

# Definitive test of the $R_h = ct$ universe using redshift drift

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## ABSTRACT

The redshift drift of objects moving in the Hubble flow has been proposed as a powerful model-independent probe of the underlying cosmology. A measurement of the first- and second-order redshift derivatives appears to be well within the reach of upcoming surveys using as the Extremely Large Telescope high resolution spectrometer (ELT-HIRES) and the Square Kilometer Phase 2 Array (SKA). Here we show that an unambiguous prediction of the  $R_h = ct$  cosmology is *zero* drift at all redshifts, contrasting sharply with all other models in which the expansion rate is variable. For example, multiyear monitoring of sources at redshift  $z = 5$  with the ELT-HIRES is expected to show a velocity shift  $\Delta v = -15 \text{ cm s}^{-1} \text{ yr}^{-1}$  due to the redshift drift in  $\Lambda$ CDM, while  $\Delta v = 0 \text{ cm s}^{-1} \text{ yr}^{-1}$  in  $R_h = ct$ . With an anticipated ELT-HIRES measurement error of  $\pm 5 \text{ cm s}^{-1} \text{ yr}^{-1}$  after 5 yr, these upcoming redshift drift measurements might therefore be able to differentiate between  $R_h = ct$  and  $\Lambda$ CDM at  $\sim 3\sigma$ , assuming that any possible source evolution is well understood. Such a result would provide the strongest evidence yet in favour of the  $R_h = ct$  cosmology. With a 20-yr baseline, these observations could favour one of these models over the other at better than  $5\sigma$ .

**Key words:** cosmological parameters – cosmology: observations – cosmology: theory – distance scale.

## 1 INTRODUCTION

The cosmological space–time is now being studied using several techniques, including the use of Type Ia SNe as standard candles (Riess et al. 1998; Perlmutter et al. 1999), baryon acoustic oscillations (BAO; Seo & Eisenstein 2003; Eisenstein et al. 2005; Percival et al. 2007; Pritchard, Furlanetto & Kamionkowski 2007), weak lensing (Refregier 2003), and also cluster counts (Haiman, Mohr & Holder 2000); the range of observations is actually much larger than this. Most of these methods, however, depend on integrated quantities, such as the angular or luminosity distances, that depend on the assumed cosmology. Such data cannot easily be used for unbiased comparisons of different expansion histories. For example, in the case of Type Ia SNe, at least three or four ‘nuisance’ parameters characterizing the SN light curve must be optimized along with the model’s free parameters, making the data compliant to the underlying cosmology (Melia 2012; Wei et al. 2015).

None the less, recent progress comparing predictions of the  $R_h = ct$  universe (Melia 2007; Melia & Shevchuk 2012; Melia 2016, 2017) and  $\Lambda$  cold dark matter ( $\Lambda$ CDM) versus the observations suggests that the  $R_h = ct$  cosmology may be a better fit to the data, both at high redshifts where, e.g. the angular correlation function in the cosmic microwave background disfavors the stan-

dard model (Melia 2014b) and the premature appearance of galaxies (Melia 2014a) and supermassive quasars (Melia 2013b; Melia & McClintock 2015) may be more easily explained by  $R_h = ct$  than  $\Lambda$ CDM, and at low and intermediate redshifts, as seen, e.g. in the Type Ia SN Hubble diagram (Wei et al. 2015) and the distribution of BAO (Melia & López-Corredoira 2016 and references cited therein).

The Alcock–Paczynski test based on these BAO measurements is particularly noteworthy because it does not depend on any possible source evolution, and is strictly based on the geometry of the Universe through the changing ratio of angular to spatial/redshift size of spherically symmetric distributions with distance. With  $\sim 4$  per cent accuracy now available for the positioning of the BAO peak, the latest measurements appear to rule out the standard model at better than  $\sim 2.6\sigma$  (Melia & López-Corredoira 2016). This in itself is rather important, but is even more significant because, in contrast to  $\Lambda$ CDM, these same data suggest that the probability of  $R_h = ct$  being correct is close to unity.

Many of these model comparisons, however, are not yet conclusive when the astrophysical processes underlying the behaviour of the sources are still not fully understood. For example, in the case of the premature appearance of supermassive black holes (at redshifts  $\sim 6$ – $7$ ), we may simply be dealing with the creation of massive seeds ( $\sim 10^5 M_\odot$ ) in the early Universe, or episodic super-Eddington accretion, rendering their timeline consistent with the predictions of the standard model. Similarly, the early appearance of galaxies (at  $z \sim 10$ – $12$ ) may itself be due to anomalously high star formation

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rates during a period, e.g. the transition from Population III to Population II stars, that is still in need of further theoretical refinement.

So we now find ourselves in a situation where some tension is emerging between the predictions of the standard cosmology and the high precision measurements (e.g. the Alcock–Paczyński effect based on BAO peak localization), which suggests that either the underlying cosmological model needs to evolve – perhaps in the direction of  $R_h = ct$  – or that we need to refine our understanding of the physics responsible for the behaviour of the sources we use for these measurements. There is therefore ample motivation for finding new, even better and more precise means of comparing different models.

The purpose of this letter is to demonstrate that a measurement of the redshift drift over a baseline of 5–20 yr should provide the best test yet for differentiating between  $\Lambda$ CDM and  $R_h = ct$ , with an eventual preference of one cosmology over the other at a confidence level approaching  $3\sigma$ – $5\sigma$ .

## 2 REDSHIFT DRIFT

The redshift drift of objects in the Hubble flow is a direct non-geometric probe of the dynamics of the Universe that does not rely on assumptions concerning gravity and clustering (Sandage 1962). It merely requires the validity of the cosmological principle, asserting that the Universe is homogeneous and isotropic on large scales. For a fixed comoving distance, the redshift of a source changes with time in a Universe with a variable expansion rate, so its first and second derivatives may be used to distinguish between different models (Corasaniti, Huterer & Melchiorri 2007; Quercellini et al. 2012; Martins et al. 2016). High-resolution spectrographs (Loeb 1998; Liske et al. 2008), such as the Extremely Large Telescope high resolution spectrometer (ELT-HIRES) (Liske et al. 2014), will allow measurements in the approximate redshift range  $2 \lesssim z \lesssim 5$ . Below  $z \sim 1$ , measurements may be made with the Square Kilometer Phase 2 Array (SKA) (Kloekner et al. 2015), and possibly also with 21 cm experiments, such as the Canadian Hydrogen Intensity Mapping Experiment (CHIME) (Yu, Zhang & Pen 2014).

The relevant equations for the first (and second) time derivatives of the redshift in the context of  $\Lambda$ CDM have been derived by, e.g. Weinberg (1972), Liske et al. (2008) and Martins et al. (2016), and we here simply quote their key results. Defining the cosmological redshift,  $z$ , between the time of emission,  $t_e$ , when the expansion factor  $a(t)$  had the value  $a_e$ , and the time of observation,  $t_0$ , when  $a(t) = a_0$ , as

$$1 + z = \frac{a_0}{a_e}, \quad (1)$$

we may write its derivative with respect to the current cosmic time as

$$\frac{dz}{dt_0} = [1 + z(t_0)]H(t_0) - \frac{a_0}{a_e^2} \frac{da_e}{dt_e} \frac{dt_e}{dt_0}, \quad (2)$$

where, by definition,

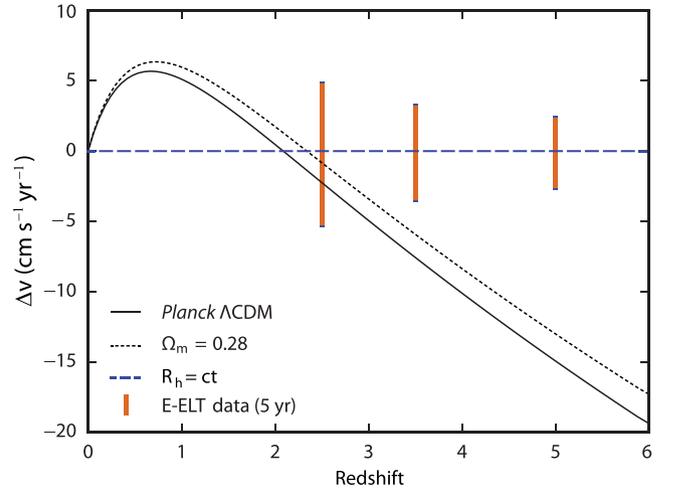
$$H(t) = \frac{1}{a(t)} \frac{da(t)}{dt} \quad (3)$$

is the Hubble constant at time  $t$ . But

$$dt_0 = [1 + z(t_0)] dt_e, \quad (4)$$

so

$$\frac{dz}{dt_0} = [1 + z]H_0 - H(z). \quad (5)$$



**Figure 1.** Expected velocity shift  $\Delta v$  associated with the redshift drift for the *Planck*  $\Lambda$ CDM model ( $k = 0$ ,  $\Omega_m = 0.3$ ,  $H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ; solid black), a slight variation with  $\Omega_m = 0.28$ , and the  $R_h = ct$  universe (blue long dash), in which  $\Delta v = dz/dt_0 = 0$  at all redshifts. Also shown are the expected  $1\sigma$  errors (red) at  $z = 2.5, 3.5$ , and  $5.0$  with the ELT-HIRES after 5 yr of monitoring (adapted from Martins et al. 2016).

Here, we may write

$$H(z) = H_0 E(z), \quad (6)$$

where  $H_0 \equiv H(t_0)$  is the Hubble constant today, and

$$E^2(z) = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_{de}(1+z)^{3(1+w_{de})} + \Omega_k(1+z)^2. \quad (7)$$

In this expression,  $\Omega_m$ ,  $\Omega_r$  and  $\Omega_{de}$  are the ratios of energy density  $\rho_m$  (matter),  $\rho_r$  (radiation) and  $\rho_{de}$  (dark energy) to the current critical density  $\rho_c \equiv 3c^2 H_0^2 / 8\pi G$ , and  $w_{de}$  is the dark-energy equation-of-state parameter,  $w_{de} \equiv p_{de} / \rho_{de}$ . In addition, the ratio  $\Omega_k \equiv -kc^2 / (a_0^2 \rho_c)$  is non-zero when the Universe is spatially curved, i.e. when  $k \neq 0$ .

During a monitoring campaign, the surveys measure the spectroscopic velocity shift  $\Delta v$  associated with the redshift drift  $\Delta z$  over a time interval  $\Delta t$ . These two quantities are related via the expression

$$\Delta v = \frac{c \Delta z}{1+z} = \frac{c \Delta t}{1+z} \frac{dz}{dt_0}. \quad (8)$$

In Fig. 1, we plot  $\Delta v$  as a function of  $z$  for the *Planck*  $\Lambda$ CDM model (defined by the parameter values  $k = 0$ ,  $\Omega_m = 0.3$ ,  $H_0 = 67.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $w_{de} = -1$ ; solid black curve), and a slight variation (short dash black curve) with  $\Omega_m = 0.28$  to illustrate the change expected with redshift and alternative values of the parameters.

In  $R_h = ct$ , the situation is much simpler. Since  $a(t) = t/t_0$  in this universe (see e.g. Melia 2007, 2016, 2017; Melia & Shevchuk 2012), we have

$$H(t) \equiv \frac{\dot{a}}{a} = \frac{1}{t}. \quad (9)$$

But from equation (1), we also see that  $(1+z) = t_0/t_e$ . Therefore,  $(1+z) = H(t_e)/H(t_0)$ , or

$$H(t_e) = H(t_0)[1 + z(t_0)]. \quad (10)$$

Thus, we conclude from equation (5) that

$$\frac{dz}{dt_0} = 0, \quad (11)$$

and therefore

$$\Delta v = 0 \quad (12)$$

at all redshifts, shown as the blue long-dashed line in Fig. 1.

According to Liske et al. (2008), the ELT-HIRES is expected to observe the spectroscopic velocity shift with an uncertainty of

$$\sigma_{\Delta v} = 1.35 \frac{2370}{S/N} \sqrt{\frac{30}{N_{\text{QSO}}}} \left( \frac{5}{1+z_{\text{QSO}}} \right)^\alpha, \quad (13)$$

where  $\alpha = 1.7$  for  $z \leq 4$ , and  $\alpha = 0.9$  for  $z > 4$ , with an S/N of approximately 1500 after 5 yr of monitoring  $N_{\text{QSO}} = 10$  quasars in each of three redshift bins at  $z = 2.5, 3.5$  and  $5.0$ . These uncertainties (shown in red) are approximately 12, 8 and  $5 \text{ cm s}^{-1} \text{ yr}^{-1}$  at these redshifts. With a baseline of 20 yr, the uncertainties are reduced to approximately 6, 4 and  $3 \text{ cm s}^{-1} \text{ yr}^{-1}$ , respectively. Thus, we may already see a  $\sim 3\sigma$  difference between the predicted velocity shifts in these two models at  $z = 5$  after only 5 yr. The difference increases to  $\sim 5\sigma$  after 20 yr.

### 3 CONCLUSION

Unlike many other types of cosmological probes, a measurement of the redshift drift of sources moving passively with the Hubble flow offers us the possibility of watching the Universe expand in real time. It does not rely on integrated quantities, such as the luminosity distance, and therefore does not require the pre-assumption of any particular model. Over the next few years, it will be possible to monitor distant sources spectroscopically in order to measure the velocity shift associated with this redshift drift. An accessible goal of this work ought to be a direct one-on-one comparison between the  $R_h = ct$  universe and *Planck*  $\Lambda$ CDM. The former firmly predicts zero redshift drift at all redshifts, easily distinguishable from all other models associated with a variable expansion rate. Over a 20-yr baseline, these measurements will strongly favour one of these models over the other at a  $\sim 5\sigma$  confidence level. However, the velocity shifts predicted by these two models at  $z \sim 5$  are so different ( $-15 \text{ cm s}^{-1} \text{ yr}^{-1}$  for the former versus  $0 \text{ cm s}^{-1} \text{ yr}^{-1}$  for the latter), that even after only 5 yr the expected  $\sigma_{\Delta v} \sim 5 \text{ cm s}^{-1} \text{ yr}^{-1}$  accuracy of the ELT-HIRES may already be sufficient to rule out one of these models relative to the other at a confidence level approaching  $3\sigma$ .

Should  $R_h = ct$  emerge as the correct cosmology, the consequences are, of course, quite profound. The reason is that, although certain observational signatures, such as the luminosity distance, can be somewhat similar for these two models at low redshifts (see fig. 3 in Melia 2015), they diverge rather quickly in the early Universe. So much so that while inflation is required to solve the horizon problem in  $\Lambda$ CDM, it is not needed (and probably never happened) in  $R_h = ct$  (Melia 2013a).

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### REFERENCES

- Corasaniti P.-S., Huterer D., Melchiorri A., 2007, *Phys. Rev. D*, 75, 062001  
 Eisenstein D. J. et al., 2005, *ApJ*, 633, 560  
 Haiman Z., Mohr J. J., Holder G. P., 2000, *ApJ*, 553, 545  
 Kloeckner H.-R. et al., 2015, *Proc. Sci.*, Advancing Astrophysics with the Square Kilometre Array (AASKA14). p. 027, available at: <http://pos.sissa.it/cgi-bin/reader/conf.cgi?confid=215>  
 Liske J. et al., 2008, *MNRAS*, 386, 1192  
 Liske J., the E-ELT Project Science Team, 2014, Top Level Requirements For ELT-HIRES, Document ESO 204697 Version 1  
 Loeb A., 1998, *ApJ*, 499, L111  
 Martins C. J. A. P., Martinelli M., Calabrese E., Ramos M. P. L. P., 2016, *Phys. Rev. D*, 94, 043001  
 Melia F., 2007, *MNRAS*, 382, 1917  
 Melia F., 2012, *AJ*, 144, 110  
 Melia F., 2013a, *A&A*, 553, A76  
 Melia F., 2013b, *ApJ*, 764, 72  
 Melia F., 2014a, *AJ*, 147, 120  
 Melia F., 2014b, *A&A*, 561, A80  
 Melia F., 2015, *Ap&SS*, 356, 393  
 Melia F., 2016, *Frontiers Phys.*, 11, 119801  
 Melia F., 2017, *Frontiers Phys.*, 12, 129802  
 Melia F., López-Corredoira M., 2016, *Proc. R. Soc. A*, preprint (arXiv:1503.05052)  
 Melia F., McClintock T. M., 2015, *Proc. R. Soc. A*, 471, 20150449  
 Melia F., Shevchuk A., 2012, *MNRAS*, 419, 2579  
 Percival W. J., Cole S., Eisenstein D. J., Nichol R. C., Peacock J. A., Pope A. C., Szalay A. S., 2007, *MNRAS*, 381, 1053  
 Perlmutter S. et al., 1999, *ApJ*, 517, 565  
 Pritchard J. R., Furlanetto S. R., Kamionkowski M., 2007, *MNRAS*, 374, 159  
 Quercellini C., Amendola L., Balbi A., Cabella P., Quartin M., 2012, *Phys. Rep.*, 521, 95  
 Refregier A., 2003, *ARA&A*, 41, 645  
 Riess A. G. et al., 1998, *AJ*, 116, 1009  
 Sandage A., 1962, *ApJ*, 136, 319  
 Seo H.-J., Eisenstein D. J., 2003, *ApJ*, 598, 720  
 Wei J.-J., Wu X., Melia F., Maier R. S., 2015, *AJ*, 149, 102  
 Weinberg S., 1972, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. Wiley, New York, p. 451  
 Yu H.-R., Zhang T.-J., Pen U.-L., 2014, *Phys. Rev. Lett.*, 113, 041303

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