Design Examples of Tilted-Component Telescopes (TCT's)  
(A Class of Unobscured Reflectors)  

Richard A. Buchroeder
DESIGN EXAMPLES OF
TILTED-COMPONENT TELESCOPES (TCT’s)
(A CLASS OF UNOBSCURED REFLECTORS)

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ABSTRACT

A tilted component telescope (TCT) is one that features no obstructions in the light path yet is appreciably simpler to build than conventional off-axis instruments. The principles of TCT design are applicable to scanning and image-stabilized optics and should allow improvements in that field.

The author has collected and computer-evaluated designs representative of existing art: Schiefspiegler, Yolo, catadioptric Herschelian, and Schupmann. It is expected that these evaluations will enable optical scientists to appraise the merits of the TCT approach and will stimulate the development of second-generation designs.
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INTRODUCTION

A telescope with reflecting rather than refracting surfaces has considerable appeal since it is perfectly achromatic. In any sufficiently complex reflector, of course, the limiting defect is the problem of one mirror obscuring another. The only way to avoid this obscuration is to have the center of view run obliquely through the collection of mirrors. The same applies to catadioptrics. The purpose of this report is to describe one class of unobscured reflectors—the "TCT’s" (tilted-component telescopes)—and to describe and evaluate some examples of TCT designs.

For purposes of discussion, most existing unobscured reflectors may be classed into one of three types (Figs. 1a–c).

First is the eccentric pupil class, an ordinary coaxial design used with an eccentric stop. Its center of view coincides with the parent axis. Common examples are the "off-axis" paraboloid and "off-axis" Schmidt.

In the off-axis class, the center of view coincides not with the parent axis but with a principal (chief) ray of a nominally coaxial design. The Herschelian telescope is of this form. We have excluded the "off-axis" paraboloid from this class. Although the distinction is perhaps only semantic, a designer approaches the problem differently than he approaches the problem of an eccentric.
Third is the tilted-component class, the subject of this report. In the TCT, each component is treated as being rotationally symmetric in its own coordinate system, and it is centered on its vertex, which will define the optical axis of the telescope. Each component may be assigned, in its own coordinate system, a marginal and a principal ray, with the principal ray passing through the vertex. The components are then assembled so that their principal rays define the optical axis of the completed telescope. The axial aberration is described by the summation of the axial and field aberration of each component. Since each component is centered on its vertex, the system may be assembled by conventional methods such as boresighting and autocollimation.

For a single element this is identical to the off-axis class. Some designers suspect that it is possible to relate all TCT's to a sufficiently complex off-axis design. We have not yet resolved the question. However, if any of our three-mirror TCT's are reduced to an equivalent coaxial off-axis design, the asphericities are extraordinarily large. Indeed, none are of common form in regard to pupil position and principal ray path. None are similar to the usual coaxial three-mirror reflectors; our formulas show that these cannot give a corrected "axis" on the assumption that the local vertex is spherical or even coaxially aspheric. On the other hand, our formulas show that the TCT configuration gives perfect third order correction.¹

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FUNDAMENTALS OF THE TCT

The idea behind the TCT is simplicity itself. Each component is assumed to be centered on the effective optical axis, and each element is assumed to be spherical. Fig. 2 gives the geometry. In the designing process, the "axial" ray is the principal ray for each element in terms of its local coordinate system, and the Seidel aberration for each component is easily computed. The idea is to have the field aberrations of each component offset those of the other components, giving a corrected axis to the telescope. It is usually convenient to do this iteratively on a calculator rather than to attempt to solve the pertinent formulas,² which are helpful but awkward. One quickly gets a feel for successful configurations, and it can be shown without much difficulty that only certain ones are successful. Similarly, it can be shown that others are unsuccessful and are to be avoided.

\[
K = \frac{i_{pr}}{i}
\]

\[
S_1 \quad \text{SPHERICAL ABERRATION}
\]
\[
S_2 = KS_1 \quad \text{COMA}
\]
\[
S_3 = K^2S_1 \quad \text{ASTIGMATISM}
\]

Fig. 2. Local coordinate system.

Design Classes

For purposes of discussion the writer has divided into four broad groups the TCT designs that are to be described here.

Schiefspiegler. The Schiefspiegler is any TCT that has a convergent mirror objective followed by a convex secondary mirror. Like most of the examples of this type, the name Schiefspiegler was created by a German, Anton Kutter. It has no exact translation but means approximately “leaning mirror telescope.”

Yolo. The Yolo is any TCT with a convergent mirror objective followed by a concave secondary mirror. It has the general form of two mirrors, one of which is usually toroidal, but at least two all-spherical forms are possible as well. Its theoretical advantage over the Schiefspiegler is that its mirrors are cooperative in power, allowing lower sensitivity to misalignment, or, if this is sacrificed, greater relative aperture for a given sensitivity. Compared with the Schiefspiegler, it seems capable of reduced image tilt. The name Yolo is that of a county in California dear to its inventor, Arthur S. Leonard, retired professor of mechanical engineering (University of California), who is responsible for all the Yolo designs described in this report.

CHT. A CHT (catadioptric Herschelian telescope) is any TCT that has a convergent mirror objective followed by one or more tilted, nonwedged spherical lenses. Prof. Leonard has proposed the use of full-aperture lenses to seal off the tube, but we do not deal with this in the present report. A catadioptric Herschelian is not a new idea, but in the past this type has utilized wedged or noncentered elements.

Schupmann. In 1899, the German mathematician Ludwig Schupmann was granted U.S. Patent 620,978 for a unique family of catadioptric telescopes. In a preferred embodiment, the Schupmann applies the tilted-component concept to eliminate obstruction, but the concept is equally applicable to coaxial designs of higher speed.

The Schupmann consists of a convergent refractive objective (usually a singlet), a field element near its focus, and a reimaging catadioptric relay known as a Mangin mirror (which in its simplest form is a singlet). The field element images the objective onto the relay, while the overcorrect color of the Mangin corrects the undercorrect color of the simple objective. A proper disposition of the three groups, with all elements made from the same glass, allows complete elimination of secondary spectrum. The relay usually has a coma-free form, as does the objective. Either may or may not be aspheric, according to the design. In the coma-free case, the Mangin may be tilted to eliminate the conflict of the image with the light, and its resulting astigmatism may be corrected by tipping the objective. More sophisticated forms employing peculiar aspherics have been reported by Baker,3 but experience in the fabrication of these has been discouraging. The writer recommends the tilted-component approach despite the fact that these have a slight amount of chromatic astigmatism.

The Schupmann is similar to its predecessor, the Hamiltonian telescope, in using a simple objective corrected with a Mangin mirror. It differs, however, in that a field lens is used, whereas the Hamiltonian Mangin reimages a virtual rather than a real image. Consequently, the Hamiltonian has lateral color and cannot be perfectly corrected for secondary spectrum.

The Schupmann design evaluated in this report was described in a bulletin of the Schupmann Club, a group that is doing a great deal to dispel the pessimism previously associated with the difficulty of making a Schupmann telescope. Actually, although considerable skill and patience are required to extract the full capability of this form of telescope, a satisfactory (though perhaps not perfect) Schupmann telescope need be no harder to build than an ordinary refracting doublet.

**General Behavior**

In a high-acuity telescope it is important that the axial image be free of spherical aberration, coma, and astigmatism. Ideally, distortion and image tilt also would be zero, but designs that offer this advantage are usually too difficult to interest the average customer, who is an amateur astronomer. Consequently, many of the designs disregard these latter two defects since they do not harm contrast or resolution.

*Astigmatism, Coma, Distortion.* Figure 3 compares the astigmatism in a TCT with that of two ordinary coaxial designs. Note that the astigmatism in a TCT resembles the anastigmatic node of a complex lens, which is obtained by balancing higher order aberration against the Seidel terms. In the TCT this crossing is obtained entirely by third order calculation. It is found that in the simple three-mirror Schiefspiegler this astigmatism is linear with field (in the meridional plane) rather than increasing with the square of the image height, as is usual in a normal lens. This simple variation of astigmatism suggests that it can be cured by appropriate aspheric figuring. What is needed is a parabolically varying deformation on one of the components—in this case the tertiary mirror for best results. One side should be turned up, the other turned down. This is well approximated by an off-axis section of an aspheric mirror, which itself could be further approximated by a toroid. The implications are controversial and will not be dwelt upon here.

![Fig. 3. Comparison of astigmatism.](image-url)
Three axially symmetric elements allow the simultaneous elimination of astigmatism, coma, and distortion. The successful configurations are the three-mirror Schießspiegel and three-mirror Yolo. These always require that the third mirror reflect the light back toward the incoming axis. Tilting of the third mirror in the opposite direction gives an unsuccessful configuration.

With only two axially symmetric elements, astigmatism and coma cannot be simultaneously eliminated. However, it is possible to use one less mirror if one of the two mirrors is toroidal. Some of the best designs described here use toroidal mirrors. Because a toroid is hard to obtain by usual optical fabrication techniques, A. S. Leonard has spent a number of years developing a simple means of mechanically warping and holding a nominally spherical mirror. He has built a number of such devices, and they seem to work satisfactorily. Figure 4 shows how a successful version works. Prof. Leonard has a more complex version in use for a 12½-in. telescope of the same form.

Clearly, one could temporarily distort the mirror, figure it as a sphere, then release the distortion when finished and use the mirror in the ordinary fashion. The point is, toroidal mirrors have been made and are successful, so there is no great objection to using them, at least in professional instruments.

Fig. 4. Warping harness for 8-in. Yolo.

**Image Tilt and Anamorphic Distortion.** Figure 5 shows how image tilt, which is a fairly natural occurrence in any "off-axis" telescope, can be corrected by an additional element, either a lens or a mirror. Our experience suggests that if only axially symmetric components are used, four elements will be required for a perfectly corrected axis in a TCT. However, we have not proved that three elements cannot do the same job. Using toroids, it is possible to achieve a fully correct axis with three elements; with only two elements, one achieves a sufficient approximation to perfection.

When we change our reference system on an off-axis telescope and use some field point as the “axis,” distortion must be reevaluated. It is proper to speak of tangential and sagittal distortion, the two being mutually perpendicular. Points above and below the center (as well as left to right in a three-dimensional TCT) are not symmetric about
the center of the field. If the field of view is small compared with that chosen as the axis for the off-axis system, the rate of change over the new field of view is small and the distortion changes only slowly.

To a first approximation, a circular object will be reimaged as an ellipse with its axes proportional to the tangential and sagittal focal lengths at the center of the image. This behavior is characteristic in the TCT.

Rather than deal with paraxial quantities, we can use the centroid data for the spot diagrams at the smallest field angle, ±3.75 arc min. Here the centroids are not greatly disturbed by aberration, which can confuse one's definition of distortion.

It is difficult to define the image tilt since the image may be some complicated toroidal sheet. Since these designs are slow and have small fields of view, however, we will assume the image is flat and compute a value for it from the centroid data. The following somewhat arbitrary definitions will be used to specify anamorphic distortion (due to the difference in tangential (t) and sagittal (s) focal length) and image tilt.

Given: \( x, y, \) and \( z \) coordinates of the spot centroids for the upper, lower, left, and right image points for a field diameter of 7.5 arc min. The reference axis of the design has been adjusted so that it nearly (or exactly) follows the path of the central ray of light.

\[
\begin{align*}
\text{Distortion} & \quad D = \frac{y(\text{upper}) - y(\text{lower})}{x(\text{left}) - x(\text{right})} - \frac{x(\text{left}) - x(\text{right})}{x(\text{left}) - x(\text{right})} \\
\text{Tilt (t)} & \quad T(t) = \frac{z(\text{upper}) - z(\text{lower})}{y(\text{upper}) - y(\text{lower})} \\
\text{Tilt (s)} & \quad T(s) = \frac{z(\text{left}) - z(\text{right})}{x(\text{left}) - x(\text{right})}
\end{align*}
\]
Symmetry simplifies these for the two-dimensional designs. The observer’s perspective is that of looking in with an eyepiece, and $x$ is positive to the left. The incoming rays approach from the left and focus to the right. In situations where this doesn’t occur naturally, it is accomplished with a dummy plane reflector at the front. Consequently, all the designs give inverted and reverted images except for the Schupmann, where a dummy reflector after the Mangin gives an erect image.

Up and down, right and left, are with regard to the untilted system (i.e., when the tilts are set to zero). When these are set to their proper values, the \textit{apparent} observing angles for the designs may have different signs. A positive tangential image tilt means that the top of the field of view (not necessarily the apparent top), as seen by the observer with his eyepiece, will protrude toward his eye. Similarly, in the skewed designs, a positive sagittal tilt means that the left side protrudes.

The distortion is with respect to a plane normal to the TCT’s axis. Distortion on the inclined focal surface would be calculated by considering the cosine elongation. A negative distortion is partially corrected by a tilt of either sign, while positive distortion is always aggravated by an inclined image plane. Data for the normal (perpendicular) distortion and image inclination are presented below.

### ANAMORPHIC DISTORTION AND FIELD TILT

<table>
<thead>
<tr>
<th>Design</th>
<th>D, %</th>
<th>Tilt (t), degrees</th>
<th>Tilt (s), degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schiefspiegler</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25-cm f/20, Kutter lens</td>
<td>- .072</td>
<td>+ 7.712</td>
<td></td>
</tr>
<tr>
<td>4½-in. f/26.6, anastigmatic</td>
<td>+ .310</td>
<td>+ 4.373</td>
<td></td>
</tr>
<tr>
<td>12½-in. f/20.2, 3-mirror O’Neill</td>
<td>- .889</td>
<td>+ 9.019</td>
<td></td>
</tr>
<tr>
<td>10-in. f/19.2, 3-mirror equiradius aspheric</td>
<td>-5.609</td>
<td>+16.658</td>
<td></td>
</tr>
<tr>
<td>Yolo</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12-in. f/15, 2-mirror anamorphic</td>
<td>+ .213</td>
<td>- 1.129</td>
<td></td>
</tr>
<tr>
<td>10-in. f/15.2, 3-mirror (2D)</td>
<td>+3.341</td>
<td>- 7.000</td>
<td></td>
</tr>
<tr>
<td>10-in. f/13.3, 3-mirror (3D)</td>
<td>+ .253</td>
<td>- .833</td>
<td>-1.442</td>
</tr>
<tr>
<td>10-in. f/13.9, 3-toroid</td>
<td>.000</td>
<td>+ 1.043</td>
<td></td>
</tr>
<tr>
<td>CHT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-in. f/10.2</td>
<td>+ .381</td>
<td>+ 7.013</td>
<td></td>
</tr>
<tr>
<td>4½-in. f/10.2</td>
<td>+ .251</td>
<td>+ 5.528</td>
<td></td>
</tr>
<tr>
<td>4½-in. f/10.5, single lens</td>
<td>+ .349</td>
<td>+ 4.084</td>
<td></td>
</tr>
<tr>
<td>48-in. f/6.4, monochromat</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>10-in. f/15.1, hybrid (Schief/CHT)</td>
<td>+ .944 +16.501</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Schupmann</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-in. f/12</td>
<td>.000</td>
<td>- 2.757</td>
<td>-2.015</td>
</tr>
</tbody>
</table>

In general, image tilt is more objectionable than distortion since the eye loses its range of accommodation as it grows older. With the equiradius Schiefspiegler, it is essential to tilt the eyepiece for even a young eye, whereas with the other Schiefspiegler experiments show the tilt to be acceptable. The reaction to the inclined field of the CHT’s has been mixed, so apparently it depends on the observer.
In evaluating the designs presented here, we have used computer-generated spot diagrams. The spot diagram is a geometrical approximation of the spread function (light distribution) that an optical system will give when it is pointed toward a star. This is accomplished by tracing a large number of uniformly spaced rays into the aperture and examining their intersection with a focal surface.

Because TCT's are designed using a local coordinate system, it is best to evaluate them using the same logic as used in their design if a complicated transformation of coordinates is to be avoided. Not all lens design and evaluation programs permit this, but our version of ACCOS-IV is perfectly suited. Because the image surfaces are neither flat nor perpendicular to the optical axis, an additional feature of our program is useful: it will search in three dimensions for the focal point at which the rms spot size is a minimum. The coordinates of this centroid are printed out and can be used to derive image tilt and distortion.

We have prepared spot diagram montages by pasting a large number of computer-generated diagrams onto a larger sheet, then reducing the sheet. The largest circle corresponds to a field diameter of 0.5°. The innermost circle is for 1/8° (7.5 arc min). The center we call the axis. Except for three-dimensional designs, only half the field need be plotted since the two sides are symmetrical.

The relative field scale is only approximate, and distortion is eliminated for convenience in preparing the drawings. The individual spot diagrams have a separate angular scale indicated by a circular marker.

Next to the lens prescription of each TCT is an enlarged spot diagram of the axial region alone. Few of the designs are thoroughly optimized; however, the qualitative nature of the axis is of value. If the shape of the axial spot diagram suggests that third order aberration is dominant, then further optimization is possible. If higher order shapes prevail, no further improvement is likely unless major alterations in curvature and spacing are made.

The catadioptric designs are evaluated using rays of three different wavelength: x for helium yellow (d-light), + for hydrogen blue (F-light), A for hydrogen α red (C-light). Although usually irresolvable on the montages, these show up on the axial evaluations.

Some cautions are in order in regard to the interpretation of the spot diagrams.

First, our computer program gives a repeated square grid of rays. This regular geometry influences the patterns observed at the focus, suggesting apparent patterns that do
not really exist. Unless one is skilled at interpreting spot diagrams, he should consider only the area density of spots, this being proportional to the intensity.

Second, although spot diagrams are invaluable for dealing with such a large number of peculiar designs, they have disadvantages. For the "best focus" position on these plots, astigmatism is indistinguishable from spherical aberration. One could easily conclude that these designs are like conventional lenses. This is not the case, as was mentioned in regard to Fig. 3.

Computer Prescriptions

Readers may wish to re-examine the work in this report, and for that purpose the simple prescriptions accompanying the spot diagrams are exact only for the purely reflective designs. The information required to duplicate our traces is described below.

Four conics were encountered:

\[ K = 0 \] (spherical)
\[ K = -1 \] (paraboloid)
\[ K < -1 \] (hyperboloid)
\[ -1 < K < 0 \] (ellipsoid of revolution about the major axis; the easy kind)

The surface equation for all surfaces is

\[ z - \frac{y^2 + (K + 1)z^2}{2R} = 0.0, \]

where \( R \) = vertex radius of curvature
\( y \) = zonal radius of the optical surface
\( z \) = axial distance

For an ellipsoid, \( K = (a^2 - b^2)/b^2 \), where \( b \) and \( a \) are the semimajor and semiminor axes of the ellipse, respectively.

For a hyperboloid, \( R = (K + 1)b \), where \( b \) is the axial separation between the vertex and the asymptotic intersection of the hyperbola with the axis.

The \( x,y,z \) coordinate system is a little unusual. The \( z \) axis is positive to the right and the \( y \) axis is positive upward, but the \( x \) axis is positive out of the \( y-z \) plane, the opposite of the usual analytic geometry coordinate system. Decentrations are positive when the surface under question is displaced in the positive direction as defined for the coordinate system.

Angular tilts are positive when performed counterclockwise viewing toward the origin of coordinates; \( \alpha \) is a tilt about the \( y \) axis, \( \beta \) about the \( x \) axis, and \( \gamma \) about the \( z \) axis. Only \( \beta \) is used for two-dimensional designs; both \( \alpha \) and \( \beta \) are used in three-dimensional designs; \( \gamma \) has not been used.

ACCOS/GOALS version IV is available from the following consulting firm:

Scientific Calculations, Inc.
110 Allen's Creek Road
Rochester, New York 14618
DESIGNS

One could become callous toward the existence of small optical imperfections unless he compares his final result to that which he would have if no corrections had been made. It is instructive to see how badly aberrated the tipped objective mirror alone would be.

Figure 6 shows spot distributions from two of the fastest mirrors described in this report. The f/10 is the sort used in most of the CHT’s; the f/6.4, also a CHT, is parabolic and was included to show that an ordinary Newtonian telescope could be converted to off-axis use. Although the latter design is monochromatic and covers only a small field of view, it is apparent that a more complex version would extend its capabilities. An interesting application would be to use a semicircular array of correctors (which are only 1/6 the diameter of the mirror) to observe a number of field points simultaneously. Indeed, such correctors could observe without disturbing the normal mode (coaxial) of observation. If one tolerated an obscuration, it is conceivable that with a fairly simple mechanism controlling the tilts of the lenses, the mirror could be left fixed while the moving lenses allowed an object to be tracked for a range of at least 8°! Conversely, a stabilization scheme could be achieved for a moving platform. Fig. 6a shows the improvement obtained using only two simple lenses.

Fig. 6. Spot distributions from two mirrors.
Left: 10-in. f/10 spherical tipped 2.3365°; right: 48-in. f/6.4 paraboloid tipped 4°. Fig. 6a shows the distribution from the f/6.5 mirror with two BK-7 correcting lenses of 7-in. diameter.
All of the Schiefspieglers have f/12 or slower mirrors. These could be left spherical, but contrast will be improved if the small residual of spherical aberration is eliminated. The figuring on the primary is elliptical. In the Yolo, the individual mirrors are exceedingly slow, none faster than f/20 and many f/60 and worse. Although this slowness is an optical advantage, it presents a problem in testing. Usually the primary mirror is figured hyperbolically, for all mirrors will aggravate the spherical aberration despite their individually low speeds.

The designs are presented in the following pages. All dimensions, unless noted otherwise, are in millimeters.
Schiefspieglers
Schiefspiegler

25-cm f/20 catadioptric or lens-corrected


\[ R_1 = 6000.0 \text{ concave} \]
\[ R_2 = 6340.0 \text{ convex} \]
\[ R_3 = \text{plano} \]
\[ R_4 = 19387.5 \text{ convex} \]

\[ S_1 = 1707.0 \]
\[ S_2 = 932.0 \]
\[ T_1 = 8.0 \]
\[ S_3 = 1206.3 \text{ untilted solution} \]

\[ b_1 = 6.300 \]
\[ b_2 = -18.1333 \]
\[ b_3 = -28.000 \]

Glass: \[ N_d = 1.5168 \]
\[ V_d = 64.14 \]

NOTE: Proper aspherizing on primary mirror \( K = -.55 \) (optional).

Kutter recommends that lens be slightly wedged for best axial chromatic correction.
BK-7, d-F-C

1 arc sec
BK-7, d-LIGHT

1 arc sec

16
Schiefspiegler
4¾-in. f/26.6 anastigmatic

\[ \begin{align*}
R_1 &= 3238.5 \text{ concave} \\
R_2 &= 3238.5 \text{ convex} \\
S_1 &= 914.4 \\
S_2 &= 1248.0 \text{ untilted} \\
b_1 &= 5.58334 \\
b_2 &= -12.83334
\end{align*} \]
Schiefspiegler
12½-in. f/20.2 three-mirror O'Neill (ellipsoidal primary)
Design credit: R. A. Buchroeder

\[ R_1 = 7620.0 \text{ concave} \]
\[ (\text{aspheric: } K = -0.55) \]
\[ R_2 = 7620.0 \text{ convex} \]
\[ R_3 = 53250.0 \text{ concave} \]
\[ S_1 = 2172.0 \]
\[ S_2 = 1700.0 \]
\[ S_3 = 1128.8 \text{ untilted solution} \]

\[ b_1 = 6.300 \]
\[ b_2 = -19.210 \]
\[ b_3 = -77.090 \]
1 arc sec
Schiefspiegler
10-in. f/19.2 three-mirror equiradius aspheric (ellipsoidal primary)
Design credit: R. A. Buchroeder

\( R_1 = 6096.0 \) concave
\( R_2 = 6096.0 \) convex
\( R_3 = 6096.0 \) concave

\( S_1 = 1735.0 \)
\( S_2 = 2000.0 \)
\( S_3 = 278.6 \) untilted solution

\( b_1 = 6.300 \)
\( b_2 = -19.400 \)
\( b_3 = -90.000 \)
Yolos

All Yolo designs are by Arthur S. Leonard, 740 Elmwood Drive, Davis, Calif. 95616. (Proceedings of Western Amateur Astronomers Convention, 7th Annual (1955), 17th Annual (1965), 21st Annual (1969); personal correspondence to R. A. Buchroeder.)
12-in. f/15 two-mirror anamorphic (hyperboloidal primary, toroidal secondary)

Design credit: Arthur S. Leonard

\[ R_1 = 15240.0 \text{ concave} \]
\[ \text{(aspheric: } K = -4.16, \text{ hyperboloid)} \]

\[ R_2(y) = 15336.578 \text{ concave} \]
\[ \text{(toroidal)} \]

\[ R_2(x) = 15144.6484 \text{ concave} \]

\[ S_1 = 2540.0 \]

\[ S_2 = 3055.7 \text{ untilted solution} \]

\[ b_1 = -7.470 \]

\[ b_2 = -6.3028 \]

0.1 arc sec
Yolo
10-in. f/15.2 three-mirror (spherical)
Design credit: Arthur S. Leonard

\[ R_1 = 10499.1 \text{ concave} \]
\[ R_2 = 14614.9 \text{ concave} \]
\[ R_3 = 13174.8 \text{ convex} \]
\[ S_1 = 1905.0 \]
\[ S_2 = 1866.9 \]
\[ S_3 = 457.0 \text{ untilted solution} \]
\[ b_1 = 8.400 \]
\[ b_2 = 6.600 \]
\[ b_3 = 81.600 \]
Yolo
10-in. f/13.3 three-concave-mirror skewed 3-dimensional
also known as the "3-D" (see p. 30)
Design credit: A. S. Leonard

\[ R_1 = 16764.0 \text{ concave} \]
\[ R_2 = 16764.0 \text{ concave} \]
\[ R_3 = 12192.0 \text{ concave} \]
\[ S_1 = 1524.0 \]
\[ S_2 = 1562.1 \]
\[ S_3 = 1622.0 \text{ untilted solution} \]
\[ a_1 = 0.0 \]
\[ a_2 = 7.96762 \]
\[ a_3 = 11.04734 \]
\[ b_1 = 8.6271 \]
\[ b_2 = 2.17996 \]
\[ b_3 = -3.33270 \]

NOTE: Remember that this drawing is a two-dimensional representation of a three-dimensional design (see p. 30).
The 3-D
A Tilted-Mirror Unobstructed Reflecting Telescope
(Arthur S. Leonard)

For initial design:

\[ Q = \frac{\text{Diam. of Primary}}{9.778} \text{ or } \frac{\text{Diam. of Secondary}}{8.000} \]

whichever is the smaller.

Then:

\[ R_1 = 660Q \]
\[ R_2 = R_1 \pm 4.0 \text{ in.} \]
\[ R_3 = (8/11)R_1 \pm 1.5 \text{ in.} \]

All surface curves are spherical concave.

For final design (after the mirrors are finished):

\[ Q = \frac{R_1}{1301.33} + \frac{R_2}{1957.4} + \frac{R_3}{3016.4} \]

Coordinates of points are:

\[
\begin{array}{ccc}
\text{Point} & \text{X} & \text{Y} & \text{Z} \\
P_0 & +59.32Q \pm .30 & 0.00 & 0.00 \\
P_1 & 0.00 & 0.00 & 0.00 \\
P_2 & +59.32Q \pm 1.0 & 0.00 & +9.00Q \pm .02 \\
P_3 & -0.50Q \pm 1.0 & +8.51Q \pm .03 & -2.44Q \pm .03 \\
P_4 & +59.37Q \pm 1.0 & -12.29Q \pm .04 & +5.30Q \pm .04 \\
\end{array}
\]

For a 10-in. model:

\[
\begin{align*}
R_1 &= 660 \text{ in.} \\
R_2 &= 660 \text{ in.} \\
R_3 &= 480 \text{ in.} \\
Q &= 1.000 \\
\end{align*}
\]

Mirror blank sizes:

Primary 10 in.
Secondary 8 in.
Ternary 6 in.

Note: In order to minimize the net effect of mirror sag, locate mirror mounting lugs as shown above.
Yolo
10-in. f/13.9 three-toroid (hyperboloidal primary)
Design credit: A. S. Leonard

Dimensions in inches

\[ R_{1y} = 1162.0704 \text{ (cc)} \]
\[ R_{1x} = 1153.9438 \text{ (cc)} \]
\[ R_{2y} = 602.11964 \text{ (cc)} \]
\[ R_{2x} = 597.88782 \text{ (cc)} \]
\[ R_{3y} = 456.63452 \text{ (cc)} \]
\[ R_{3x} = 454.60733 \text{ (cc)} \]
\[ S_1 = S_2 = 66.000 \]
\[ S_3 = 80.357 \text{ untilted solution} \]
\[ b_1 = 9.594 \text{ all negative tilts} \]
\[ b_2 = 9.618 \]
\[ b_3 = 7.640 \]

Polynomial figuring:
\[ D = 4.966756 \times 10^{-9} \text{ (dummy surface)} \]
\[ E = 4.543788 \times 10^{-14} \]
\[ \text{sag} = D_{y^4} + E_{y^6} \]

Note that all the mirrors are concave toroids. The primary is a fake hyperboloid, for convenience in tracing. The secondary could also be aspheric, with a slightly different figure on the primary, to eliminate the fieldcoma noted in the spot diagrams.
CHT's
CHT

10-in. f/10.2 two-lens
Design credit: R. A. Buchroeder

\[ R_1 = 5080.0 \text{ concave} \]
\[ R_2 = 214.0 \]
\[ R_3 = 223.16 \]
\[ R_4 = 650.0 \]
\[ R_5 = 605.72 \]
\[ S_1 = 2058.0 \]
\[ T_1 = 16.0 \]
\[ S_2 = 25.582 \]
\[ T_2 = 25.4 \]
\[ S_3 = 460.4 \text{ untilted solution} \]
\[ b_1 = -4.6730 \]
\[ b_2 = 11.310 \]
\[ b_3 = -7.125 \]

Glass 1  \( N_d = 1.517 \)
\( V_d = 64.5 \)
Glass 2  \( N_d = 1.617 \)
\( V_d = 36.6 \)

Offset of axis approximately 2.1
BK-7 & F4, d-F-C

4 arc sec
$R_1 = 2159.0\text{ concave}$
$R_2 = 282.8$
$R_3 = 302.85$
$R_4 = 302.85$
$R_5 = 282.8$

$S_1 = 739.0$
$T_1 = 8.0$
$S_2 = 6.072$
$T_2 = 10.0$
$S_3 = 281.98$ untilted solution

$b_1 = -6.182$
$b_2 = 15.642$
$b_3 = -13.496$

Both glasses: $N_d = 1.517$
$V_d = 64.5$

Offset of axis approximately 1.5
BK-7, d-F-C

4 arc sec
R₁ = 2159.0 concave  
R₂ = 282.8  
R₃ = 302.85  
S₁ = 739.0  
T₁ = 8.0  
S₂ = 354.3 untilted solution  
b₁ = -6.182  
b₂ = 16.700  
Glass: Nₐ = 1.517  
Vₐ = 64.5  
Offset of axis approximately 1.5
BK-7, d-F-C

4 arc sec
CHT
48-in. f/6.4 parabolic monochromatic
Design credit: R. A. Buchroeder

\[ R_1 = 15849.6 \text{ parabolic} \]
\[ R_2 = 351.64 \]
\[ R_3 = 923.65 \text{ convex} \]
\[ R_4 = 913.74 \text{ concave} \]
\[ R_5 = 351.64 \]
\[ S_1 = 6934.2 \]
\[ T_1 = 25.4 \]
\[ S_2 = 15.704 \]
\[ T_2 = 25.4 \]
\[ S_3 = 989.5 \text{ untilted} \]

\[ b_1 = -8.000 \]
\[ b_2 = 16.400 \]

All glass: \[ N_d = 1.5168 \]
\[ V_d = 64.14 \]

Note: No effort is made to correct lateral color, which is prominent.
CHT
10-in. f/15.1 hybrid (Schiefspiegler/CHT)
Design credit: R. A. Buchroeder

\[
\begin{align*}
R_1 & = 5080.0 \text{ concave} \\
R_2 & = 5080.0 \text{ convex} \\
R_3 & = 214.0 \\
R_4 & = 223.16 \\
R_5 & = 650.0 \\
R_6 & = 605.72 \\
S_1 & = 1693.0 \\
S_2 & = 556.0 \\
T_1 & = 16.0 \\
S_3 & = 25.4 \\
T_2 & = 25.4 \\
S_4 & = 700.3 \text{ untilted solution} \\
b_1 & = 6.867 \\
b_2 & = -13.734 \\
b_3 & = 14.1980 \\
b_4 & = -8.9785 \\
\text{Glass 1:} & \quad N_d = 1.517 \\
& \quad V_d = 64.5 \\
\text{Glass 2:} & \quad N_d = 1.617 \\
& \quad V_d = 36.6 \\
\end{align*}
\]

Offset of axis approximately 2.7

\[
\begin{align*}
1 \text{ arc sec} \\
\end{align*}
\]
4 arc sec
Schupmann

Certain liberties were taken in evaluating this design, which, although it was nearly correctly traced, is not precisely what its inventor had in mind. Instead of arranging it so the axis ran through the nodal points of the objective and Mangin relay, it was convenient to tip these elements about their front vertices. This will induce lateral color and some monochromatic aberration that will vanish when the lenses are properly adjusted.

J. G. Baker has pointed out that it is preferable not to tilt the objective but rather to give the Mangin’s reflecting surface a toroidal figure. The principle of the present design is that the tipped objective corrects the astigmatism induced by the inclined Mangin; however, since the objective is refractive, whereas the Mangin is catadioptric, the color balance will fail in the extremes of the spectrum. The problem, of course, is chromatic variation of astigmatism.

In designing more complicated versions of the Schupmann, with multi-element Mangin mirrors, one must be aware of the possibility of ghost images. Schupmann himself was concerned with this problem.

The implications in the Schupmann are fairly broad, and Baker has used the concept to design color correctors for ordinary refractors that use doublet mangins to exactly balance the secondary spectrum of the main doublet objectives.
Schupmann
4-in. f/12


\[ R_1 = 689.1782 \]
\[ (\text{aspheric: } K = -.733225) \]
\[ R_2 = 7680.96 \text{ convex} \]
\[ R_3 = 1225.4992 \text{ concave} \]
\[ R_4 = 495.3 \]
\[ R_5 = 825.5 \]
\[ T_1 = 12.7 \]
\[ S_1 = 1209.3702 \]
\[ S_2 = 1219.2 \]
\[ T_2 = 12.7 \]
\[ S_3 = 1219.0 \text{ untilted solution} \]

\[ b_1 = 1.684 \]
\[ b_2 = -7.182 \]
\[ b_3 = 0.0 \]
\[ a_3 = 1.193 \]

All glass: \( N_d = 1.519 \)
\[ V_d = 64.5 \]

Parameters were taken from Schupmann Club Bull. No. 1, with the exception of the tilt and aspherization of the objective, which were not prescribed. These were supplied by the writer of this report (RAB). Since the Bulletin did not give explicit optical specifications, this raytrace may not represent the ultimate capability of the design.

Note that the refractive elements are tilted about their front vertices. This is “slight misalignment” and gives the lateral color of the spot diagrams. It is easily removed by proper centration so should be ignored here.
1 arc sec