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THE IONIZATION OF PLANETARY NEBULAE

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ABSTRACT

The ionization of the most abundant elements in planetary nebulae has been determined for a number of models of nebulae at different epochs in their expansion. The values used for the temperatures and radii of the central stars and the sizes and densities of the shells have come from Seaton's evolutionary sequence. The ionizing radiation field has been taken from model atmosphere calculations of the central stars by Gebbie and Seaton, and Böhm and Deinzer. Emission-line fluxes have been calculated for the models and compared with observations of planetary nebulae by O'Dell, Osterbrock's group, and Aller and his collaborators. Results indicate that the central stars have strong He^+ Lyman continuum excesses, similar to those predicted by Gebbie and Seaton. The mean abundance determinations for the nebulae made by Aller are confirmed, with the exception of nitrogen, which appears to be 3 or 4 times more abundant than his value. It is also seen that the electron temperatures of the nebulae are higher than previous theoretical determinations, providing better agreement with empirically derived values.

I. INTRODUCTION

The basic processes which govern the ionization structure of planetary nebulae are believed to be reasonably well understood. Ionization of the elements is caused by the far-ultraviolet radiation from the central star, hence the fundamental problem to be solved in determining the statistical equilibrium is that of the radiative transfer of this radiation within the gaseous shell. A knowledge of the state of ionization of all the elements is useful because this information can be used to predict the intensities of the emission lines of the nebulae. Comparison of these theoretical intensities with those actually observed then provides a check on the validity of the theory, and gives information on the physical parameters of the gas and the radiation field of the central star.

Hummer and Seaton (1963, 1964) have recently considered in detail the ionization of planetary nebulae for nebulae composed of hydrogen and helium, and excited by a star radiating like a black body. They did not calculate any emission-line intensities because the heavy elements were not included in their work, and also because little was known at that time about the far-ultraviolet distribution of radiation from the central stars and the physical conditions within the nebulae at different stages in their expansion. However, our present knowledge of these quantities is greatly improved. O'Dell (1963a), Harman and Seaton (1964), and Seaton (1966) have delineated an evolutionary sequence for planetary nebulae which allows us to attach some statistically meaningful size and density to these objects, in addition to the temperatures and radii of the central stars, at various stages of evolution. In addition, Gebbie and Seaton (1963) and Böhm and Deinzer (1965, 1966) have computed model atmospheres for the central stars, giving the emergent intensity of radiation at frequencies above the hydrogen Lyman limit. In view of these developments, and the improved techniques used by O'Dell (1963c), Osterbrock's group at Wisconsin, and Aller and his collaborators to obtain emission-line fluxes of planetary nebulae, it seems worthwhile to consider the ionization of the elements in these objects.

II. THEORY

The theory of the determination of the ionization and electron temperature of a gas excited by a known radiation field has been discussed by Hummer and Seaton (1963, 1964) for the elements hydrogen and helium, and extended to include the heavy elements by Williams (1967, referred to hereafter as Paper I). It will not be repeated here except to point out the modifications necessary to determine the mean intensity of ionizing radiation in the gas for a curved geometry, rather than the semi-infinite plane-parallel configuration used in Paper I.

It is well known that the equation of transfer is sufficiently simple for a plane-parallel gas such that the diffuse radiation of such a nebula may be calculated straightforwardly. The introduction of curvature terms into the transfer equation, as would be required in the case of planetary nebulae, make these calculations very difficult. The traditional approach to this problem for planetaries, therefore, has been to make the on-the-spot approximation, which assumes that the diffuse ionizing radiation produced by the gas is immediately reabsorbed. This makes it possible to equate the mean intensity of diffuse radiation at any point with the source function. This is tantamount to requiring the ionization of H and He to remain approximately constant over a distance where the gas is optically thick to the radiation causing the ionization---a situation that is fairly well satisfied in planetary nebulae (Hummer and Seaton 1963, 1964). In this approximation, the mean intensity of radiation from both the star and the gas is

$$J_\nu = \frac{R_s^2}{4r^2} I_\nu^\circ e^{-\tau_\nu} + \sum_i N_e \frac{\chi(i+1)}{\chi(i)} \frac{2h^4}{(2\pi m k T_e)^{3/2}} \frac{\nu^3}{c^2} e^{\frac{h(\nu_i-\nu)}{kT_e}}, \quad (1)$$

in the notation of Paper I, and I_ν° is the emergent intensity of radiation from the central star, R_s is the stellar radius, $\chi(i)$ is the relative abundance of the ion i, and the summation is performed over the ions H^o, He^o, and He^+.

Since the on-the-spot approximation requires detailed balancing to hold in the nebula between the ground state and continuum of the H^o, He^o, and He^+ ions, it can be shown upon substitution of equation (1) into the ionization equation [eqn (1) in Paper I] that the ionization of the elements H and He may be computed by considering photoionizations from the dilute stellar radiation to be balanced by recombinations to their excited states (cf. Hummer and Seaton 1963). This information can then be used in equation (1) to compute the radiation field in the nebula, from which the ionization of the remaining elements in the gas, which are not abundant enough to contribute to the opacity or scattered radiation field of the gas, may be determined. Except for the fact that the ionization caused by the helium resonance lines is ignored and the diffuse radiation is accounted for by the on-the-spot approximation in the present investigation, the procedure used to calculate the ionization of the elements and the electron temperature at each point in the gas is the same as in Paper I. Ion abundances are initially calculated by assuming the gas to have some fixed value of the temperature. These results are then used in the equation of thermal equilibrium [eqn (7) in Paper I] to compute the temperature at all points in the gas. Once the temperature is known, the statistical equilibrium is again determined for each of the elements. This iteration procedure is continued until the values of the $x(i)$ and T_e do not change by more than several per cent after one iteration.

III. THE MODELS

The evolutionary sequence for planetary nebulae is one in which the shell continually expands from a small, dense, optically thick object to one which becomes optically thin, while the star increases in temperature and luminosity. At a point when the nebula becomes optically thin in the hydrogen Lyman continuum, the star undergoes a contraction and decrease in luminosity until the gas eventually becomes optically thick again, in spite of its much lower density. Böhm and Deinzer (1965, 1966) have taken the effective temperatures and surface gravities of the central stars as determined by O'Dell (1963a) and Harman and Seaton (1964) at different epochs in their evolution and have computed model atmospheres for the stars. They give the emergent flux of radiation for several non-gray models at different effective temperatures, including in their calculations the contribution made to the opacity by the heavier elements. Prior to the time when the evolutionary tracks of the central stars in the H-R diagram had been outlined, Gebbie and Seaton (1963) had performed similar model atmosphere calculations. Since little was known about the surface gravity of the stars at the time of their work, they assumed g to have the minimum value capable of keeping the atmosphere stable to radiation pressure. Furthermore, unlike Böhm and Deinzer, they did not include the heavier elements as sources of opacity. As a result, the principal difference in the frequency distribution of radiation between the models of Gebbie and Seaton and those of Böhm and Deinzer is that the smaller gravity used by the former enhances the effect of electron scattering and results in a considerable He^+ Lyman excess for the hot models.

On the other hand, the latter find that the consideration of a higher gravity and the appreciable absorption of the ions of nitrogen, oxygen, and neon at high frequencies leads to a large He^+ Lyman deficiency.

We have taken both types of models and have fitted interpolation formulae to the curves giving the flux as a function of frequency, and have used these data to calculate the ionization in the shell. For lack of a better geometrical representation that can be applied generally to planetaries, we have assumed the gas in the shell to be distributed spherically symmetric at constant density. If we assume the "filling factor" ϵ has the average of the values found by O'Dell (1962) and Seaton (1966), $\epsilon = 0.67$, then for a uniform shell of gas $\epsilon = 1 - (R_1/R_2)^3$ where R_1 and R_2 are the inner and outer radii of the nebula, respectively, and we have the condition that $R_1 = 0.69\ R_2$. With the exception of the density, N, of the gas, all the parameters necessary for the calculations may be obtained from Seaton's (1966, Table IV) article. He gives the Zanstra temperature of the star T_S, the stellar radius R_S, and the outer radius of the nebular shell. At any epoch in the expansion, the density of the gas can be determined by assuming that planetary nebulae are all objects of the same mass, at different stages of expansion. Under this assumption, it is obvious that the quantity ($N_e \epsilon R_2^3$) = constant for optically thin nebulae, where the constant can be obtained from observations of the surface brightness of the nebulae. We use Seaton's value of log ($N_e \epsilon R_2^3$) = 0.47, derived for $T_e = 10^4$°K, where R_2 is in parsecs.

Two different effective temperatures of the central stars have been used in calculating the models of planetary nebulae: 63100°K, which is representative of cooler, low-excitation objects, and 100000°K, which is typical of the older, higher-excitation nebulae. The exact temperatures used were, of course, dictated by the available model atmospheres. Böhm and Deinzer have central star models at both of the temperatures given; however, Gebbie and Seaton have one at only the higher temperature. The latter have two other models published, but neither could be used in the present calculations. Their hottest model, with T_S = 200000°K, is considerably hotter than any of the temperatures normally encountered on the evolutionary track, whereas their coolest model, with T_S = 41700°K, does not give the flux above the ionization limit of He^+. Consequently, a low temperature model has been built assuming the ionizing radiation field to be Planckian. In all models the abundances of the elements have been taken to be those found by Aller (1961) for planetary nebulae, where, by number, the logarithms of the relative abundances of H, He, N, O, and Ne are 12.00, 11.18, 8.37, 8.77, and 8.05, respectively.

The results of the ionization and electron temperature calculations for four models are shown in Figures 1-5. The ionization curves give the relative abundance of the ions of each of the elements at every point in the gas. Also plotted are the optical depths of the gas at the ionization edges of the ions H^o, He^o, and He^+. With the exception of the second model, the incident ionizing stellar radiation has been taken from the model atmospheres previously mentioned. All other pertinent information concerning the nebulae have been obtained from the data given by Seaton (1966). The parameters used for each of the models are as follows: (1) Stellar radiation from Böhm and Deinzer (1966, Figure 3b) model atmosphere, T_S = 63100°K, R_S = 1.20 $R_\odot$, N = 9.0 x 10^3 cm^{-3}, R_2 = 0.080 pc; (2) Stellar radiation black body, T_S = 63100°K, remaining quantities the same as Model 1; (3) Stellar radiation from Böhm and Deinzer (1966, Figure 4b) model atmosphere, T_S = 100000°K, R_S = 0.10 $R_\odot$, N = 1.50 x 10^2 cm^{-3}, R_2 = 0.31 pc; (4) Stellar radiation from

Gebbie and Seaton (1963, Figure 1) model atmosphere, T_S = 100000°K, remaining quantities the same as Model 3.

The scale of the graphs does not permit the abundances of the neutral atoms of the heavy elements to be shown. In all of the models, the abundance ratio of singly ionized ions to neutral atoms for these elements is fairly uniform throughout the gas, and approximately equal to the value 3 x 10^3. Therefore, the fractional abundance of N^o, O^o, and Ne^o in the models varies from 10^{-7} to 10^{-5}.

IV. DISCUSSION OF THE RESULTS

The ionization curves show that for the lower temperature models the elements H and He are primarily singly ionized, while the heavy elements are predominantly doubly and triply ionized. At the higher temperature, the ionization is greater because of the shift in the radiation to higher frequencies. In general, one finds nitrogen to be more highly ionized than the other heavy elements in all of the models. This is due to the fact that nitrogen, unlike the other heavy elements, has an ionization potential for the third stage of ionization which is less than that of He^+. The strong absorption of He^+ for $\lambda < 228$ Å gives rise to a very weak radiation field for this region of the spectrum for all but the inner portions of the gas. Because the N^{+2} can be ionized by radiation with 261 Å $> \lambda >$ 228 Å, which is unaffected by any He^+ Lyman discontinuity in the central star or absorption by He^+ in the gas, the nitrogen has a large fraction of ions in the fourth stage of ionization, even for low-excitation nebulae. Of considerable interest, also, is the pronounced difference in the abundances of the higher stages of ionization of the heavy elements between Models 3 and 4. Although the effective temperatures of the central stars of both models are the same, the disparity in ionization is due to the large difference, by as much as four orders of magnitude, in the fluxes of the stellar continua at the high frequencies that cause ionization of these ions. The same effect is also evident, though to a lesser extent, in Models 1 and 2.

Because each of the models presented here is density-bounded---the nebulae do not become ionization-bounded at these stellar temperatures until $\tau_{912\,\text{Å}} \sim 100$---except for some of the more highly ionized ions, there is no pronounced stratification of the ions. This feature appears only at very early and very late evolutionary stages, when the nebulae are optically thick (cf. Seaton 1966). When this does occur, there appears a small region near the periphery of the shell, where the heavy elements are virtually all singly ionized. However, even then, this region occupies less than 20 per cent of the volume of the shell. In such cases, the heavy elements are generally in the third (and fourth, for nitrogen) stage of ionization for all but the very outer portions of the gas.

The electron temperatures of the models are presented in Figure 5. Except for Model 4, all have temperatures in the vicinity of 10000°K. These values are about 2000° higher than those obtained by Osterbrock (1965) from similar calculations. Part of this difference is due to the rough estimates Osterbrock made for the ionization of the heavy elements; however, the principle cause of the discrepancy is the use of different element abundances in the two investigations. The primary coolants in the nebulae are neon and oxygen, and our abundances of these elements are 5 and 1.5 times less, respectively, than those used by Osterbrock. In order to determine the effect this has on the resultant temperatures, some of our thermal equilibrium calculations were repeated using Osterbrock's abundances. The values of T_e obtained were in the range 7000°-9000°K, which is what Osterbrock found from his calculations.

The higher electron temperature of Model 4 is caused by the large excess in the emergent intensity of the stellar radiation for this model at high frequencies, which is very efficient in heating the gas. Evidence will be presented shortly which indicates that the He^+ Lyman excess is a common characteristic of most of the central stars. If so, the sensitivity of T_e to this feature leads us to expect electron temperatures in the range 10000°-17000°K for most nebulae. This is in satisfactory agreement with the temperatures of planetary nebulae found by Liller and Aller (1954, 1963) and O'Dell (1966) from measurements of the emission lines. Certainly the theoretical calculations of electron temperatures in the nebulae need no longer be considered significantly lower than one should expect on the basis of empirical determinations.

Little can be said about the validity of the individual models from a knowledge of only the ionization of the gas. Most of the available information concerning conditions in nebulae come from observations of the emission-line fluxes. In recent years, photoelectric techniques have been used to good advantage to secure intensities of the lines in a number of planetary nebulae. Consequently, there is sufficient data with which to compare theoretical results such that some conclusions can be drawn about the overall structure of the nebulae. We have calculated the emergent flux of radiation, πF_ν, in a number of the emission lines for the models that have been computed, using the equation which is derived for an optically thin medium possessing spherical symmetry,

$$\pi F_\nu = \frac{4\pi}{R_2^2} \int_{R_1}^{R_2} j_\nu(r)\, r^2\, dr \,, \tag{2}$$

where $j_\nu(r)$ is the volume emission coefficient in the line at a distance r from the center of the nebula. The emission coefficients used for the lines of the HI Balmer series and HeII Paschen series were those determined most recently by Pengelly (1964), while the emissivity of the HeI recombination lines and the collisionally excited lines of the heavy elements has been taken from Seaton (1960). The collision strengths used in the calculations were the same as those given in Paper I. The resultant fluxes of the various emission lines are listed in Table 1, and are given relative to Hβ, whose absolute flux is then given for each of the models as the last entry in the Table.

In order for the comparison between the theoretical and observed fluxes to be meaningful, we must have a number of observations of planetary nebulae at the same expansion epoch as those of our models. Because of the limited data available for nebulae at any given epoch, and the uncertainty in the physical parameters of the individual nebulae, the selection of specific representative objects with which to compare the models is impractical. Instead, we have chosen to plot the intensities of the lines of as many nebulae as possible for which observations exist as a function of some parameter which characterizes the stage in the evolution. The most widely determined parameter that fulfills this purpose is the Zanstra temperature of the central star. The recent evaluations of central star temperatures for a number of the nebulae by Harman and Seaton (1966) are particularly useful in this connection because of their improved accuracy. We have therefore taken those nebulae for which Harman and Seaton have determined the stellar temperatures and have plotted all observed fluxes of the lines for which calculations were made with respect to the temperature of the star, as shown in Figure 6. Also depicted, with different symbols, are the flux calculations from the models, as given in Table 1. The flux of each line is given relative to Hβ. All of the uncorrected data have been corrected for interstellar reddening using values of the reddening constant, c, given by Harman and Seaton for each nebula. With few exceptions, the observed data have been taken from the following sources: all HeI λ4471 and HeII λ4686 measurements are those given by Harman and Seaton (1966). Intensities of [OIII] λ5007 are those listed by Collins, Daub, and O'Dell (1961) and, for several cases, O'Dell (1963b). All but one of the observations of [NII] λ6584, which are few because of the relative scarcity of work done in the red region of the spectrum, come from Osterbrock, Capriotti, and Bautz (1963). The remaining line fluxes were obtained from a number of sources, including Minkowski (1942), Wyse (1942), Aller (1941, 1951), Minkowski and Aller (1956), O'Dell (1963c), and the recent series of papers by Aller and co-workers entitled "Spectrophotometric Studies of Gaseous Nebulae" (cf. Aller, Kaler, and Bowen 1966, for the most recent paper). In all instances where several observations are available for the same object, an average of the measurements has been used, except in cases where photoelectric observations have been made. In the latter event, only the photoelectric measurements are considered in the average.

The two most important factors that govern the intensity of a line are the abundance and ionization concentration of the element. Since we have no *a priori* knowledge that any of our models give a correct representation of the ionization conditions in planetary nebulae, differences between our predicted fluxes and those observed cannot be attributed simply to the use of incorrect element abundances. The relative importance of the two factors must be established. Some statement concerning the effect of abundances can be made by considering the line radiation from ions whose abundances are insensitive to different physical conditions in the gas. It is seen from the ionization curves of the models that the ions He^{+}, N^{+2}, N^{+3}, O^{+2}, and Ne^{+2} satisfy this requirement reasonably well---they are the most abundant ions of the respective elements in both high- and low-excitation objects, showing only minor variations in abundance. This fact is borne out by the similar strengths each of the lines of these ions has for the four models. Also, observational confirmation of the relative constancy in abundance of these ions in different nebulae is given by the smaller scatter of the fluxes of HeI λ4471, [OIII] λ5007, and [NeIII] λ3869 in Figure 6 compared to the other lines. Consequently, we have some assurance that a direct comparison of the theoretical and observational fluxes of these lines will yield information on the abundances of these particular elements. It is seen that there is excellent agreement between the observed and predicted fluxes of the predicted HeI and [OIII] lines. The agreement is likewise good for the [NeIII] line, however, it would be improved if the logarithmic abundance of neon in the models were to be increased by an amount +0.20. Therefore, we find general agreement with the abundances of these elements as derived by Aller (1961) for the nebulae. Because there are no strong NIII or NIV lines in the visible, the abundance of nitrogen must be deduced from [NII] λ6584, which is sensitive to ionization conditions. On the other hand, the confirmation of the abundances of helium, oxygen, and neon enables the lines from other ions of these elements to be used to evaluate the ionization equilibrium of the models.

Let us consider the strength of [OII] λ3727. In spite of a considerable amount of scatter in the observations, it is evident that all of our models predict too small a flux for λ3727 by a factor of about 3, due to an underestimate of the abundance of O^{+} by a similar amount in the computations. In each instance, the models using the atmosphere calculations of Böhm and Deinzer show poorer agreement than the other models, although the differences are slight. The disparity in the O^+ abundance between our models and typical nebulae must be due to our choice of the parameters affecting the statistical equilibrium---the sizes and densities of the nebulae, or perhaps the spectral distribution of the central stars. In view of the uncertainties in the determination of these quantities, the discrepancy should not be considered too serious. It is of some interest to note in this connection, that as a result of the study of planetary nebulae in the Magellanic Clouds, Webster (1966) has criticized the value of the mean mass of the nebulae (0.6 $M_\odot$) as found by Seaton (1966). She believes this is caused by an underestimate of the mean density of several nebulae which were used by Seaton to calibrate his distance scale, resulting in an overestimate of their sizes. If this is correct, the underabundance of O^+ in the models could be explained, since model calculations made with higher densities and smaller dimensions would result in an increased abundance of the singly ionized ions, as long as there is no appreciable stratification.

It is seen from Figure 6 that the calculated strength of [NII] λ 6584 is also less than it should be, by one order of magnitude for models of both temperatures. As was true in the case of the theoretical [OII] λ 3727 fluxes, part of this discrepancy may be attributed to a deficiency in the computed abundance of N^+. Because of the similar ionization potentials of the first two stages of ionization of nitrogen and oxygen, it is probable that the abundance deficiencies of N^+ and O^+ are approximately the same. Correcting for this effect, however, accounts for only half of the difference between the fluxes. The [NII] fluxes of the models are still three times smaller than the observations call for. The only plausible explanation for the remaining disagreement that is consistent with the conclusions reached from the earlier analyses of other spectral lines is that too small a nitrogen abundance has been used in the calculations. On this basis, we suggest that the normal nitrogen abundance in planetary nebulae should be increased to the value log [N(N)/N(H)] = -3.15.

Finally, we consider the fluxes of the lines HeII λ4686 and [NeV] λ3426. The relative abundances of these two elements have already been ascertained, consequently any discrepancies between theory and observation may be attributed to the choice of incorrect parameters which govern the ionization. A great deal of variation in the line strengths of the different models is evident from Figure 6. It is caused by the widely differing intensities of the model atmospheres at frequencies above the He^+ Lyman limit, since it is the radiation from this region which produces He^{+2} and Ne^{+4}. In each of the two models which use the stellar atmospheres of Böhm and Deinzer, both lines have unacceptably small fluxes. On the other hand, at the higher temperature the strengths of the lines for the model which is based upon the atmosphere calculations of Gebbie and Seaton show very good agreement with the observations. The combination of a lack of good data and a considerable amount of scatter in the intensities of the [NeV] line at lower stellar temperatures makes it difficult to pick a representative flux of this line for the low-excitation models. The value log [F(λ3426)/F(Hβ)] = -3.0 is certainly a lower limit. The black body model at T_S = 63100°K fails to produce this much λ3426 radiation, indicating that the central stars of the nebulae must have a considerable He^+ Lyman excess <u>at high frequencies</u> even at this lower stellar temperature. The fact that the strength of λ4686 for this model agrees well with the observational data indicates that the stellar continuum does not differ markedly from that of a

Planckian distribution at wavelengths near 228 Å. Actually, this behavior of the spectral distribution is quite similar to that found by Gebbie and Seaton for the central stars. In view of this fact, the evidence suggests that the spectral distribution of the central stars of both high- and low-excitation nebulae are similar in nature for $\lambda < 228$ Å, and may best be depicted qualitatively in this spectral region by the Gebbie and Seaton model for which $T_S = 1.0 \times 10^{5}$°K.

There is an alternative explanation that might account for the observed HeII and [NeV] line strengths without requiring a strong far-ultraviolet radiation field. The reason Böhm and Deinzer's model atmospheres have so little emergent radiation below 228 Å is because this radiation is absorbed in the atmosphere by He^{+2} and Ne^{+4}, among other ions, whose abundances are high. Recently, evidence has been presented by Mathews (1966) in support of a continuous loss of mass occurring from the central stars in the form of a stellar wind. Could it be that the ions He^{+2} and Ne^{+4}, which are produced in the stars, are injected by such a wind into the nebulae, where the emission lines from these ions are then produced? In order to test this hypothesis, calculations were made to determine the maximum amount of radiation that could be produced in each of the lines in this manner, using several different sets of values for the density and velocity of the stellar wind. It was found that once the ejected gas reaches the nebular shell, recombination of the ionized material occurs in a time scale much too short to enable the required amount of radiation to be emitted. In no case was the flux of either the HeII or [NeV] line found to be within four orders of magnitude of the observed fluxes. Furthermore, the distance over which the recombination takes place is $\lesssim 10^{14}$ cm, which means the radiation would come from a very thin rim around the inside boundary of the shell. Yet, monochromatic images of planetary nebulae obtained by Wilson (1950) and Aller (1956) show that the radiation from λ 3426 and λ 4686 usually extends an appreciable distance into the shell. These considerations make it difficult to associate any HeII and [NeV] emission with a stellar wind.

From the standpoint of attempting to correctly account for the ionization structure of planetary nebulae, there is strong evidence for an ultraviolet excess at frequencies above the He^+ ionization limit. It is an entirely different matter to account for this behavior by constructing model atmospheres of the central stars. The most thorough work of this nature has been done by Böhm and Deinzer, and they predict a strong ultraviolet deficiency. Perhaps the present mass estimates of the central stars, which are uncertain, are incorrect, and therefore the values of the surface gravity are in error. It would be worthwhile to construct models of the stars taking into account the opacity due to the heavy elements, but using lower values of g in order to see what changes in the radiation field would result.

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TABLE 1

EMISSION-LINE FLUXES COMPUTED FROM THE MODELS

Model	Emergent Line Fluxes Relative to Hβ								Absolute Emergent Hβ Flux (ergs cm^{-2} sec^{-1})
	[NeV] λ3426	[OII] λ3727	[NeIII] λ3869	HeI λ4471	HeII λ4686	[OIII] λ5007	[NII] λ6584	[NeII] λ12.79μ	
1	9.95(-11)	6.73 (-2)	6.12 (-1)	6.71 (-2)	8.73 (-4)	1.14 (1)	8.16 (-3)	5.92 (-3)	7.50 (-2)
2	3.24 (-6)	1.02 (-1)	4.17 (-1)	6.67 (-2)	2.43 (-2)	8.30 (0)	2.58 (-2)	1.57 (-2)	8.03 (-2)
3	6.35 (-7)	2.00 (-1)	7.21 (-1)	6.59 (-2)	2.91 (-2)	1.37 (1)	3.45 (-2)	9.75 (-3)	8.65 (-5)
4	1.78 (-1)	2.85 (-1)	7.79 (-1)	3.71 (-2)	7.89 (-1)	1.20 (1)	8.02 (-2)	7.56 (-3)	5.76 (-5)

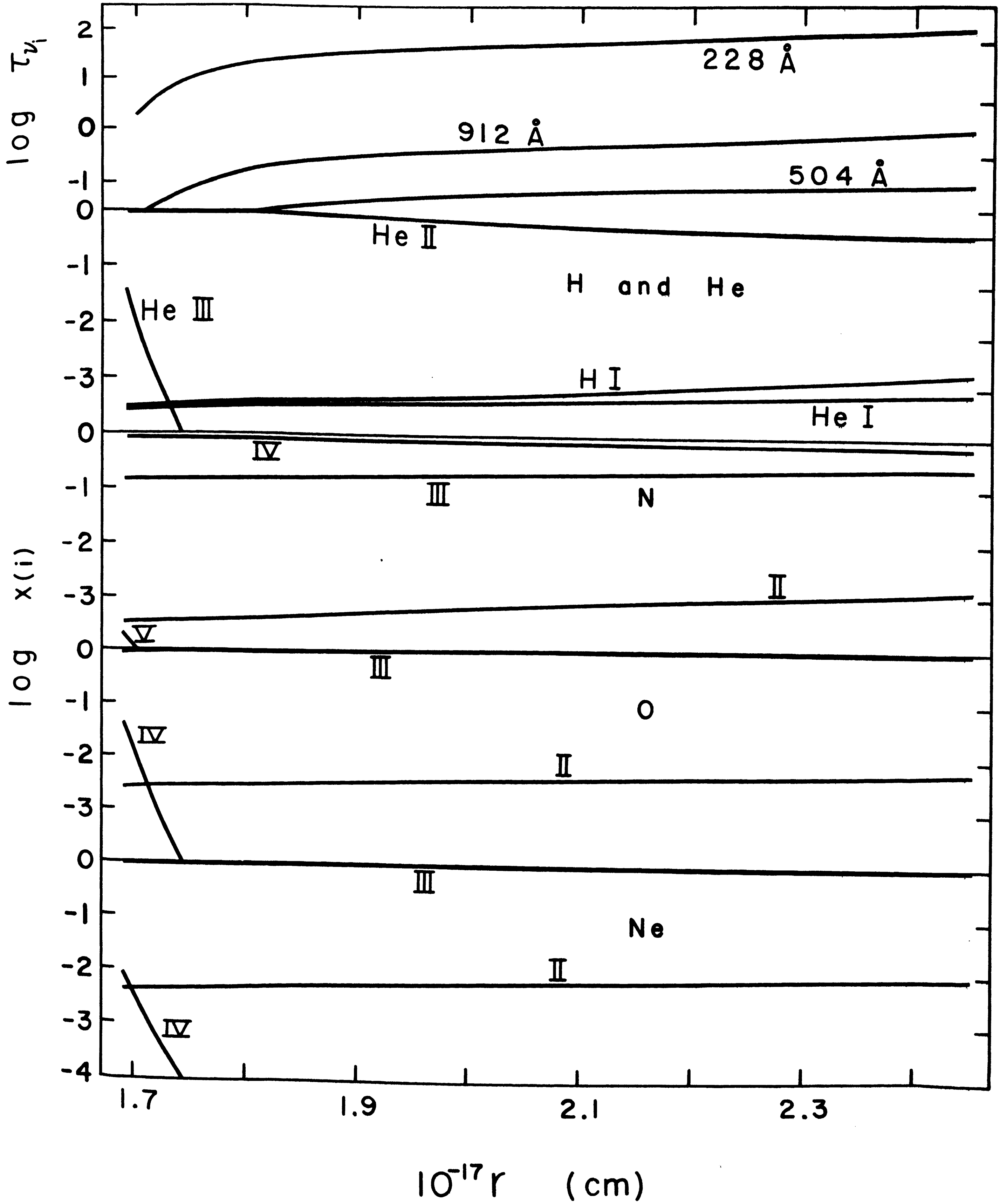

Fig. 1. - The ionization curves and optical depth of the gas for Model 1 (Böhm and Deinzer model atmosphere, T_S = 63100°K).

FIGURE 1

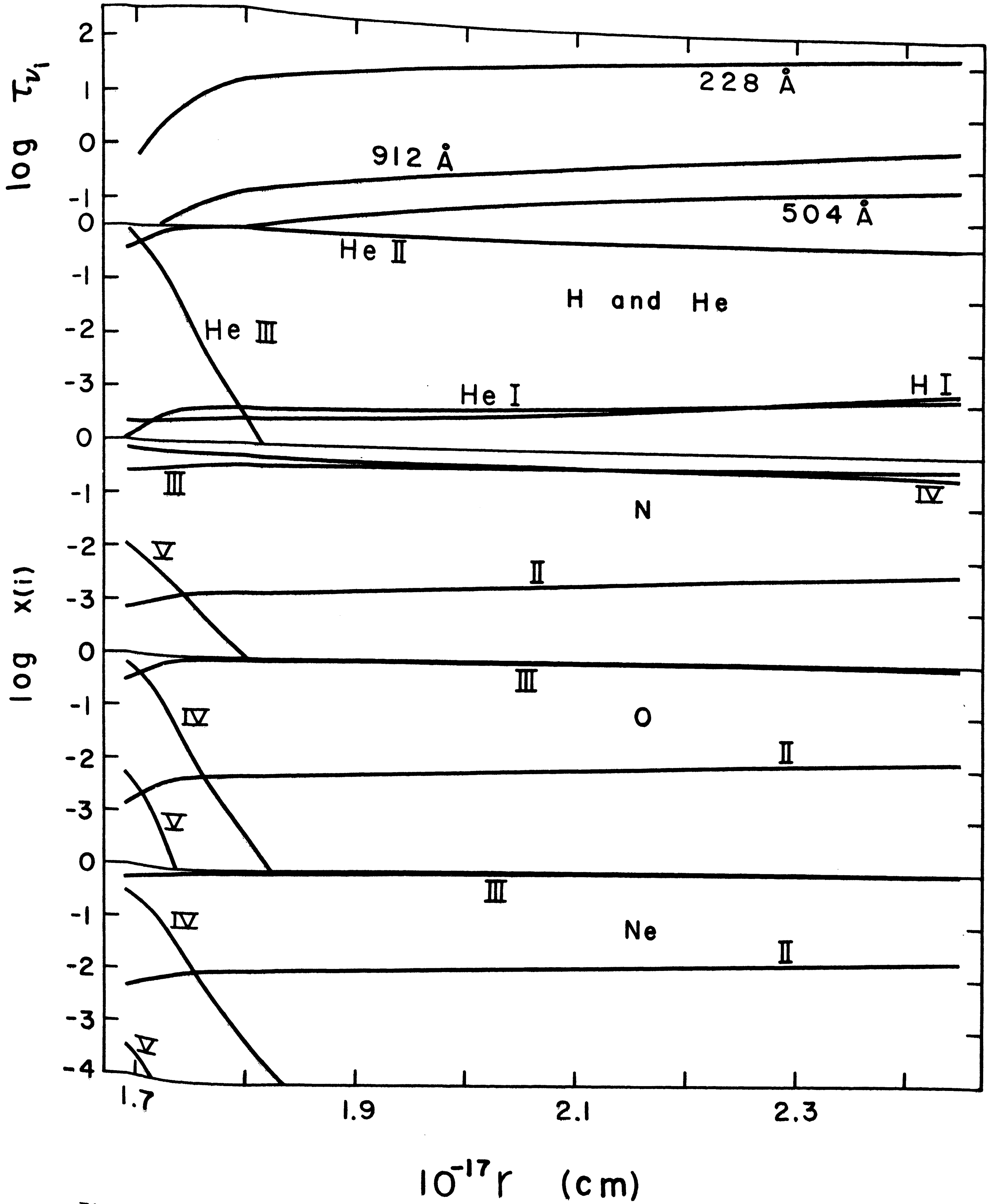

Fig. 2. - The ionization curves and optical depth of the gas for Model 2 (Black body, T_S = 63100°K).

FIGURE 2

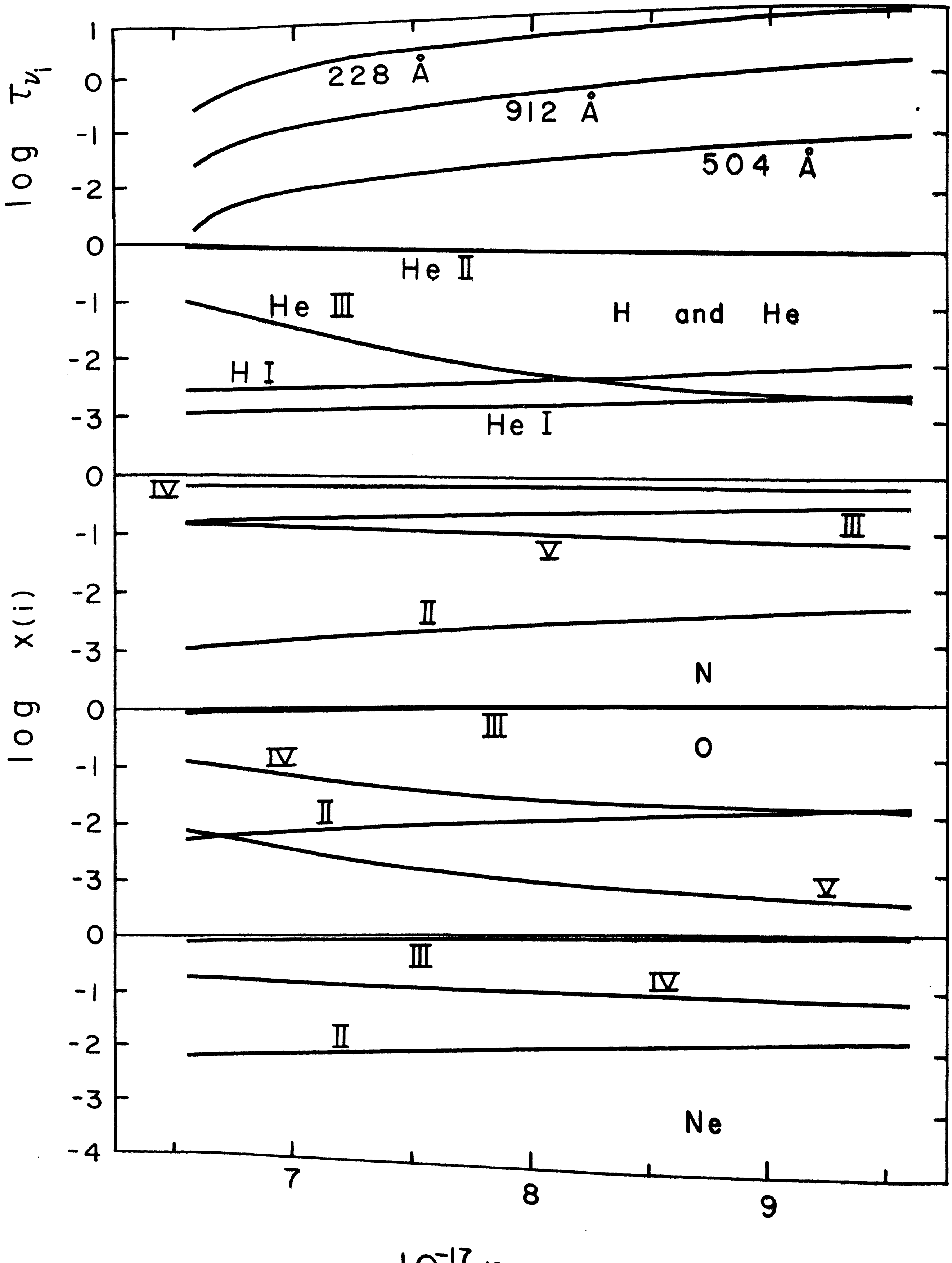

Fig. 3. - The ionization curves and optical depth of the gas for Model 3 (Böhm and Deinzer model atmosphere, T_S = 100000°K).

FIGURE 3

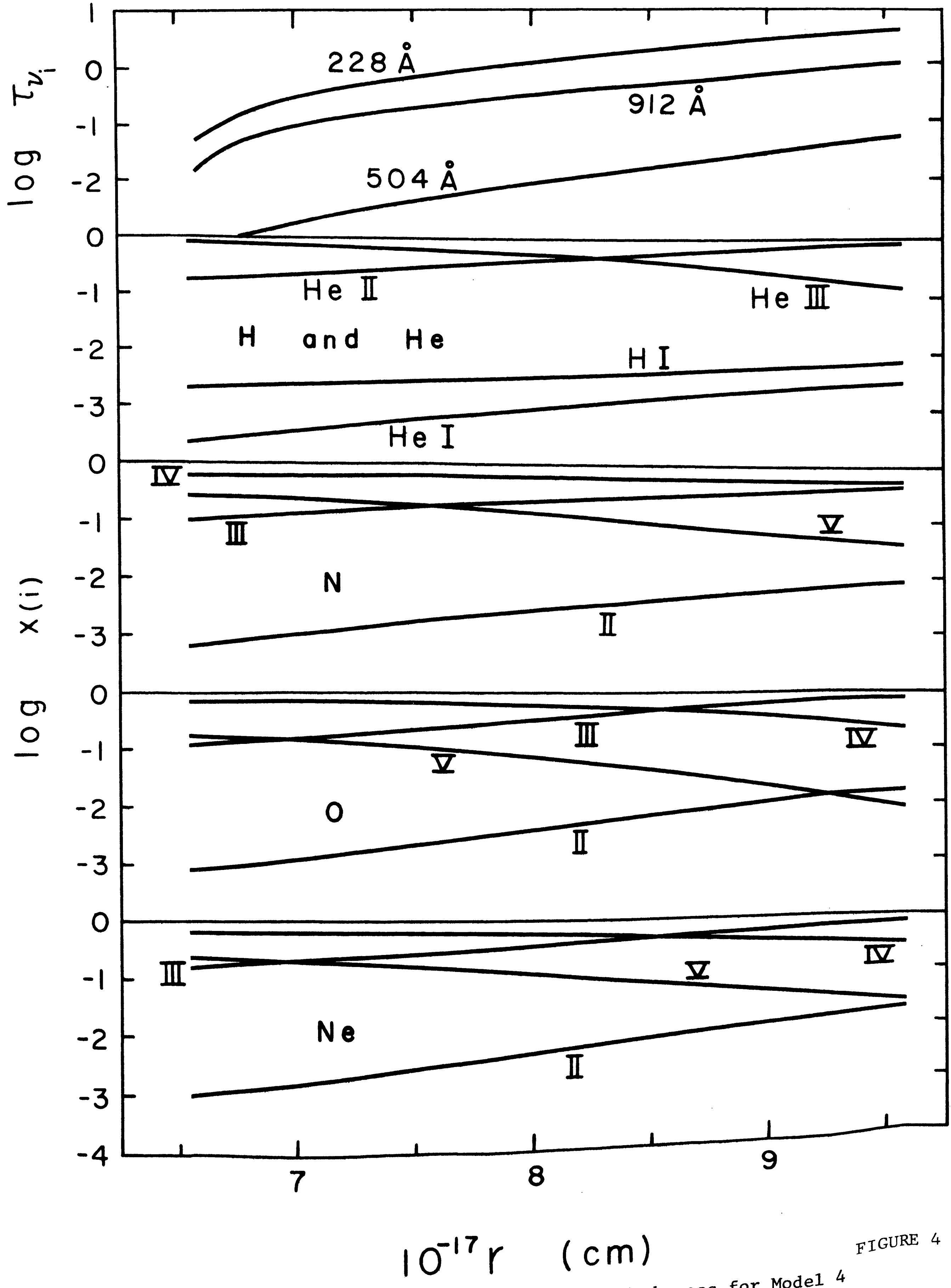

FIGURE 4

Fig. 4. - The ionization curves and optical depth of the gas for Model 4 (Gebbie and Seaton model atmosphere, T_S = 100000°K).

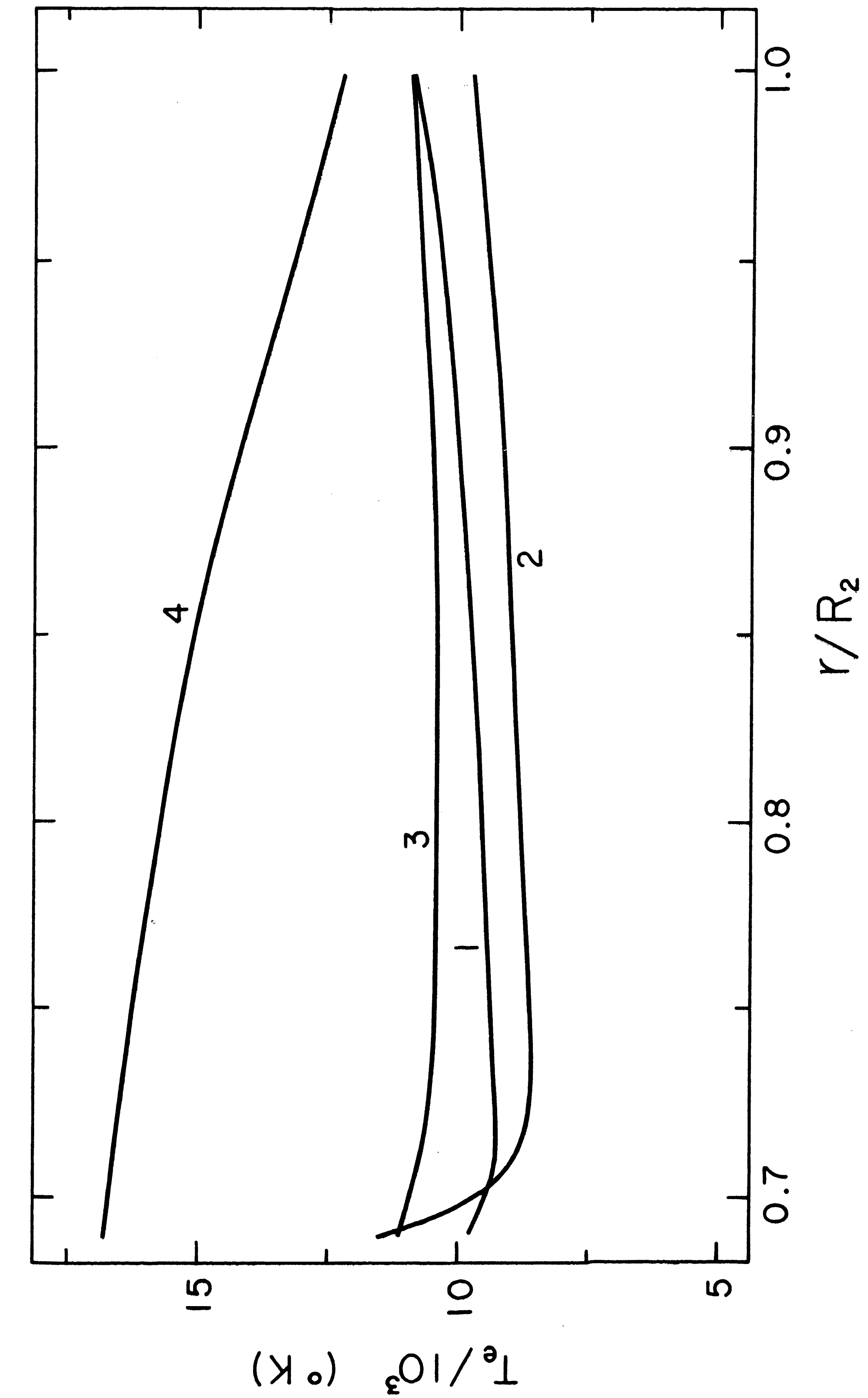

FIGURE 5

Fig. 5. - The electron temperature at each point in the gas for the four models. The distances are given relative to the outer radius of the nebular shell.

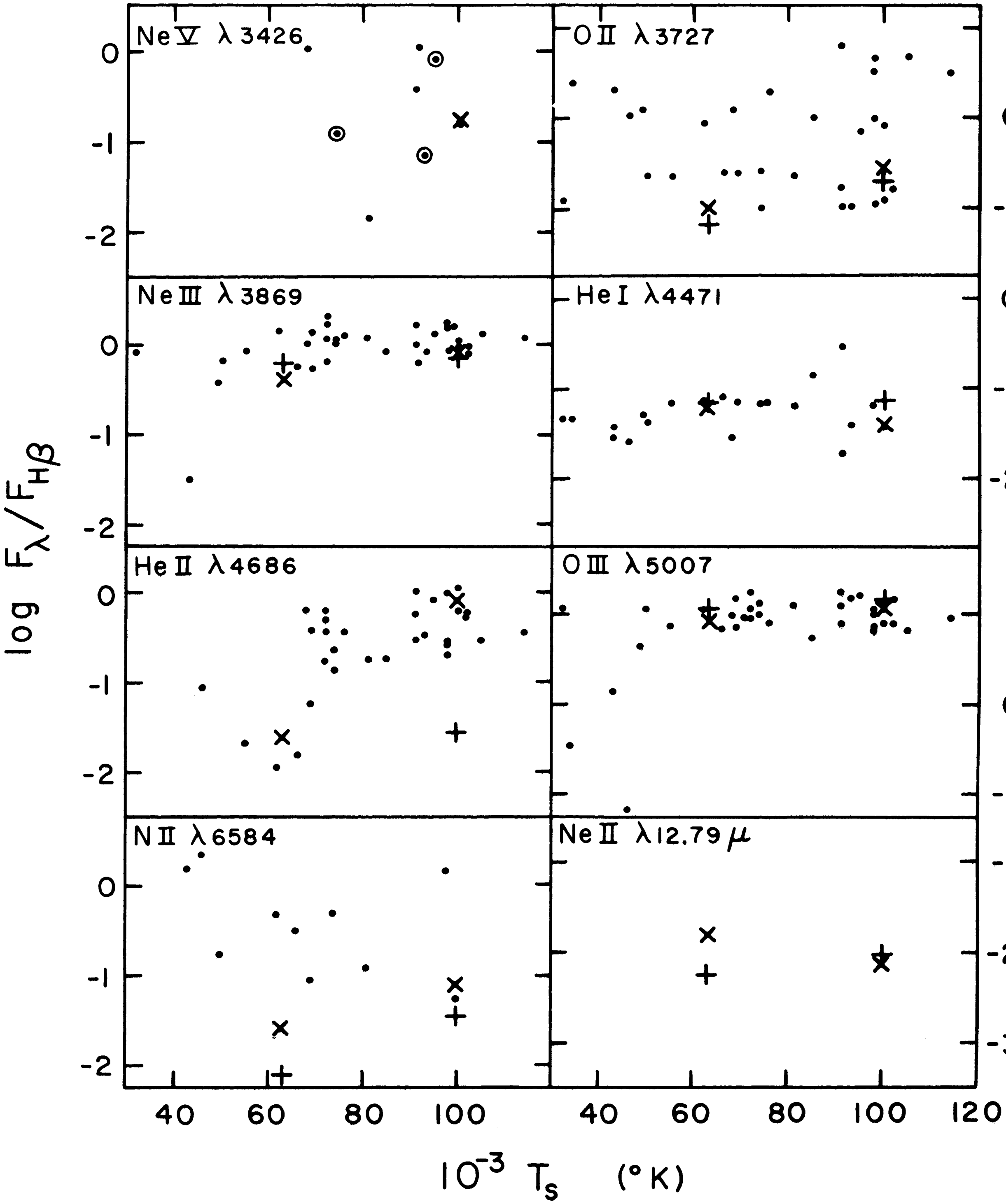

FIGURE 6

Fig. 6. - Emission-line fluxes of planetary nebulae plotted as a function of the temperature of the central star. Only those nebulae are considered for which Harman and Seaton have determined Zanstra temperatures. The filled circles denote observational data. The three measurements of λ3426 which are encircled are uncertain because of blends with OIII λ3444. The computed fluxes for the models are also plotted, using the following symbols: + for Models 1 and 3 (Böhm and Deinzer models), and X for Models 2 (Black body) and 4 (Gebbie and Seaton model). Several of the calculated fluxes for the lines λ 3426 and λ4686 fall outside the scale of the graph, and are not shown. However, for these cases the numerical values can be readily obtained from Table 1. The [NeII] λ12.79μ line is shown, although no published observations of this line are yet available.