AUTOMATIC MODULATION RECOGNITION FOR CPM

Christopher Nash
Brigham Young University

Faculty Advisor:
Dr. Michael Rice

ABSTRACT

This paper uses detection and estimation theory techniques for automatic modulation recognition of CPM signals. The CPM signals of interest are PCM/FM, SOQPSK-TG, and ARTM/CPM. The modulation recognition problem is formulated as a hypothesis test with the test statistic computed using samples of the observed signal. Using such techniques, simulation results show that correct modulation can be achieved error free at a carrier-to-noise ratio of 19 dB for PCM/FM, 50 dB for SOQPSK-TG, and 25 dB for ARTM CPM.

INTRODUCTION

The demand for finite spectrum allocation can be improved by using bandwidth efficient modulations. A software defined radio satisfies such a demand by using the modulation type as a parameter that can be altered to improve the efficiency of spectrum allocation [1]. As a result, the demand for automatic modulation recognition software has become increasingly popular.

In addition to a software defined radio, other applications of automatic modulation recognition include communication interception and cognitive radio use for aeronautical telemetry. For example, a cognitive radio that has the ability to recognize the modulation in use, could adjust the radio to optimize the spectral utilization when a number of airborne test articles are simultaneously in play [2]. Also, if an aircraft or missile is authorized to transmit using one of various modulations, the flight test engineer on the ground could use automatic modulation recognition software to verify that the transmitter is operating properly [3].

A variety of methods for modulation recognition have been explored. Such methods can be found in [2] [3]. This paper explores the application of invariant statistical tests [4] to distinguish between three modulations of increasing bandwidth efficiency as defined by IRIG 106 [5].

The paper is organized as follow: In the next section, the IRIG 106 modulations are described. In the section that follows, a meticulous approach to formulate the problem is explained. The final sections include the simulation results and conclusion.

THE IRIG 106 MODULATIONS

In applications such as aeronautical telemetry, constant envelope modulations (CPM) are used because the constant envelope does not produce "spectral regrowth" with non-linear RF power amplifier operation [2]. As a result, the three modulations of interest are CPM signals where the continuous-time complex-valued low-pass equivalent representation of the transmitted CPM signal is

$$s(t) = \exp\{j\phi(t)\} \tag{1}$$

where

$$\phi(t) = 2\pi \int_{-\infty}^{t} \sum_{k} h_k I_k g(x - kT_s) dx \tag{2}$$

where I_k is the k-th symbol, h_k is the modulation index applied during the k-th symbol, T_s is the symbol time, and $g(t)$ is the frequency pulse spanning L_i symbol times and scaled to have an area of $1/2$. Described below are the three modulations as defined in the aeronautical telemetry standard IRIG 106 [5]:

- **PCM/FM** is defined as follows:
  - $a_k \in \{-1, +1\}$,
  - $h_k = \frac{7}{10}$ for all k,
  - $g(t)$ is the length-2 raised-cosine (2RC) pulse.
- **SOQPSK-TG** is defined as follows:
  - $a_k \in \{-1, 0, +1\}$,
  - $h_k = \frac{1}{2}$ for all k,
  - $g(t)$ is the length-8 windowed, spectral raised-cosine pulse.
- **ARTM CPM** is defined as follows:
  - $a_k \in \{-3, -1, +1, +3\}$,
  - $h_k = \begin{cases} \frac{4}{16} & \text{k even} \\ \frac{5}{16} & \text{k odd,} \end{cases}$
  - $g(t)$ is the length-3 raised cosine (3RC) pulse.

The goal is to send data using one of the three modulations define above and be able to automatically recognize the modulation in use. The problem formation to achieve this goal is explained in the next section.

PROBLEM FORMULATION

The approach taken in this paper is to formulate the modulation recognition problem as a hypothesis test. The three hypotheses are

$$\begin{aligned} H_0 &: \quad \text{Modulation = PCM/FM} \\ H_1 &: \quad \text{Modulation = SOQPSK-TG} \\ H_2 &: \quad \text{Modulation = ARTM CPM.} \end{aligned} \tag{3}$$

The hypothesis test is based on a test statistic that is derived from observing the received signal

$$r(t) = e^{j\phi(t)} + z(t) \tag{4}$$

where $z(t)$ is a complex-valued wide sense stationary Gaussian random process with zero mean and autocorrelation function

$$R_z(\tau) = \mathrm{E}\left\{z(t)z^*(t-\tau)\right\} = 2N_0\delta(\tau). \tag{5}$$

The test statistic is computed from the samples of $r(t)$ as illustrated in Figure 1. The received signal is sampled at T-spaced intervals to produce the sequence

$$r(nT) = e^{j\phi(nT)} + z(nT), \qquad \text{for } n = 0, 1, \ldots \tag{6}$$

The sequence $r(nT)$ is input to a discrete time FM demodulator whose output is the sample sequence

$$x(nT) \approx \phi'(nT) + v(nT) \tag{7}$$

where $\phi'(nT)$ is the n-th sample of the time derivative of $\phi(t)$ and $v(nT)$ are samples of a noise sequence described below. Next, the symbol period T_s is estimated and used to resample the sequence $x(nT)$ to N samples/symbol:

$$x(nT_s/N) \approx \phi'(nT_s/N) + v(nT_s/N). \tag{8}$$

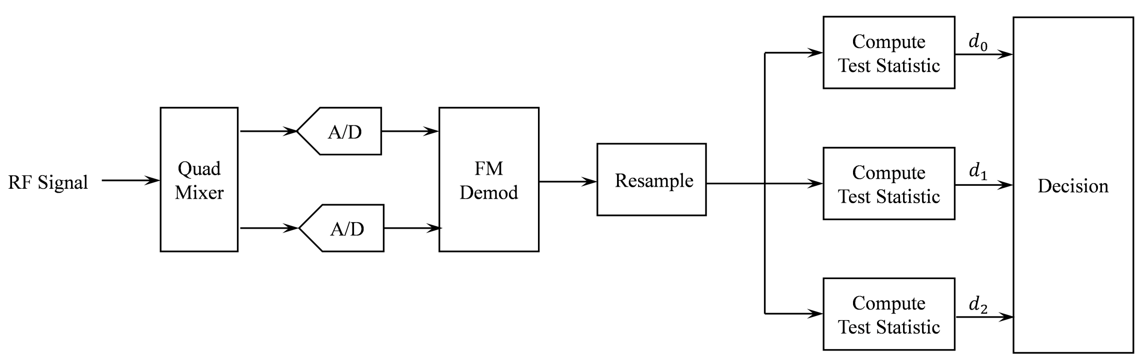

Figure 1: A block diagram outlining the signal processing used for modulation recognition.

Here, the sequence $v(nT_s/N)$ is a real-valued wide sense stationary Gaussian discrete-time random process with zero mean and autocorrelation function

$$R_v(k) = \mathrm{E}\left\{v(nT_s/N)v((n-k)T_s/N)\right\} = \begin{cases} \dfrac{\pi^2}{6} & k = 0 \\ \dfrac{(-1)^k}{k} & k \neq 0. \end{cases} \tag{9}$$

Now suppose we wish to compute the test statistic over an interval corresponding to N_I symbols. The NN_I samples corresponding to the observation interval are stacked to form the vector

$$\begin{bmatrix} x(0) \\ x(T_s/N) \\ \vdots \\ x(NN_I-1) \end{bmatrix} = \begin{bmatrix} \phi'(0) \\ \phi'(T_s/N) \\ \vdots \\ \phi'(NN_I-1) \end{bmatrix} + \begin{bmatrix} v(0) \\ v(T_s/N) \\ \vdots \\ v(NN_I-1) \end{bmatrix}. \tag{10}$$

The instantaneous frequency samples $\phi'(nT_s/N)$ are a function of the N_I data symbols I_0, I_1, …, I_{N_I-1}, the modulation index(es), and samples of the frequency pulse $g(t)$. At this point, the modulation is unknown. Consequently, the modulation index(es) and the frequency pulse are unknown. In addition, the data sequence is also unknown. Because there are only three known possibilities for the frequency pulse, the first vector on the right hand side of (10) may be expressed as

$$\begin{bmatrix} \phi'(0) \\ \phi'(T_s/N) \\ \vdots \\ \phi'(NN_I-1) \end{bmatrix} = \begin{bmatrix} g_i(0) & & & \\ g_i(T_s/N) & & & \\ \vdots & g_i(0) & & \\ g_i(NL_i-1) & g_i(T_s/N) & & \\ & \vdots & & \\ & g_i(NL_i-1) & \ddots & \\ & & & g_i(0) \\ & & & g_i(T_s/N) \\ & & & \vdots \\ & & & g_i(N-1) \end{bmatrix} \begin{bmatrix} h_0 I_0 \\ h_1 I_1 \\ \vdots \\ h_{N_I-1} I_{N_I-1} \end{bmatrix} \tag{11}$$

where $g_i(\cdot) = g_0(\cdot)$ with $L_i = L_0 = 2$ for PCM/FM, $g_i(\cdot) = g_1(\cdot)$ with $L_i = L_1 = 8$ for SOQPSK-TG, and $g_i(\cdot) = g_2(\cdot)$ with $L_i = L_2 = 3$ for ARTM CPM. The matrix on the right hand side of (11) is an $NN_I \times N_I$ matrix. The columns of this matrix are formed from the frequency pulse samples $g_i(nT_s/N)$ for $n = 0, 1, \ldots, NL_i - 1$. The k-th column for $k = 0, 1, \ldots, N_I - 1$ starts with kN zeros, followed by the samples of the frequency pulse, and ends with the number of zeros required to create NN_I rows.

Using (11), Equation (10) may be expressed in the form

$$\mathbf{x} = \mathbf{H}_i \boldsymbol{\theta} + \mathbf{v} \tag{12}$$

where $\mathbf{x}$ is the $NN_I \times 1$ vector on the left hand side of (10), $\mathbf{H}_i$ is the $NN_I \times N_I$ matrix on the right hand side of (11), $\boldsymbol{\theta}$ is the $NN_I \times 1$ vector on the right hand side of (11), and $\mathbf{v}$ is the $NN_I \times 1$

vector on the right hand side of (10). The vector $\mathbf{v}$ is a Gaussian random vector with zero mean and autocorrelation matrix

$$\mathbf{R}_v = \begin{bmatrix} R_v(0) & R_v(-1) & \cdots & R_v(-NN_I+1) \\ R_v(1) & R_v(0) & \cdots & R_v(-NN_I+2) \\ \vdots & & & \vdots \\ R_v(NN_I-1) & R_v(NN_I-2) & \cdots & R_v(0) \end{bmatrix}. \tag{13}$$

We use the notation $\mathbf{v} : N[\mathbf{0}, \mathbf{R}_v]$ to describe this. Consequently, $\mathbf{x} : N[\mathbf{H}_i\boldsymbol{\theta}, \mathbf{R}_v]$.

The vector $\boldsymbol{\theta}$ in (12) contains the unknown modulation index(es) and the unknown N_I data symbols I_0, I_1, ..., I_{N_I-1} as shown in (11). As a result, the vector $\mathbf{x}$, containing NN_I samples of the FM demodulator output, lies in the linear subspace spanned by the columns of $\mathbf{H}_i$ but its exact location is unknown because $\boldsymbol{\theta}$ is unknown. Therefore, the test statistic computed from $x(nT_s/N)$ must be invariant to a rotation in the subspace spanned by the columns of $\mathbf{H}_i$. A test statistic that satisfies this invariance requirement projects the data onto the subspace spanned by the columns of $\mathbf{H}_i$ and computes the power in the subspace by using an inner product [4]. But this is only optimal when the additive noise term is white. Therefore, the first step is to whiten the noise. This is accomplished by

$$\mathbf{y} = \mathbf{R}_v^{-1/2}\mathbf{x}. \tag{14}$$

Now we have $\mathbf{y} : N\left[\mathbf{R}_v^{-1/2}\mathbf{H}_i\boldsymbol{\theta}, \mathbf{I}\right]$. Applying Scharf's invariance principle [4] to $\mathbf{y}$ gives the desired test statistic:

$$d_i = \mathbf{y}^T\mathbf{R}_v^{-1/2}\mathbf{H}_i\left(\mathbf{H}_i^T\mathbf{R}_v^{-1}\mathbf{H}_i\right)^{-1}\mathbf{H}_i^T\mathbf{R}_v^{-1/2}\mathbf{y}. \tag{15}$$

Substituting (14) gives the final form for the test statistic

$$d_i = \mathbf{x}^T\mathbf{R}^{-1}\mathbf{H}_i(\mathbf{H}_i{}^T\mathbf{R}^{-1}\mathbf{H}_i)^{-1}\mathbf{H}_i{}^T\mathbf{R}^{-1}\mathbf{x}. \tag{16}$$

Three test statistics are computed using (16) as shown in Figure 1 with $d_i = d_0$ for PCM/FM, $d_i = d_1$ for SOQPSK-TG, and $d_i = d_2$ for ARTM CPM. The decision rule for choosing one of the three hypotheses in (3) to be true is given by

$$D = \begin{cases} H_0 \text{ (PCM/FM)}: & d_0 = \max\{d_0, d_1, d_2\}, \\ H_1 \text{ (SOQPSK-TG)}: & d_1 = \max\{d_0, d_1, d_2\}, \\ H_2 \text{ (ARTM CPM)}: & d_2 = \max\{d_0, d_1, d_2\}. \end{cases} \tag{17}$$

A simulation of the hypothesis test was performed using this decision rule and is described below in the following section.

SIMULATION RESULTS

To simulate the hypothesis test, a signal $s(t)$ was generated by modulating N_I data symbols I_0, I_1, ..., I_{N_I-1} using one of the IRIG 106 modulations. The received signal was sent through an

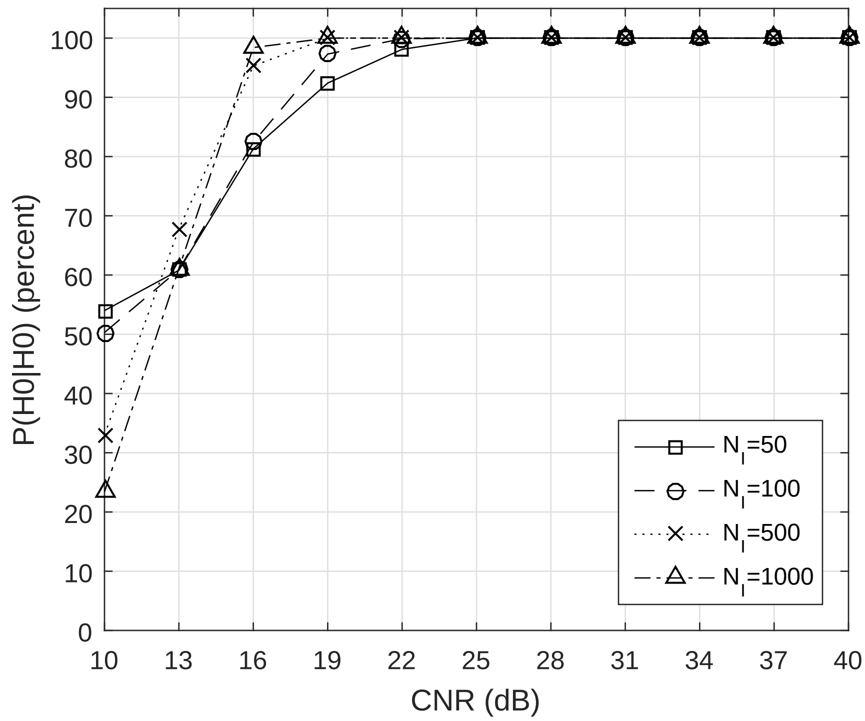

Figure 2: A plot of the probability of correctly identifying the modulation when PCM/FM is true.

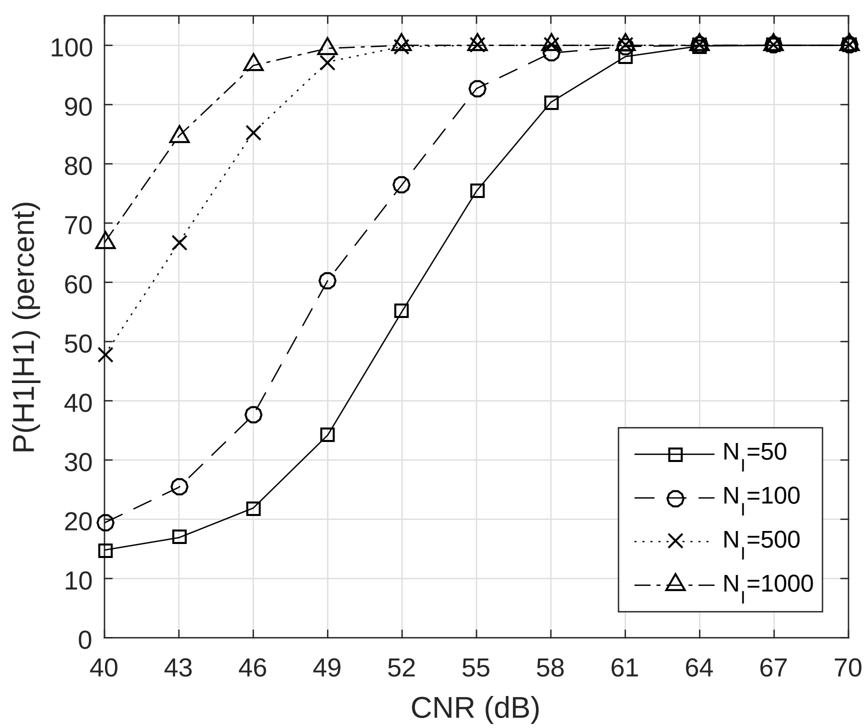

Figure 3: A plot of the probability of correctly identifying the modulation when SOQPSK-TG is true.

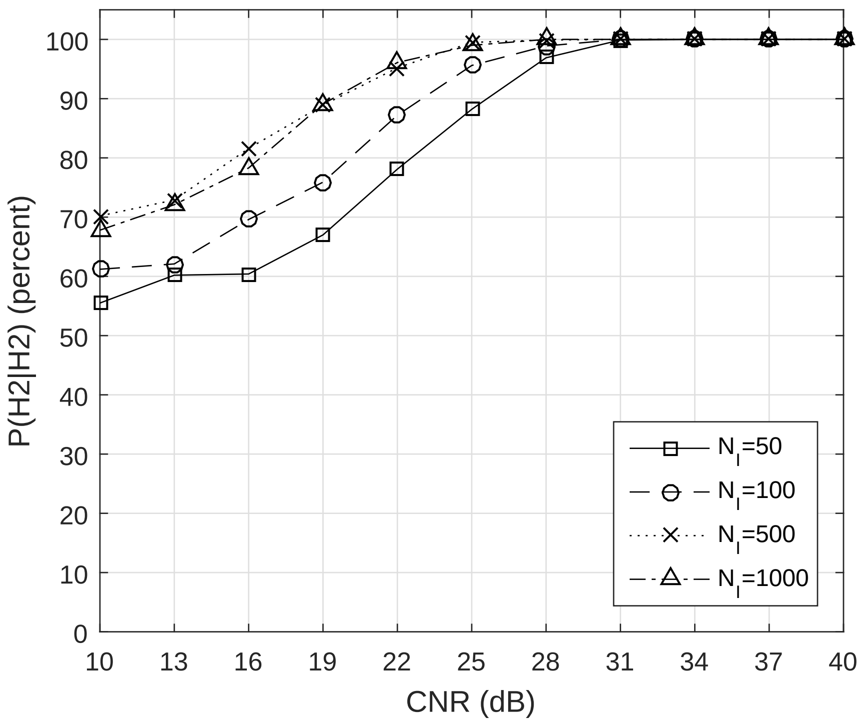

Figure 4: A plot of the probability of correctly identifying the modulation when ARTM/CPM is true.

FM demodulator and resampled to $N = 4$ samples/symbol. The autocorrelation matrix $\mathbf{R}_v$ was constructed with N_I equal to the number of sent symbols and $N = 4$ samples/symbol. Likewise, $\mathbf{H}_i$ was constructed using the appropriate frequency pulse $g_i(t)$, with N_I equal to the number of sent symbols and $N = 4$ samples/symbol. The three test statistics were computed using (16) and a decision for selecting the modulation used to send the data was made by (17).

The process described above was simulated in an iterative manner for $N_I = 50$, 100, 500, and 1000, at various values of a carrier-to-noise ratio (CNR). The results for PCM/FM, SOQPSK-TG, and ARTM CPM are shown in Figures 2, 3, and 4 respectively. In the figures, the abscissa is the value of CNR in decibels (dB) and the ordinate, given in percentage, is

$$\begin{aligned} P(H_0|H_0): &\quad \text{probability of correctly identifying the modulation when PCM/FM is true} \\ P(H_1|H_1): &\quad \text{probability of correctly identifying the modulation when SOQPSK is true} \\ P(H_2|H_2): &\quad \text{probability of correctly identifying the modulation when ARTM CPM is true.} \end{aligned} \tag{18}$$

As seen in figures 2, 3, and 4, the performance improves as the CNR increases however, each plot reaches a ceiling of 100 percent at different CNR values. The lowest CNR value for error free detection in each plot is; 19 dB for PCM/FM, 25 dB for ARTM CPM, and 50 dB for SOQPSK-TG. Each of these CNR values correspond to a symbol length of $N_I = 1000$. This shows that a test statistic computed over a larger interval of N_I symbols produces a higher probability of accurate detection. Also seen from the figures, SOQPSK-TG required a much higher CNR for error free detection as compared to PCM/FM and ARTM/CPM.

CONCLUSIONS

By using hypothesis testing in the linear statistical model, a test statistic invariant to the unknown modulation index(es) and unknown symbols was derived to detect the modulation in use. The test statistic proved to reliably distinguish PCM/FM, SOQPSK-TG, and ARTM CPM for sufficiently high carrier-to-noise ratio. Further research is needed to improve the performance for smaller CNR values.

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