OPTIMIZATION OF SYMBOL DISTANCE METRIC IN DIRECTIONAL MODULATION SYSTEMS

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ABSTRACT

This paper investigates a directional modulation (DM) method that distorts the received symbol constellations along a set of undesired directions while maintaining an undistorted constellation in the direction of intended communications. The problem is formulated in terms of minimization of the symbol distance metrics along the undesired directions. An algorithm assigns a symbol pair to each undesired direction for symbol distance minimization. Constraints for good reception in the desired direction are also included. The method involves iterations between a quadratic minimization problem and an unbalanced transportation problem. Numerical results are presented to show the bit error rate (BER) benefits of the proposed method.

INTRODUCTION

Security in wireless communications is a challenging problem and is traditionally addressed through encryption techniques. There has been significant interest recently to provide additional security through physical layer means. One such technique is the traditional beamformer which maximizes the signal-to-noise ratio (SNR) in the direction of the desired receiver, while reducing the SNR in undesired directions. However, an unauthorized receiver with sufficient proximity or sensitivity may still have sufficient SNR even in undesired directions. A DM system attempts to overcome this weakness by imparting directional dependence into the received signal constellation. By distorting the signal constellation along unintended directions, a DM system can provide secure communications benefits even with high adversary received SNR.

There are several methods proposed in the literature for generating DM signals. The work [1] presents array solutions that select combinations of switched parasitic elements to provide undistorted symbols in the desired directions. More recent techniques include antenna subset modulation (ASM) [2], which randomly selects a subset of the available array elements during the transmission of each symbol. Another group of DM techniques involves the generation of artificial noise. The noise is constructed so that it falls into the nullspace of the receiver located in the intended direction. The combining of the artificial noise with the array excitation is implemented through a variety of methods that include additive, multiplicative or orthogonal vector synthesis [3]-[5].

In this work, we present a DM system design method using a uniform linear array (ULA). We formulate a quadratic minimization problem to distort the received constellation in undesired directions. This is done by minimizing the Euclidean symbol distance between symbol pairs in each undesired direction. The assignment of symbol-pair to undesired directions is posed as an unbalanced linear transportation problem. Our proposed approach then iterates between quadratic minimization and linear transportation solutions to obtain the final solutions for the array weights. BER performance of the proposed approach demonstrates that significant DM security benefits are achievable over a traditional beamforming array. Note that the work [6] also explores minimization of the inter-symbol distances. However, the method used in [6] differs from our method, as it is based on iterations that involve an indirect weight optimization.

Notations: Bold upper-case letters denote matrices, bold lower-case letters denote vectors, a non-bold upper-case or lower-case variable is a scalar. The operator $(\cdot)^T$ denotes transpose and $(\cdot)^H$ is the Hermitian operator. $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ are the real and imaginary parts of a complex number. The notation $\binom{n}{k}$ denotes $n!/(n-k)!k!$, $\mathbf{0}_{m,n}$ is a zero matrix of size $m \times n$ and *diag*$(\cdot)$ denotes a diagonal matrix.

PROBLEM FORMULATION

Consider a ULA consisting of N antenna elements with an inter-element spacing of d as shown in Fig. 1. We desire to transmit an undistorted symbol constellation in an intended direction θ_0, while distorting it in the directions θ_i for $i = 1, 2 \ldots I$.

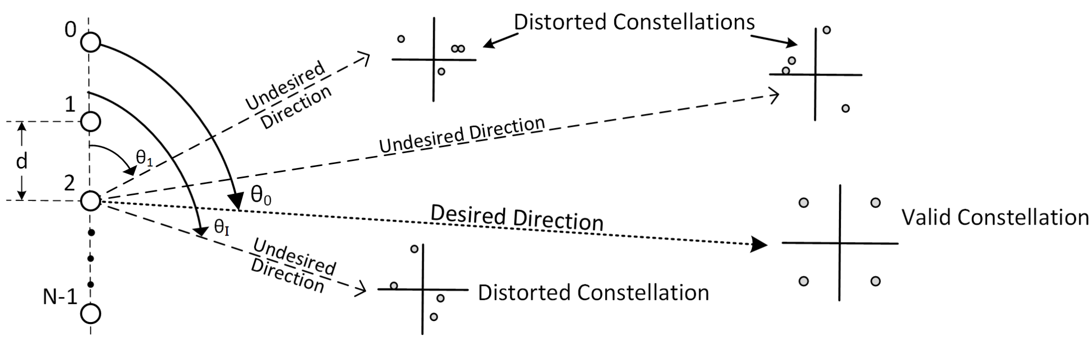

Figure 1: ULA transmitter for a DM system showing distortion of QPSK symbols in undesired directions.

Using a freespace channel model, the complex $N \times 1$ channel vector $\mathbf{h}(\theta)$, also referred to as the array manifold vector, for a receiver located in the θ direction is

$$\mathbf{h}(\theta) = \Big[1, \exp\big(j\psi_1(\theta)\big), \ldots \ \exp\big(j\psi_{N-1}(\theta)\big)\Big]^T \quad \text{with} \quad \psi_n(\theta) = nd\frac{2\pi}{\lambda}\cos\theta, \quad n = 0, 1 \ldots N-1$$

where λ is the wavelength. Consider the transmission of an M-ary symbol $a^{(m)},\ m = 1, 2, \ldots M$. Let the corresponding array transmit signal vector be $\mathbf{x}^{(m)} = \big[x_1^{(m)}, x_2^{(m)}, \ldots \ x_N^{(m)}\big]^T$. The noiseless received symbol along the direction θ is then given by

$$s^{(m)}\big(\theta\big) \ = \ \mathbf{h}^H(\theta)\mathbf{x}^{(m)} \tag{1}$$

For M-ary modulations, there are a total of $K = \binom{M}{2}$ unique unordered symbol pairs (m, p), $m = 1, 2, \ldots M-1$, and $p = m+1, m+2, \ldots M$. We call a specific symbol pair (m, p) also as the k^{th} symbol pair, where $k = M(m-1) - m(m+1)/2 + p$. Similarly for a given k, the values of (m, p) can be found by

$$m = \sum_{n=0}^{M-2} u\left[k - nM + \frac{n(n-1)}{2} - 1\right], \quad \text{and} \quad p = k - M(m-1) + \frac{m(m+1)}{2}$$

where $u[n]$ is the discrete step function so that $u[n] = 1$ for $n \geq 0$ and is zero otherwise. As an example, if $M = 8$ then $k = 25^{th}$ symbol pair implies $(m, p) = (5, 8)$. The Euclidean distance between the k^{th} pair of symbols along the direction θ is then given by $D(k, \theta) = \left|s^{(m)}(\theta) - s^{(p)}(\theta)\right|$.

In our method, the received constellation distortion along any undesired direction is obtained by minimizing the Euclidean distance between a specific pair of symbols in that direction. Therefore, there are two steps in our problem.

1. *Symbol Pair assignment*: Which symbol pair out of the K pairs should be assigned to a given undesired direction?
2. *Transmit vector optimization*: Given that each direction is assigned a symbol pair, how to obtain the signal transmit vectors $\mathbf{x}^{(m)}$, for $m = 1, 2, \ldots M$?

These issues are addressed in the following section.

Proposed DM Method

A. Symbol Pair Assignment (SPA)

One way to solve the symbol-pair assignment problem is to perform an exhaustive search by assigning each possible pair along each undesired direction. This results in a highly complex tree search. Therefore, we adopt a low complexity approach. We formulate the assignment of symbol pairs to each direction θ_i as an unbalanced transportation problem that is then solved as an integer linear program. The transportation problem is setup to supply K symbol pairs to the I undesired directions. We then introduce a dummy direction θ_{I+1} to balance the available supply and demand. Let $\mathbf{Y}$ be the $K \times (I+1)$ decision matrix, whose elements are integers with $y_{k,i} \in \{0, 1\}$ for $i = 1, 2, \ldots I$, where a 1 represents the selection of the k^{th} symbol pair to the θ_i direction. We allow $y_{k,I+1} \in \{0, 1, \ldots S_{max}\}$, where S_{max} is the maximum number of times each pair can be assigned, and all excess supply is absorbed by the dummy direction θ_{I+1}. We require that one symbol pair is assigned to each direction $\theta_i, i = 1, 2, \ldots I$, and excess supply to θ_{I+1}. This can be easily generalized to the case of more than one symbol pair assignment per direction. Therefore, the constraints are

$$\sum_{k=1}^{K} y_{k,i} = 1 \quad \text{for} \quad i = 1, \ldots I, \qquad \text{and} \qquad \sum_{k=1}^{K} y_{k,I+1} = KS_{max} - I \tag{2}$$

A symbol pair may remain unassigned, or can be assigned up to a max of S_{max} directions, yielding

$$\sum_{i=1}^{I+1} y_{k,i} = S_{max} \quad \text{for} \;\; k = 1, \ldots K \tag{3}$$

Let $\tilde{\mathbf{C}}$ be the cost matrix for the transportation problem. Each element $c_{k,i}$ of $\tilde{\mathbf{C}}$ is given by $c_{k,i} = D(k, \theta_i)$ for $i = 1, 2 \ldots I$, and $c_{k,i} = 0$ for $i = I + 1$. Thus, the decision matrix $\mathbf{Y}$ and the cost matrix $\tilde{\mathbf{C}}$ have the forms

$$\mathbf{Y} = \begin{bmatrix} y_{1,1}, & \cdots & y_{1,I+1} \\ \vdots & \ddots & \vdots \\ y_{K,1}, & \cdots & y_{K,I+1} \end{bmatrix}, \qquad \text{and} \qquad \tilde{\mathbf{C}} = \begin{bmatrix} c_{1,1}, & \cdots & c_{1,I}, & 0 \\ \vdots & \ddots & \vdots & \vdots \\ c_{K,1}, & \cdots & c_{K,I}, & 0 \end{bmatrix}$$

Denoting the i^{th} column of $\tilde{\mathbf{C}}$ as $\mathbf{c}_i$, we form the total cost vector $\mathbf{c} = [\mathbf{c}_1^T, \mathbf{c}_2^T, \ \ldots \ \mathbf{c}_I^T]^T$ by stacking the columns of $\tilde{\mathbf{C}}$. We similarly form the decision vector $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \ \ldots \ \mathbf{y}_I^T]^T$ by stacking the columns of $\mathbf{Y}$. Then the transportation minimization problem is

$$\min_{\mathbf{y}} \ \mathbf{y}^T \mathbf{c} \quad \text{subject to the constraints given in (2) and (3)} \tag{4}$$

The solution of (4) gives us the assignment of symbol pairs to each undesired direction θ_i.

B. *Transmit Vector Optimization (TVO)*

Given $\mathbf{Y}$, how do we find $\mathbf{x}^{(m)}$ for $m = 1, 2, \ldots M$? Consider

$$\left|D(k, \theta)\right|^2 \ = \ \left|s^{(m)}(\theta) - s^{(p)}(\theta)\right|^2 \tag{5}$$

Using (1), we can rewrite (5) as

$$\left|D(k, \theta)\right|^2 = \ \left(\mathbf{x}^{(m)} - \mathbf{x}^{(p)}\right)^H \mathbf{h}(\theta)\, \mathbf{h}^H(\theta) \left(\mathbf{x}^{(m)} - \mathbf{x}^{(p)}\right) \tag{6}$$

Assume that the k^{th} symbol pair is assigned to the i^{th} direction. We define $\mathbf{z}_i$ as a real $2N \times 1$ vector containing the stacked real and imaginary parts of the inter-symbol difference between the k^{th} pair of transmit symbol vectors as

$$\mathbf{z}_i \ = \ \left[\ \mathrm{Re}\left(\mathbf{x}^{(m)} - \mathbf{x}^{(p)}\right)^T, \ \ \mathrm{Im}\left(\mathbf{x}^{(m)} - \mathbf{x}^{(p)}\right)^T \right]^T \tag{7}$$

Next define the $2N \times 2N$ real matrix

$$\tilde{\mathbf{B}}(\theta) \ = \ \begin{bmatrix} \mathrm{Re}\left(\mathbf{h}(\theta)\, \mathbf{h}^H(\theta)\right), & -\mathrm{Im}\left(\mathbf{h}(\theta)\, \mathbf{h}^H(\theta)\right) \\ \mathrm{Im}\left(\mathbf{h}(\theta)\, \mathbf{h}^H(\theta)\right), & \mathrm{Re}\left(\mathbf{h}(\theta)\, \mathbf{h}^H(\theta)\right) \end{bmatrix} \tag{8}$$

Use (7) and (8) to rewrite (6) as

$$\left|D(k, \theta)\right|^2 \ = \mathbf{z}_i^T \, \tilde{\mathbf{B}}(\theta) \, \mathbf{z}_i$$

Define $\boldsymbol{\theta} \ = \ [\theta_1, \theta_2, \ldots \ \theta_I]$, and create a real $2IN \times 2IN$ block diagonal matrix $\mathbf{B}$ whose i^{th} block equals $\tilde{\mathbf{B}}(\theta_i), \ \ i = 1, 2 \ldots I$. Also form a real $2IN \times 1$ vector $\mathbf{z}$ as $\mathbf{z} = \left[\ \mathbf{z}_1^T, \ \ \mathbf{z}_2^T, \ \ \ldots \ \ \mathbf{z}_I^T \right]^T$. The minimization of the assigned inter-symbol distances along $\boldsymbol{\theta}$ then becomes

$$\min_{\mathbf{z}} \ \mathbf{z}^T \, \mathbf{B} \, \mathbf{z} \tag{9}$$

In order to explicitly show the dependence of the minimization (9) on the transmit signal vector, define the total transmit signal vector $\mathbf{x}$ as

$$\mathbf{x} \ = \ \left[\mathrm{Re}(\mathbf{x}^{(1)T}), \ \ \mathrm{Im}(\mathbf{x}^{(1)T}), \ \ \ldots \ \ \mathrm{Re}(\mathbf{x}^{(M)T}), \ \ \mathrm{Im}(\mathbf{x}^{(M)T})\right]^T$$

Also define a $2N \times 2MN$ real matrix $\mathbf{A}_i, i = 1, 2, \ldots I$, associated with the assignment (k, θ_i). The n^{th} row of $\mathbf{A}_i$ for $n = 1, 2, \ldots 2N$ is given by $\left[\, \mathbf{0}_{1,2N(m-1)+n-1}, \quad 1, \quad \mathbf{0}_{1,2N(p-m)-1}, \quad -1, \quad \mathbf{0}_{1,2N(M-p+1)-n} \right]$. Let $\mathbf{A} = \left[\mathbf{A}_1^T, \quad \mathbf{A}_2^T, \quad \ldots \quad \mathbf{A}_I^T\right]^T$. We can then write $\mathbf{Ax} = \mathbf{z}$, which is substituted in the cost function (9) to obtain

$$\mathbf{z}^T \mathbf{B} \mathbf{z} \;=\; \left(\mathbf{Ax}\right)^T \mathbf{BAx} \;\;=\; \mathbf{x^T C x}$$

with $\mathbf{C} \;=\; \mathbf{A}^T \mathbf{BA}$. The final TVO minimization is then

$$\min_{\mathbf{x}} \;\; \frac{1}{2} \mathbf{x}^T \left(\mathbf{C}^T + \mathbf{C}\right) \mathbf{x} \tag{10}$$

The minimization (10) is constrained so that each received symbol $s^{(m)}(\theta_0), m = 1, 2, \ldots M$, along the desired direction θ_0 is equal to a scaled version of the transmitted symbol $a^{(m)}$. The amplitude reduction η is a key DM parameter that represents the efficiency of the DM system in the intended direction. A lower efficiency allows additional degrees of freedom and increases the extent of the DM symbol distortion. The constraint along θ_0 is proposed as

$$\begin{bmatrix} \mathrm{Re}(s^{(m)}(\theta_0)) \\ \mathrm{Im}(s^{(m)}(\theta_0)) \end{bmatrix} \;-\; N\eta \begin{bmatrix} \mathrm{Re}(a^{(m)}) \\ \mathrm{Im}(a^{(m)}) \end{bmatrix} \;=\; \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{for } \; m = 1, 2 \ldots M$$

Define the real $2M \times 2MN$ block diagonal matrix

$$\mathbf{H} \;=\; \mathit{diag}(\mathbf{H}_0, \mathbf{H}_0, \ldots, \mathbf{H}_0), \quad \text{where} \quad \mathbf{H}_0 = \begin{bmatrix} \mathrm{Re}(\mathbf{h}^T(\theta_0)), & \mathrm{Im}(\mathbf{h}^T(\theta_0)) \\ -\mathrm{Im}(\mathbf{h}^T(\theta_0)), & \mathrm{Re}(\mathbf{h}^T(\theta_0)) \end{bmatrix} \tag{11}$$

Let the real $2M \times 1$ total symbol vector be given by

$$\mathbf{a} \;=\; \left[\mathrm{Re}(a^{(1)}), \quad \mathrm{Im}(a^{(1)}), \quad \mathrm{Re}(a^{(2)}), \quad \mathrm{Im}(a^{(2)}), \quad \ldots \quad \mathrm{Re}(a^{(M)}), \quad \mathrm{Im}(a^{(M)})\right]^T$$

This allows us to re-write the constraint along θ_0 as

$$\mathbf{Hx} - N\eta\, \mathbf{a} = \mathbf{0}_{2M,1}$$

Additionally, we limit the fluctuation of each element excitation $x_n^{(m)}$. Therefore, the constrained TVO problem now becomes

$$\begin{aligned} & \min_{\mathbf{x}} \;\; \frac{1}{2} \mathbf{x}^T \left(\mathbf{C}^T + \mathbf{C}\right) \mathbf{x} \\ \text{s.t.} \quad & 1) \quad \mathbf{Hx} \;-\; N\eta\, \mathbf{a} \;= \mathbf{0}_{2M,1} \\ & 2) \quad -1 \;\leq\; \mathrm{Re}(x_n^{(m)}) \;\leq\; 1 \\ \text{and} \quad & 3) \quad -1 \;\leq\; \mathrm{Im}(x_n^{(m)}) \;\leq\; 1 \\ \text{for} \quad & m = 1, 2, \ldots M, \;\; \text{and} \quad n = 1, 2, \ldots N \end{aligned} \tag{12}$$

C. DM Algorithm

We propose to iteratively solve the TVO and SPA as shown in Fig. 2. We initialize the cost matrix by selecting a single symbol pair and direction combination (k, θ_i) and run the TVO. The solution from TVO gives $\mathbf{x}^{(m)}$ for $m = 1, 2, \ldots M$. These values are used calculate the cost metric $\tilde{\mathbf{C}}$ which is passed to the SPA block. The SPA block then runs producing assignments of new symbol pairs to the undesired directions. The new symbol assignment is again passed to TVO, and the iterations continue until the symbol pair assignments do not change. The algorithm given in Fig. 2 is run with multiple initializations by assigning different symbol pairs in each direction. In our work, the solution that gives the smallest value in (12) is taken as the final solution. However, it is also possible to choose the final solution based on the best BER performance.

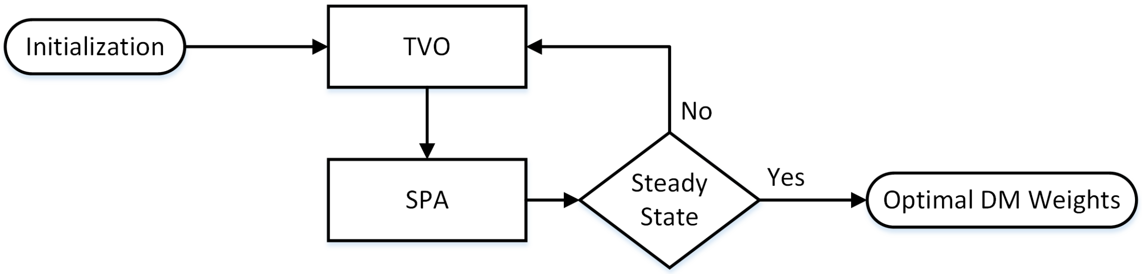

Figure 2: DM Optimization Algorithm

NUMERICAL RESULTS AND DISCUSSION

We consider a ULA with $N = 5$ and half wavelength inter-element spacing. The modulation is QPSK, which implies $M = 4$ symbols and $K = 6$ unique symbol pairs. The desired direction is the array broadside at $\theta_0 = 90°$ unless otherwise specified. The undesired directions are distributed on both sides of θ_0 as $\boldsymbol{\theta} = [\boldsymbol{\theta}_l, \boldsymbol{\theta}_h]$ with $\boldsymbol{\theta}_l = [\theta_1, \theta_2, \dots \theta_t]$ and $\boldsymbol{\theta}_h = [\theta_{t+1}, \theta_{t+2}, \dots \theta_I]$, where $\theta_1 = 30°$, $\theta_t = \theta_0 - 15°$, $\theta_{t+1} = \theta_0 + 15°$ and $\theta_I = 150°$ yielding a total of $I = 20$ undesired directions. We set $\eta = 0.5$ and $S_{max} = 8$. The SNR is defined as E_b/N_o, where

$$E_b = \frac{1}{M \log_2 M} \sum_{m=1}^{M} \left| s^{(m)}(\theta_0) \right|^2$$

and N_o is the noise power spectral density for a given E_b/N_o along θ_0. In our figures, the BER analysis results are obtained using the nearest neighbor approximation [7] to each constellation point for a fixed direction as

$$P_b \approx \frac{1}{M \log_2 M} \sum_{m=1}^{M} N_m Q\left(\frac{d_{min}^{(m)}}{\sqrt{2N_0}} \right) \quad \text{with} \quad d_{min}^{(m)} = \min_{\substack{p=1,\dots M \\ p \neq m}} \left| s^{(m)}(\theta) - s^{(p)}(\theta) \right|$$

where N_m is the number of nearest neighbors for $s^{(m)}(\theta)$, and $Q(x) = 1/\sqrt{2\pi} \int_x^\infty \exp(-y^2/2)\, dy$.

Figure 3 shows the array magnitude response and QPSK constellations for a traditional beamforming array with the weight vector $\mathbf{x}_{BF}$, which represents equal weights at each antenna element. Observe that the constellations along the desired direction $\theta_0 = 90°$ and along an undesired direction $\theta_i = 75°$ differ only by a scaling factor and rotation. In Fig. 4, we show the same results using the optimal DM transmit signal vector after the SPA and TVO optimizations. The received constellation is distorted. Although the constellation received due to the DM transmission along $\theta_i = 75°$ retains the significant amplitude expected from an angle close to the mainbeam direction, the minimum Euclidean distance between symbol pairs is greatly reduced. The array transmit signal vectors are given in Table 1. The transmit vectors are normalized to give the same magnitude response in the desired direction. For the chosen value of $\eta = 0.5$, the DM method requires 2.03 dB more power than the traditional beamformer.

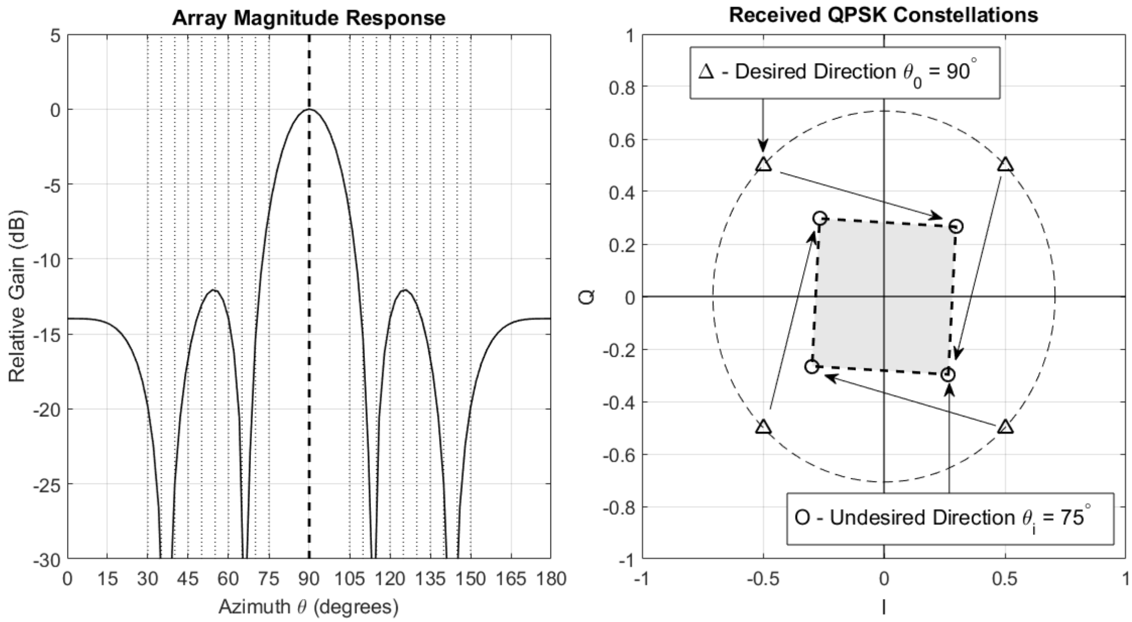

Figure 3: Array Pattern and the received QPSK Constellations for a traditional array.

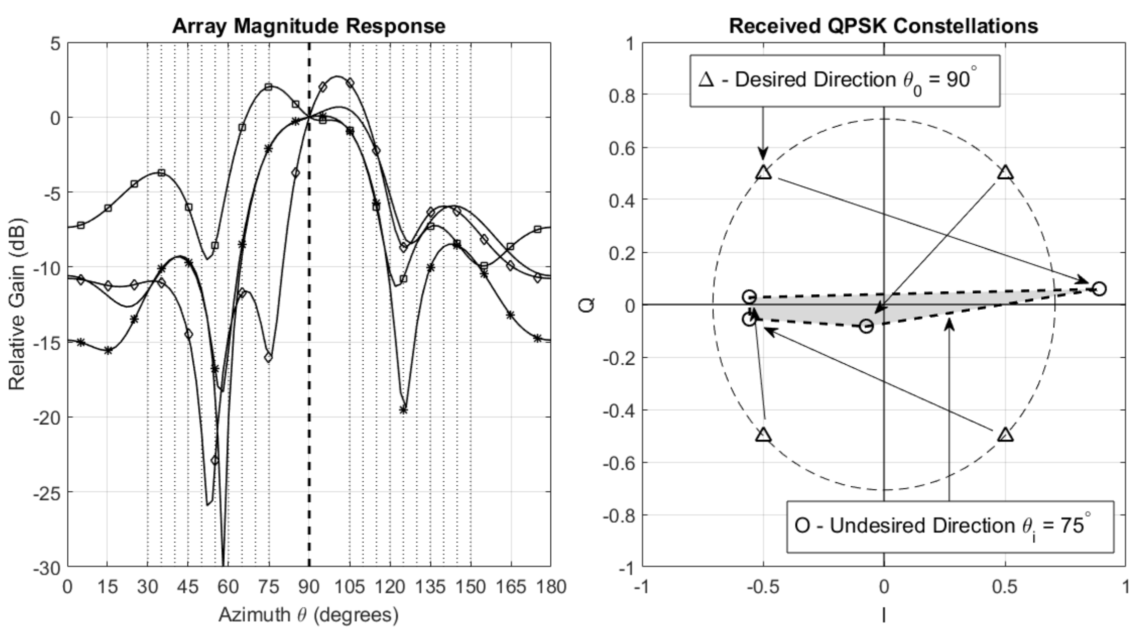

Figure 4: Array Patterns and the received QPSK Constellations for the proposed DM system.

Table 1: Transmit Signal Vectors for Figs. 3 and 4

Symbol	Antenna 1	Antenna 2	Antenna 3	Antenna 4	Antenna 5
$\mathbf{x}^{(1)} \rightarrow 11$	$-0.735 + j0.936$	$0.756 + j1.000$	$0.831 + j0.460$	$1.000 + j0.168$	$0.649 + j0.064$
$\mathbf{x}^{(2)} \rightarrow 01$	$0.704 + j0.891$	$-0.363 + j1.000$	$-0.934 + j0.961$	$-1.000 + j1.000$	$-0.907 + j0.432$
$\mathbf{x}^{(3)} \rightarrow 00$	$-0.732 + j0.890$	$-1.000 - j1.000$	$-0.540 - j0.494$	$-0.547 - j0.367$	$0.318 + j0.259$
$\mathbf{x}^{(4)} \rightarrow 10$	$-0.645 + j0.313$	$1.000 - j0.156$	$0.831 - j0.962$	$0.764 - j1.000$	$0.550 - j0.695$
$\mathbf{x}_{BF}$	0.707	0.707	0.707	0.707	0.707

Figure 5 gives BER performance results for two cases: $\theta_0 = 90°$ and $\theta_0 = 120°$. The SNR is fixed at 10 dB along θ_0 and the same N_o is used in all directions. The receiver in unintended directions are assumed to know the distorted QPSK constellation symbols perfectly, along with the exact mapping of the bit groups to each distorted symbol. So the physical layer security benefit is only due to the reduced inter-symbol distances. The figure shows that the DM method can successfully adjust to the desired θ_0 direction when θ_0 differs from the array broadside.

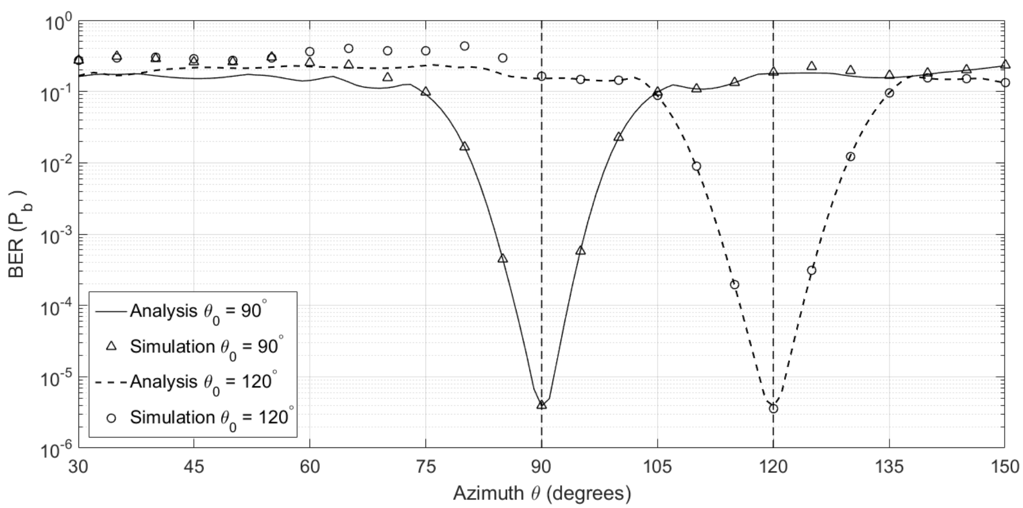

Figure 5: BER performance of the proposed DM system for an SNR of 10 dB.

Figure 6 compares the BER performance of a DM solution for $\theta_0 = 90°$ against a traditional beamformer for 10 dB SNR. The proposed technique provides a more focused response along the desired direction than the traditional beamformer. Below a BER of 10^{-4}, the angle span around θ_0 is $7°$ for DM while it is $16°$ for the traditional beamformer.

To show the effect at high SNR, the BER performance of a DM solution is compared against a traditional beamformer in Fig. 7. The SNR is 25 dB. Two benefits can be observed: 1) DM provides a more focused response along the desired direction than the traditional beamformer. 2) In undesired directions that correspond to the traditional beamformer's sidelobes, the DM method provides significant security benefits. In the case of DM, the BER increases to 10^{-2} in directions where the traditional beamformer provides values lower than 10^{-7}.

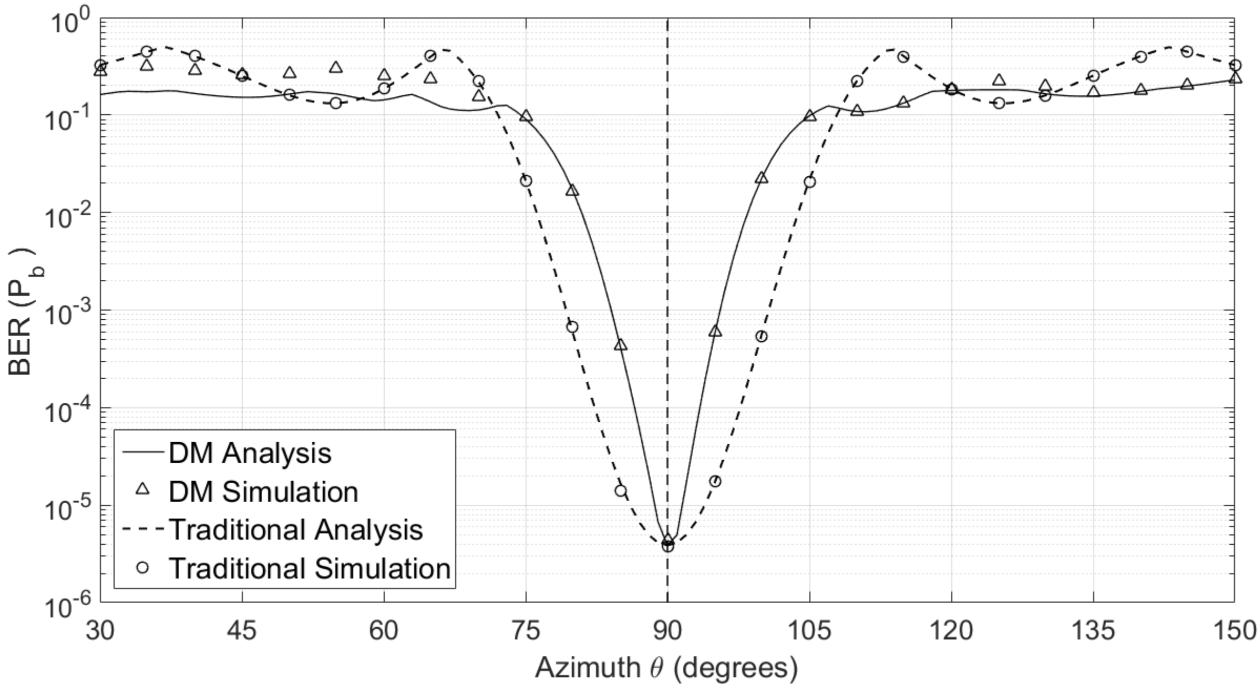

Figure 6: Comparison of the proposed DM system with a traditional beamformer for an SNR of 10 dB.

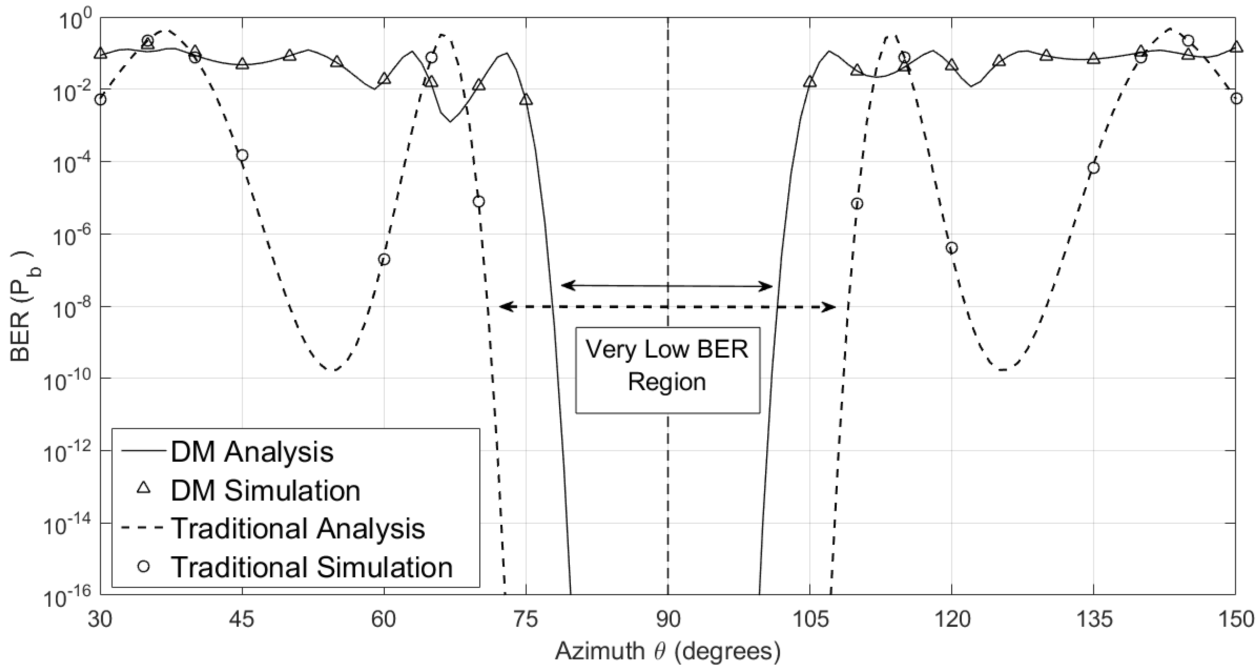

Figure 7: Comparison of the proposed DM system with a traditional beamformer for an SNR of 25 dB.

CONCLUSION

This work presents a DM system design method to distort the received constellations in undesired directions to provide physical layer security. This is accomplished by minimizing the Euclidean distance between symbol pairs in each undesired direction. BER performance of the proposed approach demonstrates significant DM security benefits over a traditional beamformer. For low SNR, the proposed method's BER performance is similar to a traditional beamformer in undesired directions, while providing a more focused mainbeam. For higher SNR, the proposed method outperforms a traditional beamformer in both undesired and desired directions. At an SNR of 25 dB, a traditional beamformer can still provide a BER of 10^{-8} in several undesired directions, while the DM method ensures a BER higher than 10^{-3}.

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REFERENCES

[1] A. Babakhani, D. B. Rutledge, and A. Hajimiri, "Transmitter architectures based on near-field direct antenna modulation," *IEEE Journal of Solid-State Circuits*, vol. 43, pp. 2674–2692, Dec 2008.

[2] N. Valliappan, A. Lozano, and R. W. Heath, "Antenna subset modulation for secure millimeter-wave wireless communication," *IEEE Transactions on Communications*, vol. 61, pp. 3231–3245, August 2013.

[3] S. Goel and R. Negi, "Guaranteeing secrecy using artificial noise," *IEEE Transactions on Wireless Communications*, vol. 7, pp. 2180–2189, June 2008.

[4] Y. Ding and V. F. Fusco, "A vector approach for the analysis and synthesis of directional modulation transmitters," *IEEE Transactions on Antennas and Propagation*, vol. 62, pp. 361–370, Jan 2014.

[5] M. Daly, *Physical Layer Encryption Using Fixed and Reconfigurable Antennas*. PhD thesis, University of Illinois at Urbana-Champaign, 2012.

[6] M. P. Daly and J. T. Bernhard, "Directional modulation and coding in arrays," in *Proc. IEEE International Symposium on Antennas and Propagation (APSURSI)*, pp. 1984–1987, July 2011.

[7] A. Goldsmith, *Wireless Communications*. New York, NY, USA: Cambridge University Press, 2005.