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REDSHIFT QUANTIZATION IN THE LYMAN-α FOREST AND THE MEASUREMENT OF q_0

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ABSTRACT

We present evidence for redshift quantization in the Lyman-α forest of several QSOs. The Ly-α data are at redshifts z from 1.89 to 3.74, and the theory of redshift quantization proposed by Cocke (1983, 1985) is used to scale the quantization interval (24.15 km s^{-1}) to these high redshifts. The scaling depends on the deceleration parameter q_0, and the quantization is present at a statistical significance of greater than 99% for $q_0 = 1/2$. This may be taken as confirming the inflationary model of the early history of the universe. The significance of the quantization is highest at $q_0 \approx 0.48$, and the width of the peak is about 0.03 . The result can also be seen as providing confirmatory evidence for both the theory of the redshift quantization and the above value of q_0, but at a significance of only 93%. The scenario proposed for the relativistic generalization of the theory is that of fermion wavefunctions and quantum operators in a background Riemannian spacetime satisfying Einstein's field equations.

I. INTRODUCTION

A recurrent barrier to the acceptance of redshift quantization by the astronomical community has been the apparent lack of a satisfactory theory or model for the phenomenon. The evidence for the quantization seems incompatible with conventional dynamics (Newtonian or relativistic), and attempts to model the effect with large-scale quantum operators have not been viewed sympathetically by the scientific community.

In this paper we use existing theories of the quantization to show clumping in nearest-neighbor differences in the redshifts in the Ly-α forests of several QSOs. In spite of their different points of view, three theoretical treatments all claim that the basic quantization interval (presumably 12.05 km s^{-1}) should be proportional to the square-root of the local value of the Hubble "constant" $H(t)$, here a function of time and therefore of redshift $z(t)$. Thus, one may in effect measure the ratio $\sqrt{H(z)/H(0)}$ at different epochs by detecting and measuring redshift quantization at those epochs. This is tantamount to measuring the deceleration parameter q_0, as we show in the next section.

One of the plausible interpretations of the Ly-α clouds is that they are proto-dwarf galaxies (Sargent 1988), and so one might expect the quantization to be in multiples of $2 \times 12.05 \approx 24.15$ km s^{-1}, as shown by Tifft and Cocke (1984) for dwarf irregulars in the Fisher-Tully catalogue. This is in fact what is found in the Ly-α data.

The statistical analysis of the result takes two different directions, the choice of which depends on one's *a priori* assumptions: (1) If one assumes the validity of the inflationary model of the early universe, so that $q_0 = 1/2$, then the data confirm the quantization and lend confidence to the general ideas implied by the theoretical treatments.

(2) One could assume the reality of the quantization (and the $\sqrt{H(z)/H(0)}$ dependence of the intervals), and the data would then provide a measurement of $q_0 = 0.48 \pm 0.03$, confirming one of the main predictions of the inflationary model. The statistical level of conclusions (1) and (2) is quite high.

(3) If one is willing to assume neither $q_0 = 1/2$ nor the validity of the redshift quantization, then the level of significance is difficult to determine, but we estimate it to be about 93%, which is just below the level required for formal acceptance. In any case, the data and its component subsamples show a higher probability of being drawn from a nonrandom (quantised) population than from a uniform population.

The theory and the data are compatible with representing galaxy components as fermion wavefunctions and dynamical variables as quantum operators, having Riemannian space-time as a background metric. We discuss this in more detail in Section VI.

II. THEORETICAL PRELIMINARIES

The theoretical basis for redshift quantization has been investigated by Cocke (1983 and 1985), Dersarkissian (1985), and Buitrago (1988). These treatments differ considerably in their details, but they are all based on the idea that a new quantum mechanical regime is needed to explain the observed redshift quantization.This quantization has been shown to occur in a number of different contexts, and it does not seem possible to explain these observations by envoking classical dynamics.

Nieto (1986) has considered a variety of relativistic wave equations as candidates for redshift operators. His main result is that first-order relativistic equations have many desirable properties. Among these properties is the possibility of an eigenspectrum that is proportional to the quantum number n, for large n. In particular, an equation involving

the absolute value of the momentum operator yields a redshift operator with this sort of spectrum. This equation seems to have been discussed first by Iachello (1970. See further references in Nieto 1986).

Nieto's redshift operator, however, does not have a part readily identifiable with the Hubble expansion, although it does have a potential term which he takes to be an infinite square well of radius R. In the limit of large n, one can show that the "quantization interval" is proportional to R^{-1}. One might identify R with the age of the universe or with the scale factor $R(t)$ of the Robertson-Walker metric, but there seems no good way to associate this directly with the Hubble parameter $H(t) := \dot{R}/R$. The point is that the Hubble expansion parameter itself should enter directly into the redshift operator.

Some of the theoretical work, when applied to the basic redshift interval $c\Delta z \approx 12.07$ km/s, implies a Planck's constant of about $\hbar_g \approx 2.4 \times 10^{73}$ erg s (Cocke 1983). In a paper that seems at first glance to be unrelated to redshift quantization, Liu, Deng, and Cao (1985) derive a value of 4.2×10^{73} erg s as one of a series of "characteristic actions" in astronomy.

All the quantization treatments referred to at the beginning of this section conclude that the basic redshift interval Δz should be proportional to the square root of the Hubble constant H_0. The above value for $\hbar_g$ found by Cocke (1983) was calculated by assuming $H_0 \approx 90$ km/s/Mpc and a characteristic galaxy component mass of $M \approx 5 \times 10^{10} M_\odot \approx 10^{44}$ g.

The conclusion that $\Delta z \propto \sqrt{H_0}$ at the present epoch implies that information about $H(t)$ as a function of time can be obtained by detecting quantization at past epochs. In the context of standard Einsteinian cosmology, this would amount to a determination of q_0, the deceleration parameter. Lyman-α forest data seem ideal for such a study, for there is a wide range of redshifts available, and many of the data are of high quality.

There is a simple relation (see, for example, Schutz 1986) between $H(t)/H_0$, the redshift $z(t)$, and q_0. One has, for a standard matter-dominated universe with zero cosmological constant, $H(t) = H_0(1+z)\sqrt{2q_0z+1}$, where q_0 is the *current* value of the deceleration parameter. The theory then implies that

$$\Delta z(t) = \Delta z_0\sqrt{H(t)/H_0} = \Delta z_0\sqrt{1+z}(2q_0z+1)^{1/4} \tag{1}$$

Since there is presently no global, relativistic theory of redshift quantization, this paper is limited to studying the differences δz between neighboring Ly-α lines. These differences must be divided by $1+\bar{z}$ to get back to the rest frame between the two lines. Here, of course, if $\delta z = z_{k+1} - z_k$ is the difference between two adjacent redshifts, then $\bar{z} = (z_{k+1}+z_k)/2$. One may then study a normalized velocity difference

$$\begin{aligned}\widetilde{\delta v} &= \sqrt{H_0/H(t)}c\delta z/(1+\bar{z}) \\ &= (1+\bar{z})^{-3/2}(2q_0\bar{z}+1)^{-1/4}c\delta z \quad .\end{aligned} \tag{2}$$

It is plausible that the Ly-α forest clouds are proto–dwarf galaxies (Sargent 1988). Since extreme dwarf irregulars are known to have their redshifts quantized at intervals of 24.15 km s^{-1}, (Tifft and Cocke 1984) we would expect to find some quantization of this sort in the Ly-α redshifts. A histogram of the $\widetilde{\delta v}$s should show the quantization at 24.15 km s^{-1} directly, since the dependence on $H(t)$ has been normalized out. Note that this technique does not depend on knowing the actual value of H_0.

III. THE DATA

The data used in this study were obtained from two sources. The first set was provided by R. F. Carswell and J. K. Webb. These were simply lists of Ly-α redshifts and the

associated standard deviations for the quasistellar objects Q0207-398, Q0420-388, Q1101-264, Q1158-187, Q1358+1134, Q1448-232, Q2204-573, Q2206-199N, and P2000-330. The data had been obtained and reduced as in Carswell *et al.* (1987). Although some Ly-α lines had corresponding Ly-β identifications, only the Ly-α lines were employed.

We used all of the Ly-α redshifts in these lists, except those for which the standard deviations (in z) were greater than 0.00020. This criterion eliminated about 10% of the lines. Of the rest of the lines, a representative standard deviation is about 0.00010, corresponding to a standard deviation in δz of 0.00014. Using equation (2) with $z \approx 2.5$ and $q_0 = 1/2$ leads to a standard deviation in $\widetilde{\delta v}$ of about 5.0 km s^{-1}. This is well within the $\sigma \leq P/3$ required to make a valid test of a periodicity of $P = 24.15$ km s^{-1} with 400 data points (Tifft 1982).

The second set of data were provided by C. B. Foltz in the form of digital spectra of the QSOs UM673A and PHL957; spectra of the latter were published in graphical form by Black, Chaffee, and Foltz (1987), but the data for UM673A are unpublished. Both the spectra were acquired at the Multiple-Mirror Telescope. We used the Steward Observatory IRS data reduction software package to find wavelengths for all spectral features identifiable as lines, longward of the wavelength of Ly-β at the QSO redshift. Metal lines at the redshift of an intervening galaxy were removed, and the measured wavelengths were converted to redshifts. The data acquisition and reduction are described in more detail by Black *et al.* (1987).

The two data sets comprise a sample of 893 redshifts covering a range of z from about 1.89 to 3.74, and just under 400 of their differences were in the proper range to be used for the tests which we describe in the next section.

IV. PROBABILITY ANALYSIS AND THE MEASUREMENT OF q_0

We do the quantization analysis in the context of the inflationary model of the "big bang", which implies that the deceleration parameter should be very nearly 1/2 (Guth 1981). We show that the data are strongly quantized at $q_0 = 1/2$, thus confirming either the redshift quantization or the inflationary model. The data are marginally strong enough to confirm both, provided that one accepts $0 \leq q_0 \leq 1$ as a working hypothesis.

Since the global, relativistic form of the redshift quantization law is not known, we constructed a histogram of the normalized velocity differences $\widetilde{\delta v}$ defined by equation (2), with $q_0 = 1/2$.

The bins in the histogram were 12.075 km s^{-1} wide, designed to look for periodicities of 24.15 km s^{-1}, in accordance with the fact that the redshifts of dwarf irregular galaxies are observed to be quantized at that period (Tifft and Cocke 1984).

The probability analysis follows the same lines as in previous work (for example, Cocke and Tifft 1983). The particular version that we use here is that of the classical coin-toss problem which treats the number of $\widetilde{\delta v}$s that fall in bins near multiples of 24.15 km s^{-1} as "heads" and those that fall in the other bins as "tails". It is easy to show that the probability $P(\geq N_h, N_T)$ of getting N_h *or more* heads out of a total number of tosses $N_T = N_h + N_t$ is

$$P(\geq N_h, N_T) = 2^{-N_T} \sum_{m=N_h}^{N_T} \binom{N_T}{m} \quad , \qquad (3)$$

where $\binom{n}{m}$ is the binomial coefficient.

Note that in this type of test we are evaluating the likelihood of a given h/t ratio against an *assumed* uniformity; a result becomes significant as this probability approaches

zero. Its validity as a test for periodicity depends on N_T and the intrinsic scatter σ_i in the data, as discussed by Tifft (1982). It is possible for a real, existing periodicity to be undetectable simply because N_T is not large enough to compensate for the inherent scatter. The appropriate method of testing at small N_T is to test *for* an expected nonuniform h/t ratio; the probability approaches unity as periodicity is confirmed. This method depends much less strongly on N_T. But if N_T is not large, neither uniformity nor periodicity can be formally rejected. One can, however, show that one hypothesis is more likely than the other.

In the present case, the h/t ratio is observed to be 1.3 for the total sample. This ratio corresponds to $P/\sigma_i \approx 3$ (Tifft 1982, Fig. 3). For a period of 24.15 km s^{-1}, we get $\sigma_i \approx 8$ km s^{-1}. As shown above, this is more than is implied by the known measurement uncertainty in z. We presume that the effects of a possible mixture of periods and any other inherent velocity structure or noise in the Ly-α forest would increase the effective σ, here denoted σ_i. We discuss this further in Section V.

At a P/σ_i ratio of 3, a sample must have $N_T \geq 200$ before a valid test comparing against uniformity is possible. For the total sample the uniformity test is therefore valid. For subsamples or for any of the individual QSOs there are simply not enough points for valid uniformity tests. In such cases we can, however, test for consistency with an expected h/t ratio of 1.3, as appropriate for $\sigma_i \approx 8$ km s^{-1}.

We carried out the probability analysis with the bin boundaries at 48.30, 54.34, 66.41, 78.49, 90.56, 102.64, 114.71, and 120.75 km s^{-1}. The first and last bins are half-bins, chosen to equalize the expected random probabilities corresponding to an overall uniform slope of the distributions starting at the peak near 48 km s^{-1}. The overall shapes of the various histograms (calculated for various values of q_0) make it impossible to justify using *points below about 36 km s*$^{-1}$, where spectral resolution begins to influence the data to an unacceptable extent.

To justify an upper limit of 120.75 km s^{-1} for the intervals, we note that differential studies have demonstrated quantization only for the first few multiples of the basic intervals. In particular, the triplets test described by Tifft and Cocke (1989) shows general 24.15 km s^{-1} quantization in close pairings up to differentials somewhat greater than 100 km s^{-1}. In order to limit the Ly-α test to the range *known* to show quantum effects in pairs we have therefore limited the sample to $\widetilde{\delta v} \leq 120.75$ km s^{-1}. In Section V, we note the diminished quantization found in the range 120.75 to 169.05 km s^{-1}.

Table 1 lists h/t ratios and the corresponding probabilities that the ratios were drawn from random, unquantized populations (*i. e.*, from tosses of an unbiased coin) *or* that the ratios are consistent with quantization ($h/t = 1.3$) . The entries are, first, for the sample as a whole, which is compared against uniformity, and then for the four indicated subsamples. The Carswell *et al.* data divided naturally into two subsamples; namely, the data with $1.89 \leq z \leq 3.07$ (145 differences) and those with $3.32 \leq z \leq 3.74$ (38 differences). The latter group contains the lines from P2000 only. The Foltz data are listed separately for the two QSOs PHL957 (157 differences) and UM673A (46 differences).

None of the subsamples meet the minimum N_T requirement of 200 points. However, all show an h/t ratio of approximately 1.3 . Futhermore, for the two largest samples the $h = t$ probability, shown in parentheses, is at about the minimum possible for the sample size. Thus while the individual subsamples cannot formally reject the uniformity hypothesis, they are clearly more consistent with quantization than with uniformity.

When combined as a whole, however, the data *do* have $N_T > 200$, and the data show nonrandomness at a very high level of significance; *i. e.*, at $(1 - .0063) \times 100\% = 99.37\%$. Thus one can say that the quantization confirms the $q_0 = 1/2$ prediction of the inflationary

cosmological models. One could also invert the logic and say that accepting the inflationary model confirms the redshift quantization and the theoretical framework summarized in Section II.

It is interesting to inquire about other values of q_0. Since the $\widetilde{\delta v}$s are functions of q_0, so are the N_hs and N_Ts. Thus each value of q_0 has a probability $P(\geq N_h, N_T)$ that the corresponding distribution of $\widetilde{\delta v}$s occurred randomly, *i. e.*, with no quantization.

Figure 1 shows a plot of $-\log_{10} P(\geq N_h, N_T)$ as a function of q_0 for all the data together. The peak at $q_0 \approx 0.48 \pm 0.03$ is sharp and unambiguous, and the corresponding *peak* probability is about $10^{-2.47} \approx 0.0034$. The heads/tails numbers for this peak are $219/165 = 1.33$.

If one accepts *a priori* neither the inflationary model nor the redshift quantization and its associated theory, then it is difficult to calculate the statistical significance of the result in Figure 1. One may, however, estimate it as follows: The correlation length associated with the curve is about 0.1 in terms of the variable q_0, and the number of correlation lengths in the curve as a whole is thus about 10. Therefore the peak probability *averaged* over a correlation length should be multiplied by the square-root of the number of correlation lengths to get $10^{-1.8} \times \sqrt{10} \approx .050$, which is just at the accepted formal level of significance.

Monte Carlo simulation of diagrams like Figure 1 were done by assuming a statistical distribution of δzs that results from smoothing the real distribution. This process was simplified by the fact that the density of Ly-α lines per unit redshift z is, in the present sample, roughly independent of z.

In these simulations, redshift samples were built up in fixed redshift intervals by starting at the lower boundary of an interval and adding randomly chosen δzs until the upper boundary was reached. The samples were then analyzed in the same way as were the real data, and a series of graphs were produced. Out of 20 graphs like Figure 1, three showed peaks at about the same probability level as the real data. The simulations therefore indicate that the significance of the peak in Figure 1 is poorer than our crude correlation-length analysis would suggest. Since we have little prior knowledge of the real distribution of Ly-α lines (as a function of z), it seems impossible to say anything more definite about the overall significance of the peak in Figure 1 at this time. We emphasize, however, that in the context of the inflationary model ($q_0 = 1/2$ *a priori*) the significance level of 99.4% for the quantization seems to be a firm result. All four subsamples are consistent with this hypothesis.

Note that the "length" of the graph is arbitrary and depends on one's assessment of the acceptable range for q_0. Obviously q_0 should be positive, and one can say that the observed density of luminous matter implies a lower limit for q_0 of 0.1. An upper limit of $q_0 = 1.0$ seems necessary to prevent the age of the universe from being too short.

V. ERRORS AND UNCERTAINTIES

In addition to measurement errors, there are two effects that might hinder the detection of quantization in the Ly-α forest: (1) Tifft and Cocke (1989) have shown that 72.45 km s^{-1} quantization in pairs of galaxies can be contaminated by the presence of a nearby third companion; *i. e.*, when the alleged pair is actually a triple. In the present Ly-α study we have no way of assessing the effect of such dynamical complications on the Ly-α clouds. (2) Tifft and Cocke (1984) show in a study of the global quantization of field galaxies that the 24.15 km s^{-1} quantization begins to be diluted and replaced by 36.22 km s^{-1} in galaxies whose 21cm profile widths are $\geq$ about 80 km s^{-1}. Thus, if there are appreciable numbers of protogalaxies in the Ly-α forest other than those corresponding to the extreme dwarf irregulars, one can expect dilution of the 24.15 km s^{-1} quantization.

In fact there is in the data a fairly strong signal at 36.22 km s^{-1}, for $q_0 = 1/2$. This may contaminate and weaken the 24.15 km s^{-1} effect in various ways, but if so, one might expect reinforcement at 72.45 km s^{-1}. This does not appear to be the case. It would be interesting to analyze a subsample consisting of the narrower Ly-α lines to see whether any contamination is reduced. Unfortunately, the line widths in this data set are dominated by the instrumental resolution.

As mentioned above, the analysis in Section IV was also done for a wider interval of $\widetilde{\delta v}$, with an upper limit of 169.05 km s^{-1}. The result shows that the quantization is somewhat reduced in the upper end, and the statistical significance of the effect at $q_0 = 1/2$ is lowered from 99.4% to 99.2%. The causes of this reduction are unknown.

One might want to introduce a correction for curvature of the overall shape of the histograms. This would correct for the possibility that the sum of the expected areas of the "heads" bins not be equal to that of the "tails" bins for the nonuniformity test; *i. e.*, the coin might be biased. To estimate the magnitude of this effect, the relevant part of the histogram for $q_0 = 1/2$ was fitted by a quadratic. This resulted in a "heads" probability $p_h = 0.5033$. The appropriate generalization of Equation (3) is then

$$P(\geq N_h, N_T; p_h) = \sum_{m=N_h}^{N_T} p_h^m (1-p_h)^{N_T - m} \binom{N_T}{m} \quad . \tag{4}$$

Using the Table 1 values $N_h = 218$ and $N_T = 386$ in this expression yields the probability 0.0090. This effect raises the estimated probability associated with the peak in Figure 1 to about 0.068. Thus if one assumes neither inflation nor the validity of the quantization, the significance of the result is lowered from 95% to 93%.

VI. CONCLUSIONS

We conclude that the assumption $q_0 = 1/2$ leads to the discovery of quantization of the redshifts in the Ly-α forest lines of QSOs. Conversely, if we assume *ab initio* that the redshifts should be quantized at 24.15 km s^{-1} (as reduced back to the present epoch *via* Equation (2)), then we have in effect measured $q_0 \approx 0.48 \pm 0.03$, confirming one of the main predictions of the inflationary model of the early universe. As stated above, the confidence level of this result is above 99%.

If neither the inflationary model nor the redshift quantization is assumed *a priori*, then the result is less certain, and the formal significance of the peak in Figure 1 is roughly at the 93% confidence level. Clearly it would be good to try to verify this result with an independent data set.

We may infer that the theoretical framework for redshift quantization consists of quantum operators and wavefunctions embedded in a Riemannian background metric satisfying Einstein's field equations. In a formal sense, this would be valid from the standpoint of conventional atomic quantum mechanics, since galaxies are known to be made of ordinary electrons and baryons. The only change required by the observed (large) value of the redshift quantization interval is the hypothesis that there is yet another regime of quantum mechanics. This regime would be characterized by a much larger Planck's constant.

The most basic parameter in the theory (Cocke 1985) is the analogue of the Compton wavelength $\mu = (\Delta z)^2/2H_0 \approx 8 \times 10^{18}$ cm, for $c\Delta z = 12.075$ km s^{-1} and $H_0 \approx 90$ km s^{-1} Mpc^{-1}. The galactic Planck's constant $\hbar_g \approx 2.4 \times 10^{73}$ erg s then arises from $\hbar_g := Mc\mu$, where $M \approx 10^{44}$ gm is a galaxy component mass. This implies a gravitational "Bohr

radius" of $(Mc\mu)^2/GM^3 \approx 10^{22}$ cm ≈ 3 kpc. Individual galaxies would thus consist of two or more gravitationally bound galaxy components.

Such a quantum theory would not invalidate our conventional view of extragalactic astronomy and cosmology, but would modify it in much the same way that atomic quantum mechanics supplements Newtonian mechanics.

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Figure Caption

FIG. 1.—The $-\log_{10}$ of the probability that the distribution of the $\widetilde{\delta v}$s is drawn from a population with $h/t = 1$, as a function of the deceleration parameter q_0. All four subsamples are included in the calculation.

Table 1

Head/Tail Ratios, Numbers, and Probabilies for $q_0 = 1/2$

Sample	h/t Ratio	N_T	$P(h/t = 1)$	$P(h/t = 1.3)$	Expected h/t
All the Data	218/168 = 1.30	386	0.0063		
Q0207, ... , Q2206	83/62 = 1.34	145	(0.048)	0.87	82/63
P2000	21/17 = 1.20	38		0.87	21.5/16.5
PHL957	87/70 = 1.24	157	(0.101)	0.75	89/68
UM673A	27/19 = 1.40	46		0.77	26/20

Fig. 1