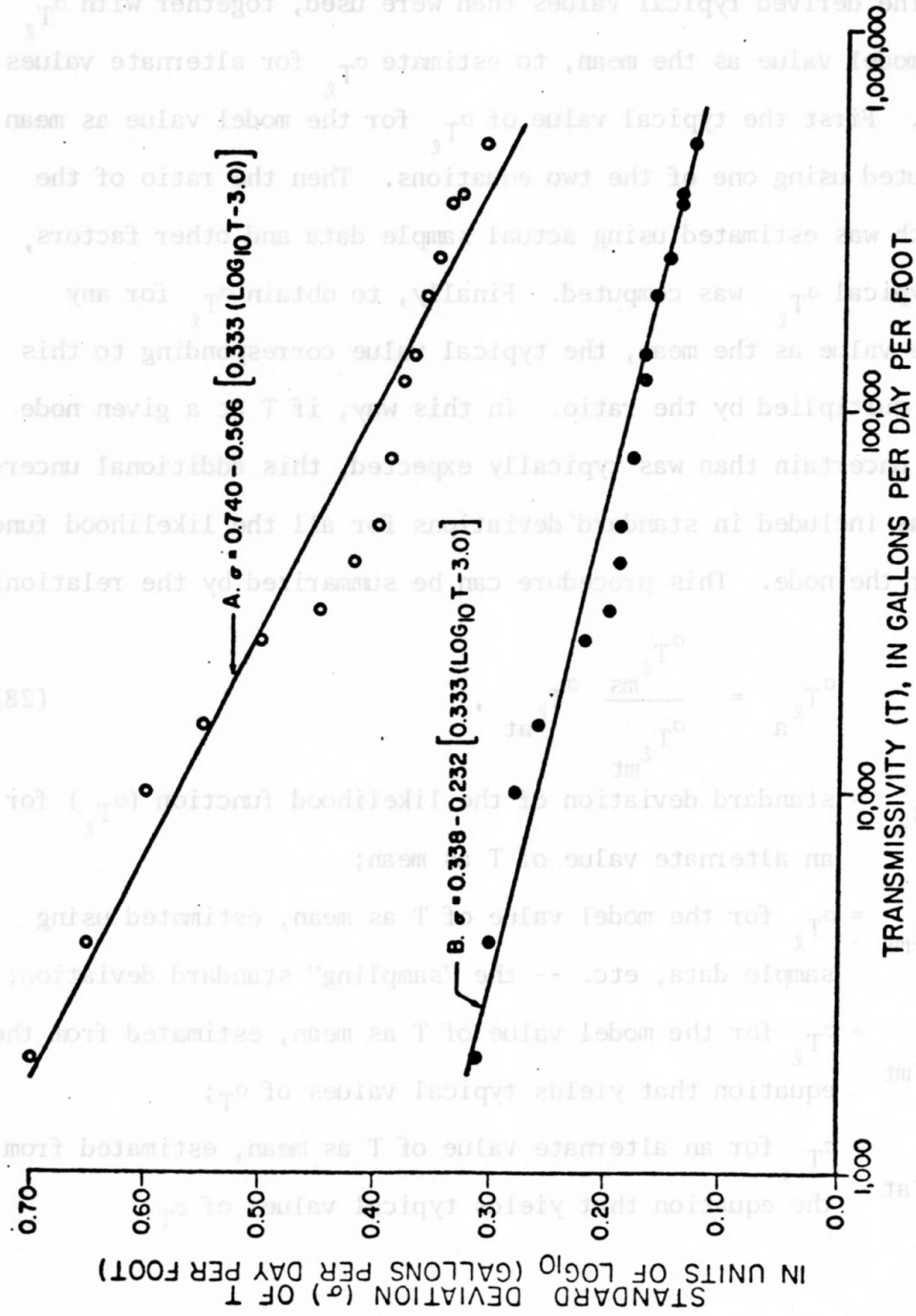


Figure 8. Graphs of estimated typical values of the standard deviation around mean values of transmissivity for (A) aquifer tests of several hours duration, and (B) aquifer tests of several days duration in the Tucson basin.

around the value, given that sampling is done by short-term or longer-term aquifer tests.

The derived typical values then were used, together with σ_{T_ℓ} for the model value as the mean, to estimate σ_{T_ℓ} for alternate values as means. First the typical value of σ_{T_ℓ} for the model value as mean was computed using one of the two equations. Then the ratio of the σ_{T_ℓ}, which was estimated using actual sample data and other factors, to the typical σ_{T_ℓ} was computed. Finally, to obtain σ_{T_ℓ} for any alternate value as the mean, the typical value corresponding to this mean was multiplied by the ratio. In this way, if T at a given node was more uncertain than was typically expected, this additional uncertainty was included in standard deviations for all the likelihood functions for the node. This procedure can be summarized by the relation:

$$\sigma_{T_{\ell_a}} = \frac{\sigma_{T_{\ell_{ms}}}}{\sigma_{T_{\ell_{mt}}}} \sigma_{T_{\ell_{at}}}, \tag{28}$$

where $\sigma_{T_{\ell_a}}$ = standard deviation of the likelihood function (σ_{T_ℓ}) for an alternate value of T as mean;

$\sigma_{T_{\ell_{ms}}}$ = σ_{T_ℓ} for the model value of T as mean, estimated using sample data, etc. -- the "sampling" standard deviation;

$\sigma_{T_{\ell_{mt}}}$ = σ_{T_ℓ} for the model value of T as mean, estimated from the equation that yields typical values of σ_T;

and $\sigma_{T_{\ell_{at}}}$ = σ_{T_ℓ} for an alternate value of T as mean, estimated from the equation that yields typical values of σ_T.

These methods of estimating parameters of the distribution of T at model nodes are not, of course, the only or are they necessarily the best possible methods that could have been used. They likely are, however, typical of the techniques that will have to be employed in this type of study, considering the quantity and reliability of the data on T that are commonly available. More research on the frequency distribution of T would be useful.

Initial Water Level

Initial water level (H) was assumed to be in 1970 for the 1970-90 simulation period. The representative value of H for a nodal area is not a quantity that can be measured directly, but it likely could be approximated best, providing the digital model is a good representation of the physical system, by averaging water levels over the node.

The frequency distribution of values of H around a nodal mean was assumed to be normal, primarily because data were insufficient to identify any other distribution. However, if only measurement errors influenced estimates of H, the distribution probably would be normal, as this distribution commonly is used to describe the distribution of measured "erroneous" values around a true value. The major sources of error in estimating H likely are interpolation error and non-representative data; and although it is not obvious that these types of error also would be normally distributed, a normal distribution is probably a sufficient description.

Although more data on H are available than on any other variable in the Tucson basin, commonly few data are within individual nodal areas. There are 311 nodes in the model in which H was sampled, or measured, in 1970, and 198 nodes with no samples. Of the sampled nodes, 109 had two samples, 45 had three samples, and 19 had more than three, the maximum being six samples at a node.

Using the same approach and computer program that was prepared to analyze the data on T, the data on H at individual nodes were studied. For H, one pseudo-sample per node was added to increase sample sizes. This added "sample" was the water level estimated to be the representative value at each node from the 1970 water-level contour map prepared by the Department of Agricultural Engineering. Values of $\bar{x}_{Hpr}$, s_{Hpr}, and s_{H_ℓ} were computed for each sampled node, and the shortest distance to a sampled node was computed for each unsampled node. The maximum water-table gradient between each node, both sampled and unsampled, and its four surrounding nodes was also computed. The maximum value of s_{Hpr} was 115 feet but $\bar{x}_{s_{Hpr}}$ was only 9.5 feet and $s_{s_{Hpr}}$ was 8.3 feet. Thus about two-thirds of the values of s_{Hpr} were between 1 and 18 feet.

As for T, an attempt was made to devise a method of estimating standard deviations of H at unsampled nodes by seeking a relation between standard deviations and (1) the magnitude of the model value of H at each node, (2) the maximum water-table gradient at each node, and (3) both of these factors taken together. Similar to T, the transformed values of H had values of $s_{s_{Hpr}}$ that were proportionally larger than $s_{s_{Hpr}}$ for the untransformed value. Thus, it was not possible to obtain a

relation between standard deviation and some measurable factor to use in estimating σ_{Hpr} and σ_{H_ℓ} at unsampled nodes. The values of $s_{s_{Hpr}}$ for transformed H, however, were closer to the original $s_{s_{Hpr}}$ than were corresponding values for T. This result may reflect the additional sample data on H and the greater certainty associated with the model value of H assigned to each node. The factors that may have prevented the definition of a usable relation between standard deviation and the other quantities were likely (1) the small sample sizes, (2) insufficient justification for the added pseudo-sample at each node, and (3) an inadequate method of estimating the maximum nodal water-table gradient.

The prior mean, or model value,of H at each node was estimated from the 1970 water-table contour map prepared by the Department of Agricultural Engineering. Values of σ_{Hpr} at each sampled node were estimated using (1) the computed values of s_{Hpr}, (2) the local variability in H and (3) the interval around the model value that includes about two-thirds of the samples. For unsampled nodes, estimates were made using (1) the σ_{Hpr} estimates at similar sampled nodes, (2) the local variability in H, and (3) the distance to the nearest sampled node. Values of σ_{H_ℓ} were estimated in a similar way. If no sample data were available for a node, σ_{H_ℓ} was set equal to σ_{Hpr}.

The standard deviations for the likelihood functions that assumed alternate nodal values of H as the mean values were assumed to be constant and equal to the "sampling" standard deviation, σ_{H_ℓ}. This procedure was equivalent to assuming that errors in H would be independent of the magnitude of H and that uncertainty would be the same for any of the alternate values.

Discharge and Recharge

Discharge -- primarily pumpage in the Tucson basin -- and recharge were tested together or considered as one variable in the worth-of-data studies. This is possible because the model treats them identically in solving the flow equations, the only difference being that pumpage and subsurface outflow are defined as positive quantities and recharge as negative. The only difference in the way discharge and recharge were treated in the worth-of-data studies is that values of recharge and subsurface outflow were assumed to be more uncertain than pumpage. Estimates of discharge and recharge were assumed to be normally distributed, as the main reason why nodal values differ from true values is the presence of measurement and estimation errors.

Discharge cannot be measured, of course, for the future period 1970-89. In many model studies the future discharge is considered to be the variable under the manager's control and is manipulated to provide the optimal combination of benefits. In this study discharge was considered only as an unknown to be estimated.

Future pumpage can be estimated based on current rates and projections of future demands derived from estimated population growth, industrial use -- which in the Tucson basin is primarily use by the mining companies in the southern half of the basin, and agricultural use. Estimation of the prior mean values of discharge was discussed in the section "Time Period Used." The prior standard deviations for each nodal value of discharge are directly proportional to the uncertainty of the estimate, and had to be estimated subjectively.

The uncertainty in pumpage is related to several factors. These factors include uncertainties in the estimates of current pumpage and in projections of population growth. Another factor is errors in estimates of industrial use, which further depend on uncertainties in future copper prices or perhaps in future environmental legislation which could reduce ore production. An additional factor is uncertainty in estimates of future agricultural demand, which is further related to uncertainty in crop and water prices and governmental subsidies. Finally, uncertainty in future pumpage also is related to uncertainties in future quantities of water available from proposed supplemental water sources such as the Central Arizona Project.

For the purposes of this study, pumpage was divided into two classes: (1) pumpage within the greater city of Tucson area and (2) pumpage in the remainder of the basin. Pumpage within the city is better known currently because most of it is metered, and it likely can be projected better into the future. Therefore, the prior standard deviations of pumpage at nodes in the city were assumed to be 25 percent of the estimated mean values, and prior standard deviations at nodes outside the city were assumed 35 percent of the means.

So little is known of current values of recharge and subsurface outflow that in the model they were considered constant with time, and future values were also assumed constant and equal to current rates. Recharge can be classified as: (1) infiltration from stream channels, (2) recharge across model boundaries, and (3) subsurface inflow through the alluvium under stream channels where channels cross the boundaries

of the model. Recharge in each of these classes generally was estimated in slightly different ways, so the uncertainty associated with each will also differ. Subsurface outflow was estimated in the same way as subsurface inflow, so they are discussed together, even though outflow is a form of discharge.

Stream-channel recharge was estimated largely from channel-infiltration studies made by the U. S. Geological Survey, and was modified during calibration of the digital model. Uncertainties in stream-channel recharge thus are related to the assumptions and measurement errors associated with the infiltration studies, including errors in estimating how much of the infiltration reaches the water table; and on the quality of model data, and especially historical water-level data, along the streams. Boundary recharge was estimated during calibration of the electrical-analog and digital models, so its uncertainties will be related to uncertainties in model data along the boundaries. Subsurface inflow and outflow largely were estimated during model calibration, although values were checked roughly by the Geological Survey (Davidson 1970, pp. 182-184) using estimates of saturated cross-section, permeability of alluvium, and hydraulic gradient at the points where channels cross boundaries. Of the three categories of recharge, stream-channel recharge likely is the least uncertain, subsurface inflow (and outflow) is intermediate, and boundary recharge is likely the most uncertain. For the purposes of this study the prior standard deviations of stream-channel recharge, subsurface inflow and

outflow, and boundary recharge were assumed to be 40, 50, and 60 percent, respectively, of the estimated nodal values. If a node had more than one class of recharge, the standard deviation corresponding to the largest component was used.

The method used to estimate the standard deviations of the likelihood functions was straightforward. First, the standard deviation of the likelihood function for the model value -- the "sampling" standard deviation -- was assumed equal to the prior standard deviation, primarily because actual sample data on discharge and recharge were not available. Then errors in estimates of discharge and recharge were assumed to be directly proportional to the quantity estimated. This assumption probably is good for measured pumpage, as errors commonly are expressed as a percentage of estimates, and the assumption is reasonable for estimated pumpage and recharge. Standard deviations of likelihood functions associated with alternate mean values of discharge/recharge were estimated by computing the ratio of the alternate value to the model value and multiplying this by the "sampling" standard deviation.

Results of Selecting and Testing Variables

The worth-of-data studies for the Tucson basin consisted of testing 91 variables -- 24 coefficients of storage, 22 transmissivities, 23 initial water levels, and 22 values of discharge or recharge -- from 61 different nodes of the small-scale digital-computer model of the Tucson basin. Variables for testing were selected from all parts of the basin, both from areas where there are relatively much

hydrogeologic data, mainly in the areas of pumpage for irrigation along the Santa Cruz River and within the city of Tucson, and from areas with few data, generally on the margins of the basin. The variables were tested for expected opportunity loss, which could be defined as expected error in predicted water levels at the end of a simulation period extending from 1970 to 1990, and expected worth of sample data, which could be defined as expected reduction in error. Values were computed in terms of feet, or other units depending on the error criterion used, over the 509 nodes of the model.

The studies were divided into two parts, 67 variables chosen because their expected errors were likely to be large and, for the purpose of comparison, 24 variables chosen because their expected errors were not likely to be large. Each of the two categories, however, includes a few nodes at which all variables were tested in order to compare expected errors associated with each variable at a single node. All of the variables at these few nodes did not fall always into either the "large-error" or "small-error" categories. Figure 9 shows the locations of the tests, and tables 8 and 9 and 10 and 11 include results of testing variables in the large- and small-error categories, respectively.

Nodes at which errors in the coefficient of storage were thought likely to produce large expected errors were not chosen on the basis of large uncertainty in the true values of S because, as previously discussed, uncertainty in S is about the same over the entire basin. During calibration of the Tucson basin model, however, changes

Figure 9. Map showing the locations of the nodes in the 509-node digital-computer model of the Tucson basin, Arizona; and locations of the variables tested in the worth-of-data studies.

Explanation

10
5 □ Node (5,10) in the 5th Row and 10th Column of the model Grid

Type of Variable Tested

- ○ Coefficient of Storage
- ● Transmissivity
- □ Initial Water Level
- ■ Discharge or Recharge
- X All Variables at a Node

Type of Sensitivity Test Made

1	Five Values per Variable
2	Normal Frequency Distribution
3	Smaller Standard Deviation for Likelihood Function
4	Worth of Data Computed at each Time-Step, Smaller Standard Deviation for Prior Distribution and Likelihood Function, and Different Spacing for Alternate Variable Values (Three Separate Sensitivity Tests)
X(S)	Coefficient of Storage Tested
X(T)	Transmissivity Tested
X(Q)	Discharge Tested
X(R)	Recharge Tested
X	No symbol indicates all variables tested

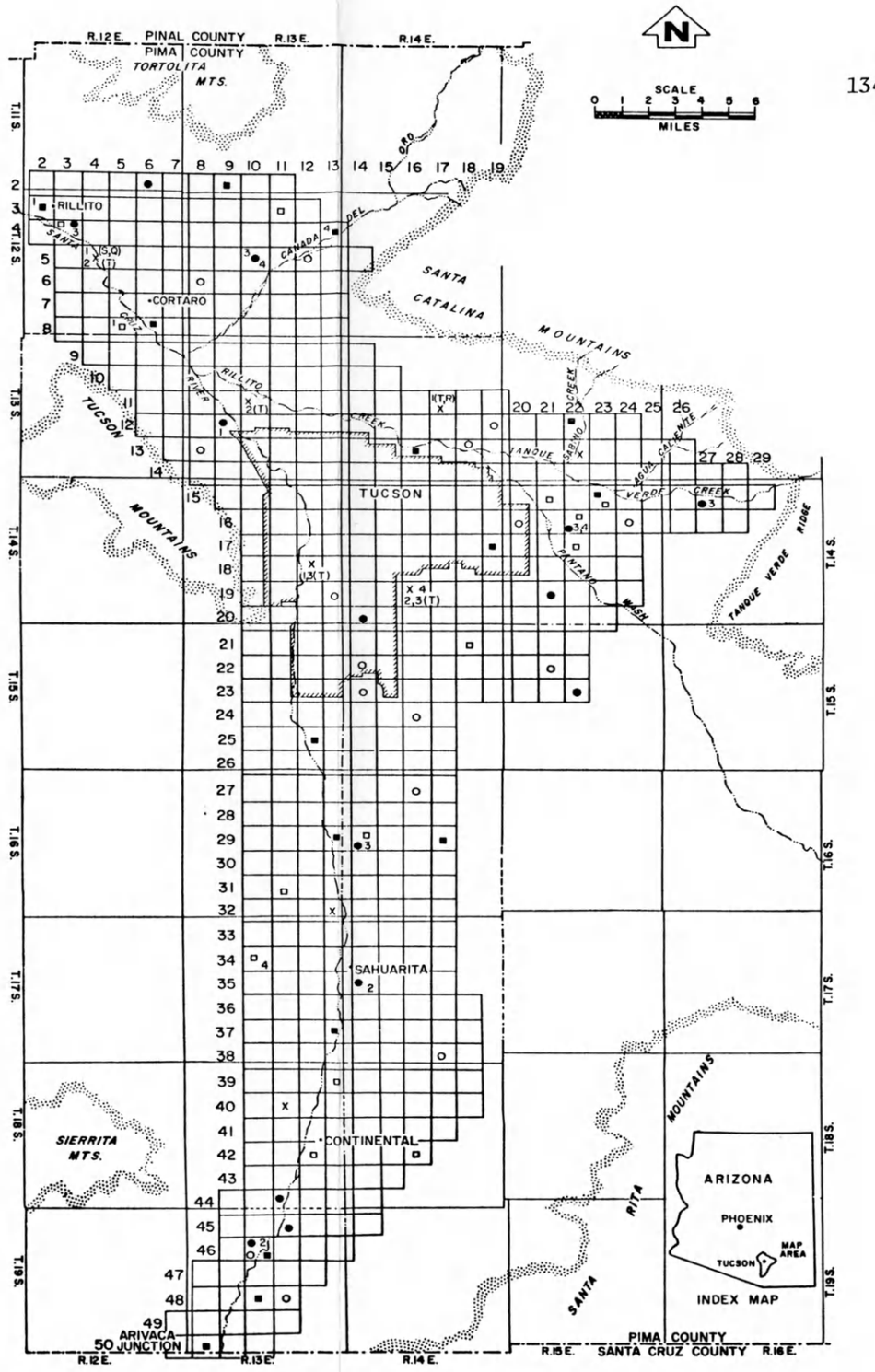


Figure 9.

in S were observed to cause the most change in predicted water levels at nodes where discharge and/or recharge were large or where water-level change itself was large over the simulation period. Nodes were selected for the large-error category of test, then, if discharge or recharge was large or if water-level change was large. Conversely, nodes at which prediction errors were expected to be less were chosen where discharge or recharge or water-level change was less. Comparison of mean values of expected error, μ_S, computed for each of the six error criteria and for each category (tables 9 and 11) generally shows that values are slightly higher for the large-error category, so that the above assumptions have some validity.

For transmissivity and initial water levels, it was assumed that if the standard deviation of the prior distribution was relatively large, the expected error would be large, and that if the standard deviation was relatively less, the error would be less. The mean expected errors for transmissivity are in fact considerably more, and for initial water levels generally are slightly more for the large-error category than for the small-error category (tables 9 and 11).

For discharge/recharge, standard deviations were assumed to be directly proportional to the magnitude of the value of discharge or recharge, as previously discussed. Values chosen because they were likely to have large expected errors were thus at nodes where discharge or recharge was relatively large, and values expected to have smaller errors were at nodes where discharge or recharge was relatively smaller.

Comparison of results in tables 9 and 11 shows that these choices were correct.

Table 8 gives results of testing variables in the large-error category. The table lists results for four of the six error criteria used -- omitting numbers of nodes in error by more than 5 feet and 25 feet, as results were similar to those from the criterion of numbers of nodes in error by more than 10 feet -- arranged in order of descending worth of sample data for absolute value of error. The data indicate that for the error criteria tabulated, discharge/recharge and transmissivity generally have the largest expected errors and sample worths, while initial water levels and storage coefficients generally have smaller values. The maximum expected error and sample worth for the absolute error criterion are 504 and 98 feet, respectively, over the 509 nodes of the model, associated with discharge (subsurface outflow) at node (3,2).

Mean values and standard deviations of expected error, expected worth of sample data, and percent improvement, the latter defined as (sample worth/expected error) x 100, were computed for each variable for each of the six error criteria, and are given in table 9. Although these means and standard deviations may be misleading because they are based on only about three percent of the possible 509 values of each variable, they are helpful in comparing the results.

Discharge/recharge has the largest mean expected error, mean expected sample worth, and mean percent improvement for four of the error criteria. For absolute value of error, for example, mean expected

Table 8. Results of Tests on Variables, Cost of Which Were Expected to Show Large Expected Errors. -- Data arranged in order of descending worth of sample data for absolute value of error; S - coefficient of storage, T - transmissivity, H - initial water level, Q - discharge (so - subsurface outflow, otherwise pumpage); R - recharge (sc - stream channel, b - boundary, si - subsurface inflow); T in gallons per day per foot; H in feet above mean sea level; and Q and R in acre-feet per 20 years.

Variable	Location in Model		Model Value	Standard Deviation of the Prior Distribution	Absolute Value of Error, in feet per 509 Nodes		Square of Error, in feet squared per 509 nodes		Maximum Nodal Error, in feet		Number of Nodes in Error by More than 10 feet	
	Row	Column			Expected Error	Expected Worth of Sample Data	Expected Error	Expected Worth of Sample Data	Expected Error	Expected Worth of Sample Data	Expected Error	Expected Worth of Sample Data
Q_{so}		2	147,400	73,700	504.1	98.4	10,311.9	2,929.3	35.1	6.9	14.7	3.5
$R_{sc,b,si}$	4	13	102,105	61,500	420.2	84.2	9,544.5	2,746.2	45.5	9.1	10.1	1.9
H	34	10	2,610	160	105.8	73.0	179.7	150.5	3.4	2.4	0.0	0.0
Q	40	11	124,100	43,400	295.4	52.9	1,891.3	518.5	14.3	2.6	4.1	1.5
Q	37	13	115,300	40,300	273.9	49.1	1,319.8	361.8	9.7	1.7	1.8	0.92
T	45	11	135,000	540, 00	232.9	44.0	1,375.5	408.3	9.3	1.6	2.7	1.1
Q	5	4	69,800	24,500	169.8	30.4	619.8	170.0	4.5	0.80	0.14	0.04
$R_{sc,si}$	50	8	44,200	22,100	150.5	29.4	1,419.3	403.1	10.9	2.1	4.6	1.9
Q	32	13	101,500	25,250	169.0	28.9	503.8	133.6	7.1	1.2	0.41	0.07
T	40	11	30,000	170,000	157.9	27.6	852.9	282.6	9.3	1.5	1.8	0.67
T	16	22	3,800	100,000	258.1	22.44	2,662.8	368.2	14.6	1.3	4.5	0.36
R_b	29	17	25,800	15,500	99.3	19.9	264.0	76.0	7.3	1.5	0.40	0.13
R_{sc}	13	16	29,500	11,800	103.4	19.1	159.8	44.4	3.6	0.67	0.0	0.0
$R_{sc,b,si}$	12	22	21,100	12,650	90.5	18.2	444.3	127.9	10.2	2.1	0.81	0.35
T	11	17	12,500	80,000	96.4	17.8	943.7	303.3	15.7	2.9	1.6	0.38
T	15	27	11,300	1,000,000	310.6	17.5	20,716.5	3,161.8	27.5	2.6	5.7	0.33
T	44	11	222,500	150,000	97.0	16.3	200.9	50.6	3.0	0.50	0.0	0.0
Q	18	12	51,400	12,750	94.4	16.1	193.1	51.2	4.0	0.69	0.0	0.0
H	42	16	2,806	50	33.3	16.1	18.4	12.4	0.91	0.44	0.0	0.0
Q	17	19	43,200	10,700	94.0	16.1	126.5	33.5	2.1	0.37	0.0	0.0
R_b	11	17	16,800	10,100	77.8	15.6	126.6	116.6	12.1	2.4	0.67	0.30
T	4	3	178,700	175,000	170.7	14.5	1,532.2	214.4	20.2	1.8	2.6	0.19
T	18	12	85,000	200,000	127.6	14.2	280.9	56.3	5.2	0.58	0.071	0.0016
T	32	13	52,500	50,000	60.3	10.0	101.9	27.9	3.7	0.62	0.0	0.0
T	2	6	13,100	80,000	50.9	7.7	342.3	116.9	10.2	1.8	0.54	0.09
T	20	14	4,500	40,000	60,8	9.7	309.8	101.7	8.1	1.3	0.54	0.22
T	13	22	85,000	175,000	87.9	9.4	361.1	57.2	9.0	0.85	0.95	0.16
S	32	13	0.153	0.07	50.9	7.7	36.9	8.9	1.4	0.22	0.0	0.0
H	16	22	2,459	60	43.7	7.2	25.9	6.4	0.91	0.15	0.0	0.0
T	12	9	222,500	225,000	42.7	7.0	34.0	9.4	1.7	0.25	0.0	0.0
H	31	11	2,540	60	42.6	7.0	25.8	6.4	1.5	0.24	0.0	0.0
S	27	16	0.30	0.11	35.1	6.9	32.8	9.8	2.3	0.46	0.0	0.0
H	3	11	2,277	60	41.1	6.8	33.3	8.3	1.4	0.23	0.0	0.0
S	40	11	0.153	0.09	40.6	6.2	45.2	9.6	2.1	0.19	0.0	0.0
R_{sc}	25	12	12,600	5,040	32.0	5.9	17.9	5.0	1.1	0.21	0.0	0.0
S	12	19	0.153	0.09	34.6	5.7	42.7	10.6	2.9	0.45	0.0	0.0

Table 8--continued

Variable	Location in Model: Row	Location in Model: Column	Model Value	Standard Deviation of the Prior Distribution	Absolute Value of Error in feet per 509 Nodes: Expected Error	Absolute Value of Error in feet per 509 Nodes: Expected Worth of Sample Data	Square of Error, in feet squared per 509 nodes: Expected Error	Square of Error, in feet squared per 509 nodes: Expected Worth of Sample Data	Maximum Nodal Error, in feet: Expected Error	Maximum Nodal Error, in feet: Expected Worth of Sample Data	Number of Nodes in Error by More than 10 feet: Expected Error	Number of Nodes in Error by More than 10 feet: Expected Worth of Sample Data
R_{sc}	8	6	10,600	4,240	30.2	5.6	13.1	3.6	0.71	0.13	0.0	0.0
H_{sc}	11	17	2,430	40	32.5	5.4	16.0	4.0	1.1	0.19	0.0	0.0
H	40	11	2,688	25	30.1	5.0	22.8	5.6	.6	0.26	0.0	0.0
S	5	4	0.153	0.07	28.9	4.6	55.9	14.1	3.2	0.50	0.0	0.0
S	22	14	0.30	0.09	22.6	4.4	13.4	3.9	1.6	0.32	0.0	0.0
H	8	5	2,063	50	67.6	4.3	142.5	13.3	.4	0.28	0.0	0.0
R_{sc}	46	10	8,350	3,320	22.6	4.2	15.0	4.2	0.81	0.15	0.0	0.0
S_{sc}	11	17	0.153	0.09	24.9	4.1	40.5	10.1	3.8	0.62	0.0	0.0
S	13	18	0.153	0.07	26.1	4.0	10.3	2.5	0.87	0.13	0.0	0.0
S	24	16	0.25	0.10	24.7	4.0	17.6	4.5	1.8	0.29	0.0	0.0
S	23	14	0.26	0.08	25.9	3.9	12.4	3.0	0.99	0.15	0.0	0.0
H	42	12	2,755	20	41.8	3.8	66.9	9.6	3.3	0.30	0.0	0.0
H	15	21	2,430	55	49.5	3.5	37.5	3.9	1.7	0.12	0.0	0.0
T	5	4	250,000	150,000	61.2	3.3	115.8	10.0	2.8	0.15	0.0	0.0
S	19	13	0.153	0.07	21.3	3.2	9.1	2.2	0.85	0.13	0.0	0.0
S	22	21	0.153	0.09	19.4	3.2	18.7	4.6	1.4	0.22	0.0	0.0
H	18	12	2,318	20	18.4	3.0	5.5	1.4	0.84	0.14	0.0	0.0
S	46	10	0.153	0.07	19.2	2.9	16.5	4.0	1.4	0.20	0.0	0.0
S	13	8	0.075	0.08	25.7	2.9	54.7	7.2	3.9	0.45	0.0	0.0
H	32	13	2,550	36	30.1	2.8	12.7	1.8	1.0	0.09	0.0	0.0
S	18	12	0.153	0.07	16.8	2.6	11.6	2.9	1.5	0.23	0.0	0.0
S	13	22	0.153	0.09	15.8	2.6	11.9	3.0	1.4	0.23	0.0	0.0
H	21	18	2,402	50	36.5	2.3	16.7	1.6	0.85	0.05	0.0	0.0
S	5	12	0.153	0.09	16.1	2.3	8.5	1.7	1.1	0.13	0.0	0.0
H	13	22	2,503	15	13.5	2.2	5.1	1.3	0.67	0.11	0.0	0.0
T	29	14	250,000	250,000	15.1	2.0	7.1	1.6	0.88	0.12	0.0	0.0
S	16	20	0.075	0.06	18.1	2.0	5.5	0.88	0.56	0.05	0.0	0.0
H	5	4	1,990	15	58.2	1.6	204.3	6.5	7.5	0.20	0.55	0.04
H	17	22	2,430	45	33.0	1.3	14.3	0.71	0.78	0.03	0.0	0.0
Q	13	22	3,840	950	7.0	1.2	1.8	0.48	0.43	0.07	0.0	0.0
T	23	22	75,000	55,000	1.70	0.27	0.24	0.06	0.29	0.05	0.0	0.0

Table 9. Means and Standard Deviations of Results from the "Large-Error" Category of Tests.

	Absolute Value of Error			Square of Error			Maximum Nodal Error			Number of Nodes with Errors More than 5 feet			Number of Nodes with Errors More than 10 feet			Number of Nodes with Errors More than 25 feet		
Variable	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP
μ_S	25.9	4.1	15.6	24.7	5.7	23.6	1.8	0.28	14.9	0.10	0.03	11.4	0.0	0.0	0.0	0.0	0.0	0.0
σ_S	9.3	1.6	2.2	17.0	3.9	4.0	1.0	0.16	2.8	0.16	0.058	21.6	0.0	0.0	0.0	0.0	0.0	0.0
μ_T	114.9	14.1	13.6	1,862.3	323.1	23.5	8.8	1.1	13.7	4.0	0.79	22.1	1.3	0.22	12.6	0.26	0.043	7.9
σ_T	89.2	10.8	4.6	5,078.7	769.4	8.1	7.6	0.88	4.0	4.5	0.96	16.9	1.8	0.31	15.4	0.63	0.080	14.2
μ_H	42.4	9.1	17.4	51.7	14.6	24.4	2.0	0.33	17.4	0.27	0.074	9.1	0.034	0.003	0.49	0.0	0.0	0.0
σ_H	21.6	17.4	17.3	64.1	36.5	21.7	1.8	0.55	17.3	0.56	0.21	25.1	0.14	0.011	2.0	0.0	0.0	0.0
μ_{QR}	154.9	29.1	18.5	1,603.0	454.4	27.7	10.0	1.9	18.5	6.4	1.5	20.4	2.2	0.62	19.8	0.44	0.15	12.0
σ_{QR}	141.4	27.5	1.1	3,184.3	910.4	0.86	12.4	2.5	1.1	8.1	1.7	15.7	4.2	1.00	19.0	1.2	0.39	24.3

18 samples of S, 16 samples of T, 17 samples of H, and 16 samples of QR (discharge or recharge).

EOL - expected opportunity loss (expected error).

EWSD - expected worth of sample data.

PCIMP - percent improvement.

Mean - μ.

Standard deviation - σ.

error is 155 feet and mean expected sample worth is 29 feet. For the squared error criterion, discharge/recharge has the second largest expected error; and for nodes with errors more than five feet, it has the second largest percent improvement.

Transmissivity has the second largest mean expected error and mean expected sample worth for all criteria except squared error, where it has the largest mean expected error. For absolute value of error, mean expected error and sample worth for T are 115 and 14 feet, respectively. However, transmissivity also has the lowest mean percent improvement for absolute error (13.6%), squared error and maximum nodal error, which is because of the relatively large uncertainties, reflected in the large standard deviations of the likelihood functions, associated with transmissivity.

Together, discharge/recharge and transmissivity dominated the results in tables 8 and 9. Mean expected errors and sample worths for these two variables are as much as or more than an order of magnitude higher than for initial water level and storage coefficient for the criteria of squared error and numbers of nodes with errors more than 5, 10, and 25 feet.

Initial water level ranks third in both mean expected error and mean expected worth of sample data for all error criteria except nodes in error more than 25 feet, where results are zero for both initial water level and storage coefficient. For absolute value of error, for example, mean expected error and sample worth are 42 and 9

feet, respectively. Initial water level also has the second highest mean percent improvement for the first three error criteria.

Storage coefficient has the lowest mean expected error and sample worth and next-to-lowest percent improvement for all of the error criteria for which results were non-zero. For absolute value of error, mean expected error and sample worth for S are 26 and 4 feet, respectively.

Results of testing variables in the small-error category are given in table 10. The make-up of this table is the same as for table 8. The data show that for the error criteria tabulated, transmissivity and initial water level generally have the largest expected errors and, to a lesser extent, the largest expected sample worths; while storage coefficient and discharge/recharge have smaller values. For the small-error category the maximum expected error and sample worth for absolute value of error are 73 and 12 feet, respectively, over the 509 nodes of the model, associated with the initial water level at node (11,10). However, for this category all the expected errors and sample worths are of the same order of magnitude. Means and standard deviations for results under five error criteria (all results for the sixth, nodes in error more than 25 feet, are zero) were computed for each variable type (table 11). For these computations, however, there were only six tests per variable, so the computed statistics may not be applicable to other than the data used to compute them.

Table 10. Results of Tests on Variables Which Were Not Expected to Show Large Expected Errors. -- Data arranged in order of descending worth of sample data for absolute value of error; S - coefficient of storage, T - transmissivity, H - initial water level, Q - discharge, and R - recharge (sc - stream-channel, b - boundary, si - subsurface inflow). T in gallons per day per foot, H in feet above mean sea level, and Q and R in acre-feet per 20 years.

Variable	Location in Model		Model Value	Standard Deviation of the Prior Distribution	Absolute Value of Error, in feet per 509 nodes	
	Row	Column			Expected Error	Expected Worth of Sample Data
H	11	10	2,165	15	73.0	12.0
R_{sc}	11	10	13,100	5,240	42.1	7.8
T^{sc}	46	10	195,000	50,000	40.1	6.6
H	4	3	1,960	10	40.2	6.6
S	6	8	0.153	0.09	38.5	6.4
T	19	21	15,000	20,000	65.5	5.4
R_b	2	9	6,320	3,780	25.7	5.2
T^b	5	10	41,300	30,000	50.5	4.8
H	19	16	2,340	30	28.2	4.6
T	11	10	250,000	100,000	27.4	4.5
R_{sc}	29	13	7,680	3,080	20.0	3.7
S^{sc}	11	10	0.153	0.07	20.7	3.3
S	19	16	0.112	0.07	17.2	3.3
S	38	17	0.156	0.09	18.2	3.0
Q	48	10	6,420	2,240	15.3	2.7
T	35	14	63,800	35,000	32.9	2.6
T	19	16	52,500	50,000	47.4	2.5
H	29	14	2,530	12	47.9	2.3
Q	15	23	6,780	1,700	12.2	2.1
Q	19	16	5,270	1,326	11.1	1.9
S	48	11	0.153	0.09	10.6	1.5
S	16	24	0.153	0.09	3.3	0.41
H	15	23	2,510	15	11.6	0.31
H	39	13	2,674	6	9.3	0.25

Table 10--continued

Variable	Square of Error, in feet squared per 509 nodes: Expected Error	Square of Error, in feet squared per 509 nodes: Expected Worth of Sample Data	Maximum Nodal error, in feet: Expected Error	Maximum Nodal error, in feet: Expected Worth of Sample Data	Number of Nodes in error by more than 10 feet: Expected Error	Number of Nodes in error by more than 10 feet: Expected Worth of Sample Data
H	151.3	37.5	5.5	0.90	0.14	0.07
R_{sc}	22.6	6.3	0.80	0.15	0.0	0.0
T^{sc}	48.7	12.1	1.5	0.25	0.0	0.0
H	58.8	14.6	3.0	0.49	0.0	0.0
S	71.2	18.7	2.5	0.42	0.0	0.0
T	158.8	26.5	5.4	0.46	0.20	0.04
R_b	64.9	18.7	5.1	1.0	0.14	0.11
T^b	73.9	12.2	2.7	0.25	0.0	0.0
H	18.0	4.5	1.4	0.23	0.0	0.0
T	8.6	2.2	0.66	0.11	0.0	0.0
R_{sc}	6.6	1.8	0.54	0.10	0.0	0.0
S^{sc}	11.9	2.9	1.1	0.17	0.0	0.0
S	5.4	1.5	0.70	0.13	0.0	0.0
S	21.6	5.3	2.2	0.35	0.0	0.0
Q	9.7	2.7	0.79	0.14	0.0	0.0
T	21.0	2.6	1.4	0.10	0.0	0.0
T	76.0	5.6	4.8	0.25	0.07	0.008
H	45.7	2.9	2:1	0.10	0.0	0.0
Q	5.2	1.4	0.78	0.13	0.0	0.0
Q	2.2	0.58	0.48	0.08	0.0	0.0
S	17.9	4.4	2.3	0.28	0.0	0.0
S	1.1	0.14	0.61	0.08	0.0	0.0
H	3.3	0.11	0.51	0.014	0.0	0.0
H	2.2	0.07	0.46	0.012	0.0	0.0

Table 11. Means and Standard Deviations of Results from the "Small-Error" Category of Tests.

	Absolute Value of Error			Square of Error			Maximum Nodal Error			Number of Nodes with Errors More than 5 feet			Number of Nodes with Errors More than 10 feet			Number of Nodes with Errors More than 25 feet		
Variable	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP	EOL	EWSD	PCIMP
μ_S	18.1	3.0	15.6	21.5	5.5	23.3	1.6	0.24	15.2	0.10	0.058	23.6	0.0	0.0	0.0	(all zero)		
σ_S	11.8	2.0	2.3	25.5	6.7	5.4	0.87	0.13	2.5	0.18	0.12	36.8	0.0	0.0	0.0			
μ_T	44.0	4.40	10.6	64.5	10.2	17.2	2.7	0.24	10.7	0.32	0.043	9.6	0.046	0.008	5.3			
σ_T	13.6	1.6	4.7	53.6	9.1	7.0	2.0	0.13	4.7	0.46	0.058	16.8	0.082	0.016	8.6			
μ_H	35.0	4.36	9.9	46.5	10.0	14.5	2.2	0.29	9.9	0.23	0.081	13.9	0.024	0.012	8.6			
σ_H	24.0	4.5	7.2	56.2	14.5	11.3	1.9	0.35	7.2	0.50	0.17	22.1	0.058	0.030	21.0			
μ_{QR}	21.1	3.9	18.2	18.5	5.2	27.5	1.4	0.27	18.2	0.067	0.021	5.2	0.023	0.018	13.0			
σ_{QR}	11.6	2.2	1.1	23.8	6.9	0.88	1.8	0.37	1.1	0.16	0.051	12.8	0.055	0.043	31.7			

6 samples per variable type.

EOL - expected opportunity loss (expected error).

EWSD - expected worth of sample data.

PCIMP - percent improvement.

Mean - μ.

Standard deviation - σ.

Transmissivity has the largest mean expected error for all error criteria and the largest mean expected worth of sample data for the first two error criteria. For example, the mean expected error and sample worth for T for absolute error are 44 and 4 feet, respectively. Similar to results in table 9, transmissivity has a low mean percent improvement, ranking third for all error criteria.

Initial water level has the second largest mean expected error and either the second largest or largest mean expected sample worth over all error criteria. The mean expected error and sample worth for absolute error are 35 and 4 feet, respectively. However, initial water level ranks lowest in mean percent improvement for the first three criteria.

Storage coefficient ranks lowest in mean expected error and sample worth under the criterion of absolute value of error, and next to lowest for the criterion of squared error. For example, for absolute error, S has a mean expected error of 18 feet and a mean expected sample worth of 3 feet. However, it ranks second or first in mean percent improvement for all error criteria under which its results were non-zero. This reflects the fact that although storage coefficient generally has a small expected error, its expected sample worth is generally intermediate in magnitude.

Discharge/recharge has the next-lowest mean expected error and sample worth for the absolute error criterion and the lowest values for the criteria squared error and number of nodes in error more than 10 feet. Mean expected error and sample worth for the absolute error

criterion are 21 and 4 feet, respectively. Discharge/recharge, however, also has a high mean percent improvement, ranking first under four of the five error criteria. This is also a result of combining a small expected error with a moderately large sample worth.

The above conclusions on the results of both categories of tests probably are applicable only to the Tucson basin model, and may apply only to the specific nodes tested.

In summary, when discharge or recharge at a node is estimated to be large, more than about 1,000 - 2,000 acre-ft/yr, the expected error and expected reduction in error with sampling are larger than for any other variable. However, for smaller values of discharge/recharge, expected errors and sample worths can be the lowest of any variable. Transmissivity commonly yields a large expected error, especially if the prior standard deviation is larger than the variable value. Transmissivity also yields fairly large values of expected reduction in error with sampling; especially if the standard deviations of the likelihood functions are less than the values of the variable, which indicates that T can be sampled with relatively little uncertainty. Initial water level has intermediate values of expected error and sample worth, which can be large if the prior standard deviation is large -- more than 50 feet -- and the standard deviation of the likelihood function is smaller. However, results also seem to depend on basin dynamics or other factors, as several values of initial water level with low -- less than 20 feet -- prior standard deviations have

relatively large expected errors. Storage coefficients commonly have low expected errors and expected reductions in error.

An analysis of expected errors and expected worth of sample data such as this, however, is somewhat misleading in at least two respects. First, it evaluates expected errors in predicted water levels over the entire model due to an error in one variable at one node, with all other variables assumed true. This analysis, then, does not indicate what prediction errors might be due to errors in all variables at all nodes, which would be more realistic. At present, a study of errors in all variables using the identical approach described here would use a prohibitive amount of computer time because of the innumerable combinations of possible errors that would have to be investigated. For example, if all five variables were considered in error at each of the 509 model nodes, and seven variable values were assumed for each discrete distribution, there would be $7^{2,545}$ possible combinations of errors.

Secondly, extreme errors in predicted water levels, such as are simulated using the end members of the discrete distribution of each tested variable, likely would not be present in the model except during the early stages of calibration. Calibration would reduce these extreme errors through adjustment of model parameters, initial conditions, and input/output. Prediction errors then would be much smaller at the node which contained the data error, but might be larger at other nodes due to the effect of the adjustments. If the model had not been calibrated, however, or if it had been calibrated over only one

time period, errors in variable values as large as those represented by the end members of the frequency distributions would still remain in the model. In addition, the extreme values of the distribution are to a large extent discounted because they are assumed to have a low probability, commonly less than 0.10.

The results of this study are not in agreement with the conclusions of Bibby (1971), who made a statistical study of the effects of errors in model variables on errors in predicted water levels. The chief difference is that Bibby found that errors in initial water levels were the major cause of errors in predicted levels, while this study indicated that errors in initial water levels commonly were less significant than errors in transmissivity and discharge/recharge. Several factors may cause the difference in results, specifically, (1) the difference in the length of time simulated, (2) differences in assumed ranges in error for the variables, and (3) differences in the methods used and their assumptions.

Bibby simulated time periods on the order of months, while the writer simulated a 20-year period. The short time periods may have influenced Bibby's results markedly, because it seems intuitive that errors in initial water levels would have a great effect during the early part of a simulation period, but that the effect would be damped over long times. Bibby pointed out (1971, p. 60) that his results may not hold for longer time periods. Some of the results of this study (see the section which follows, "Sensitivity of Results to Modification

of the Assumptions of the Method") suggest that errors in initial water levels are more significant during early times.

Differences in assumed ranges in error may also be a significant source of the difference in results. For example, Bibby (1971, table A.1.1a) assumed, in one of his tests, a maximum standard deviation for hydraulic conductivity of 35 ft/day in relation to a mean value of 100 ft/day; and a maximum standard deviation for aquifer thickness of 6.25 feet in relation to a mean value of 75 feet. These assumptions are approximately equivalent to a maximum standard deviation for transmissivity (as T = hydraulic conductivity times aquifer thickness) of about 220 sq ft/day in relation to a mean value of 7,500 sq ft/day. The standard deviation of T is thus an order of magnitude less than its mean. In contrast, in this study standard deviations of T are commonly of the same order of magnitude as the means. For initial water level, however, the standard deviations used by Bibby are comparable to those used in this study. These differences might be much of the reason why errors in variables other than initial water level were more significant in the study reported here. Bibby (1971, p. 64) was careful to point out, however, that his results are applicable only to the range of errors he considered. This is of course true for this study as well.

The results of this study may be atypical because of the relatively large uncertainty in values of T for the Tucson basin. Transmissivity likely is uncertain there because T values were obtained from short-term aquifer tests in an unconfined aquifer in which delayed drainage is significant. However, this illustrates the fact that results

obtained from generalized or idealized models may not apply to every ground-water basin.

Sensitivity of Results to Modification of the Assumptions of the Method

Many gross assumptions and simplifications were made in order to estimate worth of data, chiefly in discretizing the distribution of each variable; in choosing the type of distribution function; and in estimating the parameters, primarily the standard deviations, of the distributions. In an attempt to evaluate the effects of these assumptions, several sensitivity tests of limited extent were made. These tests included: (a) computing expected errors and sample worths at the end of each time-step and summing over all steps, instead of computing results only at the end of the simulation period, or essentially approximating integration over time; (b) computing results using a variable distribution made up of five elements instead of the standard seven; (c and d) computing the seven elements of the variable distribution more closely spaced and less closely spaced than those of the original distribution; (e) computing results using prior standard deviations and standard deviations of the likelihood functions that were an arbitrary 80 percent as large as the original values; (f) computing results for transmissivities using likelihood standard deviations that represented sampling by aquifer tests of several days duration instead of the original several hours duration, the latter of which is equivalent to the tests that have been made in the Tucson basin; and (g) computing results for transmissivities assuming they are normally distributed instead

of log-normally. Results of these sensitivity tests, for the first three error criteria only, are given in tables 12a through 12g, and nodes at which these tests were made are identified on figure 9. The same eight variables, of which four were selected from the large-error category of tests and four from the small-error category, were used in sensitivity tests A, C, D, and E. Some different variables, when necessary, were substituted in tests B, F, and G.

Results from the sensitivity tests indicate that the method is quite sensitive to such assumptions as the type of distribution function, the number of elements in the function, the spacing of the elements, and the differences in assumed standard deviations for prior distributions and likelihood functions. In addition, the expected errors and sample worths are much different if results are computed over all-time steps, as was anticipated. A summary of mean values of differences between original test data and sensitivity-test data is given in table 13.

For test A, mean expected errors and sample worths obtained by summing over all three time-steps are more than twice the values computed at the end of the period for the absolute error criterion, and almost six times the original values for the squared error criterion. For individual variable types, mean expected errors and sample worths (these means were computed, however, from only two samples per variable) for the absolute error criterion range from about 1.6 (discharge/recharge) to about 3.1 (initial water level) times the original means, and mean expected errors and sample worths for the squared error

Table 12. Results of Sensitivity Tests

Variable	Location in Model: Row	Location in Model: Column	Number of Table Containing Standard Results	Standard Deviation of the Likelihood Function	Absolute Value of Error, in feet per 509 Nodes: Expected Error	Absolute Value of Error, in feet per 509 Nodes: Expected Worth of Sample Data	Square of Error, in feet Squared per 509 Nodes: Expected Error	Square of Error, in feet Squared per 509 Nodes: Expected Worth of Sample Data	Maximum Nodal Error, in feet: Expected Error	Maximum Nodal Error, in feet: Expected Worth of Sample Data	Number of Nodes in Error by more than 10 feet: Expected Error	Number of Nodes in Error by more than 10 feet: Expected Worth of Sample Data
A. Tests done by computing expected errors and sample worths at the end of each time-step												
S	32	13	8		99.4	15.2	85.3	20.5				
T	16	22	8		491.5	38.3	5,883.0	755.8				
H	34	10	8		319.1	220.1	4,925.8	4,125.7				
R_b	4	13	8		657.2	131.8	14,914.7	4,291.3	------ Not computed ------		----- Not computed -----	
S_b	19	16	10		25.8	4.90	7.80	2.17				
T	19	16	10		94.5	4.97	171.8	13.3				
H	19	16	10		88.0	14.5	126.0	31.3				
Q	19	16	10		17.8	3.06	3.43	0.91				
B. Tests done using 5 alternate variable values												
T	11	17	8		83.4	8.94	713.3	143.0	13.6	1.44	1.32	0.14
T	12	9	8		35.9	3.48	23.1	3.30	1.39	0.12	0.0	0.0
T	18	12	8		110.1	5.83	207.3	13.2	4.52	0.24	0.15	0.065
S	5	4	8		24.7	2.19	40.6	4.82	2.72	0.24	0.0	0.0
H	8	5	8		57.3	0.91	102.1	0.0	3.75	0.060	0.0	0.0
Q	5	4	8		144.0	16.5	444.1	66.4	3.79	0.43	0.0	0.0
R_{sc}	11	17	8		66.8	9.3	295.6	61.4	10.4	1.44	0.74	0.014
R_{so}	3	2	8		427.8	54.7	7,388.2	1,336.1	29.8	3.81	11.3	1.98
C. Tests done by multiplying the model value by n x 0.40 x σ_{pr} (n = 1,2,3) to obtain alternate variable values (σ_{pr} is the prior standard deviation) (the factor 0.40 is 80% of the original)												
							26.9	4.47	1.18	0.14	0.0	0.0
S	32	13	8		44.6	4.90	1,579.3	114.8	12.5	0.77	2.45	0.15
T	16	22	8		207.6	11.7	132.4	106.0	3.00	1.91	0.0	0.0
H	34	10	8		92.6	58.8	7,186.1	1,682.8	40.3	6.46	9.21	1.59
R_b	4	13	8		371.9	59.6	4.08	0.84	0.63	0.087	0.0	0.0
S_b	19	16	10		15.3	2.13	57.0	1.38	4.21	0.10	0.0	0.0
T	19	16	10		41.7	1.00	13.3	2.18	1.22	0.13	0.0	0.0
H	19	16	10		24.6	2.63	1.62	0.29	0.42	0.049	0.0	0.0
Q	19	16	10		9.71	1.12						

Table 12. Results of Sensitivity Tests--continued

Variable	Location in Model: Row	Location in Model: Column	Number of Table Containing Standard Results	Standard Deviation of the Likelihood Function	Absolute Value of Error, in feet per 509 Nodes: Expected Error	Absolute Value of Error, in feet per 509 Nodes: Expected Worth of Sample Data	Square of Error, in feet Squared per 509 Nodes: Expected Error	Square of Error, in feet Squared per 509 Nodes: Expected Worth of Sample Data	Maximum Nodal Error, in feet: Expected Error	Maximum Nodal Error, in feet: Expected Worth of Sample Data	Number of Nodes in Error by more than 10 feet: Expected Error	Number of Nodes in Error by more than 10 feet: Expected Worth of Sample Data
D. Tests done by multiplying the model value by n x 0.60 x σ_{pr} (n = 1,2,3) to obtain alternate variable values (the factor 0.60 is 120% of original)												
S	32	13	8		55.3	11.2	46.1	14.3	1.54	0.29	0.0	0.0
T	16	22	8		303.2	31.2	4,508.0	815.0	16.6	2.01	5.37	0.52
H	34	10	8		114.5	83.4	219.0	187.2	3.71	2.70	0.0	0.0
R_b	4	13	8		438.4	93.6	10,806.4	3,398.9	47.5	10.1	10.9	2.29
S	19	16	10		17.8	3.78	5.96	1.91	0.73	0.15	0.0	0.0
T	19	16	10		51.0	3.98	90.9	11.2	5.11	0.40	0.10	0.033
H	19	16	10		30.5	6.31	21.9	6.95	1.51	0.31	0.0	0.0
Q	19	16	10		12.0	2.51	2.68	0.87	0.52	0.11	0.0	0.0
E. Tests performed using standard deviations (for both prior distributions and likelihood functions) that were 80% of original estimates												
S	32	13	8		40.8	6.74	23.3	5.86	1.07	0.18	0.0	0.0
T	16	22	8		233.3	19.9	2,132.9	299.9	13.5	1.24	4.01	0.64
H	34	10	8		84.7	58.4	115.0	96.3	2.74	1.89	0.0	0.0
R_b	4	13	8		340.1	65.7	6,241.6	1,761.5	36.8	7.12	8.41	1.72
S	19	16	10		13.9	2.8	3.53	1.08	0.57	0.12	0.0	0.0
T	19	16	10		40.4	1.91	55.4	3.54	4.07	0.19	0.0	0.0
H	19	16	10		22.5	3.71	11.5	2.86	1.12	0.18	0.0	0.0
Q	19	16	10		8.87	1.49	1.41	0.37	0.39	0.065	0.0	0.0
F. Tests on transmissivities using an estimated likelihood function corresponding to an aquifer test of several days duration												
T	4	3	8	81,000	Same	74.3	Same	955.6	Same	8.97	Same	1.47
T	16	22	8	3,800		234.8		2,494.0		13.2		4.25
T	18	12	8	46,500	as	84.4	as	228.9	as	3.46	as	0.071
T	29	14	8	104,000		7.30		4.8		0.43		0.0
T	5	10	10	26,000	Original	11.0	Original	25.2	Original	0.59	Original	0.0
T	19	16	10	32,000		15.6		37.0		1.57		0.058
T	15	27	8	8,900		308.0		20,696.8		27.2		5.69
T	45	11	8	66,000		190.3		1,253.7		7.53		2.70

Table 12. Results of Sensitivity Tests--continued

Variable	Location in Model: Row	Location in Model: Column	Number of Table Containing Standard Results	Standard Deviation of the Likelihood Function	Absolute Value of Error, in feet per 509 Nodes: Expected Error	Absolute Value of Error, in feet per 509 Nodes: Expected Worth of Sample Data	Square of Error, in feet Squared per 509 Nodes: Expected Error	Square of Error, in feet Squared per 509 Nodes: Expected Worth of Sample Data	Maximum Nodal Error, in feet: Expected Error	Maximum Nodal Error, in feet: Expected Worth of Sample Data	Number of Nodes in Error by more than 10 feet: Expected Error	Number of Nodes in Error by more than 10 feet: Expected Worth of Sample Data
G. Tests on transmissivities using a normal distribution												
T	11	10	10		33.0	5.91	12.8	3.68	0.81	0.14	0.0	0.0
T	35	14	10		42.1	3.89	35.0	6.44	1.75	0.16	0.0	0.0
T	46	10	10		45.8	7.80	64.8	17.1	1.73	0.30	0.0	0.0
T	5	4	8		79.3	5.37	200.2	31.0	3.80	0.26	0.0	0.0
T	5	10	10		60.3	6.00	99.8	16.6	3.20	0.32	0.0	0.0
T	19	16	10		58.3	4.61	126.7	21.8	5.96	0.47	0.18	0.0025
T	44	11	8		123.9	21.0	349.9	99.5	3.76	0.64	0.0	0.0
T	23	22	8		2.07	0.34	0.41	0.11	0.38	0.06	0.0	0.0

Table 13. Comparison of Data from the Sensitivity Tests with Original Test Data.

Table Number of Sensitivity Data	Mean Difference Between the Sensitivity-Test Value and the Original Value (Represented as the Mean of the Percentages of the Eight Original Values), for Sensitivity Tests in Tables 12B, C, D, E, and G. Mean ratio of the Sensitivity-Test Value to the Original Value for Sensitivity Tests in Tables 12A and F.					
	Absolute Value of Error, in feet per 509 nodes		Squared Error, in feet squared per 509 nodes		Maximum Nodal Error, in feet	
	Expected Error	Expected Worth of Sample Data	Expected Error	Expected Worth of Sample Data	Expected Error	Expected Worth of Sample Data
12A	2.08	2.06	5.72	5.72	No Data	No Data
12B	-15%	-53%	-28%	-65%	-15%	-53%
12C	-13%	-39%	-27.5%	-51%	-12%	-37%
12D	+ 8.2%	+32%	+26%	+58%	+ 8.4%	+33%
12E	-18%	-18%	-33%	-33%	-18%	-16%
12F	No Difference	6.96	No Difference	4.59	No Difference	6.10
12G	+23%	+41%	+55%	+115%	+25%	+45%

criterion range from about 1.6 (discharge/recharge) to as much as 17.2 (initial water levels) times original means. Initial water levels probably have much higher expected errors because during early time-steps the errors in water levels are not damped as much as they are at the end of the simulation period.

For test B, mean expected errors computed using five elements per discrete distribution range from 15 to 28 percent below original values for the three error criteria tabulated; and mean expected worths of sample data range from 53 to 65 percent below original values. These different results suggest that five elements are not sufficient to describe the distributions. Additional tests should be run to see how many elements are necessary to stabilize the results.

For results computed using seven elements spaced more closely than the original (test C), mean expected errors are from 12 to 28 percent below, and mean expected sample worths are from 37 to 51 percent below original values. Results computed using less closely-spaced elements (test D), yielded mean expected errors from 8 to 26 percent above, and mean expected sample worths from 32 to 58 percent above original values. The probabilities for the more widely-spaced elements in the outer parts of the distribution are lower, or for more closely-spaced elements are higher, than for the original elements. These differences in probabilities should counter-balance the effects of differences in the element values and yield expected errors and sample worths that are about the same as the originals. The fact that

significant differences exist shows that the discretization did not approximate the distributions well.

For test E, mean expected errors computed with smaller standard deviations for likelihood functions for transmissivities -- the modified standard deviations averaged about a quarter as large in arithmetic units or a third as large in logarithmic units, as original estimates -- are much higher than the original results (test F), ranging from more than 4 to about 7 times original values for the three error criteria tabulated. If future sampling for transmissivity were to be done by aquifer tests of several days duration, providing assumed standard deviations were reasonably correct, the worth of additional data on transmissivity would be significantly higher than for the other three variables.

For results computed using a normal distribution for transmissivity (test G) mean expected errors are from 23 to 55 percent higher, and mean expected sample worths are from 41 to 115 percent higher, than original values. A log-normal distribution is asymmetric, towards high values of the variable, compared to a normal distribution with an equivalent mean and standard deviation. Elements of a discrete set of alternate values based on the normal distribution, then, will be of much smaller magnitude below the mean and of slightly larger magnitude above the mean. This larger spread of alternate, or erroneous, values associated with the normal distribution results in larger expected errors and sample worths.

The sensitivity tests indicate that computed values of expected error, and especially of expected worth of sample data, depend on the assumptions and techniques of the method. Therefore these values likely are correct only within an order of magnitude. As used in this dissertation, the term "within an order of magnitude" implies that the true value lies within a range from a tenth to ten times the estimated value. However, the sensitivity tests also suggest that the relative rankings of the magnitude of expected errors and sample worths remain fairly constant. Table 14 shows a comparison of the rankings, for absolute value of error only, for expected error and sample worth of original results against sensitivity-test results. Seven sensitivity tests of eight variables each were conducted; for 48 of these tests both an expected error and sample worth were computed; whereas for eight tests only expected sample worth was computed. Thus 104 possible rankings could change. Of these 104 rankings 86 stayed the same, 16 changed only one position, and 2 changed two positions.

Worth-of-Data Computations for an Idealized Management Problem

As an illustration of the potential application of worth-of-data studies, an idealized management problem was formulated for the Tucson basin. Two of the major users of ground water in the basin are the city of Tucson and the mining companies -- Pima Mining Co., American Smelting and Refining Co., Duval Corp., and the Anaconda Co.-- who pump water for mining operations in the southern part of the basin. Predictions are that the mining companies will increase their pumping

Table 14. Comparisons of Rankings of Absolute Values of Error from the Sensitivity Tests with Rankings of the Original Test Data

Variables Tested and Locations in Model Grid (Row, Column); 1 - rankings of original data, 2 - rankings of sensitivity-test data, EOL-expected error (expected opportunity loss), EWSD-expected worth of sample data

	S(32,13)		T(16,22)		H(34,10)		R_b(4,13)		S(19,16)		T(19,16)		H(19,16)		Q(19,16)	
	EOL	EWSD	EOL	EWSD	EOL	EWSD	EOL	EWSD	EOL	EWSD	EOL	EWSD	EOL	EWSD	EOL	EWSD
	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2
12A	4 4	4 4	2 2	3 3	3 3	2 1	1 1	1 2	7 7	6 7	5 5	7 6	6 6	5 5	8 8	8 8
12C	4 4	4 4	2 2	3 3	3 3	2 2	1 1	1 1	7 7	6 6	5 5	7 8	6 6	5 5	8 8	8 7
12**D**	4 4	4 4	2 2	3 3	3 3	2 2	1 1	1 1	7 7	6 7	5 5	7 6	6 6	5 5	8 8	8 8
12E	4 4	4 4	2 2	3 3	3 3	2 2	1 1	1 1	7 7	6 6	5 5	7 7	6 6	5 5	8 8	8 8
	T(11,17)		T(12, 9)		T(18,12)		S(5,4)		H (8,5)		Q(5,4)		R_{sc}(11,17)		(R_{sc}(3,2)	
12B	4 4	3 4	7 7	6 6	3 3	5 5	8 8	7 7	6 6	8 8	2 2	2 2	5 5	4 3	1 1	1 1
	T(4,3)		T(16,22)		T(18,12)		T(29,14)		T(5,10)		T(19,16)		T (15,27)		T(45,11)	
12F	No change	4 5	No change	2 2	No change	5 4	No change	8 8	No change	6 7	No change	7 6	No change	3 1	No change	1 3
	T(11,10)		T(35,14)		T(46,10)		T(5,4)		T(5,10)		T(19,16)		T(44,11)		T(23,22)	
12G	7 7	4 4	6 6	6 7	5 5	2 2	2 2	5 5	3 3	3 3	4 4	7 6	1 1	1 1	8 8	8 8

sharply during the period 1970-1975 (J. F. Rauscher, written communication, city of Tucson Department of Water and Sewers 1968; Mark Wilmer, written communication to the city of Tucson Department of Water and Sewers 1970; and Clausen 1970, p. 85).

The city of Tucson has a large well-field, the Santa Cruz well-field south of the city, from which it pumped about 30 percent of its total supply in fiscal year 1970. The city should be interested in how much the predicted increase in pumping by the mines during 1970-1989 will affect water levels in the Santa Cruz field in 1990. However, estimates of future pumping for the mines are uncertain because of factors such as uncertain estimates of future ore production, due to unexpected changes in copper prices or environmental legislation resulting in curtailing production, etc.; or the amount of water that will be recycled. The simplified management problem posed here is: what is the worth of additional data on pumpage -- actually the worth of additional studies made to estimate future pumpage, as actual data cannot be collected -- to the city of Tucson in terms of reducing errors in predicted water levels?

Figure 10 shows the approximate location of pumping for mining and the nodes in which the Santa Cruz well-field is located. Pumping for mining occurs in three general areas, one northwest of Sahuarita, one southwest of Sahuarita, and one southwest of Continental. Table 15A lists the 17 nodes in which pumping for mining occurs and gives the total estimated 1970-89 pumpage for each node. More than half of the pumpage will occur northwest of Sahuarita, the area closest to the Santa

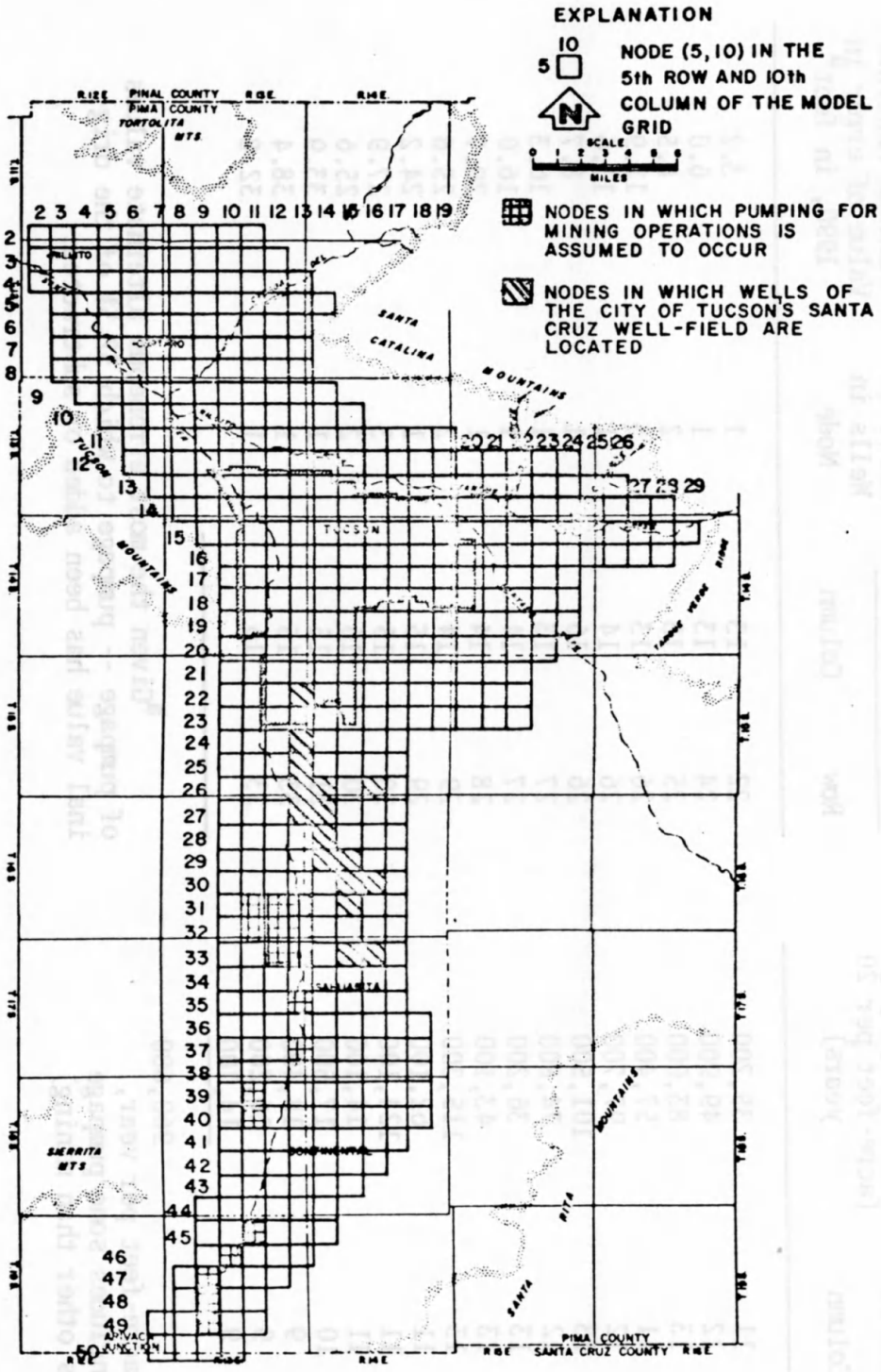


Figure 10. Map showing the approximate location of pumping for mining operations and the city of Tucson's Santa Cruz well-field.

Table 15. Data from the Idealized Management Problem.

A. Nodes from which pumping for mining operations was assumed to occur during 1970-89.

Location in Model		Estimated pumpage (acre-feet per 20 years)
Row	Column	
31	11	35,200
31	12	49,900
31	13	83,000
32	11	57,400
32	12	97,700
32	13	101,500
33	12	74,800
33	13	36,200
35	13	43,100
37	13	115,300
39	11	62,100
40	11	124,100
45	11	14,100
46	10	17,500
47	9	18,400
48	9	16,200
49	9	14,100
Sum		960,600

(48,000 acre-feet per year, which includes some pumpage for uses other than mining)

B. Nodes in which wells of the Santa Cruz well-field are located.

Location in Model		Number of Wells in Node	Maximum Absolute Value of error in 1990, in feet[a]
Row	Column		
22	13	1	3.2
24	13	1	6.0
25	13	2	8.5
26	13	3	11.9
26	14	1	11.6
26	16	1	8.7
27	13	1	16.3
27	14	1	16.0
28	14	1	20.7
29	14	2	25.6
29	15	1	24.2
30	15	3	27.9
30	16	2	25.6
31	15	1	33.9
33	15	1	38.4
33	16	1	32.8

[a]Given the most erroneous alternate values of pumpage -- pumpage to which 52.5% of the original value has been added or subtracted.

Table 15--continued

C. Results of Test. -- Values are multiplied by the number of city wells per node for absolute and squared error; or by 1.0 if there are city wells in the node or by 0.0 if not, for maximum error and number of nodes in error by more than 5, 10 and 25 feet.

	Expected Error	Expected Worth of Sample Data	Percent Improvement
Absolute Value of Error, in feet per 16 nodes times number of wells	107.6	19.3	17.9
Square of Error, in feet squared per 16 nodes times number of wells	1,725.9	473.2	27.4
Maximum nodal Error, in feet	17.1	3.1	17.9
Number of Nodes in Error by More than 5 feet	6.5	0.72	11.2
Number of Nodes in Error by More than 10 feet	4.1	0.73	17.8
Number of Nodes in Error by More than 25 feet	0.69	0.27	39.0

Cruz field. Table 15B shows the nodes in which wells of the Santa Cruz field are located and the number of wells per node.

The basic computer program for the worth-of-data studies was modified so that six alternate, or erroneous, values of pumpage were assumed at each pumping node. The criteria used to estimate the prior standard deviations and the standard deviations of the likelihood functions were the same as for the basic worth-of-data studies. The program put all the smallest alternate values of pumpage into the model, then all the next smallest values, etc., and thus tested seven separate lumped values of pumpage over all nodes. The program could have been modified to put various combinations of erroneous pumpage into the model, such as negative errors at some nodes and positive errors at others, provided the errors were assumed independent, but this would have resulted in many more than seven lumped values with an enormous increase in computer time. Specifically, 7^{17} values are possible if every combination is used. The resulting expected error and expected worth-of-sample-data computations would have been more accurate, but not enough to justify the large increase in cost.

Table 15B also shows the maximum prediction errors, using the most erroneous values of pumpage, at each node for the year 1990. These errors range from 3.2 feet at the northern end of the Santa Cruz field to 38.4 feet at the southern end. Table 15C gives the results of the worth-of-data tests. For these computations, the cost coefficient in the loss function was set equal to the number of city wells per node for the criteria of absolute error and squared error,

and set equal to 1.0 or 0.0 if city wells were in a node or not for the other criteria. The results show, for example, that the sum of expected absolute errors over the 16 nodes, including 23 wells, of the Santa Cruz field is about 108 feet-wells, or less than 10 feet-wells per node. If additional studies were made to estimate future pumpage, which were roughly equivalent in extent to studies already made, the error could be reduced by about 18 percent. As a further example, if it were judged desirable to reduce the maximum nodal error, analogous to using a minimax decision criterion, further data might reduce this error by about 18 percent.

Such studies indicate to a manager, in a qualitative sense, how much he can improve model predictions with further collection of data. The basic worth-of-data results (tables 8 and 10) also indicate, qualitatively, the worth of collecting additional data on a given variable at a given node. If a manager could assign a cost to each foot of prediction error through the cost coefficient, or cost coefficient function, if one were derived, an actual economic worth of collecting additonal data could be computed.

CHAPTER 6

STUDIES OF ERROR PROPAGATION

Some very limited studies were made of the propagation of error over the model. These consisted of printing maps of the differences between water levels, at the end of the simulation period, computed using the model value of a variable and water levels computed using selected alternate values of the variable. The eight variables used in sensitivity tests A, C, D, and E were used to obtain these maps, and table 16 summarizes the results.

Of the six possible alternate values of storage coefficient and transmissivity, data from the two variable values farthest from and the two closest to the model value are tabulated. For initial water level and discharge/recharge, data from only one outer value and one inner value are tabulated because errors for the other two values are identical to the first two, differing only in that they were opposite in sign. For all tests, the two outer values had probabilities of occurrence of either 0.06 or 0.07 and the two inner values had probabilities of 0.19, as opposed to the model or central value which had a probability of either 0.21 or 0.22.

Table 16 shows the alternate values, the associated errors at the tested nodes, the maximum errors at 1 mile, 5 miles, and 10 miles,

Table 16. Data on Error Propagation over the Model. -- A minus (-) indicates water levels computed using alternate values were above those computed using the model value; a plus (+) indicates they were below.

Variable	Location in Model Grid: Row	Location in Model Grid: Column	Model Value[a]	Selected Alternate (Erroneous) Values	Error at Node, in feet	Maximum Error at 1 mile, in feet[b]	Maximum Error at 5 miles, in feet[b]	Maximum Error at 10 miles, in feet[b]	Maximum Radius at 1 foot of error. in miles
S	32	13	0.153	0.048	-0.4	+3.3	+0.8	+0.5	4
				0.118	+0.7	+0.9	+0.2	+0.1	0
				0.188	-1.0	-0.7	-0.2	-0.1	< 1
				0.258	-3.5	-2.3	-0.7	-0.3	4
T	16	22	3,800	26.6	-2.4	-3.9	+0.5	+0.1	2
				727	-1.9	-3.1	+0.4	+0.1	2
				19,860	+5.0	+13.4	-1.6	-0.4	7
				542,500	+10.1	-68.4	-14.4	-4.9	14
H	34	10	2,610	2,370	+7.7	+6.7	+2.4	+0.2	7
				2,690	-2.6	-2.2	-0.8	-0.1	3
R_b	4	13	102,105	194,355	-103.6	-62.4	-5.3	-0.9	11
				71,355	+34.5	+20.8	+1.8	+0.3	8
S	19	16	0.112	0.007	-0.8	+1.9	+0.4	+0.0	1
				0.077	+0.0	+0.5	+0.1	0.0	0
				0.147	-0.2	-0.5	-0.1	0.0	0
				0.217	-0.7	-1.5	-0.3	-0.0	1
T	19	16	52,500	19,240	+1.5	-9.4	+0.4	+0.1	3
				37,570	+0.4	-3.5	+0.2	0.0	2
				73,360	-0.3	+3.8	-0.2	0.0	2
				143,220	-0.4	+11.5	-0.5	-0.1	3
H	19	16	2,340	2,295	+1.0	+2.4	+0.7	-0.2	2
				2,355	-0.3	-0.8	-0.2	0.0	0
Q	19	16	5,270	3,281	-1.1	-0.7	-0.2	0.0	< 1
				5,933	+0.4	+0.2	+0.1	0.0	0

[a]Units as in Tables 8 and 10.

[b]Directly north, south, east, or west only.

and the maximum radii of 1 foot of error. The first four variables in the table are from the large-error category, and the second four are from the small-error category and are all at the same node.

For the first set of four tests, the largest errors and extent of error are associated with discharge/recharge and transmissivity; and for the second set are associated with transmissivity and initial water level. These results match the results from the basic worth-of-data computations. The largest error at a tested node is about 104 feet associated with an extreme alternate recharge value at node (4,13). The largest error at 1 mile is 68 feet, at 5 miles is 14 feet, and at 10 miles is about 5 feet, and the maximum radius of 1 foot of error is 14 miles; all associated with an extreme alternate transmissivity value at node (16,22).

Of perhaps more interest, however, is that for the six tests other than on R at (4,13) and T at (16,22), errors in predicted water levels were relatively small, even for the extreme erroneous variable values. For these tests, eight of nine maximum errors at the tested node and six of nine maximum errors at 1 mile are less than 4 feet; eight of nine maximum errors at 5 miles are less than 1 foot; and all nine maximum errors at 10 miles are less than 0.5 feet. These limited results suggest that in many cases, prediction errors associated with errors in basic data are not a major problem in modeling.

Lovell (1971, p. 26-27 and Appendix B) also studied error propagation in the southern part of the Tucson basin digital model, using slightly different methods in that he observed propagation with

time. He concluded that maximum errors in predicted levels tended to stay at the tested node for storage coefficient, transmissivity, and discharge/recharge, provided the error in discharge/recharge continued over the whole simulation period; and stayed within a radius of a few nodes for initial water level. The data in table 16 generally support Lovell's conclusions, although they show that maximum errors associated with storage coefficients do not necessarily remain at the tested node.

In addition, errors in computed water levels associated with erroneous values of initial water level produced some unexpected results. A test on the initial water level at node (19,16), using an extreme erroneous value of 2,295 feet instead of the model value of 2,340 feet, yielded a computed water level at the end of the simulation period at (19,16) that was 1.0 feet below that computed with the model value. The maximum error was at node (21, 17) and represented a computed level 3.1 feet below the standard value. Eight miles directly to the north on the model boundary, however, was a secondary maximum error representing a computed water level 2.2 feet above the standard value. Errors between this node and the tested node were as little as 0.1 feet. This secondary maximum appeared at the same location in other tests at this node, although values were smaller.

Tests of errors in transmissivity at node (19, 16) produced similar, although less striking, results. Errors decreased steadily in size to a point five miles directly north of the tested node, then increased a maximum of 0.5 feet at a point seven miles to the north.

The reason for these irregularities in error propagation is not known, although it may be related to boundary effects.

CHAPTER 7

FUTURE WORK

Potential Extensions of the Method

This initial attempt to evaluate worth of ground-water data could be extended in several ways while retaining the basic approach as used here. Three types of errors in basic data: (1) in the location of discharge/recharge, (2) in the variation in discharge/recharge with time, and (3) in the position of the model boundaries, were not included in this investigation because the methods used here were not easily adaptable to their study. Conceivably, however, these types of errors could be studied without major modification of the basic approach. For variation in the location of discharge/recharge, for example, quantities of discharge or recharge could be assigned to the most likely node and to 4 or more adjacent nodes. Probabilities of discharge or recharge being at a given node could be assumed to be proportional to the distance from the most likely node. Probabilities also could be associated with various boundary configurations at a given location and to various plausible patterns of time variation of discharge/recharge. It should be pointed out, however, that to model daily or even seasonal variations in discharge/recharge over a simulation period of several years would involve using many time-steps and would be costly.

The basic approach could be modified by using continuous instead of discrete probability distributions for the variables. This procedure would involve extensive changes in the computer program and numerical integration might be necessary to compute probabilities. Also, as discussed in Chapter 4 ("Use of Statistical Decision Theory to Evaluate Worth of Ground-Water Data"), using continuous distributions would necessitate using much more computer time. If discrete probabilities are retained, however, an attempt should be made to improve the procedures used in discretizing and truncating the probability distributions.

Further Research Suggested by the Results of this Study

This section summarizes the recommendations for future research that are scattered through the text, as well as some research for which need is implied in the text by discussion of deficiencies in the digital model and in the method of computing worth of data.

During this study, areas of possible research on digital modeling became evident. More work should be done on model calibration, both to develop better objective and semi-objective calibration methods and to develop efficient techniques for trial and error calibration. Calibration based on inclusion of all model nodes should be compared to calibration based only on nodes with historical data to see which is most efficient under given conditions of areal distribution of data. More research is needed on how many time periods are necessary for the calibration process to approach a unique set of calibrated parameters, and to determine how closely the true parameter values are approximated.

The errors in a model that are caused by computation, by mathematical assumptions, and by the particular algorithm used merit more study. It would be of especial interest to determine how far the variable values of the final calibrated model are from true values because of errors caused by computation, model assumptions, and the algorithm. This will be difficult, of course, for actual ground-water basins because it normally isn't possible to determine the true variable values.

Research on whether boundary effects in a digital model are equivalent to boundary effects in a real physical system would be useful. More work also is needed on methods of determining the optimum number of time-steps for simulating the historical record for various areal distributions of aquifer parameters, initial water levels, and recharge/discharge.

Additional research could improve the method for computing worth of data that was developed in this study. The types and parameters of probability distributions of hydrogeologic variables need better definition. This should include definition of both the natural variability of hydrogeologic parameters and the variability caused by measurement errors and errors due to interpolation and non-representative data. It also would be important to determine if probability distributions are dependent or independent of the area or volume of aquifer being considered. More work could be done on using the subjective knowledge of a hydrologist to estimate parameters of probability distributions.

Another subject that needs study is the economic benefit of digital models of ground-water systems. A closely-related subject for

research is the cost of errors in water-level predictions obtained from such models. Such research would help to define more realistic loss functions, for example by specifying whether the functions are symmetric or asymmetric, and by better defining the function for cost per foot of prediction error. Costs of collecting ground-water data also need better definition.

A major improvement in the method would be to extend it to model and evaluate the effects of errors concurrently at more than one node or at all nodes in a model. Formulation of the probability distributions of variables would be difficult if variable values were not assumed independent. Research thus would be needed on joint probability distributions of variables, especially joint distributions related to errors resulting from contouring, interpolation, and non-representative data. The basic approach in evaluating worth of data would have to be modified if many or all nodes were considered in error simultaneously, so that the amount of computer time would not be prohibitive.

The studies reported here also suggest that research on the optimal design of networks for sampling ground-water data would be useful. Networks to optimally collect data for digital models and for other uses could be developed and compared.

CHAPTER 8

SUMMARY

Potential errors in the digital models of the Tucson basin were classified as errors associated with computation, errors associated with the model's mathematical assumptions, and errors associated with basic data -- the model parameters S and T, initial water levels, and values of discharge and recharge. This study focused on estimating the worth of additional basic data on a simulation period 1970-90 to the digital model. The method is most applicable in the early stages of collecting data for a basin model, prior to the time when additional field data might result in a poorer model.

Statistical decision theory was used, in a basic form, to compute expected error and expected worth of sample data over the whole model associated with uncertainty in one variable at one location. Tests were made on 91 variables at 61 different locations in the model. At 30 nodes, more than one variable was tested. Of the tests, 67 were on variables whose prior estimates generally were considered to be uncertain; the other 24 were on variables whose prior estimates were considered to be less uncertain.

Of the uncertain variables, discharge/recharge and transmissivity have the largest expected errors and worth of sample data, while initial water levels and storage coefficients have lesser values.

However, transmissivity has a low percent reduction in error because of the large uncertainty commonly associated with sampling for T. Of the variables whose prior estimates were more certain, transmissivity and initial water level generally have the largest expected errors and worth of data, while storage coefficient and discharge/recharge have smaller values. The large expected errors and worth of data associated with transmissivity may be peculiar to the Tucson basin because T's for this basin commonly are uncertain.

In general, the largest expected errors are associated with nodes at which values of discharge/recharge are large or at which prior estimates of transmissivity are very uncertain. Large worth of sample data is associated with variables which have large expected errors or which could be sampled with relatively little uncertainty. The results are generally the same for all six of the separate criteria used.

The size of the Tucson basin model necessitated the use of probability distributions composed of only seven discrete values of a given variable. In addition, most of the parameters of the distributions had to be estimated, largely on a subjective basis, because of the fact that sample data within individual nodal areas commonly were lacking.

Tests of the sensitivity of the results to the various assumptions inherent in the approach indicated that results are sensitive to all of the assumptions. For these reasons, individual values of expected error and sample worth likely are accurate only to an order

of magnitude. For example, if the expected error were computed to be 100 feet over the 509 nodes of the model, the true expected error might range from less than 50 to several hundred feet. However, the sensitivity tests indicated that the ranking of types of variables, in terms of the magnitude of expected errors and sample worths, are not sensitive to the assumptions of the approach. The general conclusions on comparison of the effects of errors in the four variable types therefore should be reasonably reliable.

The results of this study do not agree well with those of Bibby (1971), who concluded that errors in predicted water levels are largely a result of errors in initial water levels. This lack of agreement may be partly a result of the differences in the degree of uncertainty assigned to the variables, and also may be related to the different methods and assumptions of the two studies.

The approach used in this study can be applied to idealized management problems. An example application addressed the question: what is the worth of additional data on pumpage in a local area in terms of reducing errors in predicted water levels in a nearby area? In general, the method can be used by a ground-water-basin manager to indicate qualitatively how much he can improve model predictions by the collection of additional data.

Limited studies of error propagation indicated that expected errors in predicted water levels were fairly small outside of a radius of a mile around the tested node, except for those errors associated with very large values of discharge/recharge or very uncertain

values of transmissivity. This result suggests that in many cases, prediction errors associated with errors in basic data are not a major problem in digital modeling. However, this conclusion is only for errors taken one node at a time, and does not say anything about the actual condition in which all model data have some degree of uncertainty.

A drawback of the approach is that most of the required statistical parameters had to be estimated subjectively. Subjective estimation of parameters likely will be necessary for most basins, because no more hydrogeologic data are available for the Tucson basin than are available for most other basins. Therefore, worth of data evaluated by one person will not be exactly the same as that evaluated by another. However this disadvantage is in one sense somewhat of an advantage. By means of subjective estimation of the parameters of distributions, a hydrologist can enter his judgment and intuition about the uncertainty associated with basic data directly into the process of evaluating the worth of model predictions.

APPENDIX A

COMPUTER PROGRAM

The writer prepared the computer program used for the worth-of-data studies, which consists of a main program, WODATA, and seven subroutines, CYCLER, AVAL, BAYESGW, SIGMA, PROBDN, TBLKUP, and ALDIRS. Figure 11 is a simplified flow chart of the entire program. The program contains several options to control how extensive a set of worth-of-data studies will be, such as whether worth of data is computed over all time-steps or at the end of the simulation period, whether maps of errors are printed, the number of error criteria used, and the number of discrete values in each probability distribution of a tested variable. The computer time for a test will vary markedly depending on the options chosen. For example, if a test of a single variable does not involve summing worth of data over time-steps or printing maps, and uses one of the six error criteria and five discrete values for the distribution, the test takes about 13 seconds of time, exclusive of the computer time taken to compile the program, on the University of Arizona's CDC-6400 computer, at a cost of about $1.50. If, however, the program calls for maps of errors and uses seven values per distribution, the time per test increases to about 25 seconds; and if the program uses all six error criteria and seven values per

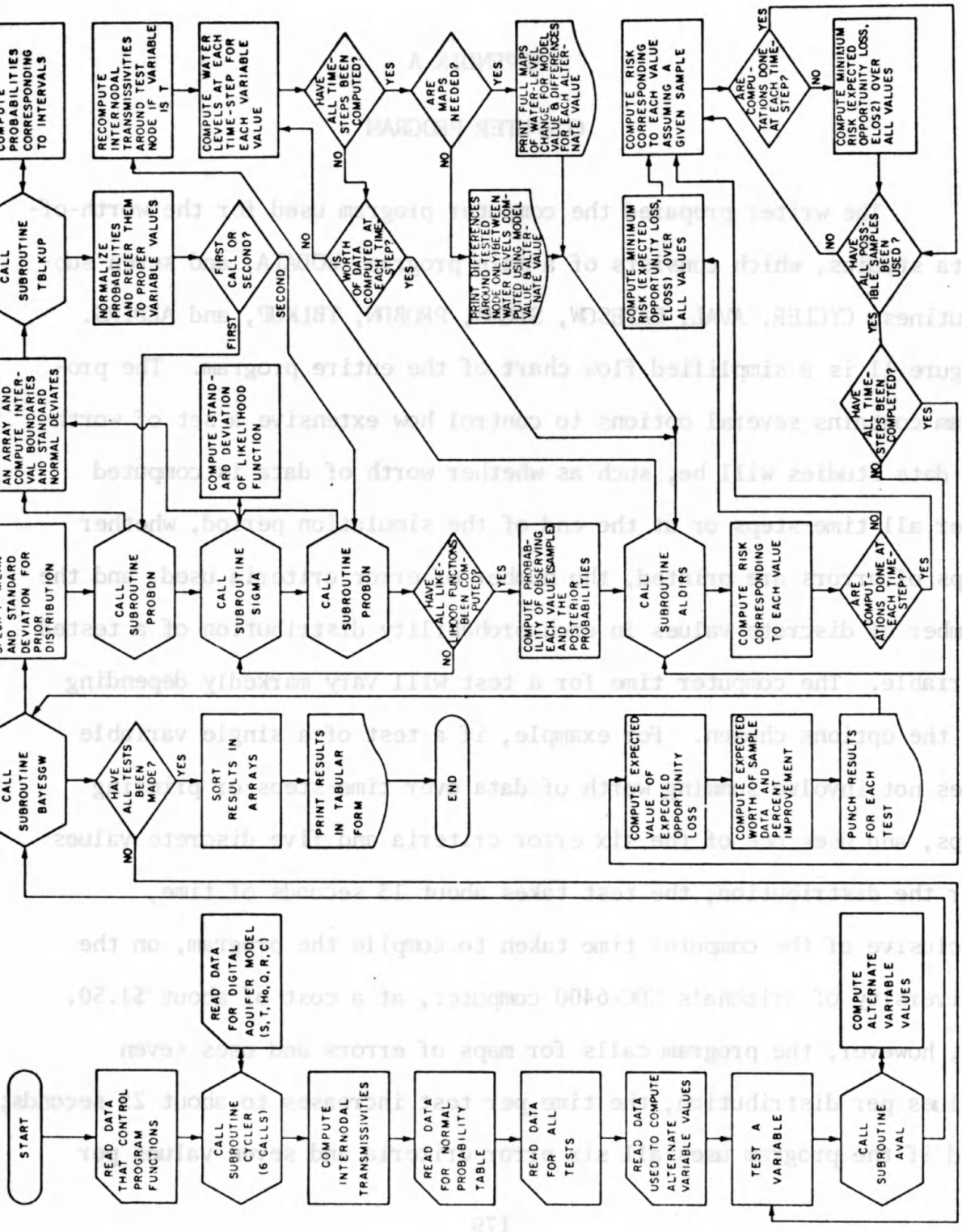


Figure 11. Simplified flow chart of the computer program WODATA and subroutines.

distribution, the time is about 30 seconds. If the program sums worth of data over three time-steps, and uses two error criteria and seven values per distribution, the time rises to about 55 seconds per test at a cost of about $6.00.

In brief, WODATA reads in all data and processes it for use by the subroutines. These data include the number of tests to be made; which operational options are selected; the type of probability distribution; the length of the simulation period; the size of the initial time-step; the size of the model grid; and data specific to each test, such as nodal location, variable type, and standard deviations of the prior distribution and initial likelihood function ($\sigma_{\ell ms}$). WODATA also converts data to proper units, computes inter-nodal transmissivities, and, at the end of a test, sorts all results in order of magnitude and prints them in tabular form.

CYCLER is an input subroutine that reads data for the variables S, T, H, Q, and R and the coefficient for cost per foot of error, converts them to proper units, and fills the rest of each data array outside the model boundary with the necessary values--0 for T, Q, and R and 1 for S and H.

AVAL takes the model value, or mean, of each variable to be tested and computes four or more alternate values, at least two larger and two smaller than the mean. These alternate values are spaced from the mean ± 1, 2, 3, 4, etc. times a specified spacing factor times the standard deviation of the prior distribution. The spacing factor for

the basic worth-of-data studies was 0.50. AVAL puts all values in ascending order, except that the mean is the first value in the array.

BAYESGW is the subroutine that actually computes expected error, or expected opportunity loss, and expected worth of sample data. First, the subroutine computes all necessary statistical data--probabilities for the discrete prior distribution of the variable, for the likelihood functions, for observing sample results, and for the posterior distributions--by using subroutines SIGMA, PROBDN, and TBLKUP. Subroutine BAYESGW then obtains water-level elevations over the model, computed using each possible variable value, by calling subroutine ALDIRS. Then BAYESGW computes risks for each variable value and selects the minimum risk for each error criteria as the expected opportunity loss (EOL). EOL can be considered the minimum expected error summed over all 509 nodes of the Tucson basin model. Then the subroutine computes risks and EOL for each possible sample result, and computes the expected value of expected opportunity loss (EEOL) over all possible samples, for each error criteria. BAYESGW finally computes expected worth of sample data (EWSD) and the percent improvement with sampling, or percent reduction in error (PCIMP) and punches all results for all error criteria on cards for future reference.

SIGMA is the subroutine that computes the standard deviations for the likelihood functions. In the first call to SIGMA the program assumes the model value is the mean of the distribution and uses it and the "sampling" standard deviation, $\sigma_{\ell ms}$, to compute the probabilities

of the first likelihood function. In subsequent calls to SIGMA alternate variable values are assumed to be means, and the subroutine derives standard deviations associated with each alternate value.

Subroutine PROBDN computes probabilities for all possible values of the variable, using one specified value as the mean and a specified standard deviation. This subroutine can compute probabilities using either arithmetic or logarithmic units. PROBDN sorts all variable values in ascending order and computes the midpoints between adjacent values. These midpoints are used to compute standard normal deviates, defined as (Midpoint value - μ)/σ . Adjacent standard normal deviates then are passed in pairs to TBLKUP which interpolates the probability of the included interval. Subroutine PROBDN then normalizes each of the probabilities by dividing by their sum, to form a discrete probability distribution.

Subroutine TBLKUP estimates the probability of the interval between two adjacent standard normal deviates by approximating the corresponding area under the standard normal probability curve. The approximation utilizes an array which contains a condensed table, including 65 values, of areas under the curve. The 65 values are spaced at intervals of 0.05 standard units from 0.00 to 3.00 units, and at intervals of 0.5 units from 3.0 to 5.0 units.

Subroutine ALDIRS is the alternating-direction-implicit algorithm which computes water-level elevations over the model at the end of a specified time period. The core of this subroutine is essentially the

Tucson basin model. ALDIRS takes each variable value, inserts it in the model, and recomputes internodal transmissivities around the tested node, if necessary. The subroutine then computes a set of water levels using each variable value. If worth of data is to be summed over all time-steps, the sets of water levels are passed to BAYESGW at the end of each step; if not, ALDIRS completes computations of water levels for the whole simulation period. ALDIRS then prints differences, in the vicinity of the tested node only, between water levels computed using the model value and using alternate values, or prints maps of the differences in water levels over the whole model.

The following pages are a complete listing of the computer program WODATA and subroutines.

```
      PROGRAM WODATA(INPUT,OUTPUT,PUNCH)
C     PROGRAM TO ESTIMATE WORTH OF ADDITIONAL DATA TO A DIGITAL MODEL
C     OF A GROUND-WATER BASIN
C
C     PROGRAMMED BY JOSEPH S. GATES, DEPT. OF HYDROLOGY AND WATER RESOURCES,
C     UNIVERSITY OF ARIZONA
C
C     DATE OF LAST CHANGE - JULY 6, 1971
C
000003      DIMENSION IROW(100), JCOL(100), NVARTYP(100), SPER(8), FMTRN(8),
           2 NFG(100), SIGP(100), SIGS(100), FMT1(8), FMT7(8), FMT8(8),
           3 X(100), NX(100), NRTYP(100), PVAL(10), FMT4(8)
000003      COMMON/WBA/IR, JC, NVT, VAL(10), NUMVAL, IS, IE, NSTART(51),
           2 NEND(51), LPASS, LTSF
000003      COMMON /WOOBGW/ EWSO(6,100), PCIMP(6,100), EOL(6,100), SP, SS,
           2 NFLAG, LFLAG, NRT, C(51,30), NUMTAB
000003      COMMON/WOOTLU/ TKALPH(65), TALPH(65)
000003      COMMON/WOOSIG/ SIGSTO(20), SIGSTU(20), SINT(10), NVSINT
000003      COMMON/WOOADS/FHEAD(8), TIME, T(51,30), S(51,30), JS, JE,
           2 IEM1, JEM1, A, MAPS, TIN(51,30,4), Q(51,30), IEP1, JEP1,
           3 HJ(51,30), R(51,30), TSTEP1
C     X IS ARRAY USED FOR RESULTS DURING SORTING, NX IS ARRAY USED TO
C     STORE THE ORIGINAL STORAGE LOCATIONS OF RESULTS DURING SORTING,
C     LPASS STORES THE TEST NUMBER DURING A TEST, PVAL IS THE ARRAY
C     FOR TEMPORARY STORAGE OF UNSCALED VALUES
C
C
C     READ IN GENERAL DATA TO RUN PROGRAM
C     READ RUN NUMBER AND THE NUMBER OF VARIABLES TO BE TESTED
000003      READ 5, FMTRN
000011 5    FORMAT(8A10)
000011      READ 101, NUMTEST
000017 101  FORMAT(5I10)
C     READ TOTAL NUMBER OF VALUES ALLOWED FOR EACH DISCRETE DISTRIBUTION
C     OF A VARIABLE (MODEL VALUE AND ALTERNATE (ERRONEOUS) VALUES)
000017      READ 101, NUMVALS, NUMVALT, NUMVAHJ, NUMVALQ, NUMVALR
000035      PRINT 103
000041 103  FORMAT('1',45X,'WORTH OF GROUND-WATER DATA, TUCSON BASIN')
000041      PRINT FMTRN
C     READ FORMATS FOR OUTPUT HEADING, INPUT DATA (3), AND HEADING
C     FOR PRINTED MAPS
000045      READ 5, FMT1, FMT4, FMT7, FMT8, FHEAD
C     READ FLAG FOR NORMAL (0) / LOGNORMAL (1) DISTRIBUTIONS ASSUMED
C     FOR TRANSMISSIVITY
000063      READ 101, LFG
C     READ FLAGS FOR WHETHER WORTH-OF-DATA COMPUTATIONS ARE MADE AT
C     THE END OF EACH TIME STEP (2) OR AT THE END OF THE SIMULATION
C     PERIOD (1) (LTSF), FOR WHETHER MAPS OF COMPUTED WATER LEVELS ARE
C     PRINTED (2) OR NOT (1) (MAPS), AND FOR THE NUMBER OF ERROR CRITERIA
C     TO BE USED IN COMPUTING SEPARATE TABLES OF WORTH OF DATA (NUMTAB)
000071      READ 101, LTSF, MAPS, NUMTAB
C     READ TIME PERIOD TO BE SIMULATED, IN YEARS
000103      READ 107, TPER
000111 107  FORMAT(8F10.0)
C     READ SIMULATION PERIOD (ARRAY FOR PRINTING)
000111      READ 5, SPER
C     READ ACRES PER NODAL AREA
```

```
000117          READ 107, ACRES
                C     READ INITIAL TIME-STEP, IN YEARS
000125          READ 107, TSTEP1
000133          PRINT FMT1
                C     TIME IS TOTAL SIMULATION PERIOD, IN DAYS
000137          TIME=365.25*TPER
                C     IS AND IE, JS AND JE ARE THE NUMBERS OF THE FIRST AND LAST ROWS AND
                C     COLUMNS, RESPECTIVELY, WITHIN THE MODEL BOUNDARIES
000141          READ 2, IS, IE, JS, JE
000155    2     FORMAT(20I4)
000155          IEM1=IE-1
000157          JEM1=JE-1
000161          IEP1=IE+1
000162          JEP1=JE+1
                C     NSTART IS THE COLUMN NUMBER OF THE LEFT-HAND (WESTERN) BOUNDARY
                C     NODE IN EACH ROW, NEND IS THE COLUMN NUMBER OF THE RIGHT-HAND
                C     BOUNDARY NODE IN EACH ROW
000163          READ 2,(NSTART(I), I=1,IEP1), (NEND(I), I=1,IEP1)
                C
                C     READ DATA FOR THE DIGITAL BASIN MODEL, INPUT IS A ROW AT A TIME,
                C     FROM LEFT BOUNDARY TO RIGHT BOUNDARY
                C     HJ IS INITIAL WATER LEVEL (BEGINNING OF SIMULATION PERIOD), IN FEET
                C     ABOVE MEAN SEA LEVEL
                C     SET HJ ARRAY TO 1.0 - AT NODES OUTSIDE BOUNDARY, WATER LEVELS ARE
                C     CALCULATED EQUAL TO 1.0 AUTOMATICALLY.  IF HJ ARRAY NOT EQUAL TO 1.0,
                C     MODEL WILL INDICATE CHANGE OUTSIDE BOUNDARY.
000203          CALL CYCLER(HJ,NSTART,NEND,51,30,FMT4,1.0,1.0)
                C     Q IS DISCHARGE (PUMPAGE), A POSITIVE QUANTITY, R IS RECHARGE, NEGATIVE
                C     FOR BOUNDARY AND STREAM-CHANNEL RECHARGE AND SUBSURFACE INFLOW AND
                C     POSITIVE FOR SUBSURFACE OUTFLOW, BOTH Q AND R ARE IN ACRE-FEET PER
                C     LENGTH OF SIMULATION PERIOD
                C     CONVERT RECHARGE AND DISCHARGE FROM ACRE-FEET PER TOTAL TIME PERIOD
                C     TO CUBIC FEET PER DAY (QRF IS THE CONVERSION FACTOR)
000213          QRF=43560./TIME
000215          CALL CYCLER(Q,NSTART,NEND,51,30,FMT8,QRF,0.0)
000225          CALL CYCLER(R,NSTART,NEND,51,30,FMT8,QRF,0.0)
                C     T IS TRANSMISSIVITY, IN THOUSANDS OF GALLONS PER DAY PER FOOT
                C     CONVERT T TO SQUARE FEET PER DAY (TF IS THE CONVERSION FACTOR)
000235          TF=1000./7.48
000237          CALL CYCLER(T,NSTART,NEND,51,30,FMT4,TF,0.0)
                C     S IS THE COEFFICIENT OF STORAGE (DIMENSIONLESS)
                C     SET S ARRAY EQUAL TO 1.0 - S MUST BE NONZERO SO GAM IS NONZERO, IF NOT,
                C     DIVISION BY ZERO WILL RESULT.
000245          CALL CYCLER(S,NSTART,NEND,51,30,FMT7,1.0,1.0)
                C
                C     READ COST COEFFICIENTS FOR NODAL ERRORS
000256          CALL CYCLER(C,NSTART,NEND,51,30,FMT8,1.0,0.0)
                C
                C     THIS SEQUENCE OF INSTRUCTIONS COMPUTES ALL INTERNODAL TRANSMISSIVITIES
000266          DO 150 I=IS,IE
000270          JSTART=NSTART(I)
000272          JEND=NEND(I)
000273          DO 150 J=JSTART,JEND
                C     BASET IS TRANSMISSIVITY OF NODE IJ (NODE BEING COMPUTED)
000275          BASET=T(I,J)
000301          K=1
000302          IF(T(I,J-1).EQ.0.)151,152
000307    151   TIN(I,J,K)=0.
```

```
000315          K=K+1
000316          GO TO 153
000316    152   TIN(I,J,K)=(BASET+T(I,J-1))/2.
000327          K=K+1
000330    153   IF(T(I,J+1).EQ.0.)154,155
000337    154   TIN(I,J,K)=0.
000345          K=K+1
000346          GO TO 156
000346    155   TIN(I,J,K)=(BASET+T(I,J+1))/2.
000357          K=K+1
000360    156   IF(T(I-1,J).EQ.0.)157,158
000367    157   TIN(I,J,K)=0.
000375          K=K+1
000376          GO TO 159
000376    158   TIN(I,J,K)=(BASET+T(I-1,J))/2.
000407          K=K+1
000410    159   IF(T(I+1,J).EQ.0.)160,161
000417    160   TIN(I,J,K)=0.
000425          GO TO 150
000425    161   TIN(I,J,K)=(BASET+T(I+1,J))/2.
000436    150   CONTINUE
          C
          C     A IS NODAL AREA IN SQUARE FEET
000443          A=ACRES*43560.
          C     READ VALUES FROM NORMAL PROBABILITY TABLE (K ALPHA AND ALPHA, FROM
          C     HILLIER AND LIEBERMAN, INTRODUCTION TO OPERATIONS RESEARCH, P. 623)
000445          READ 105,(TKALPH(I), I=1,65)
000457    105   FORMAT(20F4.2)
000457          READ 108,(TALPH(I), I=1,65)
000471    108   FORMAT(5F12.10)
          C
          C     READ ROW LOCATION, COLUMN LOCATION, VARIABLE TYPE, RECHARGE TYPE,
          C     PRIOR STANDARD DEVIATION OF VARIABLE, STANDARD DEVIATION OF LIKELIHOOD
          C     FUNCTION FOR MODEL VALUE (VALUE ASSUMED TRUE IN THE MODEL), AND FLAG
          C     FOR WHETHER MODEL VALUE IS RELATIVELY CERTAIN OR UNCERTAIN (S AND T
          C     ONLY), FOR EACH VARIABLE TO BE TESTED (VALUES ARE READ IN SAME UNITS
          C     AS MODEL DATA)
000471          READ 102,(IROW(I), JCOL(I), NVARTYP(I), NRTYP(I), SIGP(I), SIGS(I)
                2, NFS(I), I=1,NUMTEST)
000520    102   FORMAT(4I5,10X,2F10.3,I10)
          C     CONVERT STANDARD DEVIATIONS TO PROPER UNITS
000520          DO 109 I=1,NUMTEST
000522          NVT=NVARTYP(I)
000524          GO TO(109,113,109,114,114)NVT
000535    113   SIGP(I)=SIGP(I)*1000./7.48
000540          SIGS(I)=SIGS(I)*1000./7.48
000541          GO TO 109
000542    114   SIGP(I)=SIGP(I)*43560./TIME
000545          SIGS(I)=SIGS(I)*43560./TIME
000547    109   CONTINUE
          C     READ NUMBER OF INTERVALS FOR S VALUES FOR WHICH SIGMAS (STANDARD
          C     DEVIATIONS) ARE ASSUMED CONSTANT AND THE INTERVAL LIMITS
000552          READ 101, NVSINT
000557          READ 106,(SINT(I), I=1,NVSINT)
000572          NVSP1=NVSINT+1
          C     READ THE SIGMAS CORRESPONDING TO EACH INTERVAL (BOTH FOR RELATIVELY
          C     CERTAIN AND UNCERTAIN S VALUES)
000574          READ 106,(SIGSTC(I), SIGSTU(I), I=1,NVSP1)
```

```
000611   106   FORMAT(10F8.6)
         C     READ THE FACTOR WHICH IS MULTIPLIED BY PRIOR SIGMA TO OBTAIN
         C     ALTERNATE VARIABLE VALUES
000611         READ 106, FACT
000617         PRINT SPER
000623         PRINT 64
000627   64    FORMAT(////,25X,*VARIABLE DESIGNATIONS*,//,30X,*1 = STORAGE COEFFI
              2CIENT*,/,30X,*2 = TRANSMISSIVITY*,/,30X,*3 = INITIAL WATER LEVEL*,
              3/,30X,*4 = PUMPAGE*,/,30X,*5 = RECHARGE*,///,25X,*RECHARGE DESIGNA
              4TIONS*,//,30X,*1 = STREAM-CHANNEL*,/,30X,*2 = BOUNDARY*,/,30X,*3 =
              5 SUBSURFACE INFLOW*,/,30X,*4 = SUBSURFACE OUTFLOW (ACTUALLY A FORM
              6 OF DISCHARGE)*,/,34X,*(0 INDICATES NON-RECHARGE VARIABLE)*)
000627         GO TO(65,67)LTSF
000635   65    PRINT 66
000641   66    FORMAT(////,20X,*COMPUTATIONS OF EXPECTED OPPORTUNITY LOSSES, EXPE
              2CTED WORTHS OF SAMPLE DATA,*,/,22X,*AND PERCENT IMPROVEMENT ARE DO
              3NE AT THE END OF THE SIMULATION PERIOD ONLY*)
000641         GO TO 70
000642   67    PRINT 68
000646   68    FORMAT(////,20X,*COMPUTATIONS OF EXPECTED OPPORTUNITY LOSSES, EXPE
              2CTED WORTHS OF SAMPLE DATA,*,/,22X,*AND PERCENT IMPROVEMENT ARE DO
              3NE AT THE END OF EACH TIME STEP AND SUMMED*,/,24X,*FOR EXAMPLE, FO
              4R THREE TIME STEPS THE EXPECTED MAXIMUM NODAL ERROR*,/,22X,*DOES N
              5OT REPRESENT A SINGLE ERROR, BUT THE SUM OF THREE EXPECTED ERRORS,
              6*,/,47X,*ONE FOR EACH TIME STEP*)
         C
         C
         C     START TESTING VARIABLES - THIS SEQUENCE CYCLES ONCE FOR EACH VARIABLE
         C     TESTED.  NVT, NRT, IR, JC, SP, SS, AND NFLAG TEMPORARILY HOLD ARRAY
         C     VALUES DURING A TEST.  LFLAG=0 INDICATES NORMAL PROBABILITY
         C     DISTRIBUTIONS UNLESS OTHERWISE INDICATED BY LFG
000646   70    DO 10 LMAIN=1,NUMTEST
000650         LPASS=LMAIN
000651         NVT=NVARTYP(LMAIN)
000653         NRT=NRTYP(LMAIN)
000654         IR=IROW(LMAIN)
000656         JC=JCOL(LMAIN)
000657         SP=SIGP(LMAIN)
000661         SS=SIGS(LMAIN)
000662         NFLAG=NFG(LMAIN)
000664         LFLAG=0
         C
         C     THIS SEQUENCE PUTS THE INDICATED MODEL VALUE OF THE VARIABLE TO BE
         C     TESTED IN VAL(1), INDICATES THE VARIABLE TYPE FOR PURPOSES OF
         C     COMPUTING ALTERNATE VALUES, AND CALLS THE SUBROUTINE TO COMPUTE THEM
000665         GO TO(51,52,53,54,55)NVT
000675   51    NUMVAL=NUMVALS
000700         VAL(1)=S(IR,JC)
000704         MVARTYP=1
000705         CALL AVAL(MVARTYP,NUMVAL,VAL,FACT,LFLAG,SP,TIME)
000713         GO TO 60
000714   52    NUMVAL=NUMVALT
000716         LFLAG=LFG
000717         VAL(1)=T(IR,JC)
000723         MVARTYP=2
000724         CALL AVAL(MVARTYP,NUMVAL,VAL,FACT,LFLAG,SP,TIME)
000733         GO TO 60
000734   53    NUMVAL=NUMVAHJ
```

```
000736          VAL(1)=HJ(IR,JC)
000742          MVARTYP=3
000743          CALL AVAL(MVARTYP,NUMVAL,VAL,FACT,LFLAG,SP,TIME)
000751          GO TO 60
000752    54    NUMVAL=NUMVALQ
000754          VAL(1)=Q(IR,JC)
000760          MVARTYP=4
000761          CALL AVAL(MVARTYP,NUMVAL,VAL,FACT,LFLAG,SP,TIME)
000767          GO TO 60
000770    55    NUMVAL=NUMVALR
000772          VAL(1)=R(IR,JC)
000776          MVARTYP=5
          C     RECHARGE TYPE 4 (SUBSURFACE OUTFLOW) IS TREATED AS A FORM OF DISCHARGE
000777          IF(NRT.EQ.4) MVARTYP=4
001002          CALL AVAL(MVARTYP,NUMVAL,VAL,FACT,LFLAG,SP,TIME)
001011    60    CONTINUE
          C
          C     THIS SEQUENCE PRINTS THE UNSCALED MODEL VALUE AND ALTERNATE VALUES
001011          PRINT 63, NVT, IR, JC
001023    63    FORMAT(*1*,20X,*THE VARIABLE BEING TESTED IS TYPE*,I3,/,25X,*AND I
                2S AT LOCATION (*,I3,*,*,I3,*)*)
001023          GO TO(98,92,98,94,94)NVT
001034    92    DO 91 I=1,NUMVAL
001036    91    PVAL(I)=VAL(I)*7.48
001042          GO TO 99
001043    94    DO 93 I=1,NUMVAL
001045    93    PVAL(I)=VAL(I)*TIME/43560.
          C     IF ALL VARIABLE VALUES FOLLOWING THE INITIAL (MODEL) VALUE ARE NOT
          C     IN ASCENDING ORDER, THE COMPUTATIONS OF WORTH OF DATA ARE NOT VALID
001053    99    PRINT 61
001057    61    FORMAT(///,15X,*VARIABLE VALUES FOR THE FREQUENCY DISTRIBUTION*)
001057          PRINT 62,(PVAL(I), I=1,NUMVAL)
001072          GO TO 97
001073    98    PRINT 61
001077          PRINT 62,(VAL(I), I=1,NUMVAL)
001112    62    FORMAT( 23X,F12.3)
001112    97    CONTINUE
          C
          C     CALL THE SEQUENCE OF SUBROUTINES THAT COMPUTES WORTH OF DATA
001112          CALL BAYESGW(TIME)
001114    10    CONTINUE
          C
          C
          C     THIS SEQUENCE SORTS ALL RESULTS IN TABLES IN DESCENDING ORDER, AND
          C     PRINTS THE SORTED TABLES
001117          DO 200 II=1,NUMTAB
001120          DO 100 K=1,2
          C     INT RECORDS THE POSITION OF THE LAST INTERCHANGE (ALL NUMBERS
          C     BEYOND X(INT) ARE SORTED), LIM IS THE LIMIT TO WHICH THE SORTING
          C     SHOULD EXTEND IN ANY GIVEN SORTING PASS
001121          LIM=NUMTEST-1
001123          IF(K.EQ.1)301,302
001126    301   DO 303 I=1,NUMTEST
001130          NX(I)=I
001131    303   X(I)=EWSD(II,I)
001137          GO TO 305
001137    302   DO 304 I=1,NUMTEST
001141          NX(I)=I
```

```
001142   304   X(I)=PCIMP(II,I)
001150   305   INT=1
001151         IF(NUMTEST.EQ.1)GO TO 320
001153         DO 310 I=1,LIM
001155         IF(X(I+1).LE.X(I)) GO TO 310
001160         TEMP1=X(I+1)
001161         TEMP2=NX(I+1)
001163         X(I+1)=X(I)
001164         NX(I+1)=NX(I)
001166         X(I)=TEMP1
001167         NX(I)=TEMP2
001171         INT=I
001172   310   CONTINUE
001175         IF(INT.EQ.1) GO TO 320
001177         LIM=INT-1
001177         GO TO 305
001200   320   GO TO(321,322)K
001206   321   PRINT 323
001212   323   FORMAT(*1*,20X,*TABLE OF SORTED VALUES OF EXPECTED WORTH OF SAMPLE
              2 DATA (DESCENDING ORDER)*)
001212         GO TO 325
001213   322   PRINT 324
001217   324   FORMAT(*1*,20X,*TABLE OF SORTED VALUES OF PERCENT IMPROVEMENT IN E
              2XPECTED OPPORTUNITY LOSS*/,41X,*WITH SAMPLING (DESCENDING ORDER)*)
001217   325   GO TO(331,332,333,334,335,336)II
001231   331   PRINT 341
001235   341   FORMAT(/,20X,*(ABSOLUTE VALUE OF ERROR, IN FEET (TIMES A COST COEF
              2FICIENT) PER 509 NODES)*)
001235         GO TO 350
001236   332   PRINT 342
001242   342   FORMAT(/,20X,*(SQUARE OF ERROR, IN FEET SQUARED (TIMES A COST COEF
              2FICIENT) PER 509 NODES)*)
001242         GO TO 350
001243   333   PRINT 343
001247   343   FORMAT(/,29X,*(MAXIMUM NODAL ERROR, IN FEET (COST COEFFICENT = 1 O
              2R 0))*)
001247         GO TO 350
001250   334   PRINT 344
001254   344   FORMAT(/,20X,*(NUMBER OF NODES IN ERROR BY MORE THAN  5 FEET (COST
              2 COEFFICIENT = 1 OR 0))*)
001254         GO TO 350
001255   335   PRINT 345
001261   345   FORMAT(/,20X,*(NUMBER OF NODES IN ERROR BY MORE THAN 10 FEET (COST
              2 COEFFICIENT = 1 OR 0))*)
001261         GO TO 350
001262   336   PRINT 346
001266   346   FORMAT(/,20X,*(NUMBER OF NODES IN ERROR BY MORE THAN 25 FEET (COST
              2 COEFFICIENT = 1 OR 0))*)
001266   350   GO TO(351,353)K
001274   351   PRINT 352
001300   352   FORMAT(///,10X,*VARIABLE TYPE*,5X,*RECHARGE TYPE*,5X,*ROW*,5X,*COL
              2UMN*,5X,*EXPECTED OPPORTUNITY LOSS*,5X,*EXPECTED WORTH OF SAMPLE D
              3ATA*)
001300         GO TO 355
001301   353   PRINT 354
001305   354   FORMAT(///,25X,*VARIABLE TYPE*,5X,*RECHARGE TYPE*,5X,*ROW*,5X,*COL
              2UMN*,5X,*PERCENT IMPROVEMENT*)
001305   355   GO TO(356,358)K
```

```
      C     LAT IS THE ORIGINAL STORAGE LOCATION OF EACH RESULT
001313   356   DO 360 I=1,NUMTEST
001315         LAT=NX(I)
001317   360   PRINT 357, NVARTYP(LAT), NRTYP(LAT), IROW(LAT), JCOL(LAT), EOL(II,
              2LAT), X(I)
001343   357   FORMAT(/,15X,I2,16X,I2,11X,I3,6X,I3,10X,F12.4,20X,F12.4)
001343         GO TO 100
001343   358   DO 370 I=1,NUMTEST
001345         LAT=NX(I)
001347   370   PRINT 359, NVARTYP(LAT), NRTYP(LAT), IROW(LAT), JCOL(LAT), X(I)
001367   359   FORMAT(/,30X,I2,16X,I2,11X,I3,6X,I3,10X,F9.4)
001367   100   CONTINUE
001371   200   CONTINUE
001374   400   CONTINUE
001374         STOP
001376         END
```

```
                SUBROUTINE CYCLER(VAR,NS,NE,II,JJ,FMT,FACT,FILL)
          C     SUBROUTINE TO READ IN ALL VARIABLE VALUES (VAR) OF A DIGITAL MODEL
          C
000013            DIMENSION VAR(II,JJ), NS(II), NE(II), FMT(8)
          C     FMT IS THE READ-IN FORMAT, FACT IS THE SCALING FACTOR FOR EACH VARIABLE,
          C     AND FILL IS THE VALUE TO BE STORED IN THE PARTS OF THE ARRAY
          C     OUTSIDE THE MODEL BOUNDARIES
          C
000013            DO 10 I=1,II
000014            DO 10 J=1,JJ
000015      10    VAR(I,J)=FILL
000027            IIM1=II-1
000031            DO 1 I=2,IIM1
000032            JSTART=NS(I)
000034            JEND=NE(I)
000036            READ FMT,(VAR(I,J), J=JSTART,JEND)
000060            DO 2 J=JSTART,JEND
000066      2     VAR(I,J)=VAR(I,J)*FACT
000076      1     CONTINUE
000101            RETURN
000101            END
```

```
      SUBROUTINE AVAL(MVT,NUMVAL,VAL,FACT,LFLAG,SP,TIME)
C     SUBROUTINE TO COMPUTE THE ALTERNATE (ERRONEOUS) VALUES OF THE VARIABLE
C     TO BE TESTED AND STORE THEM IN ASCENDING ORDER (AFTER THE MODEL VALUE
C     IS STORED IN VAL(1)).  THE VALUES ARE COMPUTED BY ADDING (AND
C     SUBTRACTING) 1, 2, 3, ETC TIMES A FACTOR TIMES THE PRIOR STANDARD
C     DEVIATION TO (FROM) VAL(1)
C
      DIMENSION VAL(10)
C     LIM IS THE NUMBER OF ASSUMED VALUES ABOVE AND BELOW THE MEAN,
C     AND MUST BE 2 OR MORE
C
      LIM=(NUMVAL-1)/2
      LP1=LIM+1
      MVP1=LIM+2
      L=2
C
C     THIS SEQUENCE COMPUTES ALTERNATE VALUES (EXCEPT FOR T, LOG-NORMAL)
      IF(MVT.EQ.2.AND.LFLAG.EQ.1) GO TO 2
    1 DO 10 I=1,LIM
      VAL(L)=VAL(1)-FLOAT(LIM+1-I)*FACT*SP
      L=L+1
   10 CONTINUE
      DO 16 I=1,LIM
      VAL(L)=VAL(1)+FLOAT(I)*FACT*SP
      L=L+1
   16 CONTINUE
C
C     THIS SEQUENCE CHECKS TO SEE IF ANY ALTERNATE VALUES OF THE VARIABLE
C     ARE LESS THAN 0 (OR GREATER THAN 0 FOR RECHARGE WHICH HAS BEEN
C     DESIGNATED A NEGATIVE QUANTITY). THOSE LESS THAN 0 ARE RECOMPUTED
C     SPACED EQUALLY BETWEEN 0.01 AND 0.0 FOR S, BETWEEN THE LOWEST NON-0
C     VALUE AND 0.0 FOR T (NORMAL DISTRIBUTION), LEFT AS NEGATIVE VALUES
C     FOR HJ, AND SET EQUAL TO +10 OR -10 FOR DISCHARGE OR RECHARGE
      GO TO(11,12,30,14,15)MVT
C     SI AND TI ARE SPACING INTERVALS FOR S AND T
   11 IX=0
      DO 13 I=2,LP1
      IF(VAL(I).LT.0.0)17,13
   17 IX=IX+1
   13 CONTINUE
      IF(IX.EQ.0)30,18
   18 L=2
      SI=0.01/FLOAT(IX)
      DO 19 I=1,IX
      VAL(L)=0.01-FLOAT(IX-I)*SI
      L=L+1
   19 CONTINUE
      GO TO 30
   12 IX=0
      DO 22 I=2,LP1
      IF(VAL(I).LT.0.0)23,22
   23 IX=IX+1
   22 CONTINUE
      IF(IX.EQ.0)30,24
   24 L=2
      TI=VAL(2+IX)/FLOAT(IX+1)
      DO 26 I=1,IX
```

```
000159          VAL(L)=VAL(2+IX)-FLOAT(IX-I+1)*TI
000164          L=L+1
000166    26    CONTINUE
000171          GO TO 30
000171    14    DO 40 I=2,LP1
000173          IF(VAL(I).LE.0.0) VAL(I)=10.0**43560./TIME
000201    40    CONTINUE
000204          GO TO 30
000204    15    DO 41 I=MVP1,NUMVAL
000206          IF(VAL(I).GE.0.0) VAL(I)=-10.0**43560./TIME
000214    41    CONTINUE
000217    30    RETURN
              C
              C     THIS SEQUENCE COMPUTES ALTERNATE VALUES FOR TRANSMISSIVITY (LOG-
              C     NORMALLY DISTRIBUTED)
000220     2    V1L=ALOG10(VAL(1))
000227          SPL=ALOG10(SP+VAL(1))-V1L
000241          DO 20 I=1,LIM
000242          VLF=FLOAT(LIM+1-I)*FACT*SPL
000246          VAL(L)=10.0**(V1L-VLF)
000255          L=L+1
000256    20    CONTINUE
000261          DO 25 I=1,LIM
000263          VLF=FLOAT(I)*FACT*SPL
000265          VAL(L)=10.0**(V1L+VLF)
000275          L=L+1
000276    25    CONTINUE
000301          RETURN
000302          END
```

```
                SUBROUTINE BAYESGW(TIME)
              C SUBROUTINE TO COMPUTE THE EXPECTED WORTH OF GROUND-WATER DATA,
              C USING BAYESIAN STATISTICAL DECISION THEORY.
              C
000003          DIMENSION ELOSS(6,10), ELOSSM(6), EEOL(6), VMXE(10),
                2 NGT5(10), NGT10(10), NGT25(10), ELOSS2M(10,6), PRP(10),
                3 PREI(10), PRPE(10,10), PSTPE(10,10), PRPV(10), ELOS2(6,10,10)
000003          COMMON/WBA/IR, JC, NVT, VAL(10), NUMVAL, IS, IE, NSTART(51),
                2 NEND(51), LPASS, LTSF
000003          COMMON /WODBGW/ EWSD(6,100), PCIMP(6,100), EOL(6,100), SP, SS,
                2 NFLAG, LFLAG, NRT, C(51,30), NUMTAB
000003          COMMON/BGWADS/ H(51,30,7)
              C EWSD IS EXPECTED WORTH OF SAMPLE DATA, PCIMP IS PERCENT IMPROVEMENT,
              C ELOSS IS EXPECTED LOSS (BEFORE SAMPLING), ELOSSM IS MINIMUM ELOSS
              C (EXPECTED OPPORTUNITY LOSS OR EOL), ELOS2 IS EXPECTED LOSS GIVEN
              C SAMPLING HAS OCCURRED, ELOSS2M IS MINIMUM ELOS2, EEOL IS EXPECTED VALUE
              C OF EXPECTED OPPORTUNITY LOSS, VMXE IS THE VALUE OF THE MAXIMUM NODAL
              C ERROR, NGT5, 10, 25 ARE THE NUMBERS OF NODES WITH ERRORS MORE THAN
              C 5 FEET, 10 FEET, AND 25 FEET, PRP IS PRIOR PROBABILITY, PRPE IS
              C LIKELIHOOD, PREI IS PROBABILITY OF OBSERVING A GIVEN SAMPLE, PSTPE IS
              C POSTERIOR PROBABILITY, PRPV IS AN ARRAY FOR TEMPORARY STORAGE OF
              C LIKELIHOODS COMPUTED FOR A GIVEN ASSUMED MEAN, VARMUP IS UNSCALED
              C MODEL VALUE (MEAN OF PRIOR DISTRIBUTION), SPU IS UNSCALED SP,
              C SIG IS THE STANDARD DEVIATION FOR THE GIVEN TEST AND VARIABLE VALUE,
              C SIGUSC IS UNSCALED SIG, LCALL INDICATES A CALL FOR PRIOR PROBABILITIES
              C (1) OR LIKELIHOODS (2), SIGFACT IS THE FACTOR BY WHICH ALTERNATE VALUES
              C OF Q AND R ARE MULTIPLIED TO GET THEIR STANDARD DEVIATIONS (FOR
              C LIKELIHOOD FUNCTIONS) (SIGMA IS ASSUMED PROPORTIONAL TO THE MAGNITUDE
              C OF Q AND R), VMU IS ASSUMED MEAN
              C
              C THIS SEQUENCE SETS VAL(1) EQUAL TO THE MEAN AND, USING THE SPECIFIED
              C PRIOR STANDARD DEVIATION, CALLS THE SUBROUTINE THAT COMPUTES THE PRIOR
              C PROBABILITY DISTRIBUTION, AND PRINTS THE RESULTS
000003          VARMUP=VAL(1)
000005          IF(NVT.EQ.2.AND.LFLAG.EQ.1)4,5
000014    4     VARMUPU=VARMUP*7.48
000016          SPU=SP*7.48
000020          SIG=ALOG10(VARMUPU+SPU) - ALOG10(VARMUPU)
000030          PRINT 67, SIG
000036    67    FORMAT(/,32X,*SIGMA (LOG) = *,F6.3)
000036          GO TO 70
000040    5     SIG=SP
000042          GO TO(71,72,71,73,73)NVT
000053    71    PRINT 69, SIG
000061    69    FORMAT(/,33X,*SIGMA = *,F10.3)
000061          GO TO 70
000063    72    SIGUSC=SIG*7.48
000065          GO TO 75
000066    73    SIGUSC=SIG*TIME/43560.
000071    75    PRINT 69, SIGUSC
000077    70    LCALL=1
000100          CALL PROBON(NUMVAL,PRP,10,VAL,10,VARMUP,SIG,LFLAG,NRT,LCALL,NVT)
000113          PRINT 61
000117    61    FORMAT  (/,15X,*PRIOR PROBABILITIES OF THE STATES REPRESENTED BY T
                2HE VALUES (VAL(I))*,/,21X,*OF A GIVEN VARIABLE AT A GIVEN NODE (IR
                3,JC)*,/)
000117          PRINT 62,(I,PRP(I), I=1,NUMVAL)
```

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000134    62    FORMAT  (22X,*VALUE(*,I2,*)*,F10.4)
          C
          C     THIS SEQUENCE SETS EACH VARIABLE VALUE EQUAL TO THE MEAN IN TURN AND,
          C     PASSING THE SPECIFIED STANDARD DEVIATION OF THE LIKELIHOOD FUNCTION
          C     (FOR MEAN = MODEL VALUE), CALLS THE SUBROUTINE THAT COMPUTES SIGMA,
          C     CALLS THE SUBROUTINE THAT COMPUTES THE LIKELIHOOD FUNCTION, AND
          C     PRINTS THE RESULTS
000134          PRINT 68
000140    68    FORMAT (//,20X,*STANDARD DEVIATIONS AND NON-NORMALIZED TOTAL PROBA
               2BILITIES*,/)
000140          SIGFACT=SS/VAL(1)
000142          LCALL=2
000143          DO 50 I=1,NUMVAL
000146          IX=I
000147          VMU=VAL(I)
000151          CALL SIGMA(SS,NVT,VMU,SIG,SIGFACT,NFLAG,IX,TIME,LFLAG)
000162          CALL PROBDN(NUMVAL,PRPV,10,VAL,10,VMU,SIG,LFLAG,NRT,LCALL,NVT)
000175          DO 49 J=1,NUMVAL
000200    49    PRPE(J,I)=PRPV(J)
000211    50    CONTINUE
000213          PRINT 41
000216    41    FORMAT  (/,15X,*PRIOR PROBABILITIES OF EVENTS E(I) GIVEN STATES VA
               2L(J)  (LIKELIHOODS)*)
000216          PRINT 42,(I,I=1,NUMVAL)
000230    42    FORMAT(/,9X,*VAL(J)*,10I10)
000230          DO 46 I=1,NUMVAL
000233          PRINT 43, I,(PRPE(I,J), J=1,NUMVAL)
000251    46    CONTINUE
000255    43    FORMAT(10X,*E(*,I2,*)*,10F10.4)
          C
          C     THIS SEQUENCE COMPUTES THE PROBABILITY OF OBSERVING A GIVEN VARIABLE
          C     VALUE IN A SAMPLE, AND COMPUTES THE PROBABILITY THAT A GIVEN VALUE IS
          C     TRUE GIVEN A SPECIFIED SAMPLE HAS BEEN OBSERVED (USING BAYES RULE AND
          C     THE PREVIOUSLY COMPUTED PRIOR PROBABILITIES, LIKELIHOODS, AND
          C     PROBABILITIES OF OBSERVING GIVEN SAMPLES)  (OR COMPUTES PREI AND PSTPE)
000255          DO 10 I=1,NUMVAL
000256          PREI(I)=0.0
000260          DO 10 J=1,NUMVAL
000261          PREI(I)=PREI(I) + PRP(J)*PRPE(I,J)
000271    10    CONTINUE
000275          DO 11 J=1,NUMVAL
000277          IF(PREI(J).EQ.0.0)15,16
000304    15    DO 17 II=1,NUMVAL
000306          PSTPE(II,J)=0.0
000312    17    CONTINUE
000315          GO TO 11
000315    16    DO 18 I=1,NUMVAL
000317          PSTPE(I,J)=(PRP(I)*PRPE(J,I))/PREI(J)
000333    18    CONTINUE
000336    11    CONTINUE
000341          PRINT 54
000344    54    FORMAT(///,15X,*PROBABILITIES OF EVENTS E(I)*,/)
000344          PRINT 55,(I,PREI(I), I=1,NUMVAL)
000361    55    FORMAT  (10X,*E(*,I2,*)*,F7.5)
000361          PRINT 44
000365    44    FORMAT(///,15X,*POSTERIOR PROBABILITIES OF STATES VAL(I) GIVEN EVE
               2NTS E(J)*)
000365          PRINT 48,(I,I=1,NUMVAL)
```

```
000377    48     FORMAT(/,11X,*E(J)=*,10I10)
000377           DO 47 I=1,NUMVAL
000402           PRINT 45, I,(PSIPE(I,J), J=1,NUMVAL)
000420    47     CONTINUE
000424    45     FORMAT(8X,*VAL(*,I2,*)=*,10F10.4)
               C
               C
               C     THIS SEQUENCE COMPUTES EXPECTED LOSSES (EXPECTED ERRORS) USING SIX
               C     SEPARATE ERROR CRITERIA
000424           DO 7 N=1,NUMVAL
000425           DO 7 NN=1,NUMTAB
000426           ELOSS(NN,N)=0.0
000432           DO 7 K=1,NUMVAL
000433    7      ELOS2(NN,N,K)=0.0
               C     KCALL1 IS THE FLAG INDICATING WHETHER WATER LEVELS FOR ALL TIME-STEPS
               C     HAVE BEEN COMPUTED BY ALDIRS, KCALL2 COUNTS THE NUMBER OF CALLS TO
               C     ALDIRS, ADIFF IS ABSOLUTE DIFFERENCE, SQDIFF IS SQUARE OF DIFFERENCE
               C     BETWEEN WATER LEVELS COMPUTED USING TWO DIFFERENT VALUES OF THE VARIABLE,
               C     ECDIF, ECSDIF REPRESENT ECONOMIC DIFFERENCES (ADIFF, SQDIFF TIMES
               C     A COST COEFFICIENT)
000450           KCALL1=1
000451           KCALL2=0
               C     CALL TO THE SUBROUTINE THAT COMPUTES PREDICTED WATER LEVELS OVER THE
               C     MODEL FOR EACH GIVEN VARIABLE VALUE
000452    132    CALL ALDIRS(KCALL1,KCALL2)
000454           DO 2 N=1,NUMVAL
000457           DO 2 M=1,NUMVAL
000460           GO TO(302,302,303,304,305,306)NUMTAB
000472    306    NGT25(M)=0
000474    305    NGT10(M)=0
000476    304    NGT5(M)=0
000500    303    VMXE(M)=0.
000502    302    DO 300 I=IS,IE
000504           JSTART=NSTART(I)
000506           JEND=NEND(I)
000510           DO 300 J=JSTART,JEND
000512           ADIFF=ABS(H(I,J,N) - H(I,J,M))
000526           GO TO(311,312,313,314,315,316)NUMTAB
000540    316    IF(ADIFF.GT.25.0.AND.C(I,J).NE.0.0) NGT25(M)=NGT25(M)+1
000555    315    IF(ADIFF.GT.10.0.AND.C(I,J).NE.0.0) NGT10(M)=NGT10(M)+1
000574    314    IF(ADIFF.GT.5.0.AND.C(I,J).NE.0.0) NGT5(M)=NGT5(M)+1
000612    313    IF(ADIFF.GT.VMXE(M).AND.C(I,J).NE.0.0) VMXE(M)=ADIFF
000630    312    SQDIFF=ADIFF**2
000632           ECSDIF=SQDIFF*C(I,J)
000636           ELOSS(2,N)=ELOSS(2,N) + ECSDIF*PRP(M)
000645    311    ECDIF=ADIFF*C(I,J)
000652           ELOSS(1,N)=ELOSS(1,N) + ECDIF*PRP(M)
000661    300    CONTINUE
000666           GO TO(2,2,323,324,325,326)NUMTAB
000700    326    ELOSS(6,N)=ELOSS(5,N) + FLOAT(NGT25(M))*PRP(M)
000711    325    ELOSS(5,N)=ELOSS(5,N) + FLOAT(NGT10(M))*PRP(M)
000722    324    ELOSS(4,N)=ELOSS(4,N) + FLOAT(NGT5(M))*PRP(M)
000733    323    ELOSS(3,N)=ELOSS(3,N) + VMXE(M)*PRP(M)
000742    2      CONTINUE
000747           IF(KCALL1.EQ.1)112,111
               C
               C     THIS SEQUENCE SEARCHES FOR THE MINIMUM EXPECTED LOSS (EOL) FOR
               C     EACH ERROR CRITERIA
```

```
000753    111    NVM1=NUMVAL-1
000755           DO 8 II=1,NUMTAB
000757    8      ELOSSM(II)=ELOSS(II,1)
000765           DO 3 II=1,NUMTAB
000767           DO 3 I=1,NVM1
000770           IF(ELOSSM(II).GT.ELOSS(II,I+1))52,3
001000    52     ELOSSM(II)=ELOSS(II,I+1)
001005    3      CONTINUE
001012           DO 13 II=1,NUMTAB
001014    13     EOL(II,LPASS)=ELOSSM(II)
          C
          C      THIS SEQUENCE COMPUTES EXPECTED LOSSES FOR EACH GIVEN SAMPLE RESULT,
          C      USING THE SIX ERROR CRITERIA, AND SEARCHES FOR THE MINIMUM EXPECTED
          C      LOSS FOR EACH GIVEN SAMPLE AND ERROR CRITERIA
001025    112    DO 20 K=1,NUMVAL
001027           DO 22 N=1,NUMVAL
001030           DO 22 M=1,NUMVAL
001031           GO TO(352,352,353,354,355,356)NUMTAB
001043    356    NGT25(M)=0
001045    355    NGT10(M)=0
001047    354    NGT5(M)=0
001051    353    VMXE(M)=0.
001053    352    DO 350 I=IS,IE
001055           JSTART=NSTART(I)
001057           JEND=NEND(I)
001061           DO 350 J=JSTART,JEND
001063           ADIFF=ABS(H(I,J,N) - H(I,J,M))
001077           GO TO(361,362,363,364,365,366)NUMTAB
001111    366    IF(ADIFF.GT.25.0.AND.C(I,J).NE.0.0) NGT25(M)=NGT25(M)+1
001127    365    IF(ADIFF.GT.10.0.AND.C(I,J).NE.0.0) NGT10(M)=NGT10(M)+1
001145    364    IF(ADIFF.GT.5.0.AND.C(I,J).NE.0.0) NGT5(M)=NGT5(M)+1
001163    363    IF(ADIFF.GT.VMXE(M).AND.C(I,J).NE.0.0) VMXE(M)=ADIFF
001201    362    SQDIFF=ADIFF**2
001203           ECSDIF=SQDIFF*C(I,J)
001207           ELOS2(2,N,K)=ELOS2(2,N,K)+ECSDIF*PSTPE(M,K)
001221    361    ECDIF=ADIFF*C(I,J)
001226           ELOS2(1,N,K)=ELOS2(1,N,K)+ECDIF*PSTPE(M,K)
001240    350    CONTINUE
001245           GO TO(22,22,373,374,375,376)NUMTAB
001257    376    ELOS2(6,N,K)=ELOS2(6,N,K)+FLOAT(NGT25(M))*PSTPE(M,K)
001274    375    ELOS2(5,N,K)=ELOS2(5,N,K)+FLOAT(NGT10(M))*PSTPE(M,K)
001311    374    ELOS2(4,N,K)=ELOS2(4,N,K)+FLOAT(NGT5(M))*PSTPE(M,K)
001326    373    ELOS2(3,N,K)=ELOS2(3,N,K)+VMXE(M)*PSTPE(M,K)
001341    22     CONTINUE
001346           IF(KCALL1.EQ.1)20,121
001352    121    DO 340 II=1,NUMTAB
001354    340    ELOSS2M(K,II)=ELOS2(II,1,K)
001367           DO 23 II=1,NUMTAB
001370           DO 23 I=1,NVM1
001371           IF(ELOSS2M(K,II).GT.ELOS2(II,I+1,K))53,23
001405    53     ELOSS2M(K,II)=ELOS2(II,I+1,K)
001416    23     CONTINUE
001423    20     CONTINUE
          C
          C      THIS SEQUENCE COMPUTES EXPECTED VALUE OF EXPECTED OPPORTUNITY LOSS,
          C      EXPECTED WORTH OF SAMPLE DATA, AND PERCENT IMPROVEMENT, AND PUNCHES
          C      THE RESULTS
001426           IF(KCALL1.EQ.1)132,131
```

```
001432   131    DO 30 II=1,NUMTAB
001434          EEOL(II)=0.0
001436          DO 30 I=1,NUMVAL
001437          EEOL(II)=EEOL(II) + ELOSS2M(I,II)*PREI(I)
001447   30     CONTINUE
001454          DO 14 II=1,NUMTAB
001456          EWSO(II,LPASS)=EOL(II,LPASS) - EEOL(II)
001465          IF(EOL(II,LPASS).EQ.0.)391,392
001472   391    PCIMP(II,LPASS)=0.
001476          GO TO 19
001477   392    PCIMP(II,LPASS)=(EWSO(II,LPASS)/EOL(II,LPASS))*100.
001510   19     PUNCH 395, IR, JC, NVT, NRT, EOL(II,LPASS), EWSO(II,LPASS)
001536          PUNCH 395, IR, JC, NVT, NRT, PCIMP(II,LPASS)
001556   395    FORMAT(4I5,10X,2F20.5)
001556   14     CONTINUE
001562   100    CONTINUE
001562          RETURN
001563          END
```

```
                SUBROUTINE SIGMA(SS,NVT,VMU,SIG,SGFCT,NFLAG,IX,TIME,LFLAG)
      C     SUBROUTINE TO COMPUTE A STANDARD DEVIATION OF A LIKELIHOOD FUNCTION
      C
000014          COMMON/WODSIG/ SIGSTC(20), SIGSTU(20), SINT(10), NVSINT
000014          GO TO(1,2,3,4,5)NVT
      C
      C     THIS SEQUENCE SELECTS THE PROPER SIGMA FOR STORAGE COEFFICIENT FROM
      C     TWO ARRAYS (EITHER FOR RELATIVELY CERTAIN OR UNCERTAIN S)
000024  1       DO 16 I=1,NVSINT
000026          IZ=I
000027          IF(VMU.LE.SINT(I))17,16
000035  16      CONTINUE
000041          IF(VMU.GT.SINT(NVSINT)) IZ=NVSINT+1
000045  17      GO TO(11,12)NFLAG
000053  11      SIG=SIGSTC(IZ)
000056          GO TO 15
000056  12      SIG=SIGSTU(IZ)
000061  15      PRINT 10, SIG
000067  10      FORMAT(33X,*SIGMA = *,F10.3)
000067          RETURN
      C
      C     THIS SEQUENCE COMPUTES A STANDARD SIGMA FOR TRANSMISSIVITY (LOG-NORMAL),
      C     ASSUMING SAMPLING TECHNIQUES THAT YIELD EITHER RELATIVELY CERTAIN OR
      C     UNCERTAIN RESULTS, THEN COMPUTES THE RATIO (SF) OF THE SPECIFIED
      C     LIKELIHOOD SIGMA (FOR MODEL VALUE = MEAN) TO THE STANDARD VALUE.
      C     IN SUBSEQUENT CALLS THE SUBROUTINE ADJUSTS EACH STANDARD SIGMA
      C     CORRESPONDING TO EACH ASSUMED MEAN BY MULTIPLYING BY SF
      C     VMUU AND SSU ARE UNSCALED VALUES OF VMU AND SS
000070  2       VMUU=VMU*7.48
000075          SSU=SS*7.48
000077          IF(LFLAG.EQ.1)6,7
000104  6       VMUL=ALOG10(VMUU)
000106          IF(IX.EQ.1)27,26
000116  27      GO TO(21,22)NFLAG
      C     THIS EQUATION YIELDS AN APPROXIMATE STANDARD VALUE OF STANDARD DEVIATION
      C     (LOGARITHMIC) FOR A DISTRIBUTION OF T, ASSUMING AN AQUIFER TEST OF
      C     SEVERAL DAYS DURATION
000124  21      SIGTLC=0.338-0.232*((ALOG10(VMUU)-3.0)*0.333)
000132          IF(IX.EQ.1)25,26
000142  25      SF=SSU/(10.**(VMUL+SIGTLC)-VMUU)
000152          GO TO 29
      C     THIS EQUATION ASSUMES AN AQUIFER TEST OF SEVERAL HOURS DURATION
000152  22      SIGTLU=0.740-0.506*((ALOG10(VMUU)-3.0)*0.333)
000160          IF(IX.EQ.1)65,66
000170  65      SF=SSU/(10.**(VMUL+SIGTLU)-VMUU)
000200  29      SIG=ALOG10(SSU+VMUU)-VMUL
000212          GO TO 23
000212  28      GO TO(21,22)NFLAG
000221  26      X=(10.**(VMUL+SIGTLC)-VMUU)*SF
000230          GO TO 67
000230  66      X=(10.**(VMUL+SIGTLU)-VMUU)*SF
000240  67      SIG=ALOG10(X+VMUU) - VMUL
000252  23      PRINT 20, SIG
000260  20      FORMAT(32X,*SIGMA (LOG) = *,F6.3)
000260          RETURN
      C
      C     THIS SEQUENCE COMPUTES A STANDARD SIGMA FOR T (NORMAL DISTRIBUTION)
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```
      C     AND THE SF RATIO
000261  7      IF(IX.EQ.1)73,74
000271  73     SIG=SS
000272         GO TO(71,72)NFLAG
      C     THIS EQUATION YIELDS AN APPROXIMATE STANDARD VALUE OF STANDARD DEVIATION
      C     FOR A DISTRIBUTION OF T, ASSUMING AN AQUIFER TEST OF SEVERAL DAYS
      C     DURATION
000300  71     SIGTC=10.**(3.114+0.783*(ALOG10(VMUU)-3.0))
000312         IF(IX.EQ.1)75,76
000322  75     SF=SSU/SIGTC
000324         GO TO 77
      C     THIS EQUATION ASSUMES AN AQUIFER TEST OF SEVERAL HOURS DURATION
000325  72     SIGTU=10.**(3.792+0.667*(ALOG10(VMUU)-3.0))
000337         IF(IX.EQ.1)78,79
000347  78     SF=SSU/SIGTU
000351         GO TO 77
000352  74     GO TO(71,72)NFLAG
000360  76     SIG=SIGTC*SF/7.48
000363         GO TO 77
000363  79     SIG=SIGTU*SF/7.48
000366  77     SIGUSC=SIG*7.48
000370         PRINT 10, SIGUSC
000375         RETURN
      C
      C     THE SIGMA FOR EACH ASSUMED MEAN FOR INITIAL WATER LEVEL IS
      C     ASSUMED CONSTANT
000376  3      SIG=SS
000403         PRINT 10, SIG
000410         RETURN
      C
      C     THIS SEQUENCE ASSUMES THE SIGMA FOR EACH ASSUMED MEAN DISCHARGE (OR
      C     RECHARGE) IS PROPORTIONAL TO THE MAGNITUDE OF THE MEAN, AND THUS
      C     SIGMA IS COMPUTED BY MULTIPLYING BY THE RATIO OF THE SPECIFIED SIGMA
      C     FOR THE MODEL VALUE DIVIDED BY THE MODEL VALUE (SIGFACT RATIO)
000411  4      CONTINUE
000411  5      SIG=VMU*SGFCT
000416         SIGUSC=SIG*TIME/43560.
000420         PRINT 10, SIGUSC
000426         RETURN
000427         END
```

```
                SUBROUTINE PROBDN(NP,PROB,MA,PVAL,MB,PMU,PSIG,LFLAG,NRT,LCALL,NVT)
      C         SUBROUTINE TO COMPUTE A DISCRETE (EITHER NORMAL OR LOG-NORMAL)
      C         PROBABILITY DISTRIBUTION
      C
000015            DIMENSION PROB(MA), PVAL(MB), PVAL2(10), PROB2(10), XVAL(12)
      C         PMU IS THE MEAN, NP IS THE NUMBER OF VALUES, MVAL IS MIDDLE
      C         VALUE (MODEL VALUE), LFLAG IS THE FLAG FOR NORMAL (0) OR LOG-NORMAL
      C         (1) DISTRIBUTION, PROB IS THE PROBABILITY ARRAY, PVAL IS THE VALUE
      C         ARRAY, PROB2 STORES PROBABILITIES OF THE SORTED VARIABLE VALUES
000016            GO TO(1,2)LCALL
000024      1     NPM1=NP-1
000025            NPP1=NP+1
000026            MVAL=(NP+1)/2
000030            MVM1=MVAL-1
000031            MVP1=MVAL+1
      C
      C         THIS SEQUENCE SORTS ALL VARIABLE VALUES (INCLUDING THE MODEL VALUE)
      C         IN ASCENDING ORDER AND STORES THEM IN PVAL2
000032            L=2
000033            PVAL2(MVAL)=PVAL(1)
000036            DO 60 I=1,MVM1
000037            PVAL2(I)=PVAL(L)
000042            L=L+1
000043      60    CONTINUE
000046            DO 65 I=MVP1,NP
000050            PVAL2(I)=PVAL(L)
000053            L=L+1
000054      65    CONTINUE
000055            IF(LFLAG.EQ.1)30,10
000062      30    DO 35 I=1,NP
000064      35    PVAL2(I)=ALOG10(PVAL2(I)*7.48)
      C
      C         THIS SEQUENCE COMPUTES THE LOWER AND UPPER BOUNDARIES OF THE INTERVAL
      C         AROUND EACH VARIABLE VALUE.  IF BOUNDARIES ARE COMPUTED LESS THAN
      C         0 (OR GREATER THAN 0 FOR RECHARGE, EXCLUDING SUBSURFACE OUTFLOW) THE
      C         BOUNDARY IS SET EQUAL TO 0 (EXCEPT FOR INITIAL WATER LEVEL, WHERE
      C         NEGATIVE BOUNDARIES ARE ALLOWED)
000100      10    DO 80 I=1,NPM1
000102            XVAL(I+1)=(PVAL2(I)+PVAL2(I+1))/2.
000107      80    CONTINUE
000111            XVAL(1)=PVAL2(1)-(XVAL(2)-PVAL2(1))
000114            XVAL(NPP1)=PVAL2(NP)+(PVAL2(NP)-XVAL(NP))
000122            IF(LFLAG.EQ.0.AND.NRT.EQ.0.OR.NRT.EQ.4)25,27
000135      25    IF(NVT.EQ.3) GO TO 27
000140            DO 26 I=1,MVAL
000141            IF(XVAL(I).LT.0.0) XVAL(I)=0.0
000145      26    CONTINUE
000150      27    IF(NRT.NE.0.AND.NRT.NE.4)28,2
000157      28    DO 29 I=MVP1,NPP1
000161            IF(XVAL(I).GT.0.0) XVAL(I)=0.0
000165      29    CONTINUE
000170      2     IF(LFLAG.EQ.1)21,22
000175      21    PMU=ALOG10(PMU*7.48)
      C
      C         THIS SEQUENCE COMPUTES THE STANDARD NORMAL DEVIATES (SND) FOR THE TWO
      C         BOUNDARIES OF AN INTERVAL, CALLS THE SUBROUTINE THAT GIVES THE
      C         PROBABILITY ABOVE EACH BOUNDARY, AND THEN COMPUTES THE PROBABILITY
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```
      C     OF THE INTERVAL
000204   22    DO 20 I=1,NP
000206         NFLAG=0
000207         SND1=(XVAL(I)-PMU)/PSIG
000212         SND2=(XVAL(I+1)-PMU)/PSIG
000215         IF(SND1.GE.0.0.AND.SND2.GE.0.0.OR.SND1.LE.0.0.AND.SND2.LE.0.0)
              2 NFLAG=1
000235         SND1=ABS(SND1)
000237         SND2=ABS(SND2)
000241         CALL TBLKUP(SND1,SND2,ALPH1,ALPH2)
000243         IF(NFLAG.EQ.1)3,4
000253   3     PROB2(I)=ABS(ALPH1-ALPH2)
000260         GO TO 20
000263   4     PROB2(I)=ABS(ALPH1-(1.0-ALPH2))
000266   20    CONTINUE
      C
      C     THIS SEQUENCE NORMALIZES EACH PROBABILITY BY DIVIDING BY THE SUM OF
      C     ALL THE PROBABILITIES, THEN REFERS EACH PROBABILITY TO ITS RESPECTIVE
      C     VARIABLE VALUE
000271   50    PROBT=0.0
000272         DO 7 I=1,NP
000274   7     PROBT=PROBT+PROB2(I)
000300         PRINT 90, PROBT
000306   90    FORMAT(15X,*THE TOTAL PROBABILITY BEFORE NORMALIZING IS *,F7.4)
000306         DO 8 I=1,NP
000313   8     PROB2(I)=PROB2(I)/PROBT
000317         PROB(1)=PROB2(MVAL)
000322         L=2
000323         DO 70 I=1,MVM1
000325         PROB(L)=PROB2(I)
000330         L=L+1
000331   70    CONTINUE
000334         DO 75 I=MVP1,NP
000336         PROB(L)=PROB2(I)
000341         L=L+1
000342   75    CONTINUE
000344   40    RETURN
000345         END
```

```
                SUBROUTINE TBLKUP(XA,XB,ALPH1,ALPH2)
            C   SUBROUTINE TO APPROXIMATE THE CUMULATIVE PROBABILITY THAT A VALUE
            C   WILL BE ABOVE A SPECIFIED STANDARD NORMAL DEVIATE (K ALPHA).  THE
            C   APPROXIMATION IS MADE BY INTERPOLATING WITHIN AN ABRIDGED TABLE OF
            C   AREAS UNDER THE NORMAL PROBABILITY CURVE (FROM K ALPHA TO INFINITY),
            C   USING VALUES OF ALPHA CORRESPONDING TO EVERY 0.05 INCREMENT IN
            C   K ALPHA FROM K ALPHA=0.50 TO 3.00 AND CORRESPONDING TO EVERY 0.5
            C   INCREMENT FROM K ALPHA=3.0 TO 5.0
            C
000007          COMMON/WODTLU/ TKALPH(65), TALPH(65)
            C   XA AND XB ARE COMPUTED STANDARD NORMAL DEVIATES, TKALPH IS THE TABLE
            C   OF STANDARD NORMAL DEVIATES, ALPH1 AND ALPH2 ARE COMPUTED PROBABILITIES,
            C   TALPH IS THE TABLE OF PROBABILITIES, MXA AND MXB ARE THE LOCATIONS
            C   OF THE COMPUTED STANDARD NORMAL DEVIATES IN THE TABLE
            C
000007          IF(XA.GT.5.00)1,2
000014      1   ALPH1=TALPH(65)
000015          GO TO 3
000016      2   IF(XA.GT.3.00)42,41
000023      41  XINT=0.05
000024          MXA=(XA*20.0)+1.0
000032          GO TO 43
000030      42  XINT=0.50
000031          MXA=(XA*2.0)+55.0
000035      43  ALPH1=((XA-TKALPH(MXA))/XINT)*(TALPH(MXA+1)-TALPH(MXA))+TALPH(MXA)
000045      3   IF(XB.GT.5.00)4,5
000052      4   ALPH2=TALPH(65)
000053          GO TO 60
000054      5   IF(XB.GT.3.00)52,51
000061      51  XINT=0.05
000062          MXB=(XB*20.0)+1.0
000070          GO TO 53
000066      52  XINT=0.50
000067          MXB=(XB*2.0)+55.0
000073      53  ALPH2=((XB-TKALPH(MXB))/XINT)*(TALPH(MXB+1)-TALPH(MXB))+TALPH(MXB)
000103      60  RETURN
000104          END
```

```
      SUBROUTINE ALDIRS(KCALL1,KCALL2)
C     SUBROUTINE TO COMPUTE WATER LEVELS OVER A DIGITAL MODEL OF A GROUND-
C     WATER BASIN OVER A GIVEN SIMULATION PERIOD.  SOLUTION OF THE FLOW
C     EQUATIONS FOR POTENTIALS (WATER LEVELS) IS BY THE ALTERNATING-
C     DIRECTION IMPLICIT METHOD.  BASIC PROGRAM BY R.L. KNICKERBOCKER,
C     MODIFIED BY J.S. GATES, UNIV. OF ARIZONA, DEPT. OF HYDROLOGY AND
C     WATER RESOURCES
C     BASIC UNITS ARE FEET, CUBIC FEET, AND DAYS.
C     DERIVED FROM METHODS OF PEACEMAN AND RACHFORD (PINDER AND BREDEHOEFT,
C     WATER RES. RESEARCH, V.4, NO.5, OCT. 1968)
C
000005       DIMENSION HTEM(51), AC(51), BC(51), CC(51), DC(51), W(51),
     2 G(51), TINTEM(1,1,8), DIFF(13)
000005       DIMENSION DELHC(51,30), DIFM(51,30)
000005       EQUIVALENCE (DELHC,DIFM)
000005       COMMON/WBA/IR, JC, NVT, VAL(10), NUMVAL, IS, IE, NSTART(51),
     2 NEND(51), LPASS, LTSF
000005       COMMON/WOOADS/FHEAD(8), TIME, T(51,30), S(51,30), JS, JE,
     2 IEM1, JEM1, A, MAPS, TIN(51,30,4), Q(51,30), IEP1, JEP1,
     3 HJ(51,30), R(51,30), TSTEP1
000005       COMMON/BGWADS/ H(51,30,7)
C     H IS WATER LEVEL AT EACH NODE, USED INSIDE THE ALTERNATING-DIRECTION
C     ALGORITHM, HTEM IS WATER LEVEL AFTER EACH COLUMN SWEEP OR ROW SWEEP
C     THROUGH THE NODAL ARRAY (TEMPORARY STORAGE), DELHC IS COMPUTED CHANGE
C     IN WATER LEVEL AND IS POSITIVE FOR DECLINE, DIFF IS DIFFERENCE BETWEEN
C     WATER LEVELS COMPUTED USING THE MODEL VALUE OF THE VARIABLE AND THOSE
C     COMPUTED USING ALTERNATE VALUES (AROUND THE TESTED NODE ONLY), DIFM
C     IS THE ARRAY OF DIFFERENCES OVER THE WHOLE MODEL
C     AC IS AVERAGE T (OR INTERNODAL T) BETWEEN A GIVEN NODE AND THE NODE ABOVE
C     (COLUMN SWEEP) OR TO ITS LEFT (ROW SWEEP), AND IS COEFFICIENT OF
C     UNKNOWN WATER LEVEL ABOVE OR TO LEFT, CC IS AVERAGE T BETWEEN A GIVEN
C     NODE AND THE NODE BELOW (COLUMN SWEEP) OR TO ITS RIGHT (ROW SWEEP),
C     AND IS COEFFICIENT OF UNKNOWN WATER LEVEL BELOW OR TO RIGHT, BC IS
C     A DUMMY VARIABLE (THE COEFFICIENT OF UNKNOWN WATER LEVEL AT THE
C     GIVEN NODE AND IS SUM OF AC, CC, AND GAM), DC IS A DUMMY VARIABLE (ALL
C     THE KNOWN TERMS IN THE FINITE-DIFFERENCE EQUATION), W AND G ARE
C     DUMMY VARIABLES USED IN THE THOMAS ALGORITHM FOR SIMULTANEOUS SOLUTION
C     OF THE TRIDIAGONAL SYSTEM OF EQUATIONS
C
C
C
C     THIS SUPER-SEQUENCE COMPUTES ONE SET OF WATER LEVELS FOR EACH
C     POSSIBLE VALUE OF THE VARIABLE BEING TESTED
000035       KCALL2=KCALL2+1
000036       DO 1000 L=1,NUMVAL
000010       IF(L.EQ.1.AND.NVT.EQ.2.AND.KCALL2.EQ.1) GO TO 7
000022       IF(L.EQ.1.AND.KCALL2.EQ.1) GO TO 6
C
C     THIS SUBSEQUENCE PUTS EACH ALTERNATE VALUE OF THE VARIABLE INTO THE
C     MODEL IN TURN
000031       GO TO(1,2,3,4,5)NVT
000042     1 S(IR,JC)=VAL(L)
000050       GO TO 6
000050     2 T(IR,JC)=VAL(L)
C
C     THIS SUBSEQUENCE RECOMPUTES INTERNODAL TRANSMISSIVITIES (AROUND THE
C     TESTED NODE) CORRESPONDING TO AN ALTERNATE VALUE (IF VARIABLE IS T)
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000056          BASET=T(IR,JC)
000061          K=1
000062          IF(T(IR,JC-1).EQ.0.)51,52
000070    51    TIN(IR,JC,K)=0.
000076          K=K+1
000077          GO TO 53
000100    52    TIN(IR,JC,K)=(BASET+T(IR,JC-1))/2.
000112          TIN(IR,JC-1,2)=TIN(IR,JC,K)
000117          K=K+1
000120    53    IF(T(IR,JC+1).EQ.0.)54,55
000127    54    TIN(IR,JC,K)=0.
000135          K=K+1
000136          GO TO 56
000137    55    TIN(IR,JC,K)=(BASET+T(IR,JC+1))/2.
000151          TIN(IR,JC+1,1)=TIN(IR,JC,K)
000156          K=K+1
000157    56    IF(T(IR-1,JC).EQ.0.)57,58
000166    57    TIN(IR,JC,K)=0.
000174          K=K+1
000175          GO TO 59
000176    58    TIN(IR,JC,K)=(BASET+T(IR-1,JC))/2.
000210          TIN(IR-1,JC,4)=TIN(IR,JC,K)
000215          K=K+1
000216    59    IF(T(IR+1,JC).EQ.0.)60,49
000225    60    TIN(IR,JC,K)=0.
000233          GO TO 50
000234    49    TIN(IR,JC,K)=(BASET+T(IR+1,JC))/2.
000246          TIN(IR+1,JC,3)=TIN(IR,JC,K)
000253    50    GO TO 6
          C
000254    3     HJ(IR,JC)=VAL(L)
000262          GO TO 6
000262    4     Q(IR,JC)=VAL(L)
000270          GO TO 6
000270    5     R(IR,JC)=VAL(L)
000276          GO TO 6
          C
          C     THIS SUBSEQUENCE TEMPORARILY (DURING A TEST) STORES THE ORIGINAL
          C     INTERNODAL TRANSMISSIVITIES AROUND THE TESTED NODE IN ARRAY TINTEM
000276    7     DO 8 K=1,4
000300    8     TINTEM(1,1,K)=TIN(IR,JC,K)
000313          TINTEM(1,1,5)=TIN(IR,JC-1,2)
000317          TINTEM(1,1,6)=TIN(IR,JC+1,1)
000322          TINTEM(1,1,7)=TIN(IR-1,JC,4)
000325          TINTEM(1,1,8)=TIN(IR+1,JC,3)
          C
          C
          C     THIS SEQUENCE SOLVES THE GROUND-WATER FLOW EQUATIONS
000330    6     IF(LPASS.GT.1.AND.L.EQ.1.AND.LTSF.EQ.1) GO TO 1000
000343          IF(KCALL2.EQ.1)21,117
          C     SET H EQUAL TO INITIAL WATER LEVEL
000347    21    DO 20 I=1,IEP1
000351          DO 20 J=1,JEP1
000352    20    H(I,J,L)=HJ(I,J)
000371          IF(LTSF.EQ.2.AND.L.GT.1) GO TO 117
          C     DELT IS TIME-STEP SIZE, IN DAYS
000431          DELT=TSTEP1*365.25
          C     TOT IS SUM OF TIME-STEPS, NTSTEPS IS THE NUMBER OF TIME STEPS
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000403          TOT=DELT
000404          NTSTEPS=1
                C
                C     START THE ALTERNATING-DIRECTION ALGORITHM, SWEEP BY COLUMNS FIRST
000405    117    DO 110 J=JS,JE
000407           DO 115 I=IS,IE
                C     GAM IS DUMMY VARIABLE (COEFFICIENT OF UNKNOWN WATER LEVEL AT GIVEN
                C     NODE - RIGHT SIDE OF EQUATION
000411           GAM=2.*A*S(I,J)/DELT
000417           IF(T(I,J).EQ.0.)31,32
000426    31     AC(I)=0.
000430           CC(I)=0.
000432           BC(I)=-GAM
000434           DC(I)=-GAM
000435           GO TO 115
000436    32     AC(I)=TIN(I,J,3)
000443           CC(I)=TIN(I,J,4)
000446           BC(I)=-CC(I)-AC(I)-GAM
                C     EC IS AVERAGE T BETWEEN GIVEN NODE AND NODE TO ITS LEFT (COLUMN SWEEP)
                C     OR BELOW (ROW SWEEP)
000453           EC=TIN(I,J,1)
                C     FC IS AVERAGE T BETWEEN GIVEN NODE AND NODE TO ITS RIGHT (COLUMN SWEEP)
                C     OR ABOVE (ROW SWEEP)
000457           FC=TIN(I,J,2)
000462           DC(I)=-EC*H(I,J-1,L) + (EC+FC-GAM)*H(I,J,L) - FC*H(I,J+1,L)
                2 + Q(I,J) + R(I,J)
000513    115    CONTINUE
                C
                C     START THOMAS ALGORITHM FOR SUCCESSIVE SOLUTION OF UNKNOWN WATER LEVELS
000516           W(2)=BC(2)
000517           G(2)=DC(2)/W(2)
000521           DO 120 K=3,IE
000522           W(K)= BC(K)-AC(K)*CC(K-1) /W(K-1)
000532    120    G(K)=(DC(K)-AC(K)*G(K-1))/W(K)
000542           IF(J.EQ.JS) GO TO 121
000544           H(IE,J-1,L)=HTEM(IE)
000552    121    HTEM(IE)=G(IE)
000555           IF(J.EQ.JE) H(IE,J,L)=HTEM(IE)
000566           DO 130 K=IS,IEM1
000570           KBW=IEM1-K+2
000572           IF(J.EQ.JS) GO TO 122
000574           H(KBW,J-1,L)=HTEM(KBW)
000603    122    HTEM(KBW)=G(KBW)-CC(KBW)*HTEM(KBW+1)/W(KBW)
000613           IF(J.EQ.JE) H(KBW,J,L)=HTEM(KBW)
000623    130    CONTINUE
000626    110    CONTINUE
                C
                C     START ROW SWEEP
000630           DO 80 I=IS,IE
000632           DO 85 J=JS,JE
000634           GAM=2.*A*S(I,J)/DELT
000642           IF(T(I,J).EQ.0.)41,42
000651    41     AC(J)=0.
000653           CC(J)=0.
000655           BC(J)=-GAM
000657           DC(J)=-GAM
000660           GO TO 85
000661    42     AC(J)=TIN(I,J,1)
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000666          CC(J)=TIN(I,J,2)
000672          BC(J)=-CC(J)-AC(J)-GAM
000677          EC=TIN(I,J,4)
000703          FC=TIN(I,J,3)
000706          DC(J)=-H(I+1,J,L)*EC + H(I,J,L)*(EC+FC-GAM) - H(I-1,J,L)*FC
               2 + Q(I,J) + R(I,J)
000736    85    CONTINUE
          C
000741          W(2)=BC(2)
000742          G(2)=DC(2)/W(2)
000744          DO 90 K=3,JE
000745          W(K)= BC(K)-AC(K)*CC(K-1) /W(K-1)
000755    90    G(K)=(DC(K)-AC(K)*G(K-1))/W(K)
000765          IF(I.EQ.IS) GO TO 86
000767          H(I-1,JE,L)=HTEM(JE)
000776    86    HTEM(JE)=G(JE)
001001          IF(I.EQ.IE) H(I,JE,L)=HTEM(JE)
001012          DO 100 K=JS,JEM1
001014          KBW=JEM1-K+2
001016          IF(I.EQ.IS) GO TO 87
001020          H(I-1,KBW,L)=HTEM(KBW)
001027    87    HTEM(KBW)=G(KBW)-CC(KBW)*HTEM(KBW+1)/W(KBW)
001037          IF(I.EQ.IE) H(I,KBW,L)=HTEM(KBW)
001047   100    CONTINUE
001052    80    CONTINUE
          C
          C     THIS SUBSEQUENCE CHECKS FOR THE END OF THE SIMULATION PERIOD,
          C     IF IT HAS BEEN REACHED, OUTPUT RESULTS, IF NOT IT PROCEEDS TO THE
          C     NEXT TIME STEP
001054          IF(L.EQ.1.AND.LPASS.EQ.1)901,902
001064    901   PRINT 900, DELT,TOT
001074    900   FORMAT  (/,15X,*TIME STEP SIZE IS *,F12.4,2X,*DAYS*,10X,*TOTAL ELA
               2PSED TIME IS *,F12.4,2X,*DAYS*)
          C     IF SUM OF TIME INCREMENTS EQUALS TOTAL TIME, OUTPUT RESULTS, IF NOT,
          C     DOUBLE TIME INCREMENT
001074    902   IF(TOT.EQ.TIME)GO TO 18
001100          IF(LTSF.EQ.2.AND.L.NE.NUMVAL) GO TO 17
001107          DELT=2.*DELT
          C     SUM TIME INCREMENTS
001110          TOT=TOT+DELT
001112          NTSTEPS=NTSTEPS+1
          C     IF SUM OF TIME INCREMENTS IS NOW LESS THAN TOTAL TIME, CALCULATE WATER
          C     LEVELS FOR NEW TIME STEP
001113          IF(TOT.LT.TIME)GO TO 17
          C     IF SUM OF TIME INCREMENTS IS NOW GREATER THAN TOTAL TIME, REDUCE NEW TIME
          C     STEP SO THAT WHEN ITS ADDED TO SUM OF TIME INCREMENTS THE NEW SUM
          C     EXACTLY EQUALS TIME
001115          DELT=DELT+TIME-TOT
001117          TOT=TIME
001120    17    GO TO(117,1000)LTSF
001126    18    KCALL1=2
001127          IF(L.EQ.1.AND.LPASS.EQ.1) PRINT 585, NTSTEPS
001146    585   FORMAT(/,38X,*THE NUMBER OF TIME STEPS IS*,I3,//)
001146          GO TO(1011,1011)MAPS
001154   1011   IF(L.GT.1)1021,1000
          C
          C     THIS SUBSEQUENCE PRINTS DIFFERENCES BETWEEN WATER LEVELS COMPUTED
          C     USING THE MODEL VALUE AND BY USING AN ALTERNATE VALUE (AT THE TESTED
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      C     NODE AND 12 SURROUNDING NODES)
001161  1021  DIFF(1)=H(IR,JC,1) - H(IR,JC,L)
001172        DIFF(2)=H(IR,JC+1,1) - H(IR,JC+1,L)
001200        DIFF(3)=H(IR-1,JC,1) - H(IR-1,JC,L)
001207        DIFF(4)=H(IR,JC-1,1) - H(IR,JC-1,L)
001215        DIFF(5)=H(IR+1,JC,1) - H(IR+1,JC,L)
001222        DIFF(6)=H(IR+1,JC+1,1) - H(IR+1,JC+1,L)
001227        DIFF(7)=H(IR-1,JC+1,1) - H(IR-1,JC+1,L)
001235        DIFF(8)=H(IR-1,JC-1,1) - H(IR-1,JC-1,L)
001242        DIFF(9)=H(IR+1,JC-1,1) - H(IR+1,JC-1,L)
001250        DIFF(10)=H(IR,JC+2,1) - H(IR,JC+2,L)
001254        DIFF(11)=H(IR-2,JC,1) - H(IR-2,JC,L)
001262        DIFF(12)=H(IR,JC-2,1) - H(IR,JC-2,L)
001267        DIFF(13)=H(IR+2,JC,1) - H(IR+2,JC,L)
001275        PRINT 201, DIFF(1), DIFF(2), DIFF(3), DIFF(4), DIFF(5), DIFF(6),
     2 DIFF(7), DIFF(8), DIFF(9), DIFF(10), DIFF(11), DIFF(12), DIFF(13)
001332  201   FORMAT(//,20X,*THE DIFFERENCE BETWEEN H(IR,JC,1)     AND H(IR,JC,L
     2)     =*,F7.2,/,43X,*H(IR,JC+1,1)   AND H(IR,JC+1,L)    =*,F7.2,/,
     3 43X,*H(IR-1,JC,1)    AND H(IR-1,JC,L)    =*,F7.2,/,43X,*H(IR,JC-1,1
     4)   AND H(IR,JC-1,L)    =*,F7.2,/,43X,*H(IR+1,JC,1)    AND H(IR+1,JC
     5,L)    =*,F7.2,/,43X,*H(IR+1,JC+1,1) AND H(IR+1,JC+1,L) =*,F7.2,/,
     6 43X,*H(IR-1,JC+1,1) AND H(IR-1,JC+1,L) =*,F7.2,/,43X,*H(IR-1,JC-1
     7,1) AND H(IR-1,JC-1,L) =*,F7.2,/,43X,*H(IR+1,JC-1,1) AND H(IR+1,JC
     8-1,L) =*,F7.2,/,43X,*H(IR,JC+2,1)   AND H(IR,JC+2,L)    =*,F7.2,/,
     9 43X,*H(IR-2,JC,1)    AND H(IR-2,JC,L)    =*,F7.2,/,43X,*H(IR,JC-2,1
     A)   AND H(IR,JC-2,L)    =*,F7.2,/,43X,*H(IR+2,JC,1)    AND H(IR+2,JC
     8,L)    =*,F7.2)
001332        GO TO 1000
001334  1011  IF(L.EQ.1)1002,1001
      C
      C     THIS SUBSEQUENCE PRINTS A MAP OF WATER-LEVEL CHANGES OVER THE SIMULATION
      C     PERIOD (USING MODEL VALUE) AND A MAP OF THE DIFFERENCE BETWEEN THE
      C     FIRST MAP AND A MAP COMPUTED USING AN ALTERNATE VARIABLE VALUE
001341  1002  DO 170 I=IS,IE
001343        DO 170 J=JS,JE
001345  170   DELHC(I,J)=HJ(I,J) - H(I,J,L)
001367        PRINT 700
001372  700   FORMAT(1H1,5X,*MAP OF GRID-COMPUTED WATER-LEVEL CHANGES, IN FEET (
     2FOR MEAN VALUE)*,//)
001372        GO TO 1003
001374  1001  DO 140 I=IS,IE
001376        DO 140 J=JS,JE
001400  140   DIFM(I,J)=H(I,J,1) - H(I,J,L)
001422        PRINT 701, L
001427  701   FORMAT(*1*,5X,*MAP OF GRID - COMPUTED DIFFERENCES IN WATER LEVELS,
     2 IN FEET, BETWEEN H(I,J,1) AND H(I,J,L)*,/,10X,*(FOR VALUE(*,I2,
     3 *))*,//)
001427  1003  PRINT 61,(J,J=JS,15)
001440  61    FORMAT(/,60X,14I5,///)
001440        PRINT 66,(I,(DELHC(I,J), J=JS,15), I=IS,IE)
001462  66    FORMAT(47X,I3,10X,14F5.1,//)
001462        PRINT 61, (J,J=JS,15)
001473        PRINT FHEAD
001477        IF(L.EQ.1)1004,1005
001505  1004  PRINT 700
001511        GO TO 1006
001513  1005  PRINT 701, L
001521  1006  PRINT 63,(J,J=16,JE)
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001533   63     FORMAT(/,5X,14I5,//)
001533          PRINT 68,((DELHC(I,J), J=16,JE), I, I=IS,IE)
001555   68     FORMAT(5X,14F5.1,10X,I3,//)
001555          PRINT 63, (J,J=16,JE)
001567          PRINT FHEAD
001573   1000   CONTINUE
001577          IF(KCALL1.EQ.2)317,217
         C
         C
         C
         C     THIS SEQUENCE REPLACES THE ORIGINAL (MODEL) VALUE OF THE VARIABLE AND
         C     THE INTERNODAL TRANSMISSIVITIES (IF THE VARIABLE WAS T) IN THE MODEL
001613   317    GO TO(11,12,13,14,15)NVT
001614   11     S(IR,JC)=VAL(1)
001621          GO TO 16
001621   12     T(IR,JC)=VAL(1)
001626          DO 19 K=1,4
001627   19     TIN(IR,JC,K)=TINTEM(1,1,K)
001642          TIN(IR,JC-1,2)=TINTEM(1,1,5)
001646          TIN(IR,JC+1,1)=TINTEM(1,1,6)
001651          TIN(IR-1,JC,4)=TINTEM(1,1,7)
001654          TIN(IR+1,JC,3)=TINTEM(1,1,8)
001657          GO TO 16
001660   13     HJ(IR,JC)=VAL(1)
001665          GO TO 16
001665   14     Q(IR,JC)=VAL(1)
001672          GO TO 16
001672   15     R(IR,JC)=VAL(1)
001677   16     CONTINUE
001677   217    RETURN
001700          END
```

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